Pithoprakta: The historical measures 52-59 New evidence in glissando speed formalization Theory & theoretical applications

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Assimilating glissandos to monatomic gas molecular movements and considering that molecular speeds are proportional to molecular mean kinetic energy (*temperature*), Xenakis seeks for normal distributions of sound speeds (glissandos) via *Gauss equation*. He uses a two-variable form of the equation (bi-dimensional domains) and then a three-variable one (three-dimensional domains). Through *correlation* he proves that speed distributions provided by both forms of Gauss equation are practically equivalent. Thus, temperature might be resulting from glissando execution limitations, i.e. *speed limits*. Since mean speed value becomes an invariable, temperature is meant to be a logically designated magnitude. Consequently, the introduction of *speed* as a scientific notion in composition is justified. Given that glissando directions are not stochastically preset, probabilistic determinism controls the interior of the *sound mass*, meas. 52-59, but not its overall evolving form. After having set all required parameters, Xenakis draws a general probability table that leads to 33 analytical sub-tables; 1168 theoretical and 1221 effective speeds are provided and virtually distributed to bowed instruments. Yet, both numbers remain theoretical as the majority of speeds are too quick for execution. A more realistic eventual maximum speed is applied then. Depending on speed, temperature is now close to "temperature proportional to *α*=35" stated in *Musiques formelles*. Pre-compositional applications are completed by a set of three very detailed graphic tables, through which a wider range of speeds is assigned anew to individual instruments. Due to the composer's musical intentions, a number of theoretical data are revised. Anticipating the graphic score, creative theoretical reasoning that leads to speed distributions constitutes the *scientific matrix* of the work, which is bridged with the *musical matrix* by the application of theoretical results. Xenakis's theoretical reasoning might be regarded as pure creation proportionally equivalent to composition itself.

Iannis Xenakis first offered the formalization principles of mobile sounds, i.e. glissandos, in *Gravesaner Blätter No 6*, 1956, p. 28-34 and later on in *Musiques Formelles*, 1963, p. 27-30. This historical writing, together with the corresponding graph that follows in p. 30, is focusing on meas. 52-59 of *Pithoprakta* (1956). The text gives only a short overview of sound speed formalization principles based on the Gauss normal distribution formula as applied by Maxwell-Boltzmann in gas kinetics. Precise application of that law in composition, however, remains obscure, since Xenakis gave no analytical examples of glissando speed computation. Besides, although meas. 52-59 of *Pithoprakta* have thoroughly been commented and explored by several researchers, lack of evidence let the distance between theory and its detailed application in compositional practice grow unbridgeable. The issue has been resolved, since access to *Archives Iannis Xenakis*, hosted nowadays in *Bibliothèque Nationale de France* (*BnF*), offers the possibility of systematic documentation.

Our study, based on *Archives Xenakis – BnF / Oeuvres Musicales / Pithoprakta* / *Dossier 1/13* and especially *Carnet #16*, deals strictly with theoretical principles, reasoning and planning that provide all necessary data for the composition of *Pithoprakta* meas. 52-59*.* The practical part of the subject, i.e. the graphic score composition in association with pre-compositional data, is not examined here at all.

Definitions – Theoretical considerations

Although the normal distribution is sometimes called the "Gaussian distribution" after Gauss who discussed it in 1809, it was actually already known in 1774 by LaPlace. A random variable X is said to be *normally distributed* if its density function $f(x)$ has the following form:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Mean or expected value μ is any real number, standard deviation σ is any positive number and variance σ^2 is a positive number. That is, mean, variance, and standard deviation of the nor-

mal distribution $N(\mu, \sigma^2)$ are used as the parameters in the definition of the above density function $f(x)$. [Lipschutz – Lipson, 2000, 180-1]
Assimilating moving instrumental sounds, i.e. string glissandos, to gas molecular movements in space, Xenakis used a simple form of the Gauss equation that leads to normal distributions of the absolute sound speeds v. "This chain of reasoning borrowed from Paul Lévy[1] was established after Maxwell, who, with Boltzmann, was responsible for the kinetic theory of gases. The function $f(v)$ gives the probability of the speed v; the constant α defines the 'temperature' of this sonic atmosphere. The arithmetic mean[2] of v is equal to $\alpha/\sqrt{\pi}$ and the standard deviation[3] is $\alpha/\sqrt{2}$." [Xenakis, 1992, 15] The equation used is:

$$f(v) = \frac{2}{\alpha\sqrt{\pi}} \cdot e^{-\frac{v^2}{\alpha^2}}$$

This final formulation, however, is preceded by a long theoretical research –February to May 1956–, which is mainly based on Paul Lévy's *Calcul des probabilités* (1925), Ulysse Philippi's *Connaissance du monde physique* (1947) and Emile Borel's *Éléments de la théorie des probabilités* (1950)[4]. After a short preliminary study, Xenakis starts with a form of Gauss equation valid for molecular movements in bi-dimensional domains, i.e. Cartesian plains:

$$f(v) = \frac{2v}{u^2} \cdot e^{-\frac{v^2}{u^2}}$$

v stands for the number of half-tones per time unit (speed) and u for the mean value of square speeds. Parameters:

Average square speed $u^2 = \overline{v}^2$

Average speed $\overline{v} = \frac{\sqrt{\pi}}{2} \cdot u = 0.886 \cdot u$

Standard deviation $\sigma_v = 0.463 \cdot u$ and $f(v) = 0.857/u$ for $v = \frac{u}{\sqrt{2}} = 0.707$

He draws then several speed distribution tables that are not used but for theoretical reasons, as we shall see[5]. He also defines u as "number of considered sounds", where

$$u = \sqrt{\frac{RT}{m}}$$ [Carnet #16, p. 25, 30-3-1956]

and comes to four different values of it:

(a) $u = 55.70 \rightarrow \overline{v} = 49.30$
(b) $u = 33.00 \rightarrow \overline{v} = 29.25$
(c) $u = 20.50 \rightarrow \overline{v} = 18.20$
(d) $u = 10.00$

But as the aforementioned equation "[...] is valid only for two variables; therefore, in the case of glissandos it is false." [Carnet #16, p. 25, 30-3-1956], mathematical reasoning starts all over again, till the final formulation of Gauss equation for three-dimensional domains, as stated in *Musiques formelles*:

$$f(v) = \frac{2}{\alpha\sqrt{\pi}} \cdot e^{-\frac{v^2}{\alpha^2}}$$

In theory, there is 0 probability for speeds equal to 0 intervals per time unit ($v_p = 0$), because if $v = 0$, then

$$f(v) = \frac{1}{\alpha\sqrt{\pi}} \cdot \frac{-2v}{\alpha^2} \cdot e^{-\frac{v^2}{\alpha^2}} = 0$$ [*Carnet #16*, p. 28 (31-3-1956)].

In other words, static sounds are not probable and therefore not in use[6].

He also comes up to the following results for mean speed standard deviation σ:

a) $\sigma_v = \sqrt{\frac{\pi - 2}{\pi}} \cdot u \rightarrow \sigma_v = 0.6025 \cdot u$ b) $\sigma_v = \sqrt{\frac{\pi - 2}{2\pi}} \cdot \alpha \rightarrow \boxed{\sigma_v = 0.425 \cdot \alpha}$[7]

Then, he formulates the probability $\Theta(\lambda)$ that relative deviation λ fluctuates from $-\lambda$ to $+\lambda$, where $\lambda = v/\alpha$ and $\lambda_2 < \lambda < \lambda_1$:

$$\Theta(\lambda) = \frac{2}{\sqrt{\pi}} \int_{\lambda_1}^{\lambda_2} e^{-\lambda^2} \cdot d\lambda$$ [Borel, 1950, 69][8]

Thus:

$$f_\mu(\lambda) = \frac{2}{\alpha\sqrt{\pi}} \int_0^v e^{-\frac{v^2}{\alpha^2}} \cdot dv = \frac{2}{\sqrt{\pi}} \int_0^{\frac{v}{\alpha}} e^{-\left(\frac{v}{\alpha}\right)^2} \cdot d\left(\frac{v}{\alpha}\right) = \frac{2}{\sqrt{\pi}} \int_0^{\lambda} e^{-\lambda^2} \cdot d\lambda = \Theta(\lambda)$$ [Carnet #16, p. 31]

In p. 33 (13-4-1956) all factors are finally depicted, through standard deviation $\boxed{\sigma = 24}$:

Maximum speed: $v_{max} = 4\ \sigma = 96$

Temperature[9]: $\sigma = \sqrt{\frac{\pi - 2}{2\pi}} \cdot \alpha = 0.425 \cdot \alpha \rightarrow \alpha = 24/0.425 = 56.47 \rightarrow \boxed{\boldsymbol{\alpha \sim 56}}$

Mean of square speed values: $u = \frac{\alpha}{\sqrt{2}} = 56/1.414 \rightarrow \boxed{u \sim 40}$

In a theoretical level, $\bar{v} + \sigma \rightarrow 1.4 \cdot u = 1.4 \cdot 40 \rightarrow \alpha = 56$. Since α is a standard magnitude and σ fluctuates from σ to 11σ, ten additional values of u are provided (p. 33):

$\bar{v} + \sigma$: $1.4 \cdot u = \alpha = 56 \rightarrow u = \underline{40}$

$\bar{v} + 2\,\sigma$: $2.0 \cdot u = \alpha = 56 \rightarrow u = \underline{28}$

$\bar{v} + 3\,\sigma$: $2.6 \cdot u = \alpha = 56 \rightarrow u = 21.5$

$\bar{v} + 4\,\sigma$: $3.21 \cdot u = \alpha = 56 \rightarrow u = 17.5$

$\bar{v} + 5\,\sigma$: $3.81 \cdot u = \alpha = 56 \rightarrow u = 14.7$

$\bar{v} + 6\,\sigma$: $4.41 \cdot u = \alpha = 56 \rightarrow u = \underline{12.7}$

....

