Mid-semester Exam

● Graded

Student

PALLAV GOYAL

Total Points

45.5 / 50 pts

Question 1

Question 1 — 4 / 4 pts

✔ **+ 4 pts** Correct

Question 2

Question 2 — 4 / 4 pts

✔ **+ 4 pts** Correct

Question 3

Question 3 — 4 / 4 pts

✔ **+ 4 pts** Correct

Question 4

Question 4 — 4 / 4 pts

✔ **+ 4 pts** Correct

Question 5

Question 5 — 5 / 5 pts

✔ **+ 5 pts** Correct

Question 6

Question 6 — 5 / 5 pts

✔ **+ 5 pts** Correct - The statement is false.

Question 7

Question 7

Resolved **4** / 5 pts

+ **4 pts** Question asked final states

Regrade Request **Submitted on: Oct 02**

I have indirectly also given the proof for no. of final states minimized along with minimization of total states as I have shown that inital state is accepting due to epsylem acceptance. Also I have mentioned that for strings 0 and 1 to be accepting. If we had to do in only 1 accepting states the 0 and 1 transitions would then fall onto the initial state. And then all strings have to be accepted.
Although I have not explicitly stated line containing the minimum no . of accepted states but my proof captures the fact that a single accepting state isn't possible and given a DFA for 2 accepting states

Need counterexample. +1 given

Reviewed on: Oct 04

Question 8

Question 8

4 / 4 pts

+ **4 pts** correct answer: sigma* - {a}

Question 9

Question 9

Resolved **6** / 8 pts

+ **1 pt** marks given for guessing start state

+ **5 pts** T should start simulating when the x string starts. In your construction, it starts simulating from the

Regrade Request **Submitted on: Oct 02**

As per the comment made while correction that T should start simulating when x string starts. This follows my construction as initially T is taken as phi (see the epsylem transition made to construct union) which is mentioned in the transition function for s_0. So before the string x starts T would be making phi to phi transitions. The T is changed to {T Union s} when their is a suitable y found which is suggested by string y reaching upto a final state in the original DFA.

firstly your proof is difficult to read because you have used "a" for alphabets as well as states! In the start you have used Q_1 and Q_2 as the states. There is no formal construction of the union of different NFAs. I know you are using this result from the class, but you need to state this clearly in one sentence that you are invoking the construction taught in the class, else one may think you are making a guess.

Reviewed on: Oct 05

Question 10

Question 10

Resolved **5.5 / 7 pts**

- ✔ **+ 1.5 pts** DFA Definition
- ✔ **+ 1.5 pts** DFA Transition
- ✔ **+ 2 pts** Proof of $\hat{\delta}([x], y) = [xy]$
- ✔ **+ 0.5 pts** (Only) One direction shown

Pls write the DFA definition formally.

Regrade Request **Submitted on: Oct 02**

I have given the definition of DFA as stated in the given comment could you please tell if something else is missing

Your argument shows $x \in L(M)$ implies $x \in R$. 0.5 marks is awarded for that. Proof of converse is missing.

Reviewed on: Oct 03

CS340 (2024) – Mid Semester Exam

Duration: 120 minutes, Total marks: 50, Pages: 10

- **Important note.** Answers without clear and concise explanations will not be graded.

Name: PALLAV GOYAL

Roll No: 220747

Problems

1. **(4 marks)** Let $Y = \{0000000, 1010101, 1101001, 0010110\}$ and let $G = (\{S, D, A\}, \{0, 1\}, P, S)$ be a CFG where P is as follows.

$$\begin{aligned} S &\rightarrow 0S0 \mid 1S1 \mid D \\ D &\rightarrow 1A0 \mid 0A1 \\ A &\rightarrow \varepsilon \mid 0A \mid 1A \end{aligned}$$

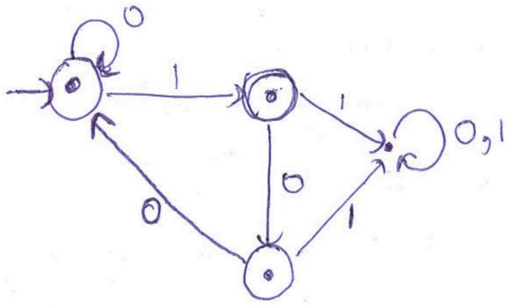
What is $Y \cap L(G)$?

