Lateral-Directional Aerodynamics Parameter Estimation using Neural Partial Differentiation

Vikalp Dongare
Research Scholar
Department of Aeronautical Engineering
M.V.J. College of Engineering
Bangalore, India
Email: vikalp.dongre@gmail.com

Dr. Majeed Mohamed
Senior Scientist
Flight Mechanics and Control Division
CSIR-National Aerospace Laboratories
Bangalore, India
Email: majeed_md_123@rediffmail.com

***Abstract*—In this paper, application of neural networks combined with partial differentiation of the neural outputs has been discussed to estimate lateral-directional flight stability and control parameter. A neural model capable of predicting generalized force and moment coefficients using measured motion and control variables can be employed to extract aerodynamic parameters from flight data. The Neural Partial Differentiation method is used for this purpose. The estimated results are compared with the parameter estimates obtained from Output Error Method. The validity of estimates has been verified by the model validation method, wherein the estimated model response is matched with the flight-test data that are not used for estimating the parameter.**

Keywords—Parameter estimation; Neural network; Aircraft modeling ;

I. Introduction

One of the important aspects of flight-testing of an aircraft is the estimation of its stability and control parameter from flight data [1], [2]. Parameter estimation is a key element of aircraft system identification, and system identification is a general procedure to match the observed input-output response of a system dynamics by a proper choice of an input output model and its physical parameters [3], [4]. Aircraft parameter estimation has become an important tool for flight test engineers to determine the aerodynamic characteristic of an aircraft from the flight data [5]. Therefore, the extraction of aerodynamic parameters from flight data has received increased attention over the past several years [3]–[7].

The higher order modeling is requisite to describe the complex system like an aircraft. The commonly used methods to estimate aircraft parameters in order to model the aircraft system dynamics are Equation Error Method (EEM), Filtering Methods (FM) and Output Error Method (OEM). The simplest method for the aircraft parameter estimation is considered as EEM, which was successfully applied to flight data of F-18 High alpha research vehicle [8], and to a gliding flight vehicle [9]. This method needs only the model structure of the aircraft dynamics and does not required a priori knowledge of parameters. However, in the presence of noise, the least squares estimates of EEM are asymptotically biased, inconsistent and inefficient. Whereas Filtering Method is used to estimate aerodynamic parameters accurately from the flight data during the change of process noise [10], but this algorithm requires high computational power and initial values of the estimates. OEM is based on the assumption of presence of measurement noise only. This method also needs a priori knowledge of dynamic model and initial values of the parameters to estimate aircraft stability and control derivatives [1], [11], [12]. The initial values of certain parameters are available from the wind tunnel data base of an aircraft. Conversely, the use of scaled version of aircraft in the wind tunnel may introduce the errors in the prediction of aerodynamic parameter. This motivates the application of Neural Networks that can provide accurate estimates of aircraft parameters without their initial values.

To overcome the difficulties posed by the classical approaches (EEM, FM, OEM), neural networks are used for the aircraft parameter-estimation problem [6], [13]–[16]. Delta and Zero Method of neural networks are commonly used to extract aircraft aerodynamic parameter from flight data [6], [15]. These methods are able to provide the estimates of aircraft parameters, but the statistics of estimates are not inferred directly. Whereas, recently introduced Neural Partial Differential (NPD) method is able to give theoretical insight into statistical information of relative standard deviation (RSTD) of estimates from noisy data [17]. This method has originated from the fact that solution of ordinary differential equation and partial differential equation can be obtained by neural networks [18]. Moreover, NPD Method can also extract parameters of dynamical systems, which are nonlinear to the states of the system. This paper extends the use of NPD Method for multi input and multi output (MIMO) aircraft Neural Model, previously it has employed only for the extraction of aerodynamics parameter from multiple input single output (MISO) aircraft system [16]. The main contributions of this paper are

- The primary investigation of aircraft lateral-directional aerodynamic parameter estimation is carried out with the simulated data of a small transport aircraft. Neural Partial Differentiation (NPD) method is used for this purpose. The results are found to be encouraging to apply with flight data.
- lateral-directional flight stability and control parameters are estimated from flight data. The results are compared with the estimates obtained from Output Error Method (OEM).
- The estimated Neural model of aircraft is validated by a complementary set of flight data.