$\bar{v} + 11\,\sigma$: $7.42 \cdot u = \alpha = 56 \rightarrow u = \underline{7.55}$

Among them, four (previously underlined) u values are used to provide four new values of α:

$u = 40$	is assigned to $\alpha = 56.47 \sim 56$	(I)
$u = 28$	" " " $\alpha = 30.40 \sim 30$	(II)
$u = 12.7$	" " " $\alpha = 17.95 \sim 18$	(III)
$u = 7.55$	" " " $\alpha = 10.67 \sim 10$[10]	(IV)

Remarks:

1. A very intriguing fact is that α values I, II, III and IV are almost equal to u values a, b, c and d mentioned above; in particular, I is very close to a and IV to d. It seems, then, that three-dimensional 'invariant' a is assimilated to bi-dimensional 'invariant' u.
2. Temperature (IV) is very low; it allows only very slow speeds[11] and, therefore, cannot be integrated in meas. 52-59.
3. Average value of temperatures (I)-(II)-(III) is (56+30+18)/3 = 104/3 = 34.666 ~ 35; it coincides with the "temperature of that 'atmosphere' proportional to $a = 35$" [Xenakis, 1963, 29]
4. Values II, III and IV neither have an apparent origin nor are used for speed computation. Nevertheless, Xenakis makes use of them to designate speed mean values $\bar{v}$ and their corresponding standard deviations σ (p. 39):

I	$\alpha = 56$	$\bar{v} = 31.5^{7}$	$\sigma = 24$
II	$\alpha = 30$	$\bar{v} = 17^{13}$	$\sigma = 13$
III	$\alpha = 18$	$\bar{v} = 10^{7}$	$\sigma = 7.5$
IV	$\alpha = 10$	$\bar{v} = 5.5^{4}$	$\sigma = 4$

Time partition – Mean numbers of instruments

As far as durations of sonic events are concerned, we should focus on the original subdivision of a 2/2 measure in 20 uneven parts. Their inequality derives from the superposition of three standard rhythm patterns, α, β and γ, which are assigned in advance to all 46 strings (24 violins, 8 violas, 8 cellos and 6 double basses)[12]:

α = 10 quintuplet eighth-notes: $Vln_{1,2,9,10,13,14,21,22}$ – $Vla_{1,4,7}$ – $Vc_{1,6}$ – $Cb_{1,4}$ (15 strings)
β = 8 eighth-notes: $Vln_{3,4,7,8,15,16,19,20}$ – $Vla_{2,5,8}$ – $Vc_{2,4,7}$ – $Cb_{2,5}$ (16 strings)
γ = 6 triplet crotchets: $Vln_{5,6,11,12,17,18,23,24}$ – $Vla_{3,6}$ – $Vc_{3,5,8}$ – $Cb_{3,6}$ (15 strings)

In Xenakis's graph paper a whole-note, i.e. a 2/2 measure, is equal to 50 mm; so:

1 quintuplet eighth-note in pattern α = 1 whole-note/10 = 50mm/10 → α = 5 mm
1 eighth-note in pattern β = 1 whole-note/8 = 50mm/8 → β = 6.25 mm
1 triplet quarter-note in pattern γ = 1 whole-note/6 = 50mm/6 → γ = 8.33 mm

Superimposing the rhythms, 20 uneven subdivisions of the measure are obtained (**Fig. 1**).

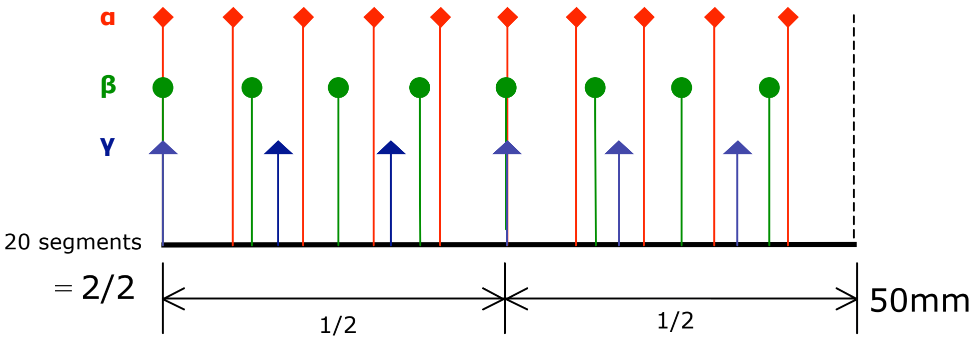

Figure 1. Rhythm patterns α, β and γ divide the measure in 20 uneven segments

The length of a particular time segment represents the distance between two consecutive events in millimeters, or in other words, the differential duration of the first of them. Computing the durations leads to the series [Antonopoulos, 2008, 112, 113]:

𝅗𝅥 : | 0.1 – 0.025 – 0.042 – 0.033 – 0.05 | 0.05 – 0.033 – 0.042 – 0.025 – 0.1 |

𝅗𝅥 : | 0.1 – 0.025 – 0.042 – 0.033 – 0.05 | 0.05 – 0.033 – 0.042 – 0.025 – 0.1 |

In meas. 52-59, however, 2/2 measure is divided in 10 time units instead of 20; consequently, all aforementioned event durations have do be doubled:

: | 0.2 – 0.05 – 0.084 – 0.066 – 0.1 | 0.1 – 0.066 – 0.084 – 0.05 – 0.2 |

Considering, then, that a whole-note is always equal to 50 mm,

1 quintuplet crotchet (α) = 10 mm or 0.20 of a whole-note
1 crotchet (β) = 12.5 mm or 0.25 " " "
1 triplet half-note (γ) = 16.66 mm or 0.33 " " "

Sound mass in meas. 52-59 is produced by the continuous glissando play of the entity of the strings, without rests. In order to avoid emphasizing the down-beat through its periodical accentuation by the whole orchestra, 46 strings, Xenakis reduces to 15 the number of new sonic events per down-beat[13], i.e. to an average instrumental density d_m assuring proportional presence of all three rhythm types: $d_m = 46/3 = 15.3 \rightarrow d_m \sim 15$. This means that only 15 instruments are starting a new glissando by producing new pitches on the down-beat, while the others are sliding pre-existing ones. As average density d_m is equal to 15 sonic events of all rhythm types per downbeat, the standard deviation σ_m of this mean value will be:

$$\sigma_m = \sqrt{d_m} = \sqrt{15} = 3.87 \rightarrow \sigma_m \sim 4$$

$d_m = 15$ should fluctuate from $|d_m + \sigma_m|$ to $|d_m - \sigma_m|$: $|d_m - \sigma_m| < d_m < |d_m + \sigma_m| \rightarrow 11 < d_m < 19$

Instrumental density[14] has to be preserved during every hole-note time unit, while the number of active strings remains unvarying and equal to 46 during 8 measures, meas. 52-59. It follows that rhythm α should be assigned to fewer instruments than rhythm β as much as rhythm β to fewer instruments than rhythm γ, because $\alpha = 5 > \beta = 4 > \gamma = 3$ events:

Rhythm α $\longrightarrow$ $d_m - \sigma_m = 15 - 4 = 11$ strings (11 sonic events)
Rhythm β $\longrightarrow$ $d_m = 15$ strings (15 sonic events)
Rhythm γ $\longrightarrow$ $d_m + \sigma_m = 15 + 4 = 19$ strings (19 sonic events)

But as the above sum is 11+15+19 = 45 instruments instead of 46 required, the lower limit of d_m which corresponds to 11 strings should be raised to 12; thus, the final sum becomes:

12+15+19 = 46 instruments.

