AST2000 - Part 7
The Landing

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By expanding the model of Buttercup's atmosphere that we made in the last study, we were able to calculate the air resistance that will affect our spacecraft and lander throughout the descent and landing process. We then went on to use this air resistance to calculate the necessary cross-sectional area of our parachute in order to perform a soft landing. Any radial velocity above 3 m/s when meeting the surface is considered a crash landing, and with this in mind, we found that we needed a cross-sectional area of at least 129.90 m^2 in order to succeed in our task. This will, in theory, ensure that it won't be required to take use of our landing thrusters.

I. INTRODUCTION

We have finally reached the part of our journey where we will land our spacecraft on our destination planet. However, preparations are needed before we are ready for the descent. We need to take height of the air resistance, as well as the gravitational pull, that will work on us throughout the landing. This is crucial for calculating the area of our parachute and deciding when and where in the atmosphere we wish to deploy it, which are important factors when trying to ensure a safe landing.

Fortunately for us, we have already made a model of the atmospheric density on Buttercup. Therefore, by expanding this model and taking the atmosphere's velocity relative to the surface into account, we will soon be ready for the last step before landing. This involves simulating our planned landing trajectory to see if we have made reasonable assumptions and calculations.

II. METHOD

First and foremost, we need to model the air resistance to decide the area of our parachute. This force is defined as

$$F_d = \frac{1}{2}\rho C_d A {v_{drag}}^2 \tag{1}$$

Where ρ is the atmospherical density at a distance r from the surface, C_d is the drag coefficient, which we will approximate as 1 in further calculations, and v_{drag} is the absolute velocity of our spacecraft with respect to the atmosphere.

We make all the same assumptions as we have done earlier, in addition to assuming that our atmosphere moves with the exact same angular velocity as Buttercup, with no other components. This angular velocity is

$$\omega = \frac{2\pi}{T} \tag{2}$$

Where T denotes Buttercup's rotational period. This means that the atmosphere moves with the rotational velocity

$$\mathbf{w} = r\omega\mathbf{e}_\phi = \frac{2\pi r}{T}\mathbf{e}_\phi \tag{3}$$

Relative to Buttercup at a distance r from the surface.

As mentioned above, we define $\mathbf{v}_{drag}$ as the velocity of the spacecraft with respect to the atmosphere. The atmosphere moves with the same velocity as the planet at surface level, and is of course non-existent below surface-level. With this in mind, we can find $\mathbf{v}_{drag}$ by using our known velocity $\mathbf{v}$ relative to Buttercup, and subtract the newly found expression for the atmosphere's velocity $\mathbf{w}$ with respect to Buttercup. We use

$$v_{drag} = |\mathbf{v} - \mathbf{w}| \tag{4}$$

In the expression for air resistance, as we're not interested in the direction.

We know that our spacecraft's velocity with respect to the planet can be written as

$$\mathbf{v} = v_r\mathbf{e}_r + v_\phi\mathbf{e}_\phi \tag{5}$$

And that as we fall, the gravitational pull from our planet will pull us toward it. The air resistance always works against the direction of motion, so it will work against gravity, but it will also slow down our velocity relative to the atmosphere. Our tangential velocity component v_ϕ will therefore become the same as $|\mathbf{w}| = w$, which means we will have zero tangential velocity with respect to the atmosphere. As a result, we'll only have a radial velocity component soon after entering the atmosphere.

As we know, we reach a terminal velocity after falling for a while. This terminal velocity v_t is constant for the rest of the fall, or at least until we launch our lander. Since the tangential velocity tends to zero, this means that the radial velocity component v_r will stabilize on a constant value, namely v_t. In this case, Newton's second law of motion tells us that the net force working on us has to be zero. The gravitational force from Buttercup is

$$\mathbf{F}_g = -\frac{GmM}{(r+R)^2}\mathbf{e}_r \tag{6}$$

Where G is the gravitational constant, m is our mass, which depends on whether we've launched our lander or not, and M and R are the mass and radius of Buttercup, respectively. Finally, r is the same as used in the aforementioned expressions. Remembering that the air resistance works against the direction of motion, it will work in the opposite direction of $\mathbf{F}_g$ by the time we reach the terminal velocity $v_t = v_r$. The sum of these two forces will have to be zero at this point, and we can use this in our quest to derive an expression for v_r (see appendix A 1):

$$v_r = \sqrt{\frac{2GmM}{(r+R)^2 \rho C_d A} - \left(\frac{2\pi r}{T}\right)^2} \tag{7}$$