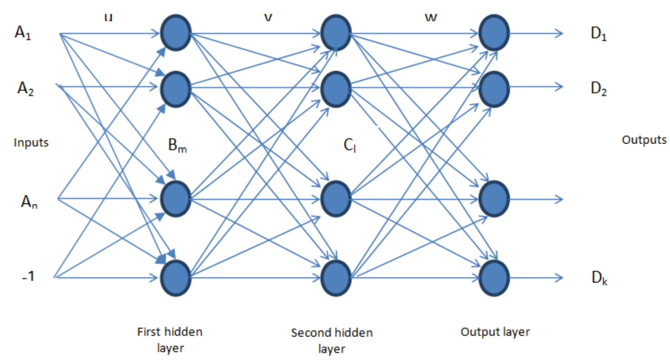


Fig. 1. Schematic of Neural Network

The paper is organized as follows: Section II of the paper describes the neural network based partial differentiation for the parameter estimation of aircraft. Lateral-Directional flight stability and control parameters estimation results are presented in section III and finally the conclusions are given in section IV.

II. Aircraft Parameter estimation

The neural networks are made up of two main components namely neuron or nodes and the connectors. The connectors have own weights between two nodes. The neural network uses the data set of input and output, to map the function on to the network in the form of weights between the internal nodes as shown in the Figure 1. The schematic structure of a three layered feed-forward neural network (NN) is consisting of two hidden layer with activation function and one output layer with summation function exempted from activation function. The weights indirectly represent the function of a given system for which the neural network is trained. The output of each node is the sum of product of the total input to the particular node and their respective weights, applied to an activation function. Back-propagation approach is used for training the neural network. The neural networks learn through input-output pair of the system and give an approximate function in the form of weights. The complexity of the network can be changed with number of neurons and/or the number of hidden layer, this decision is purely based on trial and error method. The input and output vectors of neural network are defined as $A \in \Re^{n+1}$ and $D \in \Re^{k}$, respectively. Similarly, $B \in \Re^{m+1}$ and $C \in \Re^{l+1}$ represent the first and second hidden layer of neural network. Except for the output layer all the layers contain a bias term. Thus, the output of neural network is given by

$$D = W^T C \tag{1}$$

where W is the set of weights between the second hidden layer and output layer containing the bias terms.

$$W = \begin{bmatrix} b_{w1} & \cdots & b_{wk} \\ w_{11} & \cdots & w_{1k} \\ \vdots & \ddots & \vdots \\ w_{l1} & \cdots & w_{lk} \end{bmatrix} \tag{2}$$

Similarly we define

$$\begin{cases} C = f(V^T B) \\ B = g(U^T A) \end{cases} \tag{3}$$

Where f and g are the activation function vectors and are defined as $f = [\ -1 \quad f(x_1) \quad \cdots \quad f(x_k)\ ]^T$ where $f = (x)$ is expressed as

$$f(x) = \frac{1 - e^{-\lambda x}}{1 + e^{-\lambda x}} \tag{4}$$

And the weight matrix are represented as

$$V = \begin{bmatrix} b_{v1} & \cdots & b_{vm} \\ v_{11} & \cdots & v_{1l} \\ \vdots & \ddots & \vdots \\ v_{m1} & \cdots & v_{ml} \end{bmatrix} \tag{5}$$

$$U = \begin{bmatrix} b_{u1} & \cdots & b_{um} \\ u_{11} & \cdots & u_{1m} \\ \vdots & \ddots & \vdots \\ u_{n1} & \cdots & u_{nm} \end{bmatrix} \tag{6}$$