Let us note that, in theory, the optimal downbeat disposition of instruments is:

$4_{(\alpha)}+5_{(\beta)}+6_{(\gamma)} = 15 = d_m$

The following graph, **Fig. 2**, illustrates the number of instruments per whole-note, i.e. per measure, in relation with the number of produced new pitches, i.e. glissando starting points

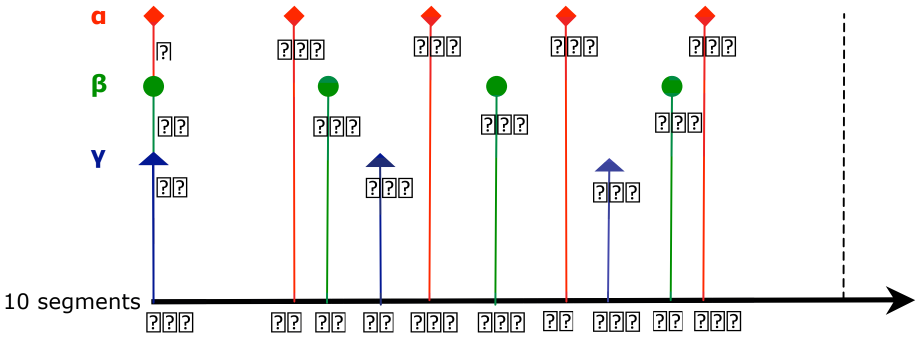

Figure 2. Number of sonic events per rhythm pattern and per measure

Sum of glissandos per measure: 1·15 + 4·12 + 3·15 + 2·19 = 15+48+45+38 = 146.
Average number of glissandos per time division: 146/10 = 14.6 ~ 15 = d_m
Glissando distribution by rhythm pattern in a measure:

α: 48 + 4 = 52
β: 45 + 5 = 50
γ: 38 + 6 = 44
Σ: 146

Glissandos by rhythm pattern for meas. 52-59 (8 measures):

α: 52 · 8 = 416
β: 50 · 8 = 400
γ: 44 · 8 = 352
Σ: 1168

Given that *speed* v means "[number of] half-tones per measure 26 MM" [Archives I.X. – BnF, Dossier 1/13, Fol. 0083], Σ: 1168 stands for the theoretical number of speeds[15], i.e. glissandos, to be calculated for meas. 52-59.

Speed computation – Probability tables

Data are now sufficient for computing the general probability table of average speeds v_μ and their distribution to rhythm patterns α, β and γ [Carnet #16, p. 33, 40] (**Tabl. 1**).

Parameters:

$\alpha = 56$ (I), $\sigma = 0.425 \cdot \alpha$ [illegible] $\sigma = 23.8$ [~ 24], $\sigma/2 = 12$, $\bar{v} = \alpha/\sqrt{\pi} = 31.6$

$v_{max} = 4\,\sigma + \bar{v} = 96 + 31.5 = 127.5$

$v_{max} = 5\,\sigma + \bar{v} = 120 + 31.5 = 151.5$

$v_{max} = 6\,\sigma + \bar{v} = 144 + 31.5 = 175.5$

	v_i	$\lambda = v/\alpha$	$\Theta(\lambda)$	P = $\Theta(\lambda_2)$ - $\Theta(\lambda_1)$	$\boldsymbol{v_\mu}$	α (5) 416	β (4) 400	γ (3) 352	α (34)		β	γ
	0	0	0.00									
				0.157	**4**	65	63	55	5.4	5	3	2
	8	0.14	0.157									
				0.232	**14**	97	93	82	7.9	8	8	9
	20	0.36	0.389									
				0.191	**26**	80	76	67	6.5	7	9	
$\bar{v}$	32	0.57	0.580									11
				0.156	**38**	65	62	55	5.4	5	6	
σ	44	0.79	0.736									6
				0.107	**50**	44	43	38	3.6	4	3	
$[\alpha]$	**56**	1.00	0.843									3
				0.070	**62**	29	28	25	2.4	2	1	
	68	1.21	0.913									
2σ				0.044	**74**	18	18	15	1.5	2	1	
	80	1.45	0.957									
				0.023	**86**	10	9	8	0.8	1	1	
	92	1.64	0.980									
3σ				0.012	**98**	5	5	4	0.4			
	104	1.86	0.992									
				0.005	**110**	2	2	2	0.17		1	
	116	2.07	0.997									
4σ				0.002	**122**	1	1	1	0.07			
	128	2.29	0.999								1	
				0.000								
	140	2.50	0.9996									
5σ												
	152	2.72	0.9999								2	
						416	**400**	**352**		34		
							Σ = **1168**					

Table 1. Average speeds v_μ and their distribution to rhythm patterns α, β and γ

- $v_\mu = (v_{i-1} + v_i)/2$. In example: $v_\mu = (8+20)/2 = 14$, $v_\mu = (20+32)/2 = 26$ etc.
- Speed average values, bold figures in column v_μ, are used as a basis for precise speed computation of various α, β and γ durations in the tables that will follow (33 tables).
- Numbers in column [α (5) 416] are obtained by multiplying the numbers of column [P = $\Theta(\lambda_2)$ – $\Theta(\lambda_1)$] by 416. The same stands for columns [β (4) 400] and [γ (3) 352]. "44" in column [α (5) 416], in example, means that average speed v_μ = 50 half-tones per measure will be used 44 times in combination with durations of rhythm pattern α*.*
- Figures in column [α 34] are rounded off; their sum is equal to 34.
- Precise positions of red figures in columns [β] and [γ] are not clear in the manuscript.
- Explanatory rough notes next to figures are omitted here, since they formally appear in individual speed calculation tables that follow in the manuscript.

The fact that no other table of this kind has been found in Xenakis's manuscripts shows that only α (I) = 56 is practically used; α (II), (III) and (IV) have been eliminated. In *Carnet #16*, 33 speed distribution sub-tables stem from the main v_μ table. In the following example of α type speeds (**Tabl. 2**), v_μ = 14 appeared first in v_μ column of the main table, but here stands for the average of v_{min} = 8 and v_{max} = 20 in v_i column.

v_μ = **14** [v_{min} = 8 – v_{max} = 20]				
α				
v		Numb. instr. (v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.	
2/0.20	10	14	15	150
3/0.20	15	30	30	450
4/0.20	20	8	8	160
5/0.40	12.5	33	35	440
7/0.40	17.5	12	12	210
	Σ:	**97**	100	1410

Table 2. An example of type α speed tables

- h.t./t.u. : half-tones per time unit of ɑ rhythm type.
- h.t./m. : half-tones per measure.
- Numb. instr. (v_μ): theoretical and effective number of instruments executing v_μ, i.e. number of v_μ; "theo" values appear also in the main distribution table.
- h.t. (last column): partial products in half-tones of v_μ numbers by h.t./m.
- Sum Σ of theoretical v_μ numbers is the same as in the main probability table.

The following analytical sub-tables (**Tabl. 3-35**) provide 112 feasible speeds as well as 1168 theoretically and 1221 effectively corresponding instruments. In other words, there are 1168 theoretical and 1221 effective speeds virtually distributed to the instruments. Yet, since the final score contains fewer speeds than indicated in the tables, both numbers remain theoretical in practice. The reason is that the majority of speeds are too quick for execution. Tables in this stage provide basic data for further elaboration. Values of theoretical and effective instrument numbers (or v_μ) are often interchanged to fit the choices of the composer and/or the demands of the composition. Data in brackets are tacitly understood since they do not figure in the manuscript. Numbers in bold refer to the main probability table. Besides, several speeds appear twice or thrice in different tables, but in a disparate probabilistic context.

v_μ = **4** [v_{min} = 0 – v_{max} = 8]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
1/0.40	2.50	38	35	87	1/0.50	2.00	21	[21]	42	1/0.66	1.50	9	10	15
2/0.40	5.00	17	15	75	2/050	4.00	21	[21]	84	2/0.66	3.00	14	15	45
3/0.40	7.50	10	10	75	3/0.50	6.00	21	[21]	126	3/0.66	4.50	14	15	67
	Σ:	**65**	60	237		Σ:	**63**	[63]	272	4/0.66	6.00	11	12	72
										5/0.66	7.50	7	8	40
											Σ:	**55**	60	239

