AST2000 - Part 3
Preparing for the Journey

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We have successfully used Stefan-Boltzmann's law to calculate the surface temperature of each of the planets in our solar system, and found that Blossom, Buttercup and Stella are within the habitable zone. We decided that we want to pay Buttercup a visit, which has a surface temperature of approximately 266.93 Kelvins. Fortunately, it receives a moderate amount of sunlight, as our lander unit's solar panel will only have to be about 1.16 m^2 in order to function on its surface. We were able to simulate a future launch from Doofenshmirtz' north pole, and derive an important expression for the minimum distance necessary when we'll eventually want to perform an orbital injection around it.

I. INTRODUCTION

Exploring the possibility of life appearing other places in the universe than on Earth is one of the most complex researches scientists have been doing for a long time. We want to model the habitable zone in our planetary system, since this includes the planets where there may be possible life as we know it. For there to exist life on a planet, its temperature needs to be within an appropriate range so that water on the planet can be liquid. We are going to analyze our planets' surface temperatures using Stephan-Boltzmann's law, which hopefully will help us decide what planet we'll choose as our destination.

There are solar panels onboard which generate electricity for the instruments we are using, so we need to calculate how large these have to be in order to work on the respective planets. We will also have to adjust our software so that its able to take our home planet's movement throughout the solar system into account when we wish to launch our rocket from other places on the planet in the future. For our spacecraft to be able to spot and photograph our destination planet in space, and afterward eventually perform an orbital injection around it, it will have to be sufficiently close. We therefore wish to find a way for us to calculate these distances so that we know whether or not we're able to perform these tasks when on our journey.

II. METHOD

Before we can determine the habitable zone within our solar system, we first need to find a way to calculate the flux received by the planets from their sun. We will assume that our star is a stable black body, which means that it absorbs all radiation coming its way, and reflects nothing. We know from Lecture Notes 1D [1] that black bodies radiate thermal energy at all frequencies, but the frequencies with the largest intensity depends on its effective temperature. As the gas gets hotter, the gas particles' velocities increase, making it more energetic. This increases the amount of thermal energy radiated away by the black body, which means that higher frequencies become more intense. We define emitted flux by any object in the following way:

$$F = \frac{dE}{dAdt} \tag{1}$$

meaning energy emitted per area per time. Specific for a black body, the emitted flux is also defined by the Stefan-Boltzmann law:

$$F_e = \sigma T^4 \tag{2}$$

where T is the black body's effective temperature and σ is the Stefan-Boltzmann constant. Furthermore, we define an object's luminosity as the energy emitted from it per time. we get:

$$\begin{aligned} L &= \frac{dE}{dt} \\ &= FdA \end{aligned} \tag{3}$$

If we now approximate our sun as a perfect sphere with surface area A_s, we can express the sun's luminosity in the following way:

$$L = \sigma T^4 4\pi R_s^2 \tag{4}$$

Where R_s is its radius. Naturally, the flux received by the respective planets will depend on their distance from the sun. For an arbitrary planet in our solar system, we can define an imaginary sphere around the sun with radius r equal to the its distance from the planet. Figure 1 shows how the energy radiated away spreads as the distance from the sun grows larger. By dividing our newfound expression for the sun's luminosity by the surface area of this sphere, we get an expression for the received flux at the distance r from the sun:

$$\begin{aligned} F_r &= \frac{L}{A_{\text{sphere}}} \\ &= \sigma T^4 \frac{4\pi R_s^2}{4\pi r^2} \\ &= \sigma T^4 \frac{R_s^2}{r^2} \end{aligned} \tag{5}$$

When we eventually land on one of the planets in our solar system, our lander unit's installed solar panel will need to be large enough to produce 40 Watts of electric power from the received sunlight. If it's too small, its instruments can't function, and we won't be able to do our exciting experiments there. Thanks to our awesome research team at the University of Oslo, the planets' day-night cycles are accounted for in this requirement, so we will look away from this. They've informed us that our lander unit's solar panels have an efficiency of 12%., so we can calculate the minimum area they can have in order to function:

$$\begin{aligned} A_{\min} &= \frac{40}{0.12 F_r} \\ &= \frac{40}{0.12}\sigma T^4 \frac{R_s^2}{r^2} \end{aligned} \tag{6}$$