Input is defined by the vector $A = [\ a_0 \quad a_1 \quad \cdots \quad a_n\ ]$, where a_0 defines bias input to the neural network. The input and output are scaled for neural network using the following equation.

$$D_{i,norm} = D_{i,norm_{\min}} + (D_{i,norm_{\max}} - D_{i,norm_{\min}}) \times \frac{D_i - D_{i,\min}}{D_{i,\max} - D_{i,\min}} \tag{7}$$

Where $D_{i,norm_{\max}}$ and $D_{i,norm_{\min}}$ denote the higher and lower limits of scaling range of D_i respectively. They are set to 0.9 and -0.9 respectively. $D_{i,max}$ and $D_{i,min}$ denote the higher and lower values of D_i.

Using the above notations, output of neural network can be written as

$$D = \{W^T f[V^T g(U^T A)]\} \tag{8}$$

A. Neural Partial Differentiation

In this method, the neural network is trained with input and output data so as to map the nonlinear function in the form of weights. The activation function holds the key for the neural partial difference method. This method does not need extra post processing as the zero and Delta method demands. Moreover, it has facility to determine the higher-order partial derivatives of nonlinear system. The partial differentiation of a system can be computed from the end of training session of neural network, and provide aerodynamic derivatives directly as follows:

The input and output of function is mapped after the training session of the neural network. Subsequently, the output variables can be differentiated with respect to input variables. Differentiate (1) and (3), we will have the form of

$$\frac{\partial D}{\partial C} = W^T \tag{9}$$

$$\frac{\partial C}{\partial B} = f'(V^T) \tag{10}$$

$$\frac{\partial B}{\partial A} = g'(U^T) \tag{11}$$

Multiplication of (9),(10), and (11) gives

$$\begin{cases} \frac{\partial D}{\partial C} \cdot \frac{\partial C}{\partial B} \cdot \frac{\partial B}{\partial A} = W^T . f'V^T . g'U^T \\ \frac{\partial D}{\partial A} = W^T . f'V^T . g'U^T \end{cases} \tag{12}$$

where $f' = diag[\ 0 \quad f'_1 \quad \cdots \quad f'_l\ ]$ and $g' = diag[\ 0 \quad g'_1 \quad \cdots \quad g'_m\ ]$ If the input and output of neural network are normalized, then

$$\frac{\partial D}{\partial A} = \frac{\partial D}{\partial D_{norm}} \times \frac{\partial D_{norm}}{\partial A_{norm}} \times \frac{\partial A_{norm}}{\partial A} \tag{13}$$

The normalized output of neural network can be de-normalized by (13). where,

$$\frac{\partial D}{\partial D_{norm}} = \begin{bmatrix} \frac{\partial D_1}{\partial D_{1,norm}} & 0 & \cdots & 0 \\ 0 & \frac{\partial D_2}{\partial D_{2,norm}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial D_k}{\partial D_{k,norm}} \end{bmatrix} \tag{14}$$

$$\frac{\partial A}{\partial A_{norm}} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{\partial A_{1,norm}}{\partial A_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial A_{n,norm}}{\partial A_n} \end{bmatrix} \tag{15}$$

The (14) and (15) can be computed from (7). The terms associated of (9) to (15) be intermediate terms of neural networks while getting it trained. Therefore, there is no extra computation required to compute the aerodynamic derivatives, and they are directly given as:

$$\frac{\partial D}{\partial A} = \begin{bmatrix} \frac{\partial D_1}{\partial A_0} & \cdots & \frac{\partial D_1}{\partial A_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial D_k}{\partial A_0} & \cdots & \frac{\partial D_k}{\partial A_n} \end{bmatrix} \tag{16}$$

