The Processing of Hyperspectral Images as Matrix Algebra Operations

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***Abstract*— The dominant, outstanding characteristic of formation vector-rotor, which provide adequate description of images was considered. It was received the algebraical expressions for determination of step vector as under direct action of rotor. It is shown as a consequence of this approach it is possible to differentiate the growth of the color vector by the individual components and establish the direction of its growth without the use of vector gradient. The inverse representation of the color vector at an arbitrary point of the environment thruogh value of vector-rotor has been written. The ratio of minimum volume or maximum compression of information can be determined for processing by first and second oder vector-rotor.**

Keywords*— *vector-rotor first and second order, comparator, step vector, processing, spectral bands, compression

I. Introduction

Hyperspectral image processing and analysis widely known approach for non-invasive removed sensing of process, objects and their surface material or structures [1-4]. Imagery received by sensors provides an important source of information for monitoring the natural and manmade features on the land surface. The hyper spectral remote sensing data after preprocessing represents usually as 2D or 3D numerous image with high spectral resolution and large amounts of volume [5-8]. This data makes it possible to differentiate, objects or structure in the scene [8]. However, the huge amounts of data, makes it difficult for compression, storage and correct classification after extraction. That why, it is very important to reduce the volume and dimension in the hyper spectral image analysis [9].

The hyperspectral principal of analysis reacquires to accumulate images in wide range of wavelength and also precisely full in close spectral bands. Consequently, the number of features given as input to a classifier can't be reduced without considerable of information losses. Application for this purpose quantization or subdivission in two discrete levels, that are recoded as two integer values (8 or 12 bits of data in a binary encoding system) is general fall into feature extraction and band selection. Feature selection techniques, search algorithm, criterion of function is usually realized in the image spatial space and feature transformation and feature extraction is used to reduce the image dimension such as the principal component analysis, absorption features extraction and spectral statistical analysis [10].

The aim of article is to further studying the properties of the vector-rotor as one of the main tools of intellectualization microchip for colour image segmentation, the construction of theoretical principles and represantation images at ordinary points and points of roundabout thruough the vector-rotor for three very close spectral bands and operations of matrix algebra.

II. Pointing the Problem and Properties of First and Second Order Vector-rotor as Instrument of Intellectualization.

A. Notifications and Defenitions

Let us consider a image (Fig. 1) which is formed due to transformation of image by rectangular matrix of $Q \times S$ CCD elements. For this image in non-dimensional Cartesian system of coordinate [11] (i vertical; - j horizontal) is digitized the certain value of physical quantity F_{ij} as element of matrix in spectrum expression and as function at arbitrary point $p(i, j)$.

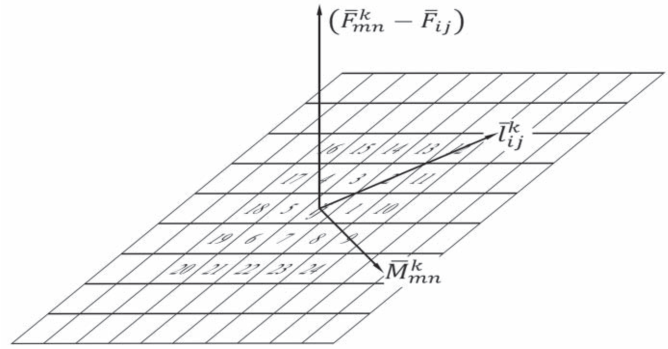

Fig. 1. Sheme of component vector-rotor orientation

image is noticed as F_f. Let introduce the vector, which starts at point $p(i, j)$ and directed to point *k(m,n)*. The modular of which is equal to length of segment between point $p(i, j)$ and *k(m,n)*. As proposed in [5-7, 12-14] the modular of moment of deviation of physical quantity $\left(\bar{F}_{mn}^{k}-\bar{F}_{ij}\right)$ will be calculated for arbitrary point $p(i,j)$ with coordinates (i,j) for each k-th points from first to eight or each from twenty four -th points from two circles of surroundings:

$$\bar{M}_{mn}^{k}=\frac{\bar{l}_{ij}^{k}}{\left|\bar{l}_{ij}^{k}\right|}\times\left(\bar{F}_{mn}^{k}-\bar{F}_{ij}\right)\sqrt{\left(x_{mn}^{k}-x_{ij}\right)^{2}+\left(y_{mn}^{k}-y_{ij}\right)^{2}}\ . \quad (1)$$