Since we're interested in finding out how big the cross-sectional area of A needs to be, we rearrange the equation (see appendix A 2) and get the following:

$$A = \frac{2GmM}{(r+R)^2 \rho C_d (v_r^2 + w^2)} \tag{8}$$

Close to the surface, the atmosphere moves almost with the same rotational velocity as Buttercup itself, meaning that w tends to zero as r tends to zero. The atmospheric density will also tend to the atmospheric density by the surface, ρ_0, provided to us by our research team. To avoid making the process of landing last unnecessarily long, we plan to deploy our parachute no higher up than 1 kilometer above the surface. At this altitude and lower, we therefore come to the conclusion that it is fair to assume the following:

$$w \approx 0$$
$$r \approx 0$$
$$\rho(r) \approx \rho_0$$

Since our landing unit is supplied with a parachute to ensure that we avoid a potential crash landing, it is only logical to assume that our velocity at around surface level does not exceed the maximum velocity v_{safe} required for a soft landing. Since we will take use of the assumptions stated above considering the low altitude at which we'll deploy our parachute, we see it fit to replace v_r with v_{safe} in the expression for A. At last, we find that the minimum cross-sectional area needed to successfully perform a soft landing is determined by

$$A = \frac{2Gm_l M}{R^2 \rho_0 C_d {v_{safe}}^2} \tag{9}$$

Where m_l is the mass of our landing unit, which will be launched from the spacecraft long before we deploy our parachute.

In case we'll be in need of it, our wonderful research team provided us with landing thrusters with a customizable thrust force that we can utilize to soften our landing. Now that we've sucessfully found an expression for the cross-sectional area of our parachute, we have all that we need to determine the most appropriate thrust force for our lander (see appendix B):

$$F_l = \frac{1}{2}\rho C_d A ({v_r}^2 - {v_{safe}}^2) \tag{10}$$

Where v_r is the constant radial velocity that we will achieve close to the surface. The direction of the thrust force will of course have to be against the direction of motion in order to soften the landing, so in case $v_r < v_{safe}$, the two velocities will switch places in the expression. However, if this is the case, we'll already be moving slow enough to perform a soft landing, meaning we in theory won't be in need of the landing thrusters. We might still choose to activate them though, as you can never be too careful.

When simulating the landing process, we will be using the same numerical integration method as always, the trusty leapfrog method. Our research team informed us about a couple things we need to pay extra close attention to during the simulation:

- The parachute will fail if the air resistance from the atmosphere exceeds 250 000 Newtons
- Our lander will be incinerated if the drag pressure from the air resistance exceeds 10^7 Pascals

With the drag pressure being defined as

$$p = \frac{1}{2}\rho {v_{drag}}^2 = \frac{F_d}{C_d A} \tag{11}$$

Because of these fatal risks that we take during the actual landing process, we plan to adjust our simulation so that it stops and reports the flaw to us in case anything goes wrong. If everything goes smoothly, we will go forward with slowing down our spacecraft so that we start coasting toward the atmosphere, and also attempt to launch out lander, deploy our parachute and activate our landing thrusters around the same altitudes as in the simulation

III. RESULTS

Our research team at the University of Oslo supplied our spacecraft with a lander with a mass $m_l = 90$ kg, and were informed that it can handle a maximum velocity of $v_{safe} = 3$m/s when landing without it being considered a crash landing. We used this mass and velocity in (9) when calculating the minimum cross-sectional area needed for our parachute:

$$A = \frac{2 \times 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times 90 \text{ kg} \times 1.78 \times 10^{26} \text{ kg}}{(40288625.50 \text{ m})^2 \times 24.82 \text{ kg/m}^3 \times 1 \times (3 \text{ m/s})^2}$$
$$\approx 129.90 \text{ m}^2$$

When simulating the landing, we chose to launch the lander from the spacecraft 10 kilometers above the ground, deploy our parachute 500 meters above the ground, and activate our landing thrusters 100 meters above the ground.

