AST2000 - Part 6
Preparing for the Landing

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(Dated: November 23, 2022)

In this part of the project we would like to be judged only on how well we have accomplished the challenges, including our code, results, discussions, conclusions etc.
We managed to enter a low, stable orbit around Buttercup, which let us create density and temperature profiles describing its atmosphere. We found that the atmosphere consisted of 33.3% CH_4 (Methane), 33.3% CO (Carbon Monoxide), and 33.3% N_2O (Nitrogen Dioxide). We were also close enough to be able to scout for landing sites, and find a way to calculate their positional coordinates as time passes by.

I. RESULTS

When attempting to lower our orbit, we wanted to go a safe route. We let our spacecraft coast for approximately 43.8 hours at a time, before giving it a minor boost of approximately 5.56% its velocity with respect to Buttercup, but in the opposite direction. During this process, which took about 110 days, we managed to take some beautiful pictures of our destination planet (see Figure 1, 2 and 3).

The flux meter installed on our spacecraft was able to gather flux data from Buttercup's atmosphere as soon as we entered the low orbit. By using the method of χ^2 minimization, we were able to find out at which minimal flux values F_{min}, Doppler shifts $\Delta\lambda$ and temperatures T there most likely was a spectral line in our data, for each of the wavelengths. The values that fit best within the range of Doppler shifts that our spacecraft would be able to observe for each gas are displayed in the table below.

Compound	λ_0	F_{min}	$\Delta\lambda$	T
O_2	632	0.896	0.0171363	150.000
O_2	690	0.896	0.0016408	339.796
O_2	760	0.859	0.0018645	150.000
H_2O	720	0.841	0.0200865	450.000
H_2O	820	0.737	0.0235265	192.857
H_2O	940	0.920	0.0174253	150.000
CO_2	1400	0.847	0.0012163	150.000
CO_2	1600	0.933	0.0291551	450.000
CH_4	1660	0.896	0.0132633	241.837
CH_4	2200	0.761	0.0625286	339.796
CO	2340	0.884	0.0657224	247.959
N_2O	2870	0.884	0.0799490	321.429

We used these values to create Gaussian line profiles for each of the spectral lines. When deciding which of the spectral lines were real, we chose the ones for CH_4 at $\lambda_0 = 2200$ nm, C0 at $\lambda_0 = 2340$ nm, and N_2O at $\lambda_0 = 2870$ nm. The Gaussian line profiles we managed to make for these spectral lines are shown in Figure 4, 5, and 6.

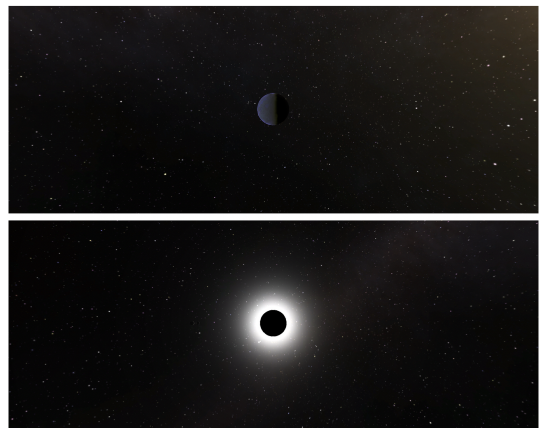

Figure 1. Pictures taken of Buttercup as we were beginning to lower our orbit.

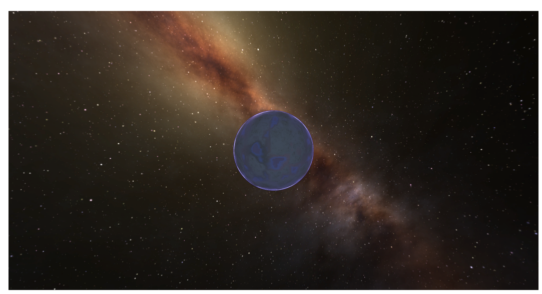

Figure 2. Picture taken of Buttercup while we were in the midst of lowering our orbit.

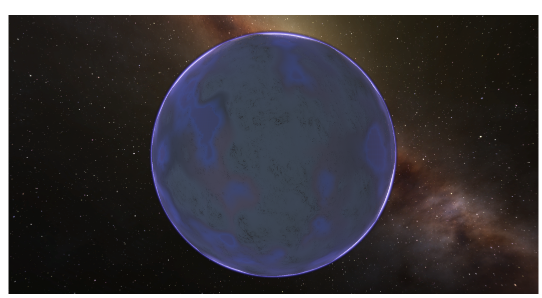

Figure 3. Picture taken of Buttercup after we were finished with lowering our orbit.