It necessary to notice, that in vector form vector rotor will be represented as the multiplication of radius vector $\left|\bar{l}_{ij}^{k}\right|$ at point k and deviation of vector of physical value $\left(\bar{F}_{mn}^{k}-\bar{F}_{ij}\right)$. For this second multiplier of vector product can be taken, for example, as one from component of color vector which is oriented perpendicular to plane of image. This definition allows to determinate direction of vector-rotor as direction of vector product of radius vector $\left|\bar{l}_{ij}^{k}\right|$ and vector of physical value $\left(\bar{F}_{mn}^{k}-\bar{F}_{ij}\right)$. It means that this vector is oriented as perpendicular to plane of two vectors multipliers at point $p(i,j)$ and tangent to contour of plane image at this point. Such definition is described vector-rotor first and second order [5-8]. Thus for first order vector-rotor it is determined as vectors sum of eights values of vector of moments for roundabout points at point $p(i,j)$. The first order vector-rotor at point $p(i,j)$ can be obtained in matrix form: Fig. 3. Print screen as result of vision spectrum analysis and determination structure of abnormal new formation

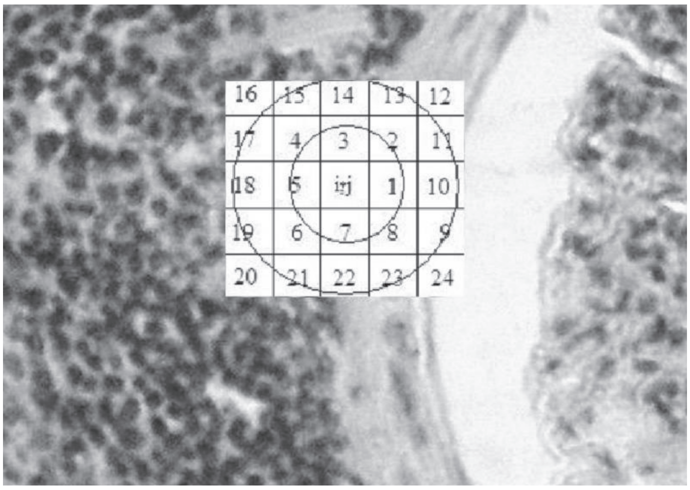

Fig. 2. Sheme point $p(i,j)$ and points of roundabout points for first and second order vector-rotor. Photo represented for analysis by Prof. Doc. of Biology Science M. S. Koziy

$$rot_{1}^{T}(i,j)=\frac{1}{\left\|\bar{F}_{ij}\right\|}\begin{bmatrix}M_{ij}^{8}+M_{ij}^{7}+M_{ij}^{6}+M_{ij}^{5}+\\ +M_{ij}^{4}+M_{ij}^{3}+M_{ij}^{2}+M_{ij}^{1}\end{bmatrix}, \quad (2)$$

where F_{ij} – value of function at point $p(i,j)$ and $M_{ij}^{1}...M_{ij}^{8}$ –noticed correspondingly values of moments for points 1-8 at central point calculated by equation (1). The second order vector-rotor, are determined as vectors sum of 24 vectors of moments at roundabout point $p(i,j)$ and rotor at point $p(i,j)$ can be obtained by matrix form:

$$rot_{2}^{T}(i,j)=\frac{1}{\left\|\bar{F}_{ij}\right\|}\begin{pmatrix}M_{ij}^{24}+M_{ij}^{23}+M_{ij}^{22}+M_{ij}^{21}+...+\\ +M_{ij}^{4}+M_{ij}^{3}+M_{ij}^{2}+M_{ij}^{1}\end{pmatrix} \quad (3)$$

Now it is easy to understand, that eight points of surroundings points for arbitrary point $p(i,j)$, make fist full circle. The second full circle start from point nine and finished at point twenties four (see fig. 2). Especially for simplification it need to chose value of radius for first and second circle of surrounding (see fig. 2) accordingly equal to 1 or 2,4 lateral length of pixel. For this assumption equation (2)-(3) will be significantly simplified.