The standard deviation of estimated parameters in (16) is computed by

$$STD = \sqrt{\frac{\sum_{p=1}^{P} \left[\sum_{m=1}^{M} (\sum_{l=1}^{L} C'_{l_p} v_{lm} w_{kl} D'_{k_p}) B'_{m_p} u_{mi} - AVG \right]^2}{P}} \tag{17}$$

where,

$$AVG = \frac{\sum_{p=1}^{P} \sum_{m=1}^{M} (\sum_{l=1}^{L} C'_{l_p} v_{lm} w_{kl} D'_{k_p}) B'_{m_p} u_{mi}}{P} \tag{18}$$

where, STD and AVG are standard deviation and average of data points, respectively. The relative standard deviation of estimates is given by

$$RSTD \quad = \quad \frac{STD}{AVG} \quad \times 100\% \tag{19}$$

B. Output Error Method

In the output error method (OEM), the unknown parameters are obtained by minimization the sum of weighted square differences between the measured outputs and model outputs. The estimation problem is nonlinear because of unknown parameter appears in the aircraft equations of motion and they are integrated to compute the states.Outputs are computed from states,control input and parameters using the measurement equation. Iterative nonlinear optimization techniques are required to solve this nonlinear estimation problem [19], [20].

III. Parameter estimation results and discussion

The online estimation of lateral-directional aircraft stability and control parameter using OEM and NPD methods was achieved. The primary investigation was carried out with simulated data of small transport aircraft. Estimated aerodynamic derivatives from simulated data are tabulated in Table I.

TABLE I. Estimated aerodynamic derivatives from simulated data

Parameters	True value	NPD	OEM
Cy_b	-1.43	-1.40 (0.1756)	-1.42 (0.49)
Cl_b	-0.109	-0.102 (0.1093)	-0.114 (1.02)
Cn_b	0.104	0.100 (0.31)	-0.114 (0.38)
Cy_p	-0.102	-0.125 (0.18)	-0.191 (19.56)
Cl_p	-0.599	-0.593 (0.07)	-0.597 (1.12)
Cn_p	-0.175	-0.189 (0.76)	-0.124 (2.04)
Cy_r	0.454	1.90 (0.23)	1.97 (2.36)
Cl_r	0.220	0.297 (0.14)	0.365 (1.57)
Cn_r	-0.141	0.388 (0.13)	-0.236 (1.13)
$Cy_{\delta a}$	-0.002	-0.20 (0.92)	-0.076 (8.76)
$Cl_{\delta a}$	-0.119	-0.107 (0.21)	-0.124 (0.85)
$Cn_{\delta a}$	-0.011	-0.007 (0.74)	-0.005 (8.95)
$Cy_{\delta r}$	0.328	0.389 (0.12)	0.248 (2.53)
$Cl_{\delta r}$	0.051	0.047 (0.42)	0.033 (2.02)
$Cn_{\delta r}$	-0.112	-0.111 (0.38)	-0.078 (0.45)

* The values in parenthesis denote relative standard deviation values in percentage.

The neural model of an aircraft system has been established by the training of input-output data. NPD can be applied to extract aerodynamic parameters from the neural model of aircraft, and their corresponding standard and relative standard deviations are able to compute. Figure 2 shows time histories of the input signals ($\beta, p, r, \delta a$ and δa) to the neural network and the output signals C_y, C_l and C_n . The flight simulation has carried out by input doublet signals of aileron and rudder for a time duration of 50 seconds. Figure 3 shows the estimates of side force aerodynamic derivatives (parameter) using the NPD method for simulated data with respect to data points. It can be observed that there is a marginal variation in the aerodynamic derivatives with respect to the different data points. The variation of side force derivative with respect to the number of iteration is shown in Fig. 4. As the number of iteration increases, the parameters attain stable value of its estimates. The time history response of flight data and estimated responses are given in Fig. 5, and found that they are in close agreement with other. This ensure that the dynamics of aircraft model have been accurately identified.Estimated aerodynamic derivatives from flight data are tabulated in Table II. The variation of parameter associated to side force, lateral- stability, directional-stability and control with respect