From our simulation of the landing trajectory we gathered the following plots to help us visualize the process:

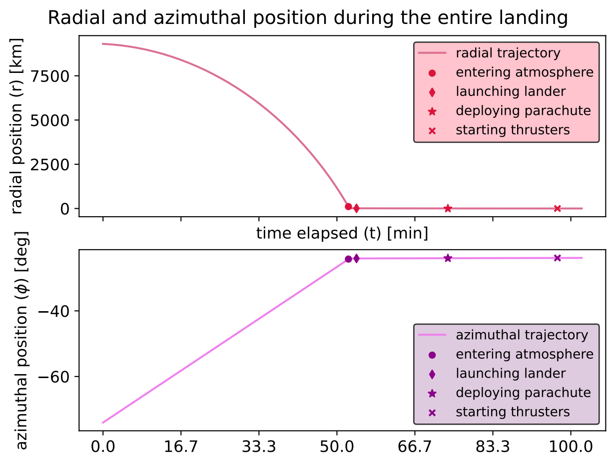

Figure 1. The radial and azimuthal position, r and ϕ respectively, during the entire simulated landing trajectory. The circle marks the point where we enter the atmosphere

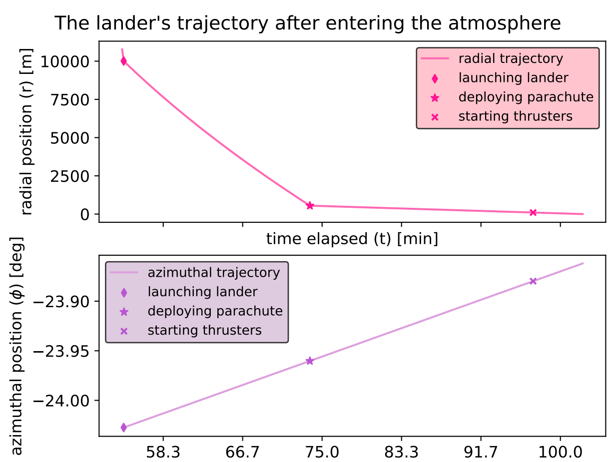

Figure 2. The simulated radial and azimuthal position, r and ϕ respectively, after entering the atmosphere

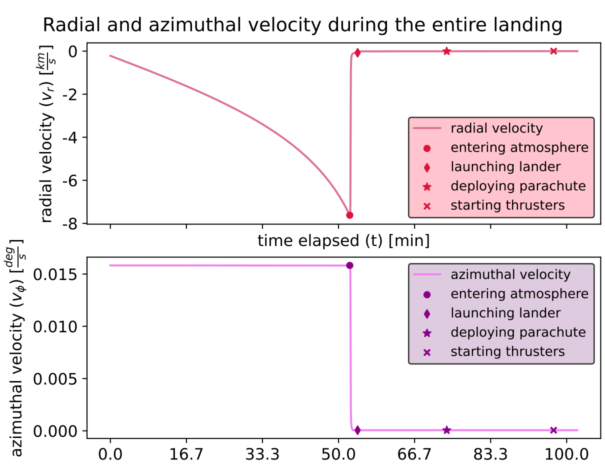

Figure 3. The radial and azimuthal velocity components, v_r and v_ϕ respectively, during the entire simulated landing trajectory. The circle marks the point where we enter the atmosphere

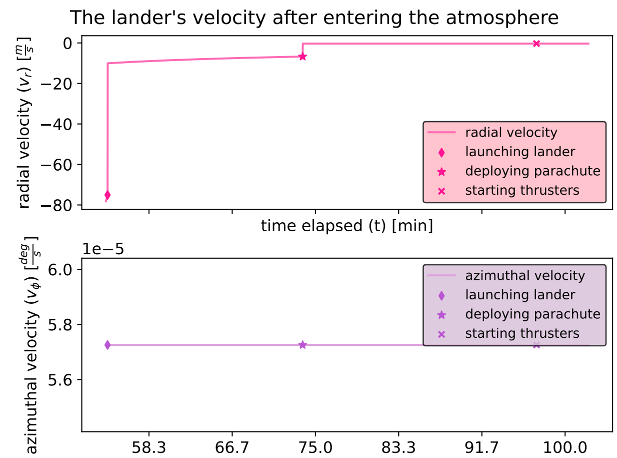

Figure 4. The lander's simulated radial and azimuthal velocity components, v_r and v_ϕ respectively, after entering the atmosphere

During the simulation, we enter the atmosphere about 52 minutes after starting the landing sequence, and launch our lander 2 minutes later. It is evident from Figure 3 and 4 that the air resistance abruptly decelerates us in this short time period. In fact, feedback from the simulation tells us that our spacecraft's radial velocity went from $v_r = 7620.42$ m/s to $v_r = 75.02$ m/s. In addition to this, its tangential velocity v_ϕ also decreases from 0.016 to approximately 5.7×10^{-5} degrees per second.