$\{1101001, 0010110\}$

$S \rightarrow 1S1 \rightarrow 1D1 \rightarrow 11A01 \rightarrow 110A01 \rightarrow 1101A01 \rightarrow 11010A01 \rightarrow 1101001$

$S \rightarrow 0S0 \rightarrow 0D0 \rightarrow 00A10 \rightarrow 001A10 \rightarrow 0010A10 \rightarrow 00101A10 \rightarrow 0010110$

2. **(4 marks)** Let $\Sigma = \{0, 1\}$ and let $\gamma = 0^*(1 + 000^*)^*0^*$ be a regular expression. Construct a DFA M with at most 4 states such that $L(\gamma) = L(M)$. Give a brief and precise justification for your answer.



The regu exp $0^* (1 + 00\overset{*}{0}) 0^*$

captures all strings ~~with~~ except the ones which have atleast one pairs of 1, 1 which are seperated by less than 2 0's

Roll No. 220747

3. **(4 marks)** Let $\Sigma = \{0, 1\}$ and let M be the automaton given in Figure 1. Give the minimal DFA M' such that $L(M) = L(M')$. Label the states of M' appropriately to indicate the states of M that are being equated.

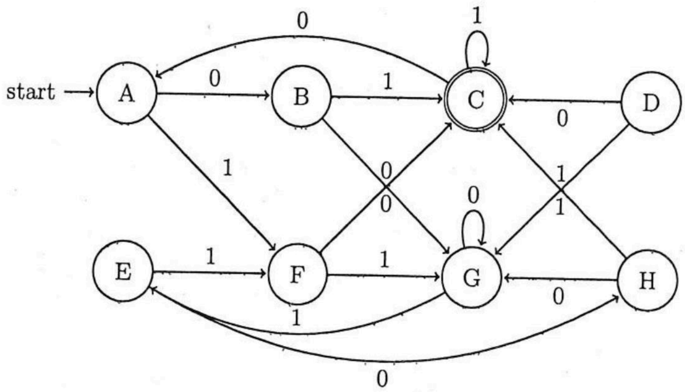
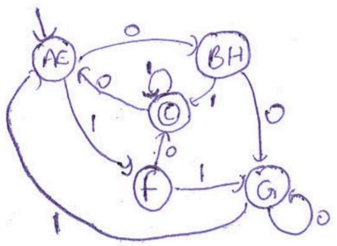


Figure 1: Automaton M

D in accessible

A
✓ B
✓ ✓ C
✓ ✓ ✓ D
— ✓ ✓ ✓ E
✓ ✓ ✓ ✓ ✓ F
✓ ✓ ✓ ✓ ✓ ✓ G
✓ — ✓ ✓ ✓ ✓ ✓ H



4. **(4 marks)** Let $\Sigma = \{a, b\}$ and let N be the NFA given in Figure 2. Using the subset construction, construct a DFA M such that $L(M) = L(N)$.

It suffices to draw a diagram of M following the standard conventions that we discussed in class. Label each state of M appropriately as the corresponding subset. You do not have to draw states which are unreachable from the initial state.

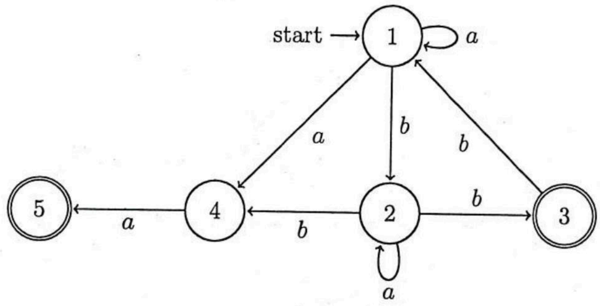
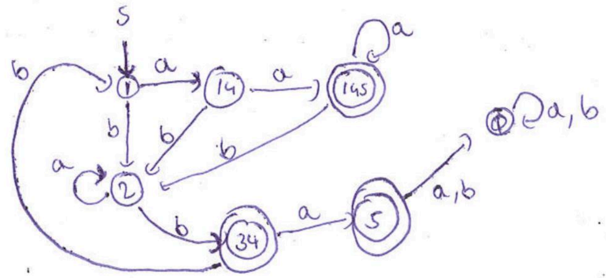