Tables 3, 4, 5. Analytical mean speed distribution sub-tables

v_μ = **14** [v_{min} = 8 – v_{max} = 20]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
2/0.20	10	14	15	150	4/0.50	8	13	14	112	6/0.66	9.00	11	15	[99.0]
3/0.20	15	30	30	450	5/0.50	10	13	14	140	7/0.66	10.50	11	13	[115.5]
4/0.20	20	8	8	160	6/0.50	12	13	14	168	8/0.66	12.00	10	12	[144.0]
5/0.40	12.5	33	35	440	7/0.50	14	15	16	224	9/0.66	13.50	10	12	[162.0]
7/0.40	17.5	12	12	210	8/0.50	16	13	14	224	10/0.66	15.00	10	12	[180.0]
	Σ:	**97**	100	1410	9/0.50	18	13	14	252	11/0.66	16.50	10	12	[198.0]
					10/0.50	20	13	14	280	12/0.66	18.00	10	12	[216.0]
						Σ:	**93**	100	1400	13/0.66	19.50	10	12	[234.0]
											Σ	**82**	100	[1348.5]

Tables 6, 7, 8. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **26** [v_{min} = 20 – v_{max} = 32]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
4/0.20	20	20	[20]	400	5/0.25	20	19	20	[400]	7/0.33	21	16	19	400
5/0.20	25	27	[27]	675	6/0.25	24	19	20	[480]	8/0.33	24	18	21	504
6/0.20	30	33	[33]	990	7/0.25	28	19	20	[560]	9/0.33	27	13	10	432
	Σ:	**80**	[80]	2065	8/0.25	32	19	20	[640]	10/0.33	30	11	13	390
						Σ:	**76**	80	[2080]	11/0.33	33	9	11	363
											Σ:	**67**	74	2089

Tables 9, 10, 11. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **38** [v_{min} = 32 – v_{max} = 44]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
7/0.20	35	38	36	1260	9/0.25	36	30	30	1080	12/0.33	36	30	[30]	1080
8/0.20	40	18	16	640	10/0.25	40	21	20	800	13/0.33	39	20	[20]	780
9/0.20	45	9	8	360	11/0.25	44	11	10	440	14/0.33	42	5	[5]	420
	Σ:	**65**	60	2260		Σ:	**62**	60	2320		Σ:	**55**	[55]	2280

Tables 12, 13, 14. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **50** [v_{min} = 44 – v_{max} = 56]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	V		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
9/0.20	45	15	[15]	[675]	11/0.25	44	11	[11]	[484]	15/0.33	45	11	[11]	[495]
10/0.20	50	15	[15]	[750]	12/0.25	48	11	[11]	[528]	16/0.33	48	9	[9]	[432]
11/0.20	55	14	[14]	[770]	13/0.25	52	11	[11]	[572]	17/0.33	51	9	[9]	[459]
	Σ:	**44**	[44]	[2195]	14/0.25	56	10	[10]	[616]	18/0.33	54	9	[9]	[486]
						Σ:	**43**	[43]	[2200]		Σ:	**38**	[38]	[1872]

Tables 15, 16, 17. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **50** [v_{min} = 44 – v_{max} = 56]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
11/0.20	55	8	[7]	385	14/0.25	56	7	[7]	[392]	19/0.33	57	7	[7]	[399]
12/0.20	60	7	[7]	420	15/0.25	60	7	[7]	[420]	20/0.33	60	7	[7]	[420]
12/0.20	65	7	[7]	455	16/0.25	64	7	[7]	[448]	21/0.33	63	7	[7]	[441]
14/0.20	70	7	[7]	490	17/0.25	68	7	[7]	[476]	22/0.33	67	4	[4]	[268]
	Σ:	**29**	[29]	1750		Σ:	**28**	[49]	[1736]		Σ:	**25**	[25]	[1528]

Tables 18, 19, 20. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **74** [v_{min} = 56 – v_{max} = 80]														
α					B					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
14/0.20	70	7	[7]	490	17/0.25	68	5	[5]	[340]	23/0.33	70	10	[7]	[700]
15/0.20	75	6	[6]	450	18/0.25	72	4	[4]	[288]	24/0.33	73	5	[7]	[365]
16/0.20	80	5	[5]	400	19/0.25	76	4	[4]	[304]	25/0.33	76	–	–	–
	Σ:	**18**	[18]	1340	20/0.25	80	5	[5]	[400]	26/0.33	79	–	–	–
						Σ:	**18**	[18]	[1332]		Σ:	**15**	[15]	[1065]

Tables 21, 22, 23. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

$\bar{v}_\mu$ = **86** [v_{min} = 80 – v_{max} = 92]					$\bar{v}_\mu$ = **72**					$\bar{v}_\mu$ = **51**				
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
16/0.20	80	4	[4]	[320]	17/0.25	68	3	[3]	[204]	16/0.33	48	3	[3]	[148]
17/0.20	85	3	[3]	[255]	18/0.25	72	3	[3]	[216]	17/0.33	51	3	[3]	[153]
18/0.20	90	3	[3]	[270]	19/0.25	76	3	[3]	[228]	18/0.33	54	2	[2]	[108]
	Σ:	**10**	[10]	[845]		Σ:	**9**	[9]	[648]		Σ:	**8**	[8]	[409]

Tables 24, 25, 26. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ). Although this table deals with average speed mean value $\bar{v}_\mu$ specified for each rhythm type, Numb. instr.(v_μ) values are as defined in the main table.

v_μ = **98** [v_{min} = 92 – v_{max} = 104]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
18/0.20	90	2	[2]	180	17/0.25	68	2	[2]	[136]	16/0.33	48	1	[1]	[48]
19/0.20	95	1	[1]	95	18/0.25	72	2	[2]	[144]	17/0.33	51	1	[1]	[51]
20/0.20	100	1	[1]	100	19/0.25	76	1	[1]	[76]	18/0.33	54	1	[1]	[54]
21/0.20	105	1	[1]	105		Σ:	**5**	[5]	[356]	19/0.33	57	1	[1]	[57]
	Σ:	**5**	[5]	480							Σ:	**4**	[4]	[210]

Tables 27, 28, 29. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **110** [v_{min} = 104 – v_{max} = 116]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
20/0.20	100	1	[1]	[100]	17/0.25	68	1	[1]	[68]	17/0.33	51	1	[1]	[51]
21/0.20	105	1	[1]	[105]	18/0.25	72	1	[1]	[72]	18/0.33	54	1	[1]	[54]
	Σ:	**2**	[2]	[200]		Σ:	**2**	[2]	[140]		Σ:	**2**	[2]	[105]

Tables 30, 31, 32. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

v_μ = **110** [v_{min} = 104 – v_{max} = 116]														
α					β					γ				
v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.	v		Numb. instr.(v_μ)		h.t.
h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.		h.t./t.u.	h.t./m.	theor.	effect.	
21/0.20	105	1	[1]	[105]	19/0.25	76	1	[1]	[76]	18/0.33	54	1	[1]	[54]
	Σ:	**1**	[1]	[105]		Σ:	**1**	[1]	[76]		Σ:	**1**	[1]	[54]

Tables 33, 34, 35. Analytical mean speed distribution sub-tables (rhythm patterns α, β and γ)

Correlation

Let us have a closer view of high theorization level of xenakien reasoning. His interest in a consistent theoretical edifice becomes clearer when after having drawn up the originating in

[a][16] $f(v) = \frac{2v}{u^2} \cdot e^{-\frac{v^2}{u^2}}$ and

[b] $f(v) = \frac{2}{\alpha\sqrt{\pi}} \cdot e^{-\frac{v^2}{\alpha^2}}$

distribution tables, he correlates forty theoretical ψ values of v deriving from equation [a] with forty real x values from equation [b] in an appropriate table using the correlation coefficient[17] equation [Carnet #16, p. 39]:

$$r = \frac{\sum (x - \bar{x})(\psi - \bar{\psi})}{\sqrt{\sum (x - \bar{x})^2} \cdot \sqrt{\sum (\psi - \bar{\psi})^2}}$$

The factor differentiating equation [a] from [b] is 'energy level' a, that determines the proportional temperature of three-dimensional domains and, consequently, the fluctuation limits of molecular speeds, i.e. sound speeds. Correlation of [a] to [b] is examined via a detailed table (**Tabl. 36**) based on square mean speed values $u^2 = \overline{v^2}$ from [b], where x^2 (instead of $\overline{x^2}$) stands for $u^2 = \overline{v^2}$ (first column). 'Real'[18] mean speeds $\bar{v}$, symbolized by x appear in the second column. All 'theoretical' mean speeds ψ provided by [a] feature in the third column[19].

