TITLE

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ABSTRACT

Aims. AIMS
Methods. METHODS
Results. RESULTS

Key words. KEYWORDS

1. Milestone I

1.1. Introduction

1.2. Theory

The Hubble parameter is

$$H = H_0 \sqrt{(\Omega_{b0} + \Omega_{\rm CDM0})\, a^{-3} + \left(\Omega_{\gamma 0} + \Omega_{\nu 0}\right) a^{-4} + \Omega_{k0} a^{-2} + \Omega_{\Lambda 0}}. \tag{1}$$

TEXT The scaled Hubble parameter is given by

$$\mathcal{H} = aH, \tag{2}$$

Thus, if we define $\Omega_{m0} = \Omega_{b0} + \Omega_{\rm CDM0}$ and $\Omega_{r0} = \Omega_{\gamma 0} + \Omega_{\nu 0}$, and use our chosen time variable $x = \log a$ instead of a, the expressions (1) and (2) can equivalently be written as

$$H = H_0 \sqrt{\Omega_{m0} e^{-3x} + \Omega_{r0} e^{-4x} + \Omega_{k0} e^{-2x} + \Omega_{\Lambda 0}}, \tag{3}$$

$$\mathcal{H} = H_0 \sqrt{\Omega_{m0} e^{-x} + \Omega_{r0} e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0} e^{2x}}. \tag{4}$$

The first and second derivatives of $\mathcal{H}$ with respect to x are then correct expressions?

$$\begin{aligned} \frac{d\mathcal{H}}{dx} &= \frac{H_0}{2} \frac{-\Omega_{m0} e^{-x} - 2\Omega_{r0} e^{-2x} + 2\Omega_{\Lambda 0} e^{2x}}{\sqrt{\Omega_{m0} e^{-x} + \Omega_{r0} e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0} e^{2x}}}, \\ &= -\frac{H_0^2}{2\mathcal{H}} \left(\Omega_{m0} e^{-x} + 2\Omega_{r0} e^{-2x} - 2\Omega_{\Lambda 0} e^{2x}\right), \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{d^2\mathcal{H}}{dx^2} &= \frac{H_0}{2} \left(\frac{\Omega_{m0} e^{-x} + 4\Omega_{r0} e^{-2x} + 4\Omega_{\Lambda 0} e^{2x}}{\sqrt{\Omega_{m0} e^{-x} + \Omega_{r0} e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0} e^{2x}}} \right. \\ &\qquad \left. + \frac{1}{2} \frac{\Omega_{m0} e^{-x} + 2\Omega_{r0} e^{-2x} - 2\Omega_{\Lambda 0} e^{2x}}{\left(\Omega_{m0} e^{-x} + \Omega_{r0} e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0} e^{2x}\right)^{3/2}} \right), \\ &= \frac{H_0^2}{2\mathcal{H}} \left(\Omega_{m0} e^{-x} + 4\Omega_{r0} e^{-2x} + 4\Omega_{\Lambda 0} e^{2x} - \frac{1}{2\mathcal{H}} \frac{d\mathcal{H}}{dx} \right). \end{aligned} \tag{6}$$

1.3. Methods

1.4. Results & Discussions

1.5. Conclusions

Sanderson & Curtin (Accessed: October 2023)

References

Sanderson, D. C. & Curtin, D. R. Accessed: October 2023, Armadillo C++ Library", https://arma.sourceforge.net/docs.html