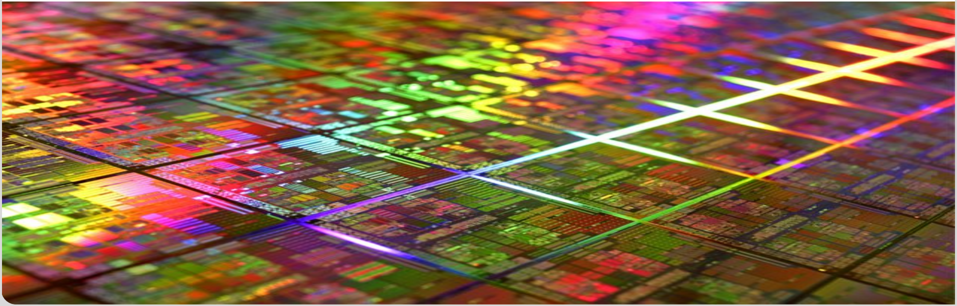
Towards Bringing Together Numerical Methods for Partial Differential Equation and Deep Neural Networks

State of the Art, Supervisor - Markus Hoffmann

Stanislav Arnaudov | December 8, 2019

CHAIR FOR COMPUTER ARCHITECTURE AND PARALLEL PROCESSING



Problem definition and motivation

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Partial differential equation (PDEs)

- used in simulations
- solutions have image representation
- hard to solve numerically

Goal: solve PDEs based on their image representation

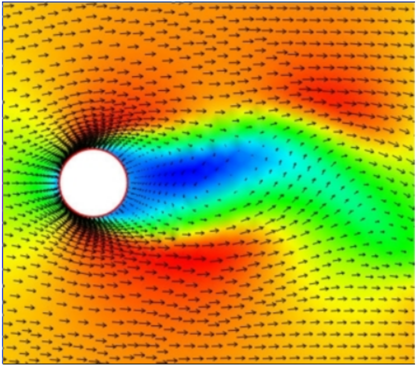


Figure: Flow Simulation[1]

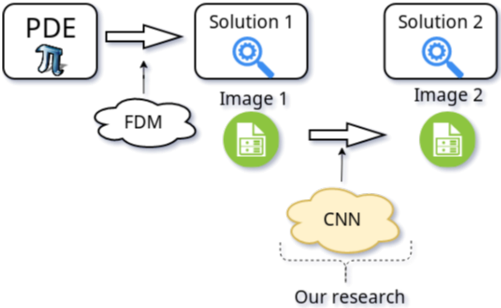
[1] "Team for Advanced Flow Simulation and Modeling", Professor Tayfun E. Tezduyar, Sunil Sathe

Problem definition and motivation

Convolutional neural networks (CNNs)

- hot topic in recent years
- impressive results in image processing

Idea: Use CNNs for the image representation of PDEs.

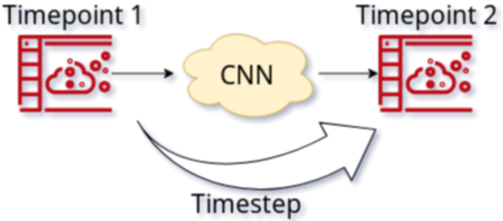


General Goals

Important Questions

1. CNNs for image-to-image mapping?

2. CNNs for numerical simulations?



CNNs for image processing

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We focus on an area where CNNs show good results:

- Image Segmentation
  - pixelwise decision about a class belonging
  - a new image is generated

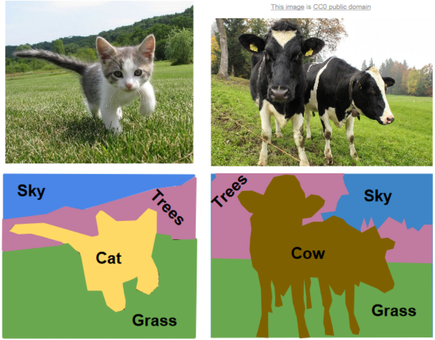

Figure: Image Segmentation[2]

[2] "Exploring Computer Vision in Deep Learning: Object Detection and Semantic Segmentation", Long et al.,SAS Institute Inc.

CNNs for image processing

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Image segmentation

- "Fully Convolutional Networks for Semantic Segmentation"[3]
  - Encoder-Decoder Architecture
  - Only convolutional layers

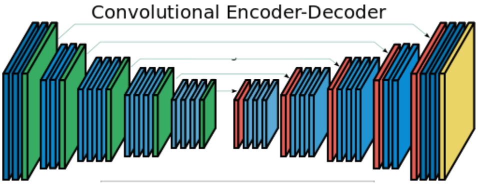

Figure: The encoder-decoder network

[3] Jonathan Long, Evan Shelhamer, and Trevor Darrell. "Fully Convolutional Networks for Semantic Segmentation". In: *CoRR* abs/1411.4038 (2014). arXiv: 1411.4038. URL: http://arxiv.org/abs/1411.4038.

CNNs for image processing

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Image segmentation

- “Semantic Segmentation using Adversarial Networks”[4]
  - Generative Adversarial Networks[5]
  - The Discriminator Network enforces contagiously segmented regions

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[4] Pauline Luc et al. “Semantic Segmentation using Adversarial Networks”. In: *CoRR* abs/1611.08408 (2016). arXiv: 1611.08408. URL: http://arxiv.org/abs/1611.08408.