Figure 2: Automaton N



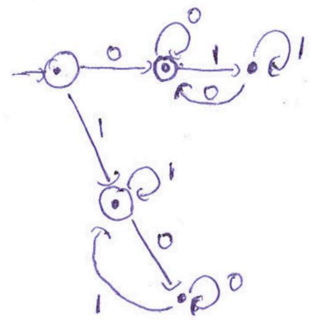
$145 = \{1,4,5\}$ $\quad$ $34 = \{3,4\}$ $\quad$ $14 = \{1,4\}$

$F = \{145, 34, 5\}$

Roll No. 220747

5. **(5 marks)** Let $\Sigma = \{0, 1\}$. For $x, y \in \Sigma^*$, let $\#_y(x)$ denote the number of occurrence of the string y as a substring in the string x. Let $A = \{x \in \Sigma^* \mid \#_{01}(x) = \#_{10}(x)\}$. Is A regular? If "Yes", construct a DFA M such that $L(M) = A$. If "No" prove using Pumping Lemma that A is not regular.

Yes



For regulkor it should either start and end at 0

OR start and end at 1

OR be null string

6. **(5 marks)** Let $h : \Sigma^* \to \Gamma^*$ be a homomorphism and let $A \subseteq \Sigma^*$. Is the following statement true? Precisely justify your answer.

Statement. If $h(A)$ is regular then A is regular.

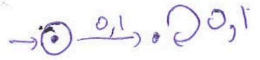
The statement is false

Let $\Sigma = \{0, 1\}$ $\Gamma = \{a, b\}$

$A = 0^n 1^n$ is not regular

Let $h(0) = \varepsilon$, $h(1) = \varepsilon$

$h(A) = \{\varepsilon\}$ is regular with DFA



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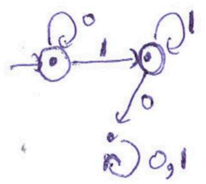
7. **(5 marks)** Let $\Sigma = \{0,1\}$ and $\gamma = 0^*1^*$ be a regular expression. What is the minimum number of final states required for any DFA M such that $L(M) = L(\gamma)$. Give a precise proof justifying your answer.

M =



minimum 3 states required

Let us assume contradictory that there is a DFA with 2 states. Initial state s has to be accepting because ϵ lies in $L(\gamma)$.

If both states are accepting then $\delta(s,x) \in Q = F$ will accept all strings and some strings not in $L(\gamma)$ will get accepted. Hence not possible.

If other state is not accepting

$\delta(s,0) \in F$ as $0 \in L(\gamma)$. Hence $\delta(s,0) = s$

$\delta(s,1) \in F$ as $1 \in L(\gamma)$. Hence $\delta(s,1) = s$.

Hence all string will get accepted which leads to contradiction

8. **(4 marks)** Let $\Sigma = \{a\}$. Let $A = \{a^{2^i} \mid i \geq 1\}$ and $B = \{a^{3^j} \mid j \geq 1\}$. Is $(A \cup B)^*$ regular? Clearly justify your answer.

~~Not~~ Regular

~~For $(A \cup B)^*$ to be regular it should accept each string a^n where n is a whole number.~~

The string a does not lie in $(A \cup B)^*$

The string ε lies in $(A \cup B)^*$

Strings a^2, a^3 are in A and B respectively thus also lie in $(A \cup B)^*$

For any ~~ad~~ other string a^n, $n \geq 3$ is ~~are~~ in $(A \cup B)^*$ because

Case ① n is even it can be written in binary representation wit ...1010...0 with 0 at end so it can be written in powers of 2 greater than 2^0,

$$n = 2^{k_1} + 2^{k_2} \dots \qquad k_1, k_2 > 0$$

Case ② n is odd, one a^3 can be used to convert into even a^n where n is odd $a^n = a^3 a^{n-3}$ where $n-3$ is even

$$(A \cup B)^* = \{a^n \mid n \geq 0, n \neq 1\}$$

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9. **(8 marks)** For $A \subseteq \Sigma^*$, let $Op(A) = \{yx \mid xy \in A\}$. Suppose $A \subseteq \Sigma^*$ is an arbitrary regular set, is $Op(A)$ a regular set? If "Yes" construct an automaton N (DFA or NFA) such that $L(N) = Op(A)$ and provide a justification. If "No" then define a set A and give a proof using Pumping Lemma.