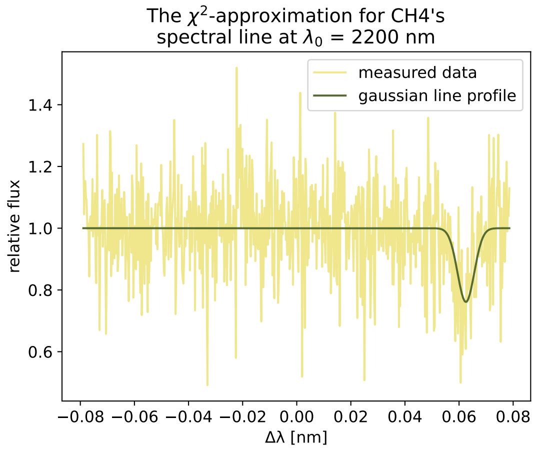

Figure 4. Relative flux measured by our flux meter along with the Gaussian line profile we made for CH_4's spectral line at at $\lambda_0 = 2200$ nm using the χ^2-method.

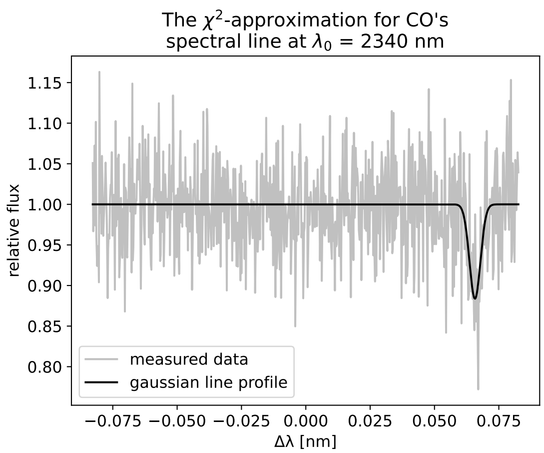

Figure 5. Relative flux measured by our flux meter along with the Gaussian line profile we made for CO's spectral line at at $\lambda_0 = 2340$ nm using the χ^2-method.

Following our assumption of which spectral lines were real, we found that our atmosphere consists of 33.3% Methane, 33.3% Carbon Monoxide, and 33.3% Nitrogen Dioxide. We used this to calculate its mean molecular mass:

$$\begin{aligned}\mu &= \frac{1}{3}\frac{m_C + 4m_H}{m_H} + \frac{1}{3}\frac{m_C + m_O}{m_H} + \frac{1}{3}\frac{2m_N + m_O}{m_H}\\ &= 5.3057562 + 9.26294518 + 14.5566757\\ &\approx 29.13\end{aligned}$$

Our wonderful research team supplied us with Buttercup's atmospheric density at surface level, while we ourselves had calculated the surface temperatures of the planets in our solar system. The density and temperature of Buttercup's atmosphere turned out to be

$$\begin{aligned}\rho_0 &= 1.1719424102973004 \text{ kg/m}^3\\ T_0 &\approx 266.93 \text{ K}\end{aligned}$$

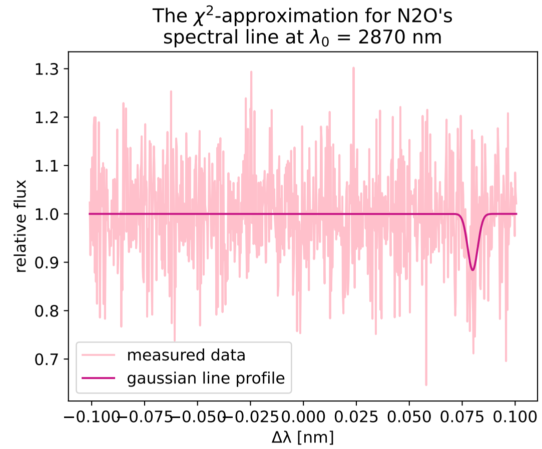

Figure 6. Relative flux measured by our flux meter along with the Gaussian line profile we made for N_2O's spectral line at at $\lambda_0 = 2870$ nm using the χ^2-method.

