Wrocław University
of Science and Technology

# Optimization Methods: Theory and Applications
# Introduction

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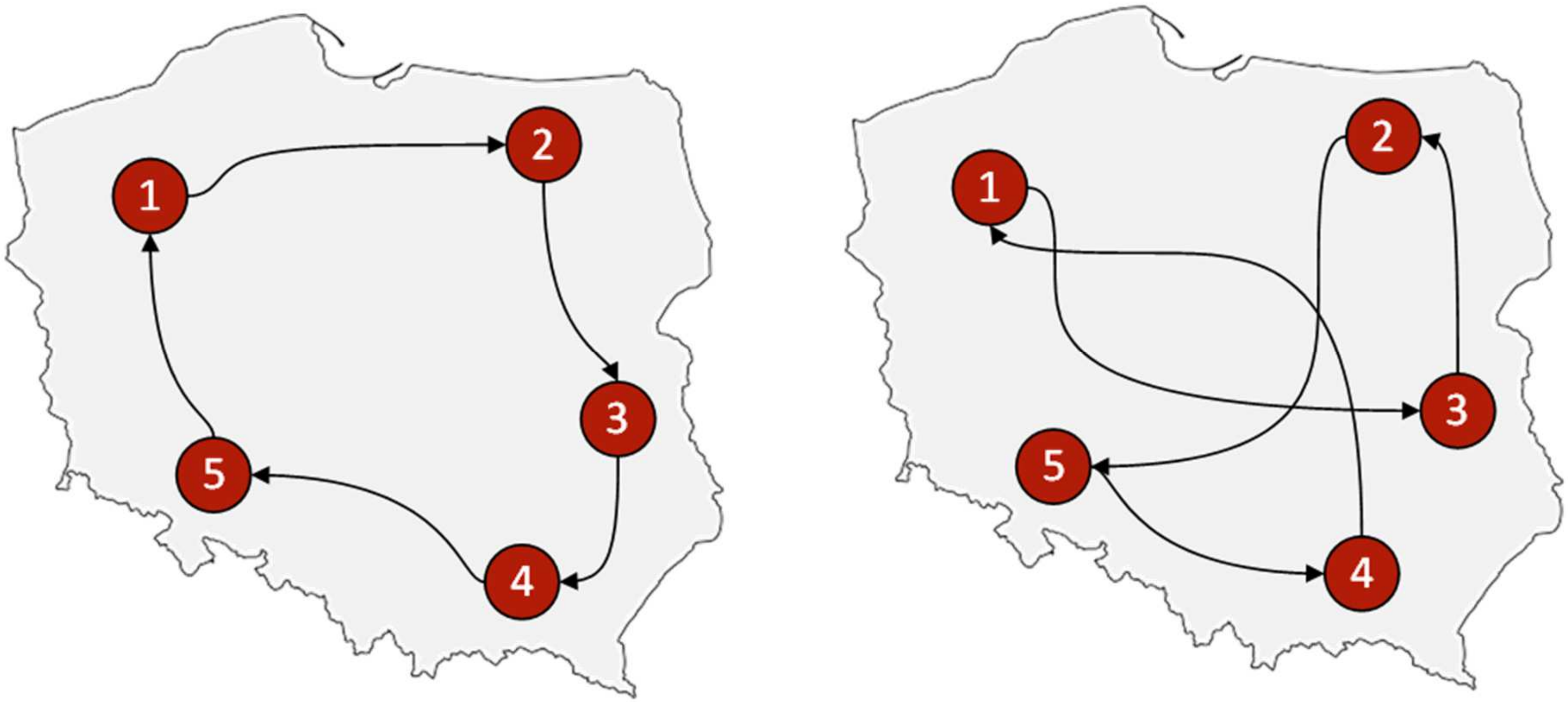
# Travelling Salesman Problem

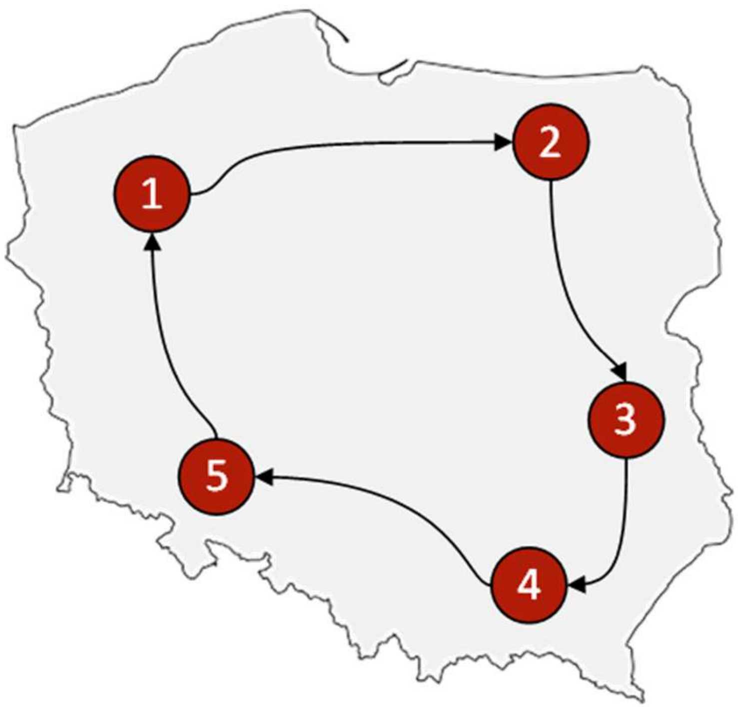
- Find the shortest route
- Visit each city exactly once