		réel a	théo.					
	x^2	x	ψ	$x - \bar{x}$	$\psi - \bar{\psi}$	$(x - \bar{x})\cdot(\psi - \bar{\psi})$	$(x - \bar{x})^2$	$(\psi - \bar{\psi})^2$
1	6	2.40	1.72	-17.80	-16.48	294.00	317	272
	36	6.00	1.80	-14.20	-16.40	233.00	202	270
	55	7.20	1.88	-13.00	-16.32	212.00	169	266
	52	7.20	2.00	-13.00	-16.20	211.00	169	262
5	54	7.38	2.14	-12.87	-16.00	207.00	166	258
…	…	…	…	…	…	…	…	…
10	137	11.70	8.00	-8.50	-10.20	87.00	72	100
…	…	…	…	…	…	…	…	…
20	337	18.35	16.00	-1.85	-2.20	4.00	3	5
…	…	…	…	…	…	…	…	…
30	625	25.00	27.50	+4.80	+9.30	45.00	23	86
…	…	…	…	…	…	…	…	…
40	3970	63.00	60.00	+42.80	+41.80	1790.00	1840	1750
	21814	808.13				7814.00	7137.00	8940
		$\bar{v}_x$ = 20.20	$\bar{v}_\psi$ = 18.20					

Table 36. Correlation of square mean speed values

Numbers herein are mostly approximated and often erroneous since Xenakis seems to be interested more in final sums than in partial precision. For that very reason, most of the entries are here omitted. Results are surprising, because $\bar{v}_\psi$ = 18.20 derives from the application of $\bar{v} = u \cdot \frac{\sqrt{\pi}}{2}$, where $\bar{v}$ stands for $\bar{v}_\psi$ and u for $\bar{v}_x \rightarrow \bar{v}_\psi = \bar{v}_x \cdot \frac{\sqrt{\pi}}{2}$. Since $\bar{v}_x$ is the grounds for further computation, then

$$\bar{v}_\psi = 20.20 \cdot \frac{\sqrt{\pi}}{2} = 20.20 \cdot 0.886 \rightarrow \bar{v}_\psi \sim 18.20.$$

Indirectly mentioned in [Carnet #16, p. 20][20] during the study of $\bar{v}$ through $f(v) = \frac{2v}{u^2} \cdot e^{-\frac{v^2}{u^2}}$, this tricky shortcut simplifies computation by supplanting all operations related to $f(v) = \frac{2v}{u^2} \cdot e^{-\frac{v^2}{u^2}}$.

Then: $r = \dfrac{\sum (x-\bar{x})(\psi-\bar{\psi})}{\sqrt{\sum (x-\bar{x})^2}\cdot\sqrt{\sum (\psi-\bar{\psi})^2}} = \dfrac{7814}{\sqrt{7137}\cdot\sqrt{8940}} = \dfrac{7814}{7985.25} = 0.97855 \rightarrow \boxed{r \sim +0.98}$

"Correlation coefficient $r = +\ 0.98$ lets me presume that both distributions are equivalent and make use of $f(v) = \dfrac{2}{\sqrt{\pi}} \cdot e^{-\lambda^2}$ $(v/\alpha = \lambda)$ [21] in the place of $f(v) = \dfrac{2v}{u^2} \cdot e^{-\frac{v^2}{u^2}}$, that is applied to bi-dimensional domains only." [Carnet #16, p.39] Thus,

'real' $u\ (u_x)$ is: $u_x^2 = \dfrac{\sum_1^{40} \overline{x^2}}{40} = \dfrac{21814}{40} \rightarrow u_x = \sqrt{545.36} \rightarrow u_x = 23.3$,

'theoretical' $u\ (u_\psi)$ is: $u_\psi = \dfrac{\sqrt{\pi}}{\sqrt{2}} \cdot \bar{v}_x = 1.253 \cdot 20.20 \rightarrow u_\psi = 25.3$,

$u_x < u_\psi$

Remark: $u_x/u_\psi = 23.3/25.3 = 0.92$ and $r \sim 0.98 \rightarrow u_x/u_\psi \sim r$

Assigning speeds – Mean speed and average 'temperature' designation

Assigning speeds to strings presupposes that none of them goes beyond the actual possibilities of each instrumental group (Vln, Vla, Vc, Cb). Xenakis took care of that technicality by defining 'feasible maximum speed' (v_{max}): "Vln: $31/x$, Vla: $33/x$, Vc: $31/x$, Cb: $36/x$, where x = 0.20, 0.25, 0.33 etc. (all chords)"[22]. This seems to be a rather queer statement if a glissando of 31 half-tones per time unit is meant to be executed on every single string, because even a 24 half-tone glissando on G or E string of a violin is hardly possible[23]. Nevertheless, via the introduction of 'average maximum speed' v_m, where $v_m = [(31+33+31+36)/4]/\ x \rightarrow v_m = 32.5/x$ and given that minimum speed is equal to $1/x$, we obtain the average speed v for each single string of bowed instruments: $v = [(32.5/x)-(1/x)]/2 = [31.5/x]/2 = 15.75/x \rightarrow v_m \sim 16/x$
Actually, 16 half-tone glissandos lie within average executing skills. This fact is not neglected by the composer, as proved by some speed graphic tables, dated 4-3-1956 in [Archives I.X.-BnF/Pithoprakta/Dossier 1/13, Fol. 0024] and mainly by the ones in [Carnet #16, p. 45] (**Tabl. 37-39**). Actually, before assigning speeds to individual instruments, Xenakis has constructed 3 transitional tables [Carnet #16, p. 45] , one per rhythm type, which are bearing on to data figuring in analytical speed tables stated before. Shadowed columns represent the theoretically expected numbers of participating instruments. Dots stand for participating (still virtual) instruments. We should keep in mind that there are no instruments assigned to speeds exceeding 14 half-tones per designated time unit[24]. These tables, holding dots instead of instruments, have been created from 27 to 29/6/1956. Three additional related tables with full instrumentation are not dated at all; they might have been constructed during or, more likely, after that period. They will be presented and commented at the end of the paragraph.

ɑ speeds

	18	15	13	10	8	5	2	0	5	10	15	20	25	30	35	40	45	50						
ν	2.5	5	7.5	10	12.5	15	17.5	20	25	30	35	40	45	50	55	60	65	70						
h.t/ t.u.	1/ 0.4	1/ 0.2	3/ 0.4	2/ 0.2	5/ 0.4	3/ 0.2	7/ 0.4	4/ 0.2	5/ 0.2	6/ 0.2	7/ 0.2	8/ 0.2	9/ 0.2	10/ 0.2	11/ 0.2	12/ 0.2	13/ 0.2	14/ 0.2	15/ 0.2	16/ 0.2	17/ 0.2	18/ 0.2	19/ 0.2	20/ 0.2	21/ 0.2
1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
2	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
3	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
4	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
5	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
6	·	·	·	·		·	·	·	·	·	·	·	·	·	·	·		·						
7	·	·	·	·		·	·	·	·	·	·	·	·	·	·	·		·						
8	·	·	·	·		·	·	·	·	·	·	·	·	·	·	·		·						
9	·	·	·	·		·	·	·	·	·	·	·	·	·	·									
10	·	·	·	·		·	·	·	·	·	·	·	·	·	·									
11	·	·	·	·		·		·	·	·	·	·	·	·										
12	·	·	·	·		·		·	·	·	·	·	·	·										
13	·	·	·	·		·		·	·	·	·	·	·	·										
14	·	·		·		·		·	·	·	·	·	·	·										
15		·		·		·		·	·	·	·	·	·	·										
16		·		·		·		·	·	·	·	·	·	·										
17		·		·		·		·	·	·	·	·	·	·										
18		·		·		·		·	·	·	·	·	·	·										
19		·		·		·		·	·	·	·	·	·	·										
20		·		·		·		·	·	·	·	·	·	·										
21		·		·		·		·	·	·	·	·												
22		·		·		·		·	·	·	·	·												
23		·		·		·		·	·	·	·	·												
24		·		·		·		·	·	·	·	·												
25		·		·		·		·	·	·	·													
26		·		·		·		·	·	·	·													
27		·		·		·		·	·	·	·													
28		·		·		·		·	·	·	·													
29		·		·		·		·	·	·														
30		·		·		·		·	·	·														
31		·		·		·		·	·															
32		·		·		·		·	·															
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34		·		·		·			·															
35		·		·		·			·															
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Table 37. Speeds of ɑ rhythm type – theoretically expected numbers of participating instruments