B. Representation of Color Vector as Frequency Dependent Vector-Function of Coordinate of Image Through a Vector-Rotor

We introduce the rule of operation of a comparator [14-17], which is defined on the range $\forall Y\in(-\infty,\infty)$ by the standards Y_1, Y_2, Y_3 and the level of allowable deviations δ_1, δ_2, δ_3:

$$D_2(Y)=\begin{cases}0, & if\ \ Y\in\left[Y_1+\delta_1, Y_2-\delta_2\right)\\ 1, & if\ \ Y\in\left[Y_2+\delta_3,\infty\right).\end{cases};D_3(Y)=\begin{cases}-1, & if\ \ Y<Y_1+\delta_1;\\ 0, & if\ \ Y\in\left[Y_1+\delta_1, Y_3-\delta_2\right];\\ 1, & if\ \ Y>Y_3-\delta_2.\end{cases} \quad (4)$$

Note that the operator D_2 and D_3 acting on the scalar function are introduced, and its image forms a scalar, which is displayed in the metric space for any value δ_1, δ_2, δ_3,, which during the adjustment can also tend to zero. Under these notation and conditions, the decomposition of the component of an arbitrary vector-function from a vector argument at the point $k(m,n)$ is given by:

$$F_i\left(\bar{x}_{ij}+\Delta\bar{x}_{mn}\right)=\left|F(\bar{x}_{ij})\right|D_3\left(F_i(\bar{x}_{mn})\right)+\sum_{j=1}^{n}\left|\frac{\partial F_i(\bar{x}_{ij})}{\partial x_j}\right|D_3\left(\frac{\partial F_i(\bar{x}_{ij})}{\partial x_j}\right)\Delta x_{mn}+$$
$$+\sum_{k=1}^{n}\sum_{j=1}^{n}\left|\frac{\partial^2 F_i(\bar{x}_{ij})}{\partial x_k\partial x_j}\right|D_3\left(\frac{\partial^2 F_i(\bar{x}_{ij})}{\partial x_k\partial x_j}\right)\frac{\Delta x_{mnk}\Delta x_{mnj}}{2}. \quad (5)$$

Such a decomposition is realized under the conditions of the existence of Frechet derivatives from the first to the third order. The adequacy and accuracy [12] of it is determined, in accordance with the theorem on the average, by maximum value of the module of the derivative of the third order [14].

Introduced product rules:

IF $F_{i+k,j+l}\geq F_{ij}$, and $F_{ij}>F_0$ or $F_{ij}<F_0$, THEN

$$\varphi(k)=sign\left(F_{mn}^{k}-F_{ij}\right)D_2\left[\frac{F_{mn}^{k}-F_{ij}}{F_{ij}}\right]\ \text{OTHER}\ \varphi(k)=0. \quad (6)$$

In equation (6) are noticed $D_n[\bullet]$ is n digital operator, which is acts on any quantitative value and each digit of it is transformed by two grade (0; 1) comparator. Thus introduced vector-rotor the first or second order with help of which we are represented in uniform information about physical value of 8 or 24 points at roundabout point $p(i,j)$.

DETERMINATION DIRECTION OF THE STEP VECTOR

Most well known numerical methods of the first order in the implementation of algorithms are based on the recurrent sequence of finding the next point. To search for it, vectors of the step are used. The gradient of the scalar function, which forms as usual mathematical definition is the main obstacle in their application. For this reason the concept of gradient in image recognition is modified for many cases due to the introduction of the operator Roberts or Sobel.