Now, since our goal was to determine which of the planets are within the habitable zone, we want to use our derived expressions for their received flux to derive a formula for an estimate of their surface temperatures. We will assume that the light rays received by the planets are parallel, and that we can approximate the planets as discs with areas equal to half their surface areas, as only half the planet will face the sun at all times. Then we can find the total energy received by each planet in the following way:

$$\begin{aligned} E_{\text{tot}} &= \frac{1}{2} A_p F_r \\ &= 2\pi R_p^2 \sigma T^4 \frac{R_s^2}{r^2} \end{aligned} \tag{7}$$

Where R_p is the planet's radius. We will assume that the planets are black bodies that absorb and emit the same amount of energy, which means that their surface temperatures will stay constant. Since they are black bodies, they will absorb all energy received from the sun, and we can therefore use Stefan-Boltzmann's law combined with (5) to find an expression for their surface temperatures:

$$\begin{aligned} T_p &= \left(\frac{F_e}{\sigma}\right)^{1/4} \\ &= \left(\frac{\sigma T_s^4 \frac{R_s^2}{r^2}}{\sigma}\right)^{1/4} \\ &= T_s\sqrt{\frac{R_s}{r}} \end{aligned} \tag{8}$$

Where we've renamed the sun's surface temperature from T to T_s so that we don't confuse it with the planet's surface temperature T_p, and used the fact that $F_e = F_r$ for a black body with stable temperature.

For a planet to be within the habitable zone, its surface temperature has to be between 260 and 390 Kelvins, with an acceptable margin error of 15 Kelvins. This is because at these temperatures, which equal about -20 to 120 degrees Celsius, water is liquid, or close to being so at least. This is a crucial necessity for life to be able to inhabit the planet. Thanks to our research team, we fortunately aren't limited to travelling to a habitable planet though, as we're equipped with isolating suits and great temperature regulating software. We will however, look into which of the planets are gas planets and which are rock, as we believe it'll be rather difficult to perform experiments and take photographs on a gas planet.

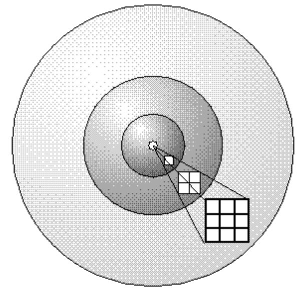


Figure 1. Radiated energy from the surface

To further plan our journey ahead, we will adjust our software that we used in our study where we modelled our rocket engine, as we'll need to be able to launch from anywhere on Doofenshmirtz. This is because we're not necessarily currently close to our soon-to-be chosen destination planet, and we therefore may have to wait a fair while until our positions are properly aligned. Thankfully, we correctly calculated all the planetary orbits in our last study, so the best time to launch at shouldn't be too hard to decide. Since all launches are pointed radially out from the surface, we may also need to launch from another point on our home planet, which our software also needs to take into consideration. So far, we've only simulated launches from the far east side of its equator, at $y = 0$ in both the sun and Doofenshmirtz' frames of reference, and $x = R$ in the latter. Since we always direct our launches radially outwards, the rocket's thrust force will be directed in the same direction as the launch site's normal vector, which is defined as

$$\mathbf{n} = \cos\phi\mathbf{x} + \sin\phi\mathbf{y} \tag{9}$$

where ϕ is the angle between our chosen launch site and the far east side of the equator, at $x = R$ and $y = 0$ in Doofenshmirtz' frame of reference.