After launching the lander, it falls through the atmosphere for 20 minutes, with the air resistance slowing down its radial velocity yet again, until it reaches $v_r = 6.71$ m/s. At this point, we're 500 meters above the surface, and the parachute deploys. In Figure 4, we can see that the lander's radial velocity immediately declines and reaches a stable radial velocity of $v_r = 0.32$ m/s after this. The lander continues to fall toward the surface for about 23 minutes, travelling 400 meters. The landing thrusters activate, although it's not really necessary, considering our lander's radial velocity has stabilized on a value well below $v_{safe} = 3$ m/s. The landing thrusters, which excel a thrust force of 677.4 Newtons, are active for the rest of the descent. This takes about five and a half minutes, making the simulated landing last approximately 102.6 minutes, or 1.71 hours.

When we scouted for landing sites in the last study, we found a way to calculate the coordinates of the sites as time passes. Our chosen landing site (see Figure 5) had the following coordinates when we first photographed it:

$$r_{ls} = 0.0 \text{ m}$$
$$\phi_{ls} = 47.5^\circ$$
$$\theta_{ls} = 90.0^\circ$$

Where r_{ls}, ϕ_{ls} and θ_{ls} are the radial, azimuthal and polar coordinates of our landing site. The r-coordinate is of course zero, as the site is on the ground, and we know that our planet only rotates in the ϕ-direction, so the θ-coordinate will stay as it is.

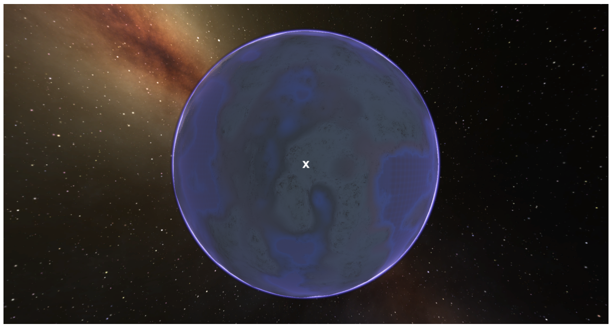

Figure 5. Our intended landing site, marked by a white x

When we were ready to initiate our landing, we were looking at Buttercup's night side, so we let the spacecraft orbit for about 17.5 hours. After this time period, our spacecraft's coordinates were the following:

$$r_{sc} = 2541.4 \text{ km}$$
$$\phi_{sc} = 286.0°$$
$$\theta_{sc} = 90.0°$$

While the landing site's ϕ-coordinate had changed to $\phi_{ls} = 49.2°$.

During our simulated landing, our ϕ-coordinate underwent a change $\Delta\phi \approx 50.1°$ in the same direction as the planet rotates. We chose to boost our lander in the direction of the predicted coordinates of our planned landing site upon launch.

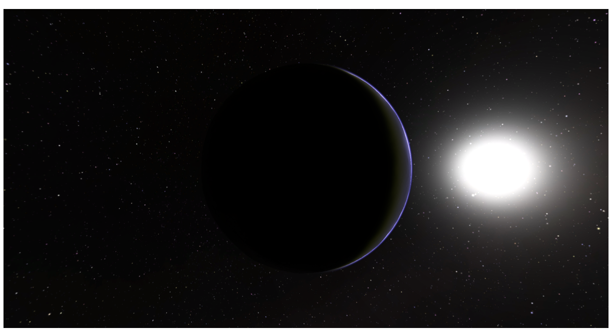

Figure 6. Our view at the moment when we slow down our spacecraft and get ready for the descent

For our spacecraft to be dragged out of our stable orbit and actually start falling toward Buttercup, we first had to decrease our velocity by one third. We let our spacecraft fall for cirka 54 minutes before we launched our lander, just like in our simulation. At this point, we were still high up in orbit, approximately 4 254 kilometers above the surface. However, thanks to the boost that our lander got, it managed to travel about 3 338 kilometers closer to the surface in just 20 minutes. At this height, we deployed our parachute, which slowed down our tremendous radial velocity of about 6617.4 m/s. Letting the lander fall for another 23 minutes or so, it was soaring through the atmosphere with a radial velocity of 17.64 m/s, about 14.2 kilometers above the surface. After another 53 minutes, our lander's radial velocity had decreased to 15.75 m/s, and it was only 107 meters above the surface by then. We activated our landing thrusters to soften the landing, which decelerated our velocity to 0.32 m/s in about five and a half minutes. Thus, our landing was successful, and nothing got destroyed in the process.