Let consider application of vector rotor in special case of determination the next point of analysis in search algorithms. We assume that the arbitrary point $p(i,j)$ is the point of contour, then coordinates of next point for analysis determine as operation with new concept of vector rotor of second order:

$$j_{k+1} = j_k + \left(\overline{x}_2^T - \overline{x}_1^T\right) rot_2(j_k, i_k);\ i_{k+1} = i_k + \left(\overline{y}_2^T - \overline{y}_1^T\right) rot_2(j_k, i_k), \quad (7)$$

where are noticed auxiliary twenty four component of vectors with five unity component:

$$\overline{x}_1 = \left[I_{1-15,21-24}^{16-20} \right],\quad \overline{x}_2 = \left[I_{1-8,13-23}^{9-12,24} \right], \quad (8)$$

$$\overline{y}_1 = \left[I_{1-11,17-24}^{12-16} \right],\quad \overline{y}_2 = \left[I_{1-19}^{20-24} \right]. \quad (9)$$

In equation (8)–(9) are used special notations - the superscript (upper) index shows number of position as element of tuple with value 1. The subscript (below) index shows number of components with values 0. The choice of values for vectors $\overline{x}$ and $\overline{y}$ are determined by the type of task, methods of its solution and properties of rotor across the horizontal and vertical line of contour [10]. The segments of straight line in 3 pixels length constrain cell element from below to upper and from left to right for a first order vector-rotor. For the vector-rotor of second order determines sum of moments inside a segment of straight line in 5 pixels length constrain cell element from below to upper and from left to right. In results of image analysis we formulate axiomatic conclusion: if point of analysis i, j moved from point of contour to inner point of image, then next value of step equal zero. We can demonstrate, that this result can be received from equation (7) base on properties of vector-rotor and notation (8)-(9) by simple addition and multiplication as algebraic operation:

$$j_{k+1} = j_k - 5 + 5 = j_k;\ i_{k+1} = i_k - 5 + 5 = i_k.$$

This result also demonstrate fact of end of search in contour search algorithm. Thus this coordinate can be inputs to memory and to begin next search of next point of contour. However, in another case of application motion continuesly in horizontal or vertical direction to inner points of image until first jump of value of rotor. The distance between two points of this class in horizontal or vertical of directions is horizontal or vertical length of image accordingly in horizontal or vertical directions. The usage values of horizontal or vertical length of image which are measured from specifically chosen points of image allows to re-determinate scaling and angle of rotation of image [16].

Determining the direction of motion to the next point for analysis of the image is presented as a result of the application of an algebraic operation over a vector-rotor. The magnitude of the angle with respect to the horizontal axis deducted counterclockwise (Fig 2) is as follows:

$$\alpha = [315, 270, 225, 180, 135, 90, 45, 0]\, rot_1(j_k, i_k) / \\ / \{[1,1,1,1,1,1,1,1]\, rot_1(j_k, i_k)\}. \quad (10)$$

Thus, the determination of the next point for analysis and direction of motion is reduced to operations on matrices in which the gradient vector is not used. However, the direction of movement to the foot of the analysis of the image determines the vector direction to the point of the eight points that surround it. This direction also coincides with the direction of the vector of the gradient, if it is not equal to zero. It is also perpendicular to the direction of the vector product or the vector of the rotor.

C. Peculiarities of the Application of the Vector-Rotor to the Processing of Color Images.

In the BMP format, are used different preservation mode of image. We will select one of them for analyzing the theoretical foundations. We consider, for example, the format number 5 – Truecolor. It uses 3 bytes to describe the colors of one pixel. Thus, each of the base colors (red, blue and green) occupies 8 bits and has 256 shades (from 0 to 255).

The pixel can have one of the 4,294,967,296 available colors. It is more convenient to use and allows you to represent a wide range of shades, reduce the number of conversions when unpacking the intensity value of each of the basic colors. Among the main drawbacks in comparison with other BMD formats, for example, the format number 4, it should be noted that the size of the graphic file is increased by a third. Consider the color image and submit the rotor point:

$$rot^{col}(i,j) = \frac{1}{\|F_{ij}\|} \begin{bmatrix} M_{ij}^8 + M_{ij}^7 + M_{ij}^6 + M_{ij}^5 + \\ + M_{ij}^4 + M_{ij}^3 + M_{ij}^2 + M_{ij}^1 \end{bmatrix},$$

$$(\ j_{k+1} = j_k - \overline{x}_1^T rot^{col}(j_k, i_k) + \overline{x}_2^T rot^{col}(j_k, i_k);$$

$$i_{k+1} = i_k - \overline{y}_1^T rot^{col}(j_k, i_k) + \overline{y}_2^T rot^{col}(j_k, i_k), \qquad (11)$$

here is noticed eight component vectors:

$$\overline{x}_1^T = [0,0,1,1,1,0,0,0], \overline{x}_2^T = [1,0,0,0,0,0,1,1],$$
$$\overline{y}_1^T = [0,0,0,0,1,1,1,0], \overline{y}_2^T = [1,1,1,0,0,0,0,0]. \qquad (12)$$

where F_{ij} – is the value of the function at the point $p(i,j)$, the RGB decomposition for the color vector row given in the form of the matrix of the row, $\|F_{ij}\|$ - its norm, which is selected as 255, is also indicated by $M_{ij}^1 ... M_{ij}^8$ – corresponding values of the points of points 1-8 relative to the center point. Thus, the rotor of color, unlike binary, of images is a matrix 8×3, that is:

$$rot^{col}(i,j) = \begin{Vmatrix} R_8, ... R_i, ... R_1 \\ G_8, ... G_i, ... G_1 \\ B_8, ... B_i, ... B_1 \end{Vmatrix}^T. \qquad (13)$$

We shall demonstrate that the next value of the point coordinate for calculating and analyzing the next value of the rotor vector, that is, for the image only for one projection of the color vector, namely projection ($A = \overline{R,G,B}$) A, will be:

$$j_{k+1,A} = j_{k,A} + 3sign\left(\left[rot^{col}(j_k, i_k)\right]_A^T (\overline{x}_2 - \overline{x}_1)\right);$$
$$i_{k+1,A} = i_{k,A} + 3sign\left(\left[rot^{col}(j_k, i_k)\right]_A^T (\overline{y}_2 - \overline{y}_1)\right), \qquad (14)$$

where the dimension matrix 3x8 of the vector-rotor of color images is indicated, which, after the projection of the color vector of the R,G,B decomposition, form eight component vector-rotor, introduced earlier for monochromatic images. As a consequence of this approach it is possible to differentiate the growth of the color vector by the individual components and establish the direction of its growth without the use of a gradient. Equally important is the found possibility of connecting two concepts of the rotor of monochromatic and color images in wide range of wavelength and also precisely full in close spectral bands

$$rot^{col}(i,j) = \begin{Vmatrix} A_{18}, ... A_{1i}, ... A_{11} \\ A_{28}, ... A_{2i}, ... A_{31} \\ A_{38}, ... A_{3i}, ... A_{31} \end{Vmatrix}^T = rot(i,j)\frac{\overline{A_1}}{|\overline{A_1}|} + rot(i,j)\frac{\overline{A_2}}{|\overline{A_2}|} +$$
$$+ rot(i,j)\frac{\overline{A_3}}{|\overline{A_3}|} = \sum_{A=A_1,A_2,A_3,..}^{n} \left[rot^{col}(j_k, i_k)\right]_A^T \frac{\overline{A_i}}{|\overline{A_i}|}.$$

Note that the projection mark is conditional because it is in fact a matrix in which rows are ordered components of a color vector, while in addition to the components of the direction of projection, they are zero. Under these conditions, the inverse representation of the color vector at an arbitrary point of the environment, for example - that (sixth) bypassing the eight, will be written:

$$F(j_k, i_k) = \|0,0,1,0,0,0,0,0\| rot^{col}(j,i) F(i,j) =$$
$$= \|0,0,1,0,0,0,0,0\| \begin{Vmatrix} R_8, ... R_i, ... R_1 \\ G_8, ... G_i, ... G_1 \\ B_8, ... B_i, ... B_1 \end{Vmatrix}^T \begin{bmatrix} \overline{R_m} \\ \overline{G_m} \\ \overline{B_m} \end{bmatrix} = \overline{R_6} + \overline{G_6} + \overline{B_6},$$

where is noticed $\overline{R_m}, \overline{G_m}, \overline{B_m}$ the value of the color vector at the point p with coordinates (i_m, j_m), and the matrix row is introduced in which all the digits contain zeros, and only the (sixth) takes the value of unity. This representation represents the algorithm of the inverse transformation and allows us to find, according to the known two values of the rotor and color vector, its value in the (in the example given for the sake of the sixth) point. It should be noted that if you allocate eight digits for each component of the color vector at the point indicated by the lower index and one digit for each component for the points of the environment, then the total amount of discharges will be 48. If you allocate eight digits for each component for the points of the environment, then the total volume of discharges will be 216.