We used these values to create a model of Buttercup's atmosphere, by making temperature and density profiles. We plotted both of these as a function of altitude up to 100 kilometers for visualization:

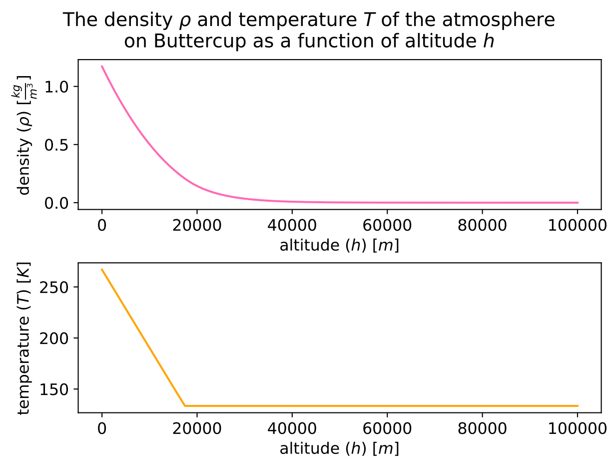

Figure 7. Density and temperature of Buttercup's atmosphere for altitudes up to 100 kilometers.

As we can see, the density is very close to zero after we get about 40 kilometers above the ground. The temperature decreases linearly until a height of about 17.42 kilometers above the surface, but is isothermal after this point and therefore remains constant at $T_0/2 = 133.465$ Kelvins.

After looking at videos of our orbit around Buttercup, we saw that there were many potential landing sites on the surface. We therefore chose to land at a spot along its equator to avoid having to worry about more than two velocity components during our landing, as our spacecraft already only moves in the r- and ϕ-directions. We decided to try to land on the landing site marked by the white cross in Figure 8.

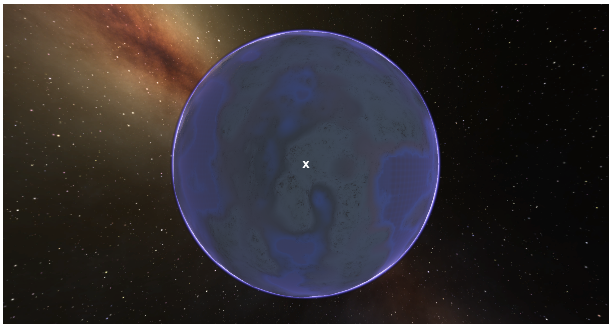

Figure 8. Our chosen landing site on Buttercup, marked by a white x.

We found that this landing site's spherical coordinates were

$$r = 0.0 \text{ m}$$
$$\phi = 47.5°$$
$$\theta = 90.0°$$

And because Buttercup only rotates in the azimuthal direction, the r- and θ-coordinates won't change.

II. DISCUSSION

When using the χ^2-method, we sliced our flux data to only look at wavelengths within the interval

$$\lambda_0 - \Delta\lambda_{max} \leq \lambda \leq \lambda_0 + \Delta\lambda_{max}$$

for each λ_0. When doing this, we essentially find where in our sliced data there most likely was a spectral line. Naturally, many of these approximated spectral lines were flukes, so we had to use our logic and intuition combined with the following criteria in order to determine which were real and not:

- The Doppler shifts of real spectral lines should be approximately the same
- Real spectral lines tend to have lower F_{min} values
- Finding one real spectral line of a gas does not necessarily mean that we'll find each of its spectral lines
- Gases may have different temperatures, but spectral lines from the same gas should have the same temperature

When looking at the data in our table of values for the approximated spectral lines, we pretty quickly ruled out the spectral lines with temperatures at 150.000 Kelvins or 450.000 Kelvins, as we deemed it highly plausible for these to be flukes as they are at either end of the expected temperature interval. We assumed this to be a sign of our software not being able to find a better approximation when using the χ^2-method. Out of the rest, there were initially five of them that stood out.

Compound	λ_0	F_{min}	$\Delta\lambda$	T
O_2	690	0.896	0.0016408	339.796
H_2O	820	0.737	0.0235265	192.857
CH_4	2200	0.761	0.0625286	339.796
CO	2340	0.884	0.0657224	247.959
N_2O	2870	0.884	0.0799490	321.429

Of course, the three we deemed as real are included here, but also the ones for O_2 at $\lambda_0 = 690$nm and H_2O at $\lambda_0 = 720$nm. The Gaussian line profiles for their approximated spectral lines are shown in Figure 9 and 10.

