HW3

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1] a] $1 - \left(\binom{20}{0} 0.01^0 \cdot (1-0.01)^{20} + \binom{20}{1} 0.01 (1-0.01)^{19}\right)$

$= 1 - (0.99^{20} + \frac{20!}{19!\,1} * 0.01 \cdot 0.99^{19} =$

$= 1 - 0.982 = 0.017$

b] 0.982

c). $\mu = n \cdot p = 20 \cdot 0.01 = 0.2$

2] $\lambda = 5$

a) $\frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-5}\, 5^0}{0!} = 0.006$

b) $P(X \geq 2) = 1 - P(X < 2) = 1 - (P(X=1) + P(X=0))$

$= 1 - (0.006 + 0.033) = 0.961$

c. $5 \cdot 5 = 25$

d. $(0,006)^5$

3. a) $N = 12 \quad K = 7 \quad n = 6 \quad x = 5$

$$\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \frac{21 \cdot 5}{924} = 0.113$$

b) $$1 - \left(\sum_{x=5}^{6} \frac{\binom{7}{x}\binom{5}{6-x}}{\binom{12}{6}} \right) =$$

$= 0.878$

c) First let's calculate std. and μ:

$$\sigma = \sqrt{6 \cdot \frac{7}{12} \cdot \frac{5}{12} \cdot \frac{6}{11}} = 0.891$$

$$\mu = 3.5$$

So need to have detective more than $\mu + \sigma = 4.391 \Rightarrow$ our x should be 5 or 6.

$$P(X > 4) = \sum_{x=5}^{6} \frac{\binom{7}{x}\binom{5}{6-x}}{\binom{12}{6}} =$$

$$= 0,121$$

4)

a) $P(X \leq 2) = \sum_{i=0}^{2} \binom{25}{i} 0.05^{i} \cdot 0.95^{25-i}$

$= 0.872$

b) $P(X \geq 5) = \sum_{i=5}^{25} 0.05^{i} \cdot 0.95^{25-i}$

$= 0,007$

c. $P(1 \leq X \leq 4) =$

$P(X \leq 4) - P(X < 1) = 0.992 - 0,277$

$= 0.715$

5. $N = 500 \quad k = 10 \quad n = 50$

a) $P(x = 0) = \dfrac{\binom{10}{0}\binom{490}{50}}{\binom{500}{50}}$

$= 0.345$

b) $1 - \sum_{x=0}^{1} \dfrac{\binom{10}{x}\binom{490}{50-x}}{\binom{500}{50}} =$

$= 1 - (0.391 + 0.345) = 0.264$

6) $P(m) = 0.5$

a) four children means 2 boys and 2 girls

$$\binom{x-1}{r-1}(1-\pi)^{x-r}\pi^r =$$

$$= \binom{3}{1} 0.5^2 \cdot 0.5^2 = 0.18$$

b) $P(x \leq) \sum_{x=2}^{<1} \binom{x+r-1}{x} \pi^r (1-\pi)^x =$

$$= \sum_{x=2}^{4} \binom{x+2-1}{x} \pi^2 (1-\pi)^x =$$

$$= 0.6875$$

c) $E(x) = \frac{r}{\pi} = \frac{2}{0.5} = 4$