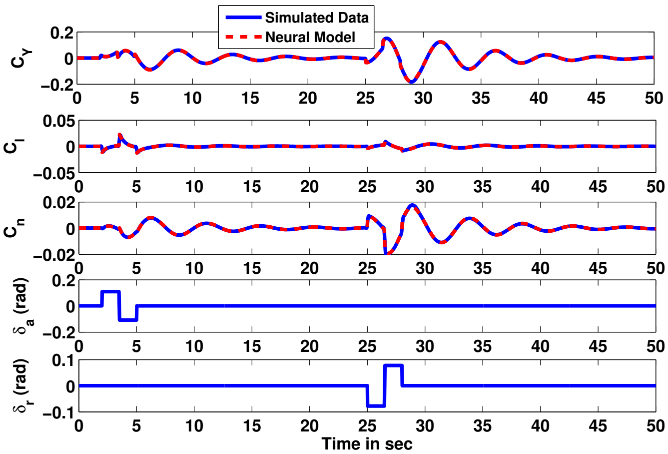


Fig. 2. Time history response of simulated data and neural model

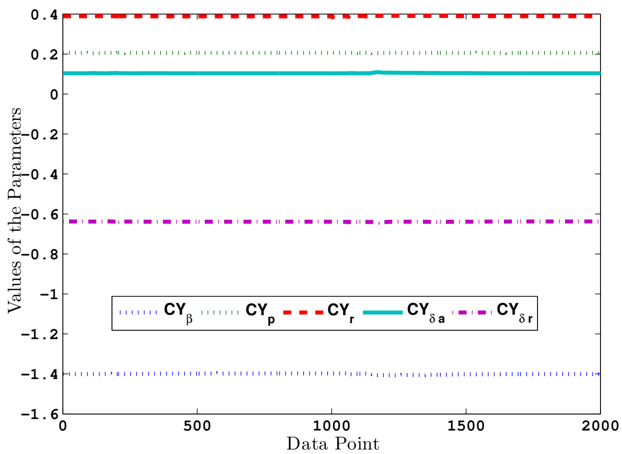


Fig. 3. Variation in parameters with respect to data points of simulated data

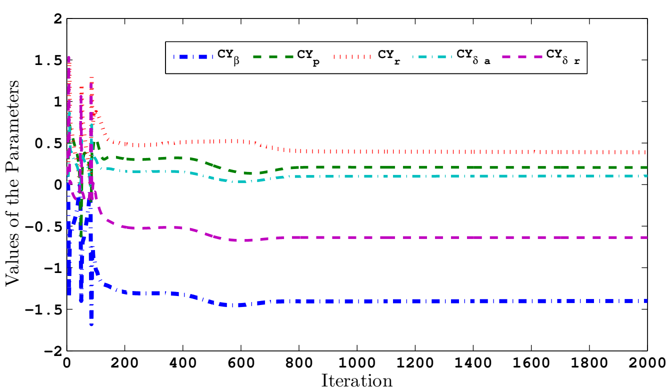


Fig. 4. Variation in parameters w.r.t. number of iterations during training for simulate data

to data points are shown in Fig. 6. The variation of these parameter with number of iterations are given in Fig. 7. This can be observed from Fig. 6 that certain parameter shows their variation due to the influence of noise. Rest of the parameter are closer to the wind tunnel values.The close agreement of