# Travelling Salesman Problem

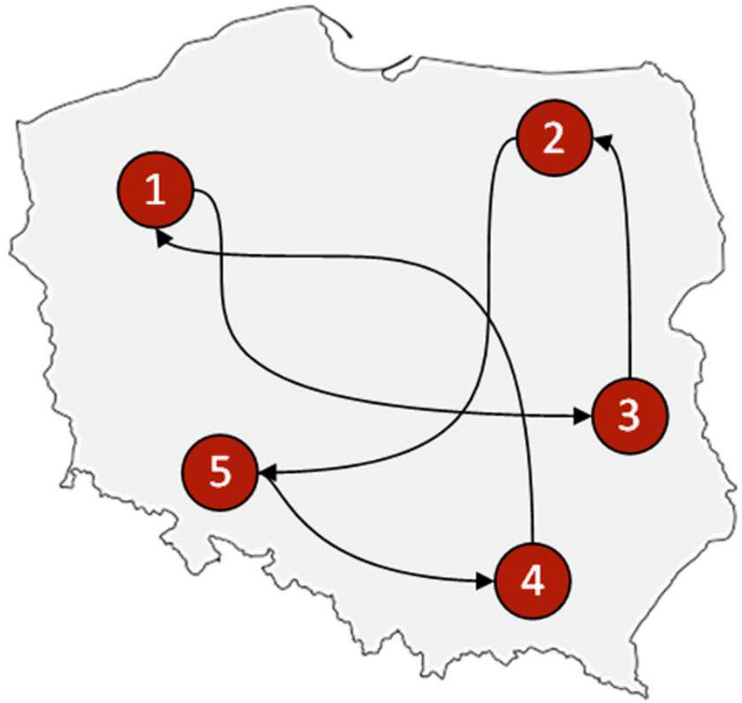



$\frac{\boldsymbol{n}!}{\boldsymbol{2n}}$ available routes (for $n = 5$ there is 12 of tchem)

# Travelling Salesman Problem




$[1, 2, 3, 4, 5]$




$[1, 3, 2, 5, 4]$

# Optimization – what do you want?

**As good solution**

as possible…

# Optimization

- Different problems have different features
- There are optimizer is optimal for every problem
- What do we want: use the features of the optimized problem (to our favour)

# Optimization – what is it?
## Traveling Salesman Problem

- Single solution:

$$\vec{x} = [x_1, x_2, x_3, x_4, x_5]$$

$$\forall_{i \in \{1,\ldots,5\}} x_i \in D_{x_i} = \{1, \ldots, 5\}$$

- The set of **all** solutions:

$$D_f = D_{x_1} \times D_{x_2} \times D_{x_3} \times D_{x_4} \times D_{x_5}$$

- The set of feasible solutions:

$$D_{\vec{x}} = \left\{\vec{x}\colon \forall_{i \in \{1,\ldots,5\}} \forall_{j \in \{1,\ldots,5\}-\{i\}} x_i \in D_{x_i} \wedge x_j \in D_{x_j} \wedge x_i \neq x_j\right\}$$

# Optimization – what is it?

## Traveling Salesman Problem

- Optimized function:

$$f : D_f \to R$$

$$f(\vec{x}) = \left(\sum_{i=1}^{4} \delta(x_i, x_{i+1})\right) + \delta(x_5, x_1)$$

where: $\delta(x, y)$ – the distance between cities $x$ and $y$

- Objective: find solution $\vec{x}^*$:

$$\vec{x}^* = \underset{\vec{x} \in D_{\vec{x}} \subseteq D_f}{\arg\min}\, f(\vec{x})$$

# Optimization – what is it?

## Various solution spaces

- **Discrete problems**

$$D_f = Z_1 \times \cdots \times Z_n$$

Example:

$$D_f = \{1,2,3\} \times \{4,5\} \times \{9,10,12,14\}$$
$$\vec{x} = [2,5,12]$$

- **Binary problems (the subset of discrete problems)**

$$D_f = \{0,1\}^n$$

Example:

$$D_f = \{0,1\} \times \{0,1\} \times \{0,1\} = \{0,1\}^3$$
$$\vec{x} = [0,1,1]$$