The landing lasted for 2.59 hours, which is about 53 minutes longer than our simulation. The ϕ-coordinate of our final landing site was $\phi_{sc} = 48.1°$. This was at a distance of about 338.4 kilometers away from our planned landing site, which by then had the ϕ-coordinate $\phi_{ls} = 51.2°$.

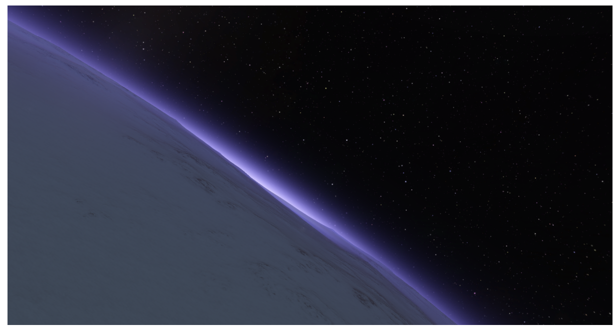

Figure 7. The surface of Buttercup

IV. DISCUSSION

After analyzing the results of our simulation, we saw it fit to go along with the same plan when initiating the actual launch. Considering neither our lander nor our parachute would've been destroyed in an actual descent, assuming our simulation was correct, this seemed reasonable. In the simulation, we saw that the lander's radial velocity was at 0.32 m/s, which is well below $v_{safe} = 3$ m/s, by the time we were 100 meters above the ground. This is the altitude where we planned to activate the landing thrusters. These results from the simulation pointed to use of the thrusters probably being unnecessary. Our choice of still utilizing them turned out to be very fortunate, as this was not the case in the actual landing. We had a radial velocity of as high as 15.75 m/s only 107 meters above the surface, meaning that if we were to ignore the landing thrusters, the turnout of the landing would've been rather tragic.

Comparing the results from our simulation and the actual landing, there are a few thing we wish to discuss. First and foremost, we want to address the fact that our actual landing process took almost an hour longer than the simulation, as well as the fact that we missed our intended landing site. There could be many reasons for this, such as round-off errors, wrong assumptions, miscalculations or other factors often encountered when simulating a physical happening or phenomena.

A very probable cause of the delay could be the fact that we had to slow down the spacecraft and let it coast for a little when trying to perform the actual descent, unlike in the simulation, where our spacecraft immediately started descending. This was the first flaw we noticed about our simulation, as it is very unrealistic. The spacecraft had already been orbiting the planet in a stable orbit multiple times beforehand, meaning it only makes sense that we would need to slow it down for it to actually be pulled out of said orbit and toward the planet. Our main hypothesis for the cause of this flaw is that, although the spacecraft's orbit around Buttercup is technically a two-body orbit, we did not approximate it as such. Considering how close we are in our orbit around Buttercup, this minuscule difference in calculations could very well be the root of the flaw. When comparing the actual landing with the simulation, we immediately noticed the fact that we're 400 times higher up than where we originally planned to launch our lander by the time we launched it in the simulation, which is 54 minutes after initiating the descent. Since the actual landing took approximately 53 minutes longer than the simulation, we deem this as a very likely source of error.

Another possible source of error in our simulation could be caused by our model of the atmosphere that we made in the last part of this study. We analyzed flux data in order to determine its chemical build-up, where the data contained large amounts of noise, making it hard to decipher which of the approximated spectral lines were flukes and which were real. Based on the three spectral lines we deemed to be real, the composition of the atmosphere is 33.3 % CH_4, 33.3 % CO, and 33.3 % N_2O. However, we also considered the possibility of all of these being false, and another two being real instead, which instead would point to the atmosphere consisting of only H_2O. These compositions are obviously very different. Thus, by possibly making the wrong assumptions, the atmospheric density profile we made may be very off. The density of the atmosphere will naturally affect important factors when descending, such as the air resistance. A more dense atmosphere would make the entire trajectory through the atmosphere last longer, meaning this possibly could have contributed to the actual landing taking longer than expected. Furthermore, we doubt this is the main cause, as an atmosphere made up of solely H_2O would point to a lower density, not higher.