TABLE II. ESTIMATED AERODYNAMIC DERIVATIVES FROM FLIGHT DATA

Parameters	Wind Tunnel value	NPD	OEM
Cy_b	-1.432	-1.648 (0.53)	-1.423 (0.49)
Cl_b	-0.109	-0.0895 (1.43)	-0.113 (1.02)
Cn_b	0.103	0.120 (0.56)	0.114 (0.38)
Cy_p	-0.102	-0.366 (0.28)	-0.190 (19.56)
Cl_p	-0.599	-0.462 (0.14)	-0.597 (1.12)
Cn_p	-0.175	-0.047 (3.25)	-0.124 (2.04)
Cy_r	0.454	1.405 (0.63)	1.975 (2.36)
Cl_r	0.221	0.02 (1.29)	0.365 (1.57)
Cn_r	-0.141	-0.371 (0.22)	-0.236 (1.13)
$Cy_{\delta a}$	-0.002	-0.037 (0.34)	-0.076 (8.76)
$Cl_{\delta a}$	-0.119	-0.111 (0.19)	-0.124 (0.85)
$Cn_{\delta a}$	-0.011	0.002 (0.83)	-0.005 (8.95)
$Cy_{\delta r}$	0.328	-0.365 (0.25)	0.248 (2.53)
$Cl_{\delta r}$	0.051	0.029 (9.26)	0.033 (2.02)
$Cn_{\delta r}$	-0.110	-0.079 (0.21)	-0.077 (0.45)

* The values in parenthesis denote relative standard deviation values in percentage.

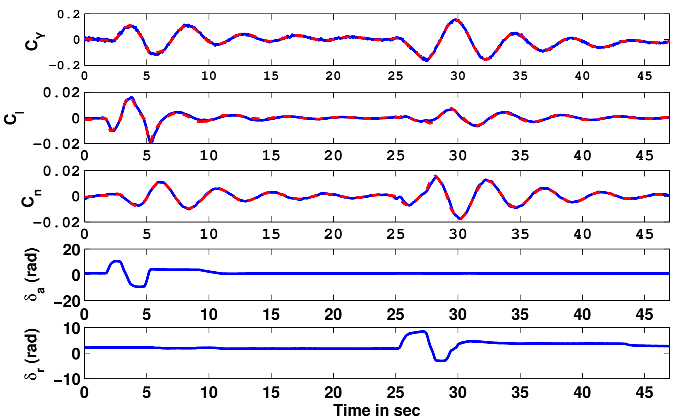
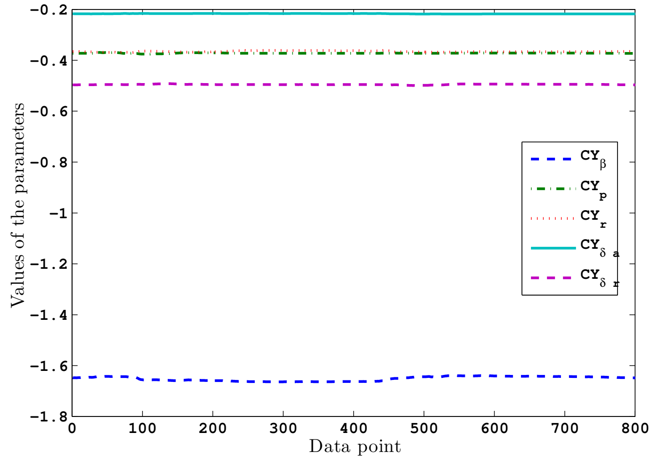
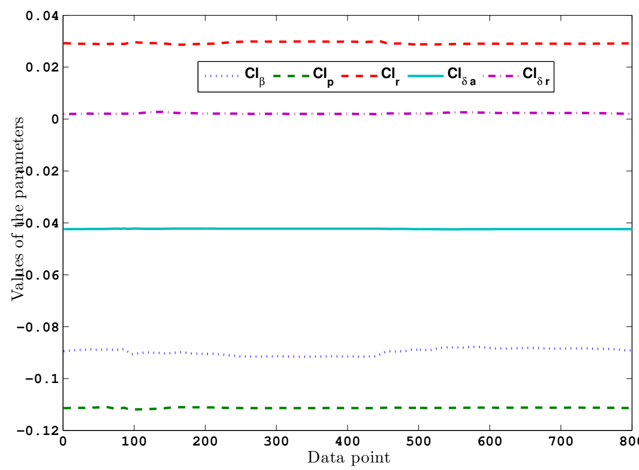