Now, as promised, our research team took a look at our rocket's engine to see if they could do something about its problematic mass loss rate to thrust force ratio. Fortunately for us, they were able to make some crucial adjustments which cut its mass loss rate down to one third. Thus, our rocket's parameters are now

$$F \approx 469.33 \text{ kN}$$
$$mlr \approx 37.85 \text{ kg/s}$$
$$m_{\text{fuel}} = 4.5 \times 10^4 \text{ kg}$$

Where F, mlr and m_{fuel} are the rocket's thrust force, mass loss rate and initial amount of fuel when launching. To test if our software is reliable, we'll first attempt to simulate an identical launch to the one we did when modelling our rocket engine, with all the same parameters, including the mass loss rate. Then, we plan to simulate the same launch again, but with the updated mass loss rate, to study the difference between the two. These launch results will hopefully also make it easier for us to see if our software is successfully able to simulate launches from any point on the planet at any point in the future. We will especially compare the duration of both simulations, as well as the amount of fuel consumed and the way the rocket's absolute velocity v changes throughout the launch sequence, as we expect all of these to be equal.

When on our way towards our destination planet, we plan to use the camera onboard our spacecraft to photograph our beautiful surroundings. As we get closer, we'll eventually want to attempt to photograph the planet as well. To avoid wasting too much time and memory on our camera, we want to find out a way to calculate if we're close enough to photograph it. We've been informed by our research team that our destination planet is considered resolved when it appears as more than one pixel. It will therefore have to cover parts of at least two adjacent pixels for us to be able to photograph it, since it's spherical and the pixels are quadratic (see Figure 2).

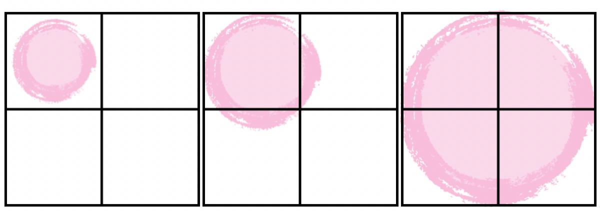


Figure 2. Our destination planet as it gets closer to our spacecraft. In the first pixel grid, it's not resolved yet. In the second, it's barely resolved, covering one pixel and parts of two adjacent pixels. The third grid shows the ideal situation.

However, because each of these pixels have one color, our planet will preferably have to cover most of a 2 × 2 pixel grid, so that we can see it as more than a monochrome blob. In this case, our destination planet's radius R will have to cover four pixels. In Appendix B, we've derived an expression for the upper limit for the distance L between our spacecraft and our destination when we want to resolve it on our camera:

$$L \lesssim \frac{RP}{F} \tag{10}$$

Where R is the planet's radius, and $P \times P$ is the amount of pixels contained within the camera's field of view $F \times F$, where F is measured in degrees.

As we finally near ourselves our destination planet, we will have to be sufficiently close to perform an orbital injection. This is a crucial step to our journey if we want to study the planet's atmosphere and plan our eventual landing sequence accordingly. To be able to perform such an orbital injection, we assume that we need to be close enough for the gravitational force working on the spacecraft from the planet to be k times larger than the gravitational force working on the spacecraft from the sun. In Appendix A we derived an expression for the distance l between the spacecraft and the planet where this task is possible, as a function of k and the spacecraft's distance to the sun:

$$l = |\mathbf{r}|\sqrt{\frac{M_p}{kM_s}} \tag{11}$$

Here, $|\mathbf{r}|$ is the distance between the spacecraft and the sun, M_s is the sun's mass and M_p is the mass of the destination planet.

III. RESULTS

We wanted to calculate the minimum solar panel necessary for our lander unit to function on the surface of each of the planets in our solar system. To do this, we had to first calculate the flux and total energy E_{tot} received by each of them. The values we were able to calculate are displayed in Table I.