Regarding the fact that we missed our landing site, this was actually to be expected. When we simulated the landing, we found that our ϕ-coordinate shifted approximately 50.1° in the same direction as the planet rotates on the way down. Our planned landing site had the ϕ-coordinate $\phi_{ls} = 49.2°$, meaning we should've initiated our descent when our spacecraft's azimuthal position was at around 359.1° instead of 286.0°. However, in an effort to compensate for this distance, we chose to attempt to boost our lander in the direction of the predicted coordinates of our chosen landing site. This was in place of the small boost of just 100 m/s in the radial direction that we chose for the simulation, which didn't do much of a difference, as evident when looking at the upper graph in Figure 4. The new boost we chose for the lander, combined with the longer duration of the landing process, actually caused us to miss the landing site we originally had in mind by only 3.1° instead of 73.1°, which signs originally pointed to.

V. CONCLUSION

At last, we were able to land on Buttercup without destroying any of our equipment, although our landing simulation had some flaws. We calculated an appropriate cross-sectional area for our parachute and a reasonable force for our landing thrusters. Together, they successfully ensured that our radial velocity decreased enough during our descent through the atmosphere to avoid a crash landing.

We did not succeed in landing at the site we originally had in mind, but our final site fortunately wasn't too far off. With this, we have completed our journey through our solar system. From modelling and launching our rocket, calculating multiple properties for numerous celestial bodies, and orienting our spacecraft in space, to analyzing and approximating the chemical build-up of our destination planet's atmosphere, and finally, performing a safe landing.

ACKNOWLEDGMENTS

A special thanks to our research team at the University of Oslo, as we would probably never been able to complete this amazing journey through space and land on our beautiful planet if it wasn't for all the help from you.

Appendix A: Deciding the cross-sectional area A of our parachute

1. Deriving an expression for v_r

We know that the following is true for v_{drag} as soon as our tangential velocity component reaches zero:

$$v_{drag} = |\mathbf{v} - \mathbf{w}| = \left| -v_r \mathbf{e}_r - \frac{2\pi r}{T}\mathbf{e}_\phi \right| = \sqrt{v_r^2 + \left(\frac{2\pi r}{T}\right)^2}$$

Combined with Newton's second law of motion telling us that the net force working on us has to be zero for us to be able to move with a constant velocity, we get the following:

$$\begin{aligned}
F_d + F_g &= 0 \\
\frac{1}{2}\rho C_d A {v_{drag}}^2 &= \frac{GmM}{(r+R)^2} \\
v_r^2 + \left(\frac{2\pi r}{T}\right)^2 &= \frac{2GmM}{(r+R)^2 \rho C_d A} \\
v_r &= \sqrt{\frac{2GmM}{(r+R)^2\rho C_d A} - \left(\frac{2\pi r}{T}\right)^2} \quad \square
\end{aligned}$$

2. Solving for A

$$\begin{aligned}
v_r^2 + \left(\frac{2\pi r}{T}\right)^2 &= \frac{2GmM}{(r+R)^2\rho C_d A} \\
(r+R)^2 \rho C_d A &= \frac{2GmM}{v_r^2 + \left(\frac{2\pi r}{T}\right)^2} \\
A &= \frac{2GmM}{(r+R)^2\rho C_d (v_r^2 + \left(\frac{2\pi r}{T}\right)^2)} \\
&= \frac{2GmM}{(r+R)^2\rho C_d (v_r^2 + w^2)} \quad \square
\end{aligned}$$

Appendix B: Deriving an expression for the landing thrusters' force

By the surface we have

$$\begin{aligned}
F_d + F_l &= F_g \\
F_l &= -\frac{1}{2}\rho C_d A {v_r}^2 - \frac{GmM}{(r+R)^2}
\end{aligned}$$

Where F_l denotes the landing thrusters' force.

We will activate the landing thrusters very close to the surface, well after we've reached the escape velocity v_r. Newton's second law of motion tells us that the force of gravity will counteract the air resistance experienced when moving with a terminal velocity v_r, meaning that $F_g = F_l$. In addition to this relation, we don't want v_r to exceed v_{safe} when landing, so we assume the air resistance to be a function of v_{safe} instead of v_r. At last, we get

$$\begin{aligned}
F_l &= -\frac{1}{2}\rho C_d A {v_{safe}}^2 + \frac{1}{2}\rho C_d A {v_r}^2 \\
&= \frac{1}{2}\rho C_d A ({v_r}^2 - {v_{safe}}^2) \quad \square
\end{aligned}$$