β speeds

	18	16	14	12	10	8	6	4	2	0	4	8	12	16	20	24	28	32	36	40	44				
v	2	4	6	8	10	12	14	16	18	20	24	28	32	36	40	44	48	52	56	60	64	68	72		
h.t/ t.u.	1/ 0.5	1/ 0.25	3/ 0.5	2/ 0.25	5/ 0.5	3/ 0.25	7/ 0.5	4/ 0.25	9/ 0.5	5/ 0.25	6/ 0.25	7/ 0.25	8/	9/	10/	11/	12/	13/	14/	15/	16/	17/	18/	19/	20/
1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
2	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·						
3	·	·	·	·	·	·	·	·		·	·	·	·	·	·	·	·	·	·						
4	·	·	·	·	·	·	·	·		·	·	·	·	·	·	·	·	·	·						
5	·	·	·	·	·	·	·	·		·	·	·	·	·	·	·	·	·	·						
6	·	·	·	·	·	·	·	·		·	·	·	·	·	·	·	·	·	·						
7	·	·	·	·	·	·	·	·		·	·	·	·	·	·	·	·	·	·						
8		·	·	·		·	·	·		·	·	·	·	·	·	·	·		·						
9		·	·	·		·	·	·		·	·	·	·	·	·	·	·		·						
10		·	·	·		·		·		·	·	·	·	·	·	·			·						
11		·		·		·		·		·	·	·	·	·	·	·			·						
12		·		·		·		·		·	·	·	·	·	·										
13		·		·		·		·		·	·	·	·	·	·										
14		·		·		·		·		·	·	·	·	·	·										
15		·		·		·		·		·	·	·	·	·	·										
16		·		·		·		·		·	·	·	·		·										
17		·		·		·		·		·	·	·	·		·										
18		·		·		·		·		·	·	·	·		·										
19		·		·		·		·		·	·	·	·		·										
20		·		·		·		·		·	·	·	·		·										
21		·		·		·		·		·	·	·	·		·										
22		·		·		·		·		·	·	·	·												
23		·		·		·		·		·	·	·	·												
24		·		·		·		·		·	·	·	·												
25		·		·		·		·		·	·	·													
26		·		·		·		·		·	·	·													
27		·		·		·		·		·	·	·													
28				·		·		·		·	·	·													
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Table 38. Speeds of β rhythm type – theoretically expected numbers of participating instruments

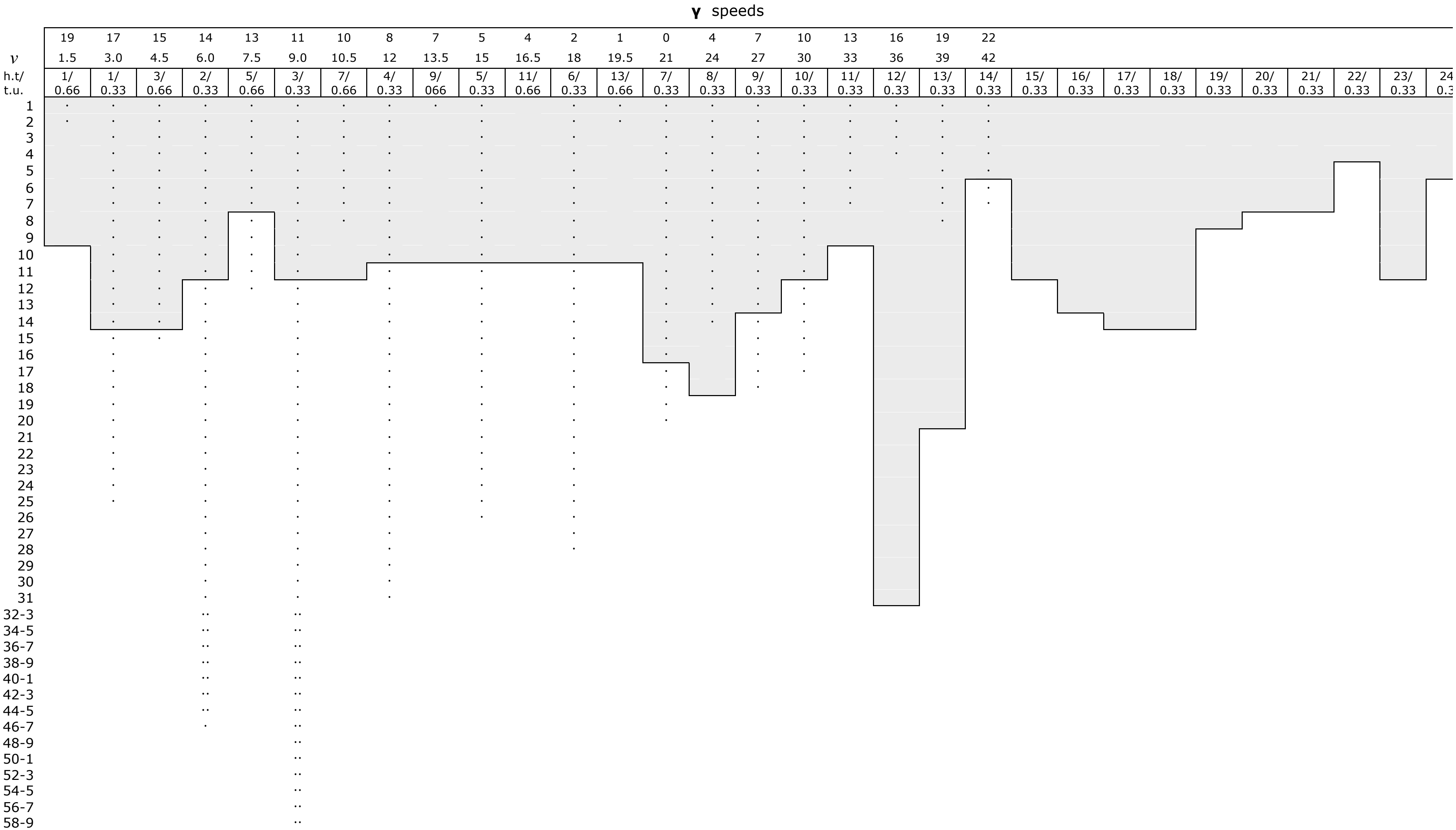

Table 39. Speeds of γ rhythm type – theoretically expected numbers of participating instruments

Essential numerical data from α, β and γ rhythm type speed tables are here collected and summarized in three new tables (**Tabl. 40, 41, 42**), as follows. Let us note that since "speed means number of half-tones per time unit", the product of a given speed (2nd column) by the number of instruments executing it (1st column) gives the total of slid half-tones (3rd column).

α data

Instr.	v	h.t.
16	2.5	40
60	5.0	300
13	7.5	97
39	10.0	390
5	12.5	63
43	15.0	645
10	17.5	175
31	20.0	620
36	25.0	900
29	30.0	870
27	35.0	945
23	40.0	920
19	45.0	855
19	50.0	950
10	55.0	550
8	60.0	480
5	65.0	325
9	70.0	630
402		**9755**

β data

Instr.	v	h.t.
7	2.0	14
27	4.0	108
10	6.0	60
44	8.0	352
7	10.0	70
49	12.0	590
9	14.0	126
47	16.0	750
2	18.0	36
30	20.0	600
31	24.0	745
32	28.0	895
24	32.0	768
15	36.0	540
21	40.0	840
11	44.0	485
9	48.0	430
7	52.0	364
11	56.0	615
393		**8388**

γ data

Instr.	v	h.t.
2	1.5	3
25	3.0	75
15	4.5	67
42	6.0	250
59	7.5	530
12	9.0	90
8	10.5	84
31	12.0	373
1	13.5	13
26	15.0	370
28	15.5	505
2	18.0	39
20	19.5	420
14	21.0	336
18	24.0	485
17	27.0	510
7	30.0	257
4	33.0	160
8	36.0	310
7	39.0	293
346		**5210**

Tables 40, 41, 42. Essential α, β and γ rhythm type speed data

Total sum of real speeds $402_{(\alpha)}+393_{(\beta)}+346_{(\gamma)}$ = 1141 differs from 1168 theoretical and 1221 effective ones in the the anticipating analytical tables. Sums are used for calculation of the general mean speed ($\bar{v}$) and the general mean temperature ($\bar{a}$):

$$\left.\begin{array}{ll} \alpha\text{-rhythm data} \rightarrow & \bar{v}_\alpha = 9755/402 = 24.3 \\ & \alpha_\alpha = \sqrt{\pi} \cdot \bar{v}_\alpha = 1.77 \cdot 24{,}3 = 43.07 \\ \beta\text{-rhythm data} \rightarrow & \bar{v}_\beta = 8388/393 = 21.3 \\ & \alpha_\beta = \sqrt{\pi} \cdot \bar{v}_\beta = 1.77 \cdot 21.3 = 37.70 \\ \gamma\text{-rhythm data} \rightarrow & \bar{v}_\gamma = 5210/346 = 15.05 \sim 15.1 \\ & \alpha_\gamma = \sqrt{\pi} \cdot \bar{v}_\gamma = 1.77 \cdot 15.1 = 26.72 \end{array}\right\} \begin{array}{l} \bar{v} = \bar{v}_{(\alpha+\beta+\gamma)}/3 = 60.7/3 \rightarrow \\ \bar{a} = \alpha_{(\alpha+\beta+\gamma)}/3 = 107.5/3 \rightarrow \end{array} \boxed{\begin{array}{l} \bar{v} = 20.23 \\ \bar{a} = 35.8 \end{array}}$$

Temperature $\bar{a}$ = 35.8 is quite near to "temperature proportional to $\alpha = 35$" [Xenakis, 1963, 27]. Last step before the graphic score composition is the final distribution of speeds to instruments. It is about three detailed graphic tables (**Fig. 3**, **4**), that derive from "α speed", "β speed" and "γ speed" tables already mentioned. It becomes clear that during elaboration Xenakis has revised several data: speed limits are now exceeding 14 half-tones per time unit and, exempting β type, totals of instruments, $413_{(\alpha)}+393_{(\beta)}+245_{(\gamma)}$ = 1051, are also diversified as a result of column subtotals differentiation. The tables in question seem to be the end of theoretical reasoning and of pre-compositional applications as well.