Planet	Flux [W/m^2]	E_{tot} [W]	Area [m^2]
Doofenshmirtz	3052.52	6.90×10^{16}	0.11
Blossom	1170.18	6.90×10^{16}	0.28
Bubbles	44.88	2.35×10^{18}	7.43
Buttercup	287.88	8.00×10^{16}	1.16
Flora	126.26	4.50×10^{18}	2.64
Stella	702.41	1.00×10^{16}	0.47
Aisha	71.58	3.60×10^{17}	4.66

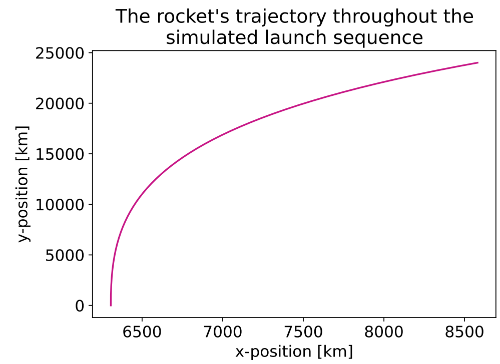
Table I. The flux and total energy received by each planet, and the necessary solar panel areas needed for our lander unit to function on them.

When then had to calculate their surface temperatures and determine whether or not they're habitable or even worth visiting (see Table II).

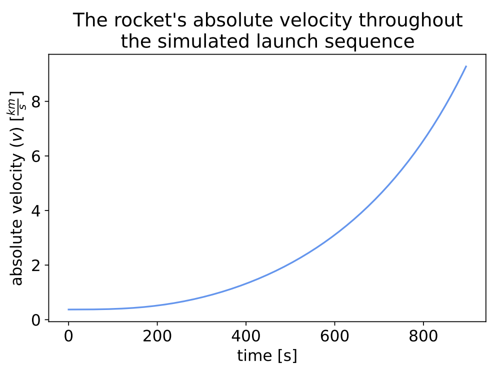
Planet	Type	Temperature [K]	Habitable
Doofenshmirtz	Rock	481.68	No
Blossom	Rock	379.02	Yes
Bubbles	Gas	167.73	No
Buttercup	Rock	266.93	Yes
Flora	Gas	217.23	No
Stella	Rock	333.61	Yes
Aisha	Gas	188.49	No

Table II. The surface temperature of each planet in our solar system, whether they are rock planets or gas planets, and whether they are habitable or not.

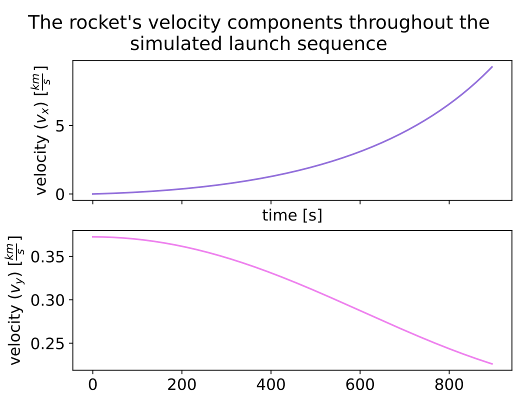
We wanted to simulate an identical landing to the one we simulated in the study where we modelled our rocket's engine, to make it easier for us to see if the updated software gave unreasonable results.



(a) Trajectory



(b) Absolute velocity



(c) Velocity components

Figure 3. The spacecraft's trajectory (a), absolute velocity (b) and velocity components (c) throughout the launch simulation with lower mass loss rate but the same initial conditions as the last time.

After this simulation, the spacecraft's positional coordinates were

$$x = 5.30 \times 10^8 \text{ km}$$
$$y = 10125.86 \text{ km}$$

Which is exactly the same as the last time we simulated a launch with the same parameters and initial position. The launch also took 392 seconds, which is only 6 seconds more than the last time. It's worth mentioning however, that our final position where our rocket reached escape velocity now was satisfyingly calculated when compared to the results that our research team at the University of Oslo got. This indicates improvements in our software, which is good news.

Because our research team was gracious enough to take a look at our engine, we once again simulated the same launch with only the mass loss rate changed, to help us compare this launch with the future launch from a new location. The spacecraft's positional coordinates, as well as its velocity components, relative to both the launch site and the sun are displayed in Table III.

	Planet frame	Sun frame
x [km]	2289.85	5.30×10^8
y [km]	280.84	24006.84
v_x [m/s]	9300.89	9292.66
v_y [m/s]	226.33	26706.24

Table III. Our spacecraft's positional coordinates relative to the launch site and to the sun when reaching escape velocity, and its velocity components relative to the planet and the sun.