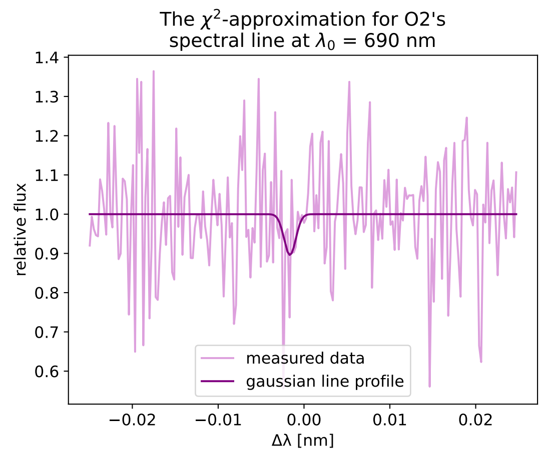

Figure 9. Relative flux measured by our flux meter along with the Gaussian line profile we made for O_2's spectral line at at $\lambda_0 = 690$ nm using the χ^2-method.

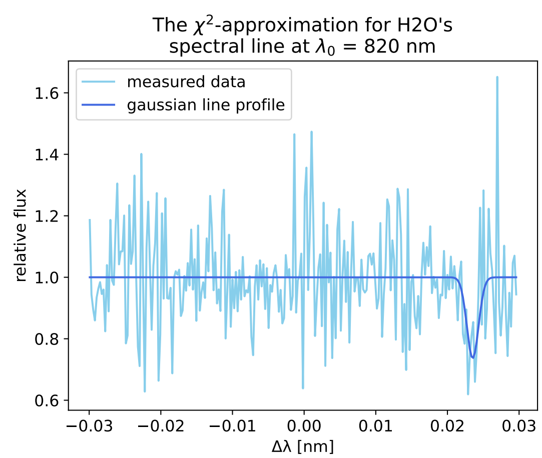

Figure 10. Relative flux measured by our flux meter along with the Gaussian line profile we made for H_2O's spectral line at at $\lambda_0 = 820$ nm using the χ^2-method.

When looking at the spectral line for O_2, we see that its F_{min} value is rather high, and its Doppler shift is not near the same as any of the others'. This indicates that if this line is real, it's most likely the only real one.

The same is true for H_2O, since it has Doppler shift of $\Delta\lambda \approx 0.0235$nm. Compared to the Doppler shifts of the spectral lines for CH_4 at $\lambda_0 = 2200$ ($\Delta\lambda \approx 0.0625$ nm), CO ($\Delta\lambda \approx 0.0657$ nm) and N_2O ($\Delta\lambda \approx 0.0799$ nm), this is relatively low. Also, its temperature is much lower than the temperature we found the aforementioned spectral lines at. Although we found its low F_{min} value to be intriguing, we saw it as highly unlikely for our destination planet's atmosphere to consist solely of Oxygen or water, and we therefore deemed these spectral lines as statistical flukes.

When looking at the data for the three remaining spectral lines, we immediately noticed how low CH_4's F_{min} value is, making it a valuable candidate. We also noticed how similar its Doppler shift is to CO's Doppler shift, which heightens the possibility of this one being real if CH_4 is. Since the temperature of the atmosphere decreases with altitude, we didn't see their difference in temperature as a big problem, as this may very well be because there's a larger concentration of Carbon Monoxide of at higher altitudes, while the concentration of Methane is larger closer to the surface. Although CO's and N_2O's F_{min} values are rather high, they are the same. In addition to this, N_2O's temperature is somewhere in the middle of the two others'. We saw these factors as possible indications of the spectral line for N_2O being real as well. Although its Doppler shift being a little bigger than that of the other two weakens this possibility, we knew that the large amounts of noise in our data could be the reason for that.

Our judgement may very well be off when determining which spectral lines were real, as we chose to divide the following intervals

$$\begin{array}{rcccl} 0.7 & < & F_{min} & < & 1 \\ \lambda_0 - \Delta\lambda_{max} & \leq & \lambda & \leq & \lambda_0 + \Delta\lambda_{max} \\ 150 & \leq & T & \leq & 450 \end{array}$$

into 30 when using the method of χ^2 minimization. This could've created too large gaps between the values, especially for large intervals like that of temperature, which means that valuable candidates may have been skipped over. In addition to this, the vast amount of noise in our data made it harder to decipher which of the spectral lines were statistical flukes and which were real. Naturally, making a mistake in these decisions will make the temperature and density profiles inaccurate. This could possibly lead to further mistakes when we attempt to plan our landing, which is the next step of our journey.