[5] Ian Goodfellow et al. “Generative Adversarial Nets”. In: *Advances in Neural Information Processing Systems 27*. Ed. by Z. Ghahramani et al. Curran Associates, Inc., 2014, pp. 2672–2680. URL: http://papers.nips.cc/paper/5423-generative-adversarial-nets.pdf.

CNNs for image processing

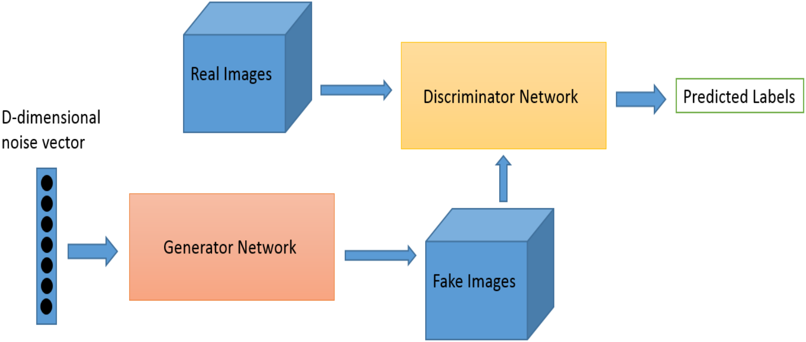

Figure: Generative Adversarial Network[6]

[6] "A Beginner's Guide to Generative Adversarial Networks (GANs)", https://skymind.ai/wiki/generative-adversarial-network-gan

CNNs for image processing

Semantic Image Synthesis

- "Semantic Image Synthesis with Spatially-Adaptive Normalization"[7]
  - Segmentation in reverse

Figure: Example of "reversed image segmentation"

[7] Taesung Park et al. "Semantic Image Synthesis with Spatially-Adaptive Normalization". In: *CoRR* abs/1903.07291 (2019). arXiv: 1903.07291. URL: http://arxiv.org/abs/1903.07291.

CNNs for numerical applications

Long standing interest in bringing together neural networks and formal mathematics.

- It is shown to be possible - “Performing basic mathematics with neurons/nets” (2002)[8]
- Even for PDEs - “Artificial Neural Networks for Solving Ordinary and Partial Differential Equations” (1998)[9]

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[8] Richard Neville. “Performing basic mathematics with neurons/nets: Part I”. In: vol. 1. Feb. 2002, pp. 589–594. ISBN: 0-7803-7278-6. DOI: 10.1109/IJCNN.2002.1005538.

[9] I. E. Lagaris, A. Likas, and D. I. Fotiadis. “Artificial neural networks for solving ordinary and partial differential equations”. In: *IEEE Transactions on Neural Networks* 9.5 (1998), pp. 987–1000. ISSN: 1045-9227. DOI: 10.1109/72.712178.

CNNs for numerical applications

Nowadays CNNs are applied to complex physical simulations.

- CNNs for calculating physical properties of objects in simulations
- CNNs in flow simulation

CNNs for numerical applications

- “Learning Soft Tissue Behavior of Organs for Surgical Navigation with Convolutional Neural Networks”[10]
  - Prediction for how organs in the human body move during operation
  - Encoder-Decoder Architecture

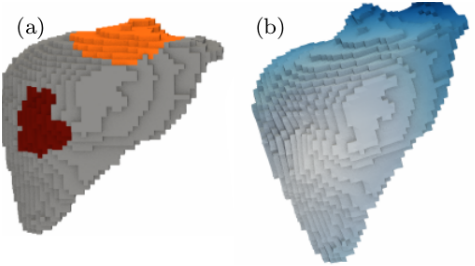
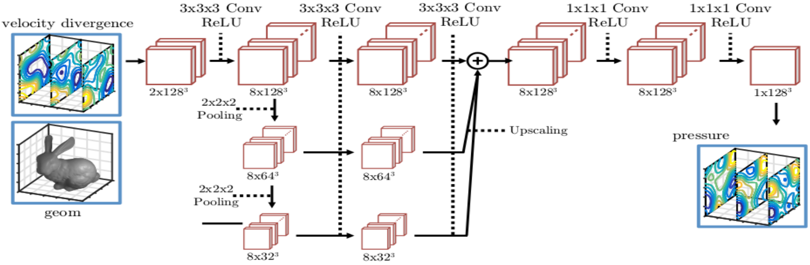


Figure: Given and predicted displacement of an organ

[10] Micha Pfeiffer et al. “Learning Soft Tissue Behavior of Organs for Surgical Navigation with Convolutional Neural Networks”. In: *CoRR* abs/1904.00722 (2019). arXiv: 1904.00722. URL: http://arxiv.org/abs/1904.00722.

CNNs for numerical applications

- "Accelerating Eulerian Fluid Simulation With Convolutional Networks"[11]
  - CNN for predicting fluid-flow
  - Calculate pressure given velocity divergence and geometry



[11] Jonathan Tompson et al. "Accelerating Eulerian Fluid Simulation With Convolutional Networks". In: *CoRR* abs/1607.03597 (2016). arXiv: 1607.03597. URL: http://arxiv.org/abs/1607.03597.

Conclusion

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To reiterate

- *Problem:* Solve PDEs
- *Approach:* Use CNNs for image-to-image mapping
- *Goal:* Efficiency and acceptable error

The conducted research illustrates:

- A research gap
- Evidence suggesting the proposed method is viable
- Typical CNN architectures that should be considered

Thank you for your attention.

Questions?