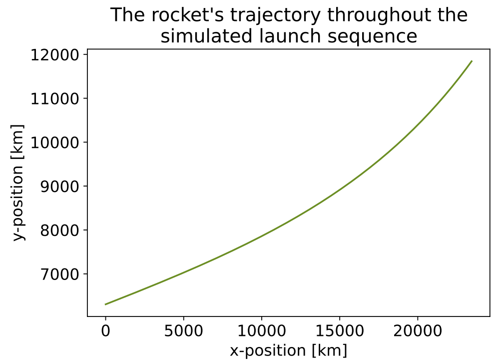
The simulation took 896 seconds, which is almost 15 minutes, and it had 12134.88 kg fuel left, which means it lost a total of 33965.12 kg fuel during the launch. When comparing our results to those of our research team, we found that their simulation took approximately 895 seconds, and that our spacecraft's final positions was correctly calculated. We've included plots displaying the spacecraft's trajectory, absolute velocity and velocity components throughout the process (see Figure 3).

We then took use of Doofenshmirtz' trajectory, which we correctly calculated in our last study, to simulate a launch from its north pole three years from now. This launch took 896 seconds, the same amount of time as the previous launch, and it was left with 12120.12 kg when reaching escape velocity, which means it lost a total of 33979.88 kg fuel during the launch. Its positional coordinates and velocity components are displayed in Table IV

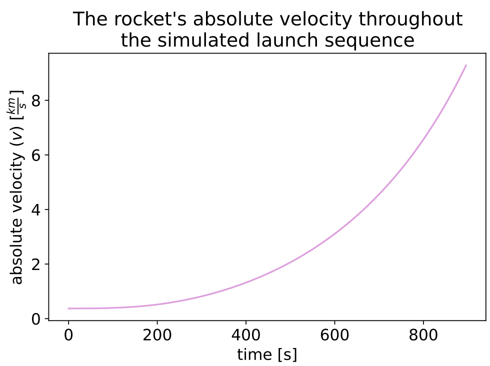
	Planet frame	Sun frame
x [km]	-280.84	7.68×10^7
y [km]	2289.81	-5.19×10^8
v_x [m/s]	-226.32	26259.74
v_y [m/s]	9304.68	12937.24

Table IV. Our spacecraft's positional coordinates relative to the launch site and to the sun when reaching escape velocity, and its velocity components relative to the planet and the sun.

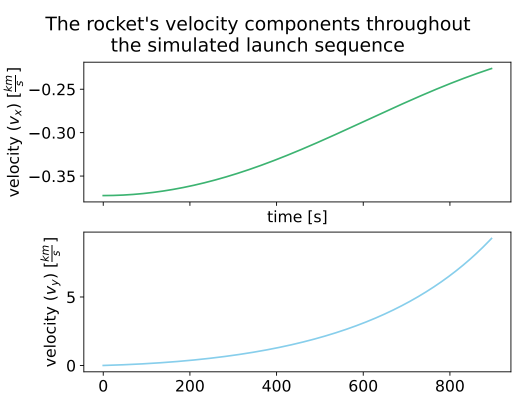
Plots visualizing the spacecraft's trajectory relative to the launch site, as well as its absolute velocity v and velocity components v_x and v_y relative to Doofenshmirtz are shown in Figure 4.



(a) Trajectory



(b) Absolute velocity



(c) Velocity components

Figure 4. The spacecraft's trajectory (a), absolute velocity (b) and velocity components (c) throughout the simulated launch from the north pole on Doofenshmirtz three years from now.

When checking with our research team whether or not our simulated launch results were accurate, we sadly found that the spacecraft's initial position relative to the sun that we calculated before the launch simulation deviated 2.77×10^5 km from where it was supposed to be.