III. CONCLUSION

We managed to lower our orbit around Buttercup, utilize our onboard equipment to gather flux data and use these to determine the atmosphere's chemical build-up. By knowing this, we were able to create density and temperature profiles for the atmosphere in preparation for our landing. We also scouted for potential landing sites by regularly taking pictures of our destination planet while completing an orbit, and chose one that looked promising. We calculated the site's coordinates, and created software that should be able to find its new coordinates as time passes by.

If we were right about which of the spectral lines were real, we doubt there exists life on Buttercup. This is mostly because, even though the presence of Methane and Nitrogen Dioxide indicates possible signs of life, there is also Carbon Monoxide in the atmosphere. This gas is very poisonous to humans and animals, and probably interplanetary life forms as well. However, we saw that there is a fair chance of its concentration decreasing as we move closer to the surface, which could possibly mean that the Carbon Monoxide levels by the surface are low enough for life to form here. It will be exciting to learn more as we finally arrive at our destination.

REFERENCES

[1] Hansen F. K. *Lecture Notes 1A*. URL: https://www.uio.no/studier/emner/matnat/astro/AST2000/h22/undervisningsmateriell/lecture_notes/part1a.pdf.

[2] Hansen F. K. *Lecture Notes 1D*. URL: https://www.uio.no/studier/emner/matnat/astro/AST2000/h22/undervisningsmateriell/lecture_notes/part1d.pdf.

Appendix A: The maximum Doppler shift our spacecraft can observe

We know that the formula for Doppler shift is

$$\Delta\lambda = \frac{v}{c}\lambda_0 \tag{A1}$$

Where v in this case is the relative velocity between our spacecraft and the particles in the gas that we're gathering flux data for, and c and λ_0 are the light speed and the wavelength at rest, of course. The faster our spacecraft is moving, the bigger the Doppler shift. We know that $v_{\max} = 10^4$ m/s is the maximum velocity that our spacecraft can have for it to be able to measure the Doppler shift. This naturally has to take the gas particle's velocities into consideration as well. We know from Lecture Notes 1D [2] that the most probable absolute velocity of a particle in a gas is

$$v = \sqrt{2\frac{kT}{m}} \tag{A2}$$

Where k is the Boltzmann constant, T is the temperature of the gas and m is the mass of the gas particle. Thus, we find that the maximum Doppler shift that our spacecraft can measure is

$$\Delta\lambda_{\max} = \left(v_{\max} + \sqrt{2\frac{kT_{\max}}{m}}\right)\frac{\lambda_0}{c} \quad \square \tag{A3}$$

Where $T_{\max} = 450$ K is the upper boundary for the temperatures of the gases in the atmosphere.

Appendix B: The standard deviation in our Gaussian line profiles

When creating Gaussian line profiles for the different spectral lines using our flux data, we had to find an expression for the standard deviation of the line profile. From Lecture Notes 1A [1], we know that the relation between the standard deviation σ and the Full Width at Half Maximum is

$$\sigma = \frac{\text{FWHM}}{\sqrt{8\ln 2}} \tag{B1}$$

In addition to this, we know from Lecture Notes 1D [2] that the FWHM of a Gaussian line profile is defined as

$$\text{FWHM} = \frac{2\lambda_0}{c}\sqrt{\frac{2kT\ln 2}{m}} \tag{B2}$$

Where λ_0 here is the wavelength we're studying, which in our case is every wavelength within the interval

$$\lambda_0 - \Delta\lambda_{max} \leq \lambda \leq \lambda_0 + \Delta\lambda_{max}$$

Thus, we find that the standard deviation of our Gaussian line profiles is

$$\begin{aligned}\sigma &= \frac{\text{FWHM}}{\sqrt{8\ln 2}} \\ &= \frac{\frac{2\lambda}{c}\sqrt{\frac{2kT\ln 2}{m}}}{\sqrt{8\ln 2}} \\ &= \frac{\lambda}{c}\sqrt{\frac{kT}{m}} \quad \square\end{aligned} \tag{B3}$$

Appendix C: The change in atmospheric temperature as a function of altitude in the adiabatic layer