Fig. 5. Time history response of flight data and neural model

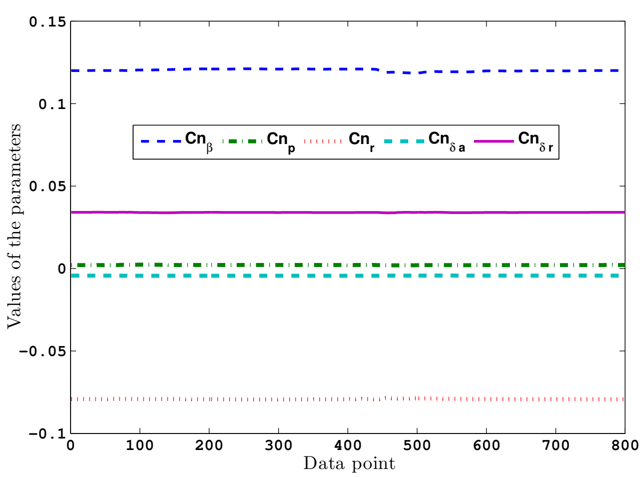
flight data with reconstructed from neural model in Fig. 8 indicates the accuracy of the estimated neural model of aircraft system. The estimated aircraft neural model is needed to be verified with complimentary flight data. For this, the neural network is trained with certain data set, and then a new data set is passed through the trained network. The output is used to compute a_y for the given input complementary flight data . The a_y can also be computed by using the side force coefficient obtained from the Wind tunnel and estimates of OEM. These computed a_y are compared with measured a_y for the same input, and comparison plot for the accelerations a_y is given in Fig. 9. A good match between these flight measured and predicted response is witnessed.