IV. DISCUSSION

As mentioned in the above section, we found that only Blossom, Stella and Buttercup were habitable. This makes sense as these are the 2$^{\text{nd}}$, 3$^{\text{rd}}$ and 4$^{\text{th}}$ planets away from the sun, respectively (see Figure 5). Although we didn't need to travel to a habitable rock planet, we still decided to travel to Buttercup. This was mostly because it's not too far away from Doofenshmirtz, and we wanted to experience a little colder temperatures. It's also a rock planet, which definitely will make it easier to do experiments and take photographs on its surface. We also saw that it got a moderate amount of sunlight, so our landing unit's solar panels don't need to be larger than 1.16 m^2 to function there, which is fortunate.

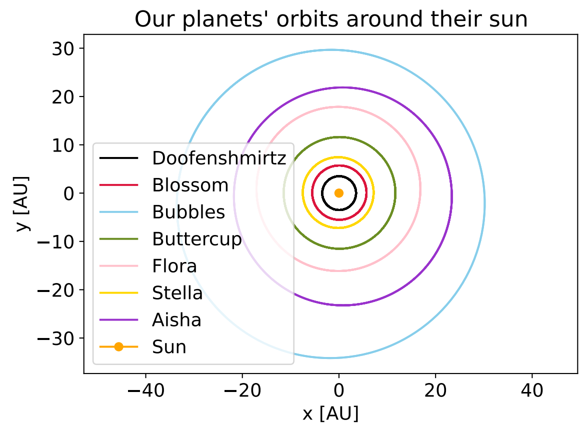


Figure 5. The planetary orbits that we calculated in the last study.

When simulating the rocket launches, we were pleased to see that there were minimal to no differences in the fuel consumption and launch duration even though we changed launch sites. The spacecraft's trajectories relative to Doofenshmirtz look reasonable, as we move

We also noticed that the spacecraft's absolute velocity v behaves in the same way when launching from another point on the planet at another point in its orbit. Studying Figures 3 and 4, we see that the spcacraft's initial velocity relative to Doofenshmirtz is in the positive y-direction when launching from the far east side of the equator, while it's solely in the negative x-direction when launching from the north pole. Since our planet rotates against the clock around the z-axis, these initial velocities seemed realistic to us. Furthermore, we see that the spacecraft's v_y-component decreases while its v_x-component increases rapidly during the launch from the equator, while the magnitude of its v_x-component decreases and its v_y-component increases rapidly when launching from the north pole. Naturally, the rapid increases in v_x and v_y respectively stem from the fact that the launches are directed radially outwards, as explained in the Method section. In the first case, our rocket launches from $\phi = 0°$, which means that $\mathbf{n} = \mathbf{x}$, when launching from the north pole, we have $\phi = 90° \rightarrow \mathbf{n} = \mathbf{y}$.

Not surprisingly, the difference in initial velocity components and launch directions also affected the spacecraft's trajectory in the two simulations. Since our rocket initially only has a positive v_y-component in the first launch, it almost only moves in the positive y-direction at the beginning of the launch. Because we accelerate in the positive x-direction, the curve bends the way it does. We see that this is the case when launching from the north pole as well, only that we have an initial v_x-component and boost in the positive y-direction. However, we did find it peculiar that this trajectory curve didn't bend the same way as the one from the last simulation, as the situations are really almost identical, except for the fact that we launched in the y-direction instead of the x-direction this time.

Although our launch simulations seemed to go very well, and our rocket's final position when simulating a launch from our current position in present time was correctly calculated, our launch results from the last simulation were sadly not satisfactory when compared to those of our research team back at the University of Oslo. However, we were surprised to see that the mistake was our rocket's initial position, which apparently was over a hundred million meters away from Doofenshmirtz' surface. We found this strange, as we used our correctly calculated planetary orbits from the last study to find our rocket's initial position in the sun's frame of reference. Since we're so far off, it seems that we must've made a mistake when finding Doofenshmirtz' position in the solar system three years from now. We tried immensely looking into our simulation software to see if we could've made a mistake, but we were unsuccessful in discovering any flaws. Fortunately, after having a discussion with the research team controlling our results, we were told that there is a possibility of their software not being completely up to date, that we therefore didn't have to worry too much about this inconsistency.