Thus, the ratio of minimum volume or maximum compression of information can be 48/216 = 0,222. With an increase in the number of discharges, the component in points of the environment from one to eight compression ratio is calculated: $k = \dfrac{24 + 3 * n * 8}{216}$ the value ranges from 0.222 to 1. It should be noted that the compression ratio for the second-order vector rotor varies from 0.28 to 1 and is calculated by the formula: $k = \dfrac{24 + 3 * n * 24}{600}$.

III. The Modeling and Discussion of Results

For modeling and realization new approach to preprocessing of color segmentation reconsider technology described in the articles [5-6, 8]. The realization equality of modeling condition for analysis, were generated artificial binary and color images, which converted after in BMP format. As result of providing this stage of modeling was simplified technology of experiment and focus was concentrated on analysis cells 3x3 or 5x5 pixels. The investigation of vector-rotor of first and second order for different images of ubnormal new formation has allow to make next conclusion. The vector rotor is sensitive for regular changes of on the contour and in specific points and wavelength for binary as well as for color images. Latest allows to investigate only individual dynamic changes at the specific point in fragment of image.

However, calibrating, testing and calculating of characteristic for conditions changing of a background leads to possibility and necessity to use different norms [12]. First that calculated under surface of object in wide range of wavelength and second under surface of background of scene in close spectral bands as specific characteristic of bio-tissue. Under application different norms and rebuilding images increases volume of information, inasmuch as changes of norm change results of logical analysis and diagnosis. For providing of single-valued results of experiment it necessary to notice, that value of pixels is determined only with multiple number 3 and noticed, that efficiency of algorithm preprocessing of images by application vector-rotor method based on will be 5

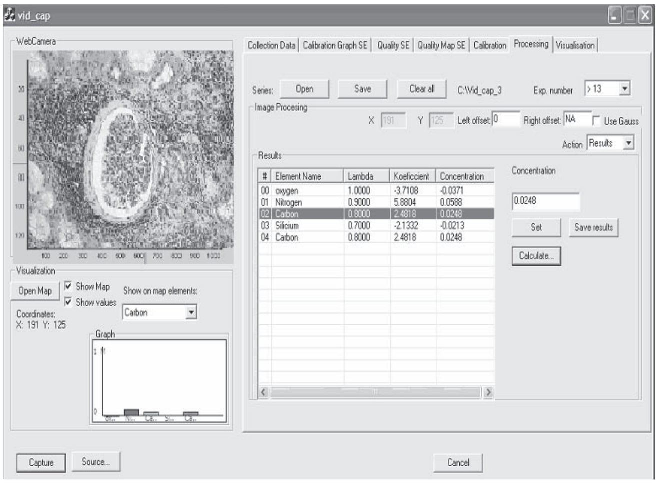

Fig. 3. Print screen as result of vision spectrum analysis and determination structure of abnormal new formation

simultaneously. It means that number of photodiodes in rows and in colons of matrix are provided by 15. It can be significantly decrease of file volume. Application vector-rotor of second order is significantly increased of compression coefficient, but increased loses of information, if not satisfied described upper multiple demand. Application four images in plane polarized fluxes defined decoding of interferrogram data in close spectral bands as specific characteristic of bio-tissue. The necessity and importance of practical application of new methods of preprocessing is demonstrated on fig. 3 (screen-shot of program). On fig. 3 is demonstrated successful attempt to determine direction of analysis by vector- rotor (right upper part on fig. 1). So red color line demonstrated, that direction of search and image analysis is perpendicular to fringes. As it shown on fig.3 the new possibility will be decreased value of error and open way for application of hyperspectral analysis.

CONCLUSSION

1. The proposed approach of vector-rotor of first and second order opens new possibility to reconstruct image in 9 or 25 points according to the value of vector-rotor first and second order, builds curves of contour, defines the image in close spectral bands as specific characteristic of bio-tissue.
2. The vector-rotor as operator and vector, which is produced in result of its action on array of imagies is also able for calculation of step vector and coordinate of points from which will be done next step of analysis.

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