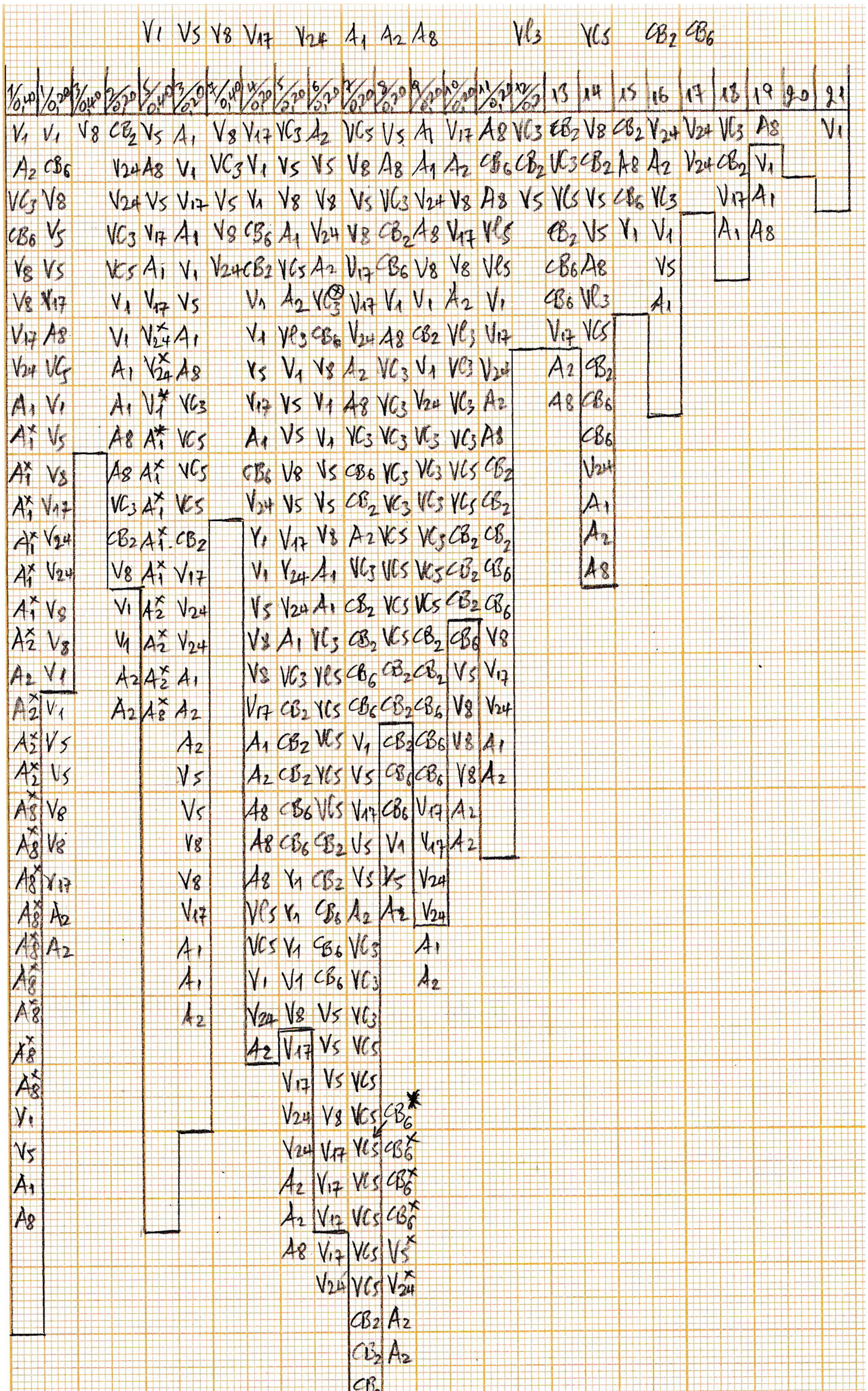

Figure 3: Attribution of α speeds to instruments [Archives I.X.-Dossier 1/13-Fol. 16] Hand copy

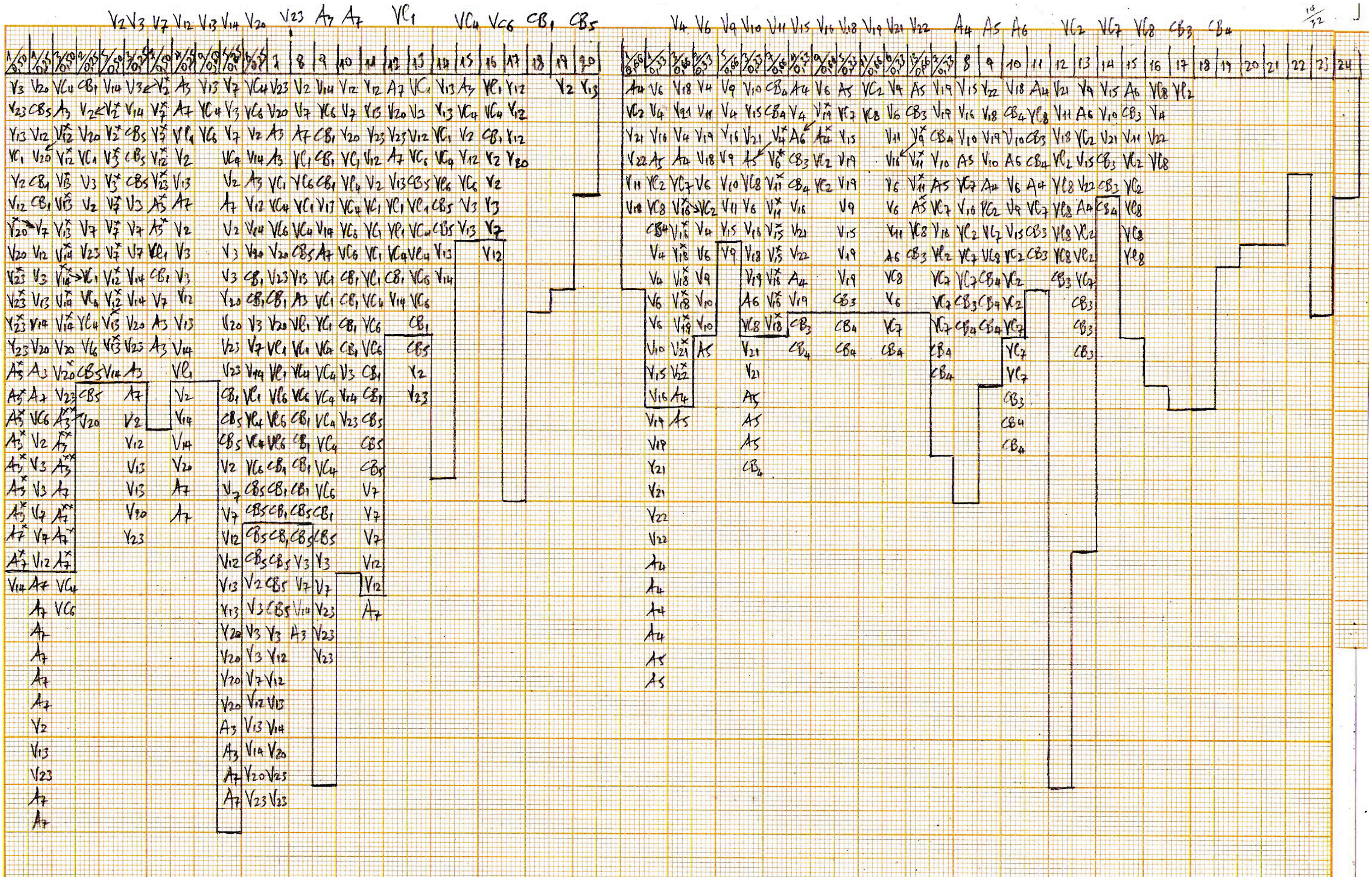

Figure 4. Attribution of β speeds (left) and γ speeds (right) to instruments [Archives I.X.-Dossier 1/13-Fol. 16] Hand copy

Synthesis - Conclusion

Our study on historical meas. 52-59 of *Pithoprakta* is attempting to clarify both postulated formalization principles and the way to their application in a pre-compositional level. In that intension, we are essentially dealing with the *scientific matrix* of the work [Antonopoulos, 2008, 52], i.e. more with theory than with the results of its application in practice. Our main source, Carnet #16 drafted by the composer for his own use, is a very demanding manuscript. If mathematical reasoning is hard to follow from the very first, arithmetical approximations and tricky shortcuts bring about additional difficulties and become a real obstacle in data cross-checking as well.