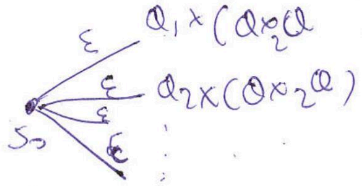
Yes regular

Let DFA of A have (Q, s, F, δ)

$Q' = (Q \times Q \times 2^Q) \cup (s_0)$

$s' = s_0$

It is union of $|Q|$ types of NFA. Each NFA is such that



Union of $|Q|$ NFA

The idea is to guess the $\delta(x, a)$ initially using ε. Then whenever second state reaches $\in F$ we say that we have got a possible y s.t $\hat{\delta}(s, xy)$ is accepted. Thus we add s to set T. If the set contains the guessed state initially. We say we got the yx

$\Delta(s_0, \varepsilon) = \{ \{Q_1, Q_1, \phi\}$ if $Q_1 \notin F$

$\{Q_1, Q_1, \{s\}\}$ if $Q_1 \in F$

$\}$

$\Delta(\{Q_1, Q_2, T\}, a) = \quad Q_1 \xrightarrow{\Delta} Q_1$

$T \in 2^Q \qquad Q_2 \xrightarrow{\Delta} \delta(Q_2, a)$

$T = \{a_1, a_2 \ldots\} \quad a_1, a_2 \in Q$

$T \to T' \quad \{\delta(a, a) \ a \in T\}$

$T' = T' \cup \{s\}$ if $\delta(Q_2, a) \in F$

Basically we ~~add~~ do same transition as original DFA for second and third terms except that when second term ~~reach~~ $\in F$ we add s to the set T. which actually means we have got a candidate y.

The accepting states will be for each NFA $\{Q_1 \times \{Q \times 2^Q\}\}$ as all states where $Q_1 \in 2^Q$

~~Basically we~~

$F = \{(Q_1, Q_2, T) \ \forall \ Q_1, Q_2$ s.t $Q_1 \in T$

10. **(7 marks)** Let $R \subseteq \Sigma^*$ and $\equiv$ be a Myhill-Nerode relation for R. Prove that we can construct a DFA $M_\equiv$ such that $L(M_\equiv) = R$.

The $\equiv$ relation is an Myhill-Nerode relation hence equivalence relation.

The equivalence relation divides Σ^* into finite ~~e~~ equivalence class.

The states of DFA Q are the equivalence classes generated by $\equiv$.

The starting state s is $[\varepsilon]$

The accepting stae f is set $\{[x] \mid x \in R\}$

The transition function $\delta([x], a) = [xa]$

Claim 1 The transition function is well defined.

By right congruence property of Myhill Nerode relation

Hence for $x \equiv y \Rightarrow xa \equiv ya$ & hence $\delta([x], a) = \delta([y], a) = [xa] = [ya]$

Claim 2. For each $x \in R$, $x \in L(M)$

~~By definition of acceptance F is $\{[x] \mid x \in R\}$,~~

to prove ~~Hence for each~~ $\hat{\delta}([\varepsilon], x) = [x]$. By induction on size of x

Base case: $\hat{\delta}([\varepsilon], \varepsilon) = [\varepsilon]$

$x = ya$

$\hat{\delta}([\varepsilon], ya) = \delta(\hat{\delta}([\varepsilon], y), a)$

$= \delta([y], a)$ I.H

$= [ya] = [x]$ Def of δ

Hence each $x \in R$, $[x] \in F$ and is accepted

claims 3 for each $x \notin R$, $x \notin L(M)$

Assume contradicotry $x \in L(M)$. By definition of F it leads to contradiction $F = \{[x] \mid x \in R\}$

Hence we found $L(M_\equiv) = R$

AC

BC

CD

CE

CF

CG

CH

AD

AF

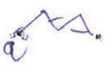
BG

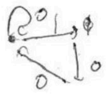
AB

$x\,y$

$s\,x$

$\delta(s,x) = a$

a  F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ϕ
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0