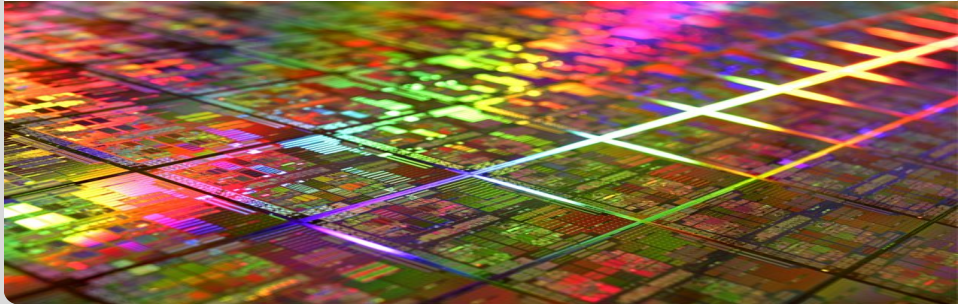# Towards Bringing Together Numerical Methods for Partial Differential Equation and Deep Neural Networks

Research proposal, Supervisor - Markus Hoffmann

Stanislav Arnaudov │ September 26, 2019

Problem definition and motivation

Partial differential equation (PDEs)

- used in simulations
- hard to solve numerically
- solutions have image representation

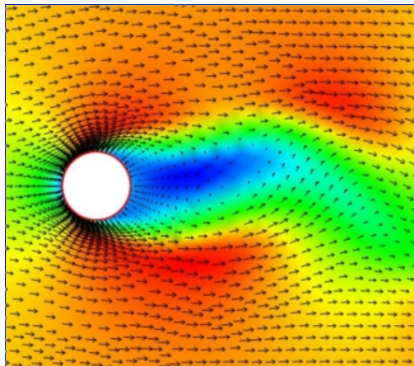**Idea**: study non-classical ways for generating solutions of PDEs

Figure: Flow Simulation[1]

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[1]"Team for Advanced Flow Simulation and Modeling", Professor Tayfun E. Tezduyar, Sunil Sathe

Problem definition and motivation

Deep neural networks (DNNs)

- hot topic in recent years
- impressive results in image processing tasks

Idea: Use DNNs in order to solve PDEs

Research topic: The applicability of DNNs in generating solutions for PDEs.

Problem definition and motivation

Deep neural networks (DNNs)

- hot topic in recent years
- impressive results in image processing tasks

Idea: Use DNNs in order to solve PDEs

Research topic: The applicability of DNNs in generating solutions for PDEs.

Problem definition and motivation

Concrete problem to study

Motivation
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Research Definition
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Problem definition and motivation

Concrete problem to study
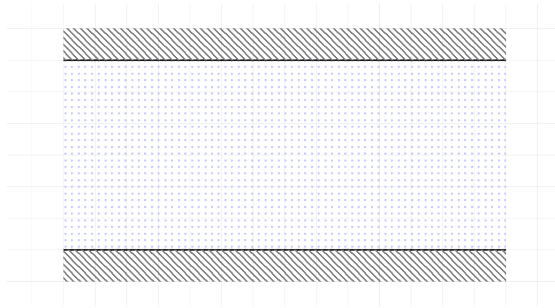
A channel with *incompressible* fluid in it.

Problem definition and motivation

Concrete problem to study

An object placed inside of the channel.

Motivation
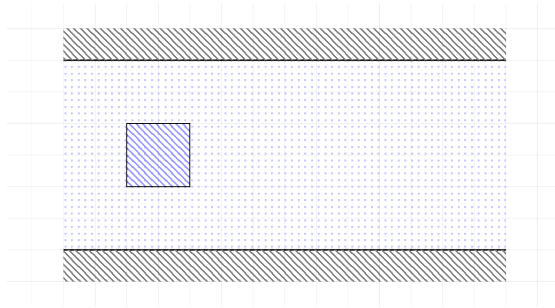○○○○●○○○○○○○○○

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Problem definition and motivation

Concrete problem to study
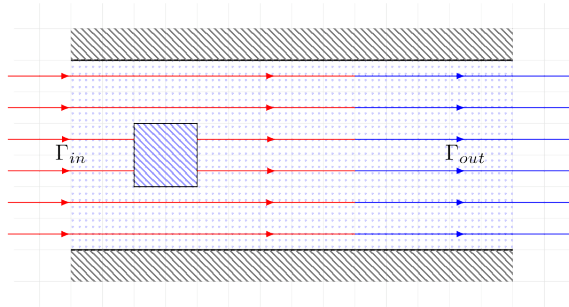
The fluid is flowing in from the one side and flowing out from the other one.