(a) side force derivative in Cy

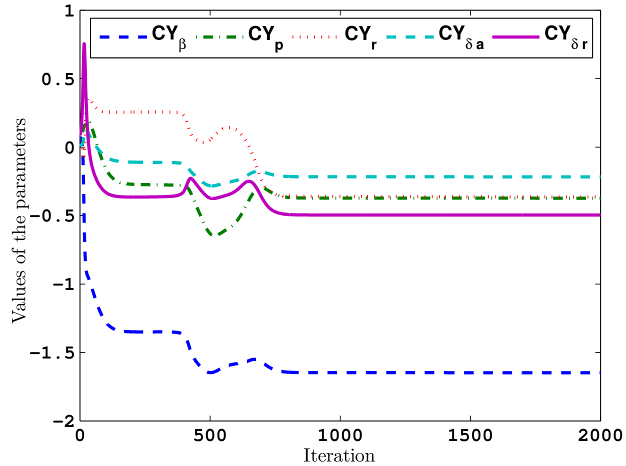


(b) lateral stability & control derivative in Cl

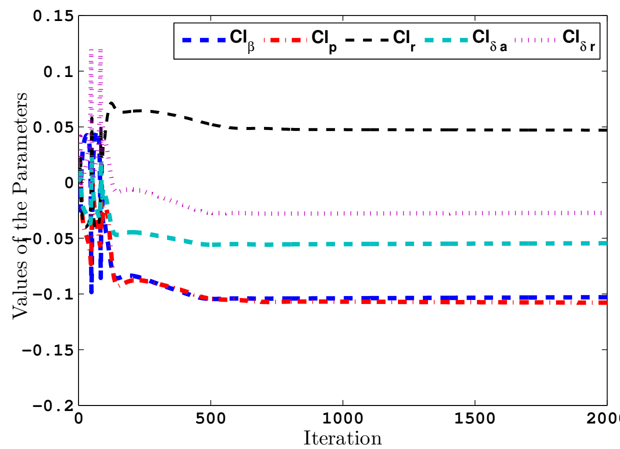


(c) Directional stability & control derivative in Cn

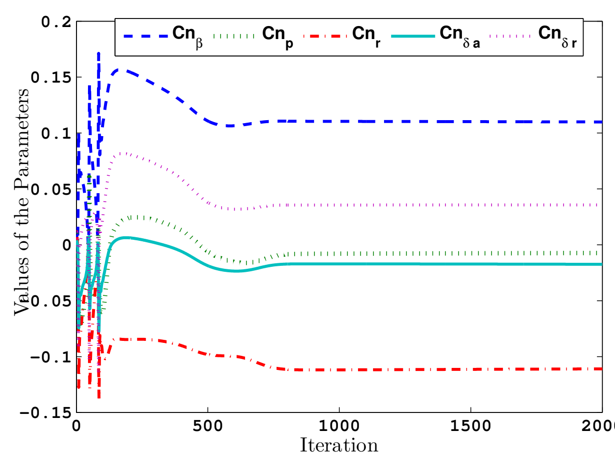
Fig. 6. Variation in parameters with respect to data points of Flight data



(a) side force derivative in Cy



(b) lateral stability & control derivative in Cl

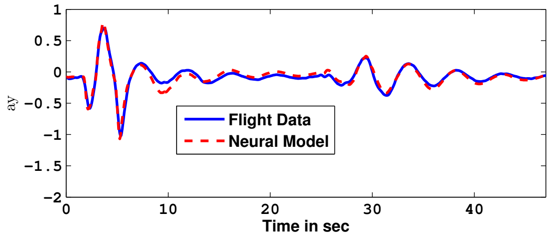


(c) Directional stability & control derivative in Cn

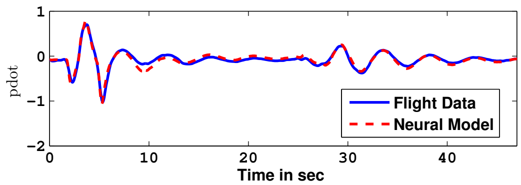
Fig. 7. Variation in parameters with respect to number of iterations during training for Flight data

IV. Conclusion

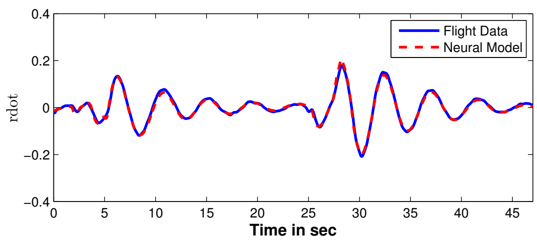
Neural Partial Differentiation (NPD) method is applied to simulated and flight data of small transport aircraft to estimate lateral-directional flight stability and control parameters. For this purpose, initially neural model of multi-input multi-output (MIMO) aircraft system is established. The primary inves-



(a) a_y



(b) $\dot{p}$



(c) $\dot{r}$

Fig. 8. Comparison of a_y, $\dot{p}$, $\dot{r}$ from Flight Data and Reconstructed from Neural Networks

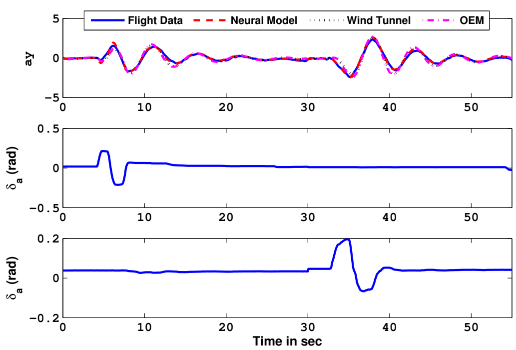


Fig. 9. Time history verification of identified neural model

tigation of lateral-directional parameter estimation is carried out from simulated data, and found that the estimates are very close to wind tunnel values. Neural Partial Differentiation method is employed to extract the Lateral-directional aerodynamic parameters from flight data, and the estimated parameters are comparable with estimates obtained from Output Error Method. Since the initial values of parameters are not available in practical situation as well as OEM requires these initial parameters, the neural network approach works well with the flight data. Finally, the identified neural model is validated by complementary flight data of small transport aircraft.

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