- **Continuous problems**

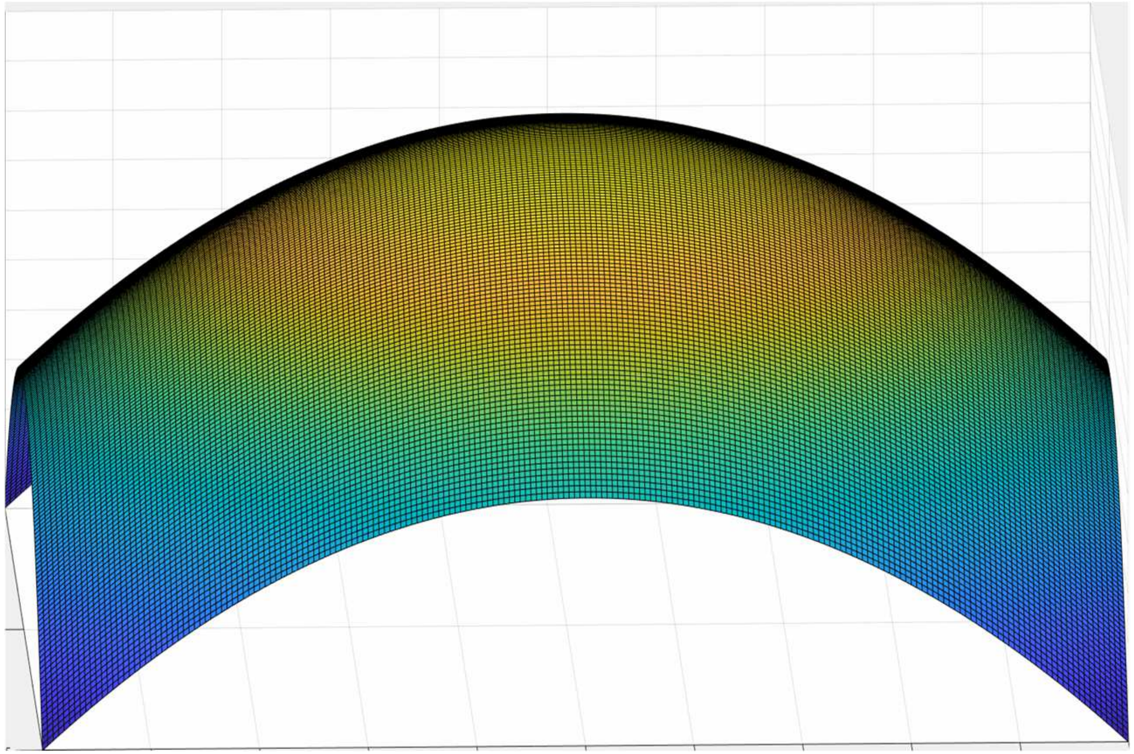
$$D_f \subseteq R^n$$

Example:

$$D_f = [-5,5] \times [-5,5] \times [-5,5] = [-5,5]^3$$
$$\vec{x} = [3.14, 2.56, 1.98]$$

# Problem nature

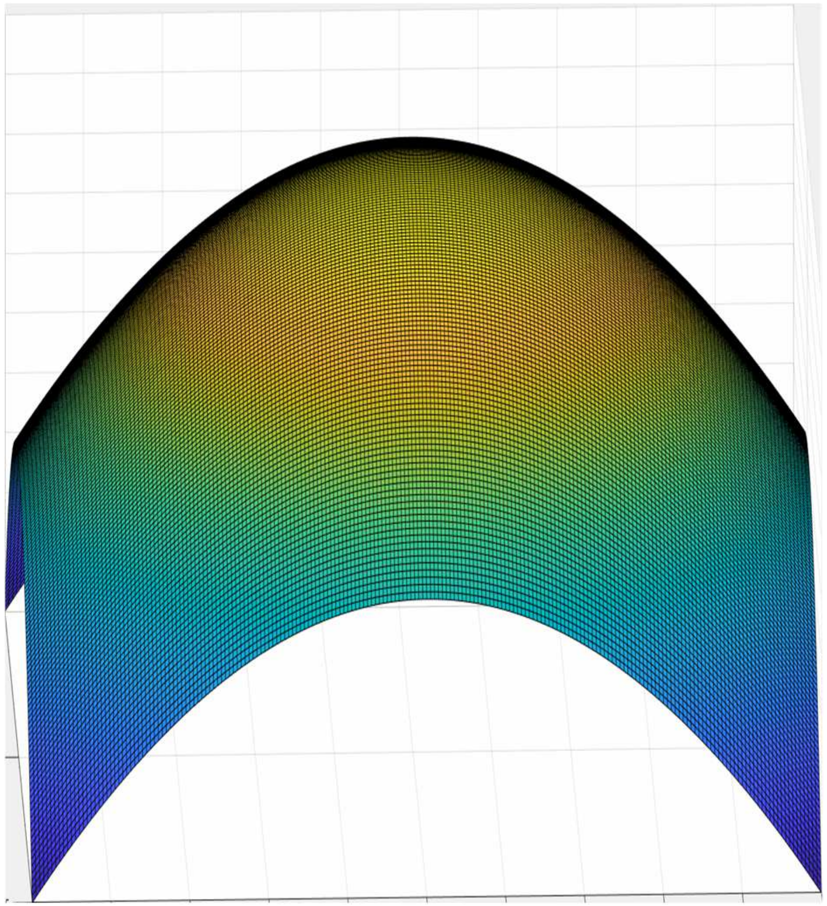
- *Hill* problem
- $\vec{x} = [x_1, x_2]$
- $f: D_f = R^2 \to R$
- $f(\vec{x}) = -x_1^2 - x_2^2$