In the 1st part of *Pithoprakta*, meas. 0-51, intervals are not slid and, so, pitch can easily be dissociated from time. Probabilistic distribution of pitches ruled by *radium formula*[25] is a process independent from the distribution of differential durations of these very pitches, which is held according to a simpler version of the same formula[26].

In contrast, meas. 52-59 in the 2nd part of Pithoprakta, deal with moving sounds (glissandos). By definition, glissando is a sonic event during which a definite pitch is sliding gradually to another definite pitch. It is about an interval covered within a certain time. Non quantization of sound motion associates slid intervals with the time elapsed till they are covered. According to that condition space measured in half-tones and time measured in time-units cannot neither be computed nor distributed independently. Hence, an appropriate new formula combining space with time is needed[27]. *Gauss equation* seems to be the optimum solution.

The main challenge in xenakien theoretical reasoning can be summarized in a single question: is energy level determination compulsory or not?

Research on bi-dimensional domains, where temperature is not required, presupposes arbitrarily set speed limits. On the other hand, speed limits in three-dimensional domains result from temperature. So, should temperature be a subjectively set magnitude or not?

Once speed limits are imposed both by the orchestral range and the technical possibilities of the instruments, four different numbers of slid sounds in the plain, symbolized by u, can be designated through standard deviation. A similar operation in three dimensional spaces leads to four energy levels (temperatures), symbolized by a. A simple comparison of u values for plains to a values for volumes shows that they are curiously similar, especially when very quick and very slow speeds are concerned. Assimilating u for plains with a for volumes and after having constructed all speed distribution tables needed, Xenakis compares probability results through correlation coefficient r and finds out that differences are actually negligible, because $r \sim +1$. So, he makes his decision for movements in three dimensional domains. Temperature, although indirectly resulting from speed limits, is considered as a logically designated magnitude. Therefore, the scientific notion of *speed* 'deserves' to be introduced in music as a 'legitimate solution' through Maxwell-Boltzmann equation.

Gauss distributions constitute a powerful tool assuring moving sound mass control: "Controlling [sound] masses means being aware of mean values and normal deviations. Opposing masses to one another means to adopt different deviations and mean values as well. a) one can create evolving sets by defining mean values, b) by this definition, an exceptional event can be rigorously determined by its extremely small probability[28]."[29] [Carnet #16, p. 37v]. But is the mass in question, *Pithoprakta* meas. 52-59, fully controlled?

It is true that speed formalization exposed above yields control over speed limits, mean values, deviations, energy levels (temperature), event densities etc. However, since glissando directions are not predetermined –and therefore not formalized– nor the form of the mass neither its evolution in time can be formalized either. The result is that even if probabilistic determinism rules movements in the interior of the mass, the latter has no direct effect on the mass shape neither on its movement as a whole.

Xenakis's trajectory beginning with the conception of glissando speed theorization up to the last instrument distribution table, i.e. from first intuition up to the completion of theoretical applications just before the graphic score elaboration, can be divided in three stages, often overlapping each other:

(a) Premises – mathematical grounding – research on Gauss equations,
(b) Elaboration of speed theoretical values – possible speed distributions – correlation,
(c) Application of theoretical results – more realistic speed and instrument distributions.

Phases (a) and (b) constitute a part of the *scientific matrix* of the work, a kind of *theory composing*. Phase (c) is a transitory one bridging scientific matrix with *musical matrix* that begins practically with the graphic score composition. At this point, pre-compositional work can easily be confused with composition, because the composer's interventions based on his musical intentions are, more often than not, leading up to differentiations from theoretically expected results. That is, esthetical choices remain a prevailing and very influential factor.

If Xenakis's compositional practice (graphic scores) implies deviations, inevitable or intended, from strict application of predetermined rules, the same stands for theoretical practice when principles are used for mere theoretical purposes. The reason is that theoretical results are not always fully compatible with each-other or with musical reality. Probabilistic deviations, 'rearrangements' or variations, then, might be regarded as a natural side-effect of theoretical reasoning in general and a part of formalization procedures in particular. Systematic musicological research cannot neglect their impact, even when results are amended through corrected or improved data. In the case of *Pithoprakta* meas. 52-59, for instance, the basic implemented equation has changed twice, speed distribution development counts already three variations and, in the case of the aforementioned precompositional phase (c), instrumentation remains unstable and, mainly, not formalized.

Establishing a distance between theory and compositional practice, musicological analysis of contemporary works usually focuses on the results of theory applications or, inversely, on theoretical extrapolations out of music score material. By *composing theory*, Xenakis minimizes that distance and proves that theoretical reasoning and expansion of rules might be regarded as proportionally equivalent to composition itself.

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Notes

[1]Paul Lévy, Paris, Gauthier – Villars et C[ie], 1925, p. 175-177. α is the so called *Gauss parameter*.

[2] $\bar{v} = \alpha / \sqrt{\pi}$. See also *Dossier 1/11* / *Archives Iannis Xenakis – BnF / Oeuvres / Pithoprakta*, p. 7 (23-1-1956).

[3] $\sigma = \alpha / \sqrt{2}$
[4] Treatises mentioned in *Carnet #16*.
[5] I*nfra*: Correlation coefficient.
[6] However, he uses it only once.
[7] See also [Xenakis, 1976, 12] and *Archives I.X.–BnF/Oeuvres Musicales/Pithoprakta/Dossier* 1/11, p. 7.
[8] Treatise on Probability Theory mentioned by Xenakis in p. 28 of *Carnet #16*. An analytical table of λ and $\Theta(\lambda)$ corresponding values appears in p. 280-282. It is most likely the one that Xenakis used for further speed computation.
[9] Temperature = energy level
[10] 10 (in the manuscript) instead of 11. 11, however, is a more accurate approximation.
[11] Carnet #16, p.45v, 5-5-56: « Ταχύτητες πολύ αργές Κοσμογονία α=10 v=5.5 »
[12] Xenakis is using different colors for elements belonging to each rhythm pattern: red = α, green = β and blue = γ.
[13] Carnet #16, p. 33: «για να αποφύγω τη σήμανση του πρώτου χρόνου, μειώνω αναλογικά το πλήθος των οργάνων ώστε ο μέσος όρος να είναι 15»
[14] In this case, instrumental density is equivalent to sonic event density.
[15] [Xenakis, 1963, 30 / 1992, 15]: "1148 speeds have been calculated." In [Xenakis, 1976, 13] 1142 speeds are mentioned. In [Gibson, 2003, 19]: 1040 speeds are actually present in the score.
[16] Since they are not formally used, tables originating in (a) equation are not presented here.
[17] See also [Xenakis, 1963, 30].
[18] « *Réel α* » in the manuscript.
[19] « *théo*[rique] » in the manuscript.
[20] Also in Carnet #16, p. 9, where $W' = \frac{\sqrt{\pi}}{2} \cdot W$. Xenakis is here referring to [Filippi, 1947, 178].
[21] Cf. supra: $\Theta(\lambda)$, for $\lambda_2 < \lambda < \lambda_1$
[22] [Carnet #16, p. 39, 10-5-56]. In Greek: «Όλες οι χορδές» and then in French: « Toutes les cordes »
[23] A long non interrupted glissando should not exceed 16 to 19 half-tones, according to violinists.
[24] « Quand aux intervalles, limités par la longueur des cordes, ils varient entre 1 et 14 demi-tons. [...] trois intervalles (0, 17 et 19 demi-tons) se situent en dehors de cet ambitus. » [Gibson, 2003, 19]
[25] $P_x = \delta \cdot e^{-\delta x} \cdot m \cdot dx$
[26] $P_x = \delta \cdot e^{-\delta x} \cdot dx$
[27] The only operator in common with definite pitch distribution, but a secondary one, would be rhythm patterns α, β,γ and the resulting time units.
[28] He is more probably than not referring to Poisson equation.
[29] « Contrôler les masses veut dire connaître la moyenne et les écarts normaux. Opposer une à autre masse veut dire admettre des écarts et des moyennes différentes. a) C'est par la définition des moyennes que l'on peut créer l'évolution des ensembles b) C'est par cette définition que l'événement exceptionnel est défini rigoureusement par sa probabilité extrêmement faible. »