Quiz 1

Graded

Student

PALLAV GOYAL

Total Points

20 / 20 pts

Question 1

Question 1

4 / 4 pts

✔ **+ 4 pts** correct

Question 2

Question 2

3 / 3 pts

✔ **+ 3 pts** Correct

Question 3

Question 3

5 / 5 pts

✔ **+ 5 pts** Correct

the dfa states are not clearly visible and also write the transition clearly

Question 4

Question 4

3 / 3 pts

✔ **+ 3 pts** Correct

Question 5

Question 5

5 / 5 pts

✔ **+ 5 pts** Correct

CS340 (2024) – Quiz 1

Duration: 40 minutes, Total marks: 20, Pages: 5.

- **Important note.** Answers without clear and concise explanations will not be graded.

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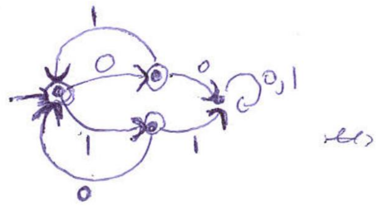
Problems

1. **(4 marks)** Let $\Sigma = \{0, 1\}$. Construct a DFA M with at most 4 states where $L(M)$ is the set of all strings $x \in \Sigma^*$ such that in every prefix of x, the number of 0s and 1s differ by at most 1. Give a brief and precise justification for your answer.