We know that while $T > T0/2$, the atmosphere is adiabatic, and the following equation holds:

$$p^{(1-\gamma)}T^{\gamma} = C \tag{C1}$$

Where C is a constant, and p denotes the atmospheric pressure. In order for us to find out how the temperature decreases with altitude, we want to take the derivative of both sides of (C1) with respect to the altitude h, as we assume the atmosphere to be spherically symmetrical. We get

$$\frac{\mathrm{d}}{\mathrm{d}h}p(h)^{1-\gamma}T(h)^{\gamma} = \frac{\mathrm{d}}{\mathrm{d}h}C$$

$$(1-\gamma)p(h)^{-\gamma}T(h)^{\gamma}\frac{\mathrm{d}p}{\mathrm{d}h} + p(h)^{1-\gamma}\gamma T(h)^{\gamma-1}\frac{\mathrm{d}T}{\mathrm{d}h} = 0$$

Solving this equation for $\mathrm{d}p/\mathrm{d}h$, we get

$$(1-\gamma)p(h)^{-\gamma}T(h)^{\gamma}\frac{\mathrm{d}p}{\mathrm{d}h} = -p(h)^{1-\gamma}T(h)^{\gamma-1}\frac{\mathrm{d}T}{\mathrm{d}h}$$

$$\begin{aligned}\frac{\mathrm{d}p}{\mathrm{d}h} &= -\frac{p(h)\gamma\frac{\mathrm{d}T}{\mathrm{d}h}}{T(h)(1-\gamma)} \\ &= \frac{p(h)\gamma\frac{\mathrm{d}T}{\mathrm{d}h}}{T(h)(\gamma-1)}\end{aligned} \tag{C2}$$

From the assumption of the atmosphere being in hydrostatic equilibrium, we know

$$\frac{\mathrm{d}p}{\mathrm{d}h} = -\rho(h)g(h) \tag{C3}$$

Where $\rho(h)$ is the density of the atmosphere, and $g(h)$ is the gravitational acceleration. Since we approximate the atmosphere as an ideal gas, we also have

$$p(h) = \frac{\rho(h)kT(h)}{\mu m_{\mathrm{H}}} \tag{C4}$$

Where μ is the mean molecular weight of the atmosphere and m_{H} is the mass of a Hydrogen atom. Solving this expression for $\rho(h)$, we get

$$\rho(h) = \frac{p(h)\mu m_{\mathrm{H}}}{kT(h)} \tag{C5}$$

Adding this expression and (C2) to (C3), and solving for $\mathrm{d}T/\mathrm{d}h$, we get

$$\begin{aligned}\frac{p(h)\gamma\dfrac{\mathrm{d}T}{\mathrm{d}h}}{T(h)(\gamma-1)} &= -\frac{p(h)\mu m_{\mathrm{H}}}{kT(h)}g(h) \\ \frac{\mathrm{d}T}{\mathrm{d}h} &= -\frac{p(h)\mu m_{\mathrm{H}}}{kT(h)}g(h)\frac{T(h)(\gamma-1)}{p(h)\gamma} \\ &= -\frac{\mu m_{\mathrm{H}} g(h)(\gamma-1)}{k\gamma} \;\square\end{aligned} \tag{C6}$$

Appendix D: The change in atmospheric density as a function of altitude

To find the density ρ as a function of altitude h, we take a look at (C4):

$$\frac{\mathrm{d}p}{\mathrm{d}h} = \frac{\mathrm{d}}{\mathrm{d}h}\frac{\rho(h)kT(h)}{\mu m_{\mathrm{H}}} \tag{D1}$$

This result comes from taking the derivative on each side of the equation with regards to h. Combining this with (C3) and (C6), we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}h}\frac{\rho(h)kT(h)}{\mu} &= -\rho(h)g(h) \\ \frac{\mathrm{d}\rho}{\mathrm{d}h}T(h) + \rho(h)\frac{\mathrm{d}T}{\mathrm{d}h} &= -\frac{\rho(h)g(h)\mu m_{\mathrm{H}}}{k} \\ \frac{\mathrm{d}\rho}{\mathrm{d}h}T(h) &= -\frac{\rho(h)g(h)\mu m_{\mathrm{H}}}{k} - \rho(h)\frac{\mathrm{d}T}{\mathrm{d}h} \\ \frac{\mathrm{d}\rho}{\mathrm{d}h} &= -\frac{\rho(h)}{T(h)}\left(\frac{g(h)\mu m_{\mathrm{H}}}{k} + \frac{\mathrm{d}T}{\mathrm{d}h}\right) \\ &= -\frac{\rho(h)\mu m_{\mathrm{H}} g(h)\left(1-\dfrac{\gamma-1}{\gamma}\right)}{T(h)k} \\ &= -\frac{\rho(h)\mu m_{\mathrm{H}} g(h)}{T(h)k\gamma} \;\square\end{aligned} \tag{D2}$$