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Problem definition and motivation

Incompressible Navier-Stokes Equation

$$-\nu\Delta u + (u \cdot \nabla)u + \frac{1}{\rho}\nabla p = 0, \quad \text{in } \Omega$$

$$\nabla \cdot u = 0, \quad \text{in } \Omega$$

$$u = g, \quad \text{on } \Gamma_{in}$$

$$(-\mathcal{I}p + \nu\nabla u) \cdot n = 0, \quad \text{on } \Gamma_{out}$$

$$u = 0, \quad \text{on } \partial\Omega/(\overline{\Gamma_{in} \cup \Gamma_{out}})$$

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Research Definition ○

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Problem definition and motivation

Incompressible Navier-Stokes Equation

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$$u = 0, \quad \text{on } \partial\Omega/(\overline{\Gamma_{in} \cup \Gamma_{out}})$$

Parameters:

- fluid viscosity and density – ρ and n
- inflow speed – g
- the object in the space

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Problem definition and motivation

Incompressible Navier-Stokes Equation

$$-\nu\Delta\,u + (\,u\cdot\nabla)\,u + \frac{1}{\rho}\nabla\,p = 0, \quad \text{in } \Omega$$

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$$u = g, \quad \text{on } \Gamma_{in}$$

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$$u = 0, \quad \text{on } \partial\Omega/(\overline{\Gamma_{in}\cup\Gamma_{out}})$$

Solutions:

- velocity field – u
- pressure field – p

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Problem definition and motivation

DNNs in the context of the described problem

- The solutions of the PDE can be visualized as images
- DNNs perform well on image processing tasks

$\Longrightarrow$ use DNNs to generate solutions of the simulation in image form

Problem definition and motivation

DNNs in the context of the described problem

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Motivation
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Problem definition and motivation

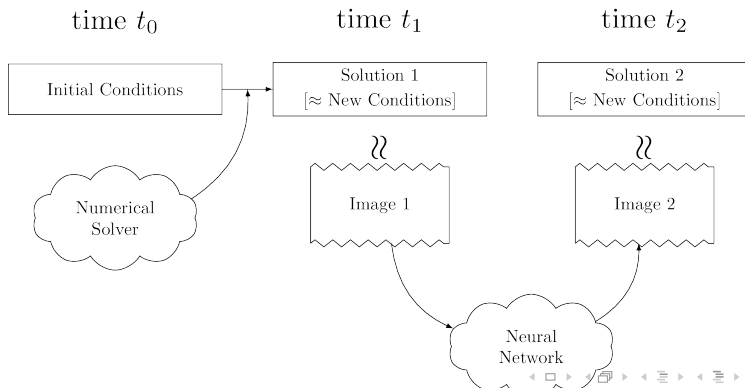DNNs in the context of the described problem

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Problem definition and motivation

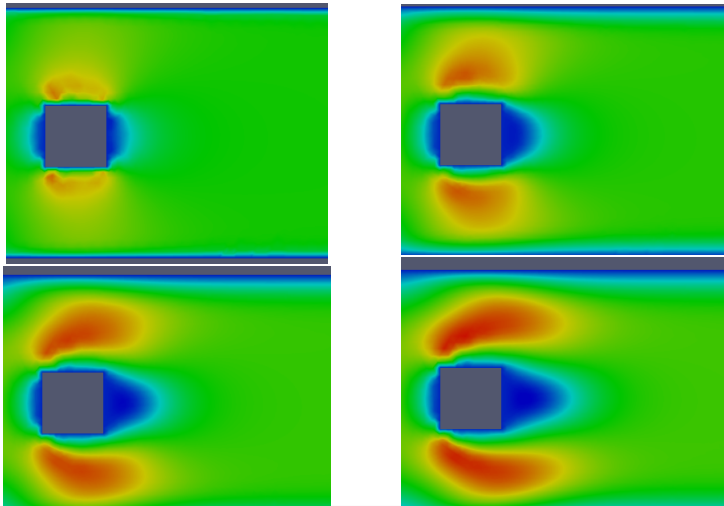DNNs in the context of the described problem

- The solutions of the PDE can be visualized as images

$\Longrightarrow$ use DNNs to generate solutions of the simulation in image form

- Why use images as input for the network?
- Why are images useful as network output?

Motivation   Research Definition   Related Work   Methodology   Workplan   Timeplan   Conclusion
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Problem definition and motivation

Images of simulations

Definition

Research topic

The applicability of DNNs in generating solutions for PDEs in numerical simulation context.

Research question

To what extend can DNNs generalize the parameters of a simulation of an incompressible fluid flow inside a channel according to the Navier-Stokes equation? The parameters of interest are:

- Fluid viscosity and density
- Inflow speed
- Object in space

To be studied is a DNN-based model that processes the image representations of the solutions of the simulation.

Related Work & State-of-the-Art

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Related Work & State-of-the-Art

DNNs in Image processing

(with focus on image-to-image mapping)

- Used in wide variety of tasks
    - Image Segmentation
    - Semantic Image Synthesis
- We build upon the work of *pix2pixHD*[2]
    - "general-purpose solution to image-to-image translation problems"
    - has not yet been applied to generation of simulation images

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[2]Ting-Chun Wang, Ming-Yu Liu, Jun-Yan Zhu, Andrew Tao, Jan Kautz, and Bryan Catanzaro. "High-resolution image synthesis and semantic manipulation with conditional gans."