- $\vec{x}^* = \underset{x \in D_{\vec{x}} = [-5,5]^2 \subseteq D_f}{\arg\max} f(\vec{x})$

# Problem nature

- The ***Hill*** problem features
  - Different than in the TSP case?
  - What makes the difference?
  - Why?
- **How would you solve it by hand**
  - TSP?
  - The ***Hill*** problem?
- What is the difference between the TSP and the Hill problem solutions?



# Problem nature

- Travelling Salesman Problem (TSP) – ***combinatorial*** *in nature*
  – We want to exchange solution fragments
  – For instacne – the „good” city sequences

- Problem Hill – ***topological*** *in nature*
  – We want to search for the better solution in the neighbourhood of the best solutions found that far
  – We shift „slightly left”, or „ slightly right”

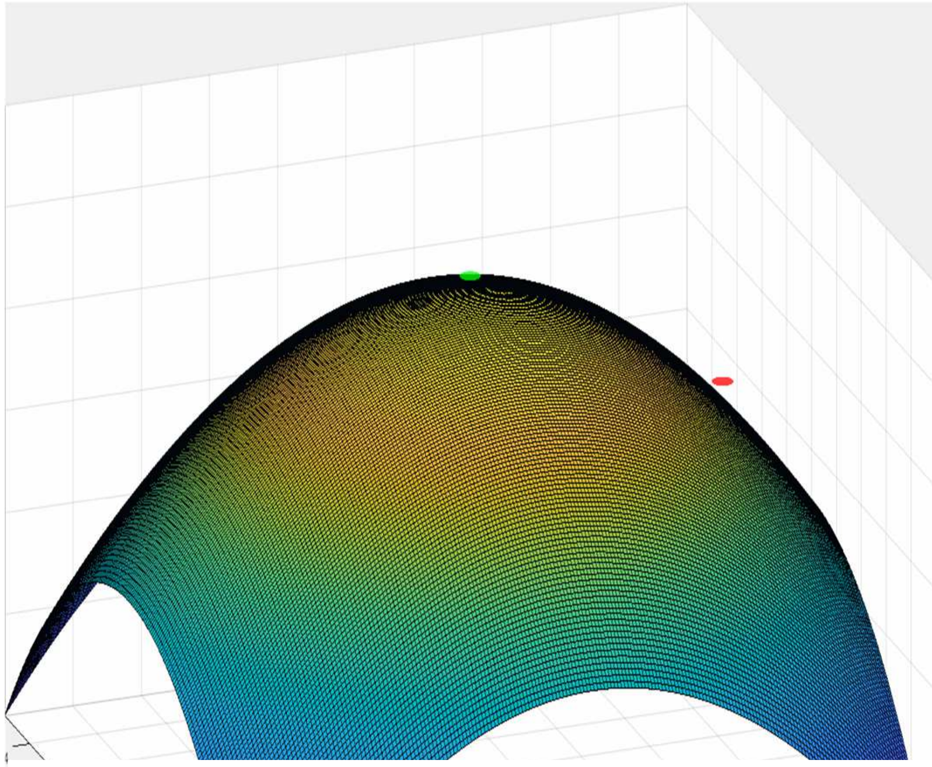
- **Different problem nature -> Different tools**

# Random Search

- Optimization – how to make it in the simplest way?
- Randomly generate and rememebr the best-found solution
- That’s it!

# Random Search

- RS – is it effective?
- What is the main drawback of RS?
- **No memory**
- **How to handle that?**
- **Next lecture**



The green point – the optimum
The black point – the best solution
The red point – the current solution