V. CONCLUSION

We were able to calculate the flux and total energy received from our sun by each of the planet's in our solar system, and use this to calculate the minimum solar panel area necessary for our lander unit to function on their respective surfaces. By applying Stefan-Boltzmann's law, we could determine the surface temperature of each of the planets, and then use this to define the habitable zone within our solar system. We found that Blossom, Stella and Buttercup were the planets included in this zone, and we decided to pay the latter a visit.

To better prepare for our upcoming journey, we successfully enhanced our software and simulated two launches, one from the site that we simulated launches from in our previous study where we modelled the rocket engine, and one from Doofenshmirtz' north pole three years from now. We were seemingly unsuccessful in correctly calculating the rocket's initial position relative to the sun in the last simulation, but were informed that this may be because of flaws in our research team's equipment. Finally, were be to find a way for us to know at what distance we are close enough to photograph Buttercup when we're on our journey through space, and how close we need to be to eventually perform an orbital injection around it.

REFERENCES

[1] Hansen F. K. *Lecture Notes 1D*. URL: https://www.uio.no/studier/emner/matnat/astro/AST2000/h22/undervisningsmateriell/lecture_notes/part1d.pdf.

Appendix A: How close to our destination planet do we need to be to perform an orbital injection?

When travelling through space, the gravitational force from our sun working on the spacecraft is defined as

$$G_s = -\gamma \frac{mM_s}{|\mathbf{r}_s|^2} \tag{A1}$$

where γ is the gravitational constant, m is the spacecraft's mass, M_s is the sun's mass and $\mathbf{r}_s$ is the spacecraft's position relative to the sun. The gravitational force working on the spacecraft from our destination planet is then

$$G_p = -\gamma \frac{mM_p}{|\mathbf{r}_p|^2} \tag{A2}$$

where M_p is the destination planet's, mass, and $\mathbf{r}_p$ is the spacecraft's position relative to it. Assuming that G_p is k times larger than G_s, we get the following relation:

$$-\gamma \frac{mM_p}{|\mathbf{r}_p|^2} = -k\gamma \frac{mM_s}{|\mathbf{r}_s|^2} \tag{A3}$$

If we now redefine $|\mathbf{r}_p|$ as l and $|\mathbf{r}_s|$ as $|\mathbf{r}|$, we get

$$\begin{aligned} -\gamma \frac{mM_p}{l^2} &= -k\gamma \frac{mM_s}{|\mathbf{r}|^2} \\ l^2 &= |\mathbf{r}|^2 \frac{M_p}{kM_s} \\ l &= |\mathbf{r}| \sqrt{\frac{M_p}{kM_s}} \ \square \end{aligned} \tag{A4}$$

Appendix B: How close to our destination planet do we need to be to photograph it?

When our spacecraft's camera is pointed at our destination planet, we can define a right triangle where the adjacent leg is the straight line from the camera to the bottom of the planet, the opposite leg is the planet's radius R, and the hypotenuse is the straight line going from the camera to the center of the planet. The tangent of the angle between the adjacent leg and the hypotenuse is then the length R of the opposite leg divided by the length L of the adjacent leg. When our spacecraft is just close enough to resolve the planet, this angle is close to zero. Thus, we know from the definition of an angle's tangent that we can approximate it in the following way:

$$\theta \approx \tan\theta = \frac{R}{L} \tag{B1}$$

Since we ideally want our destination planet to cover at least four pixels in a 2×2 pixel grid, we want its radius to cover one pixel. If we let one pixel stretch out the angle θ in each direction of the camera's field of view, we get the following relation:

$$F = P\theta \tag{B2}$$

The radius will then approximately cover one such angle θ when we're just close enough to resolve it, and we therefore get

$$F \approx P\frac{R}{L} \tag{B3}$$

Thus, we get the upper limit for the distance between the spacecraft and the planet in order to resolve it on our camera:

$$L \lesssim \frac{RP}{F} \ \square \tag{B4}$$