Related Work & State-of-the-Art

Neural Networks in numerical simulations

(with focus on Navier-Stoke problems)

- Hidden Fluid Mechanics: A Navier-Stokes Informed Deep Learning Framework for Assimilating Flow Visualization Data[3]
    - tailored DNN-model
    - maps concentration of substance (in time and space) to velocity and pressure
- Study of Deep Learning Methods for Reynolds-Averaged Navier-Stokes Simulations of Airfoil Flows[4]
    - considers the Reynolds-Averaged Navier-Stokes equation
    - maps boundary conditions to velocity and pressure fields

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[3]Maziar Raissi and Alireza Yazdani and George Em Karniadakis. (2018) "...." 1808.04327

[4]Nils Thuerey, Konstantin Weissenow, Harshit Mehrotra, Nischal Mainali, Lukas Prantl. (2018). "....". 481-490. 1810.08217.

Methodology

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Methodology

Standard machine learning project

Methodology

Standard machine learning project.

$\Longrightarrow$

1) Generate training data
2) Build and train model
3) Evaluate model

Methodology

1) Generate training data
2) Build and train model
3) Evaluate model

- Use *HiFlow* to run simulations
- Use *ParaView* to generate images based on the simulation results

Methodology

1) ~~Generate training data~~

2) Build and train model

3) ~~Evaluate model~~

- Implement a model in *PyTorch*
    - following the framework of *pix2pix*
    - *ResNet*[5] based architecture
    - *GAN* (Generative Adversarial Network) based approach
- Train with the generated data

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[5]Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. "Delving deep into rectifiers: Surpassing human-level performance on imagenet classification."

Methodology

1) Generate training data
2) Build and train model
3) Evaluate model

- Deviation from the true solution-image as an error measurement
- Two evaluation cases to consider:
  - Error when applying the model on individual data points
  - Error when applying the model recursively

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Definition

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Research question

To what extend can DNNs generalize the parameters of a simulation of an incompressible fluid flow inside a channel according to the Navier-Stoke equation? The parameters of interest are:

- Fluid viscosity and density
- Inflow speed
- Object in space

To be studied is a DNN-based model that processes the image representations of the solutions of the simulation.

$\implies$ **Goal:** Build a separate model for each of the cases being studied.
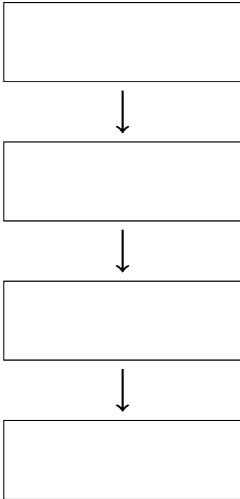
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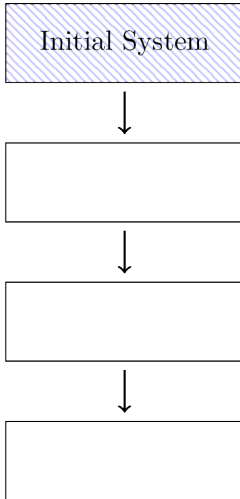- Inflow speed
- Object in space

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Methodology\Workplan

- Data generation
- Implementation of core components
    - Data loader
    - Model architecture
    - Training infrastructure
    - Evaluation infrastructure
- Training and evaluating a baseline model
    - works only with image data
    - thought of as the basis for further development

Methodology\Workplan

Initial System

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Fluid parameters

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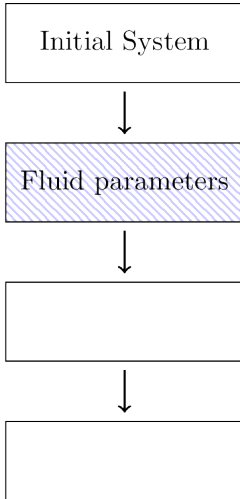
- Model modifications
  - fluid viscosity and density as input
- Training and evaluating

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Methodology\Workplan
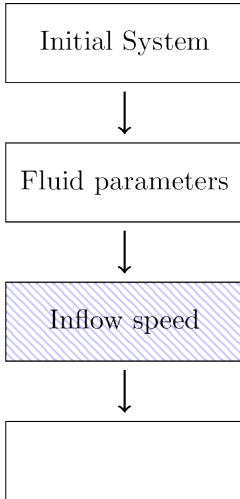
- Model modifications
  - inflow speed as input
- Training and evaluating

Motivation
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Methodology\Workplan

Initial System

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Fluid parameters

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Inflow speed

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Object in space
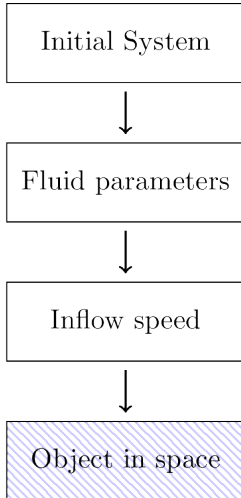
- Model modifications
  - simulation space as input
- Training and evaluating

Motivation
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Methodology\Workplan

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Timeplan

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Thank you for your attention.

Motivation   Research Definition   Related Work   Methodology   Workplan   Timeplan   **Conclusion**
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Questions?

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