Mini Project 2 : Ising Model

The 2-dimensional Ising model is described by the following Hamiltonian:

$$H = -J \sum_{<ij>} \sigma_i \sigma_j - h \sum_i \sigma_i$$

Where $J > 0$ is the coupling constant, h is the applied magnetic field and the idealised spins at every lattice site are indicated by

$$\sigma_i = \pm 1$$

Using Monte Carlo simulations on an *NxN* lattice with periodic boundary conditions answer the following questions regarding the system.

A. What is the ground state (*T=0 K*) configuration of the spins in the absence of the field? Explain your answer.
B. What is the average magnetic moment per lattice site in the above ground state (*m*)?
C. What is the ferro-paramagnetic transition temperature of the system as indicated by a plot of *m vs. T i*n the absence of an external field?
D. What is the ferro-paramagnetic transition temperature (T_C) of the system (in units of J/k_B) as indicated by a plot of χ_M *vs. T*, where χ_M is the isothermal magnetic susceptibility?
E. Compute the correlation function

$$\langle \sigma_i \sigma_{i+n} \rangle$$

   as a function of *n* (for $0 < n < N/2$) at $T/T_C = 0.1, 0.7, 1.1$.
F. Demonstrate how does the T_C change as a function of *h* by performing simulations to extract T_C at *h=0, 0.01, 0.1, 0.5* (in units of *J*).
G. Considering *N*=25 and 50, also mention the important differences you notice in the results by increasing the size of the lattice.

In at most 500 words summarise your understanding of the Ising system gained from these simulations.

Note: It is important to check the performance of your random-number generator. Provide a test for the same along with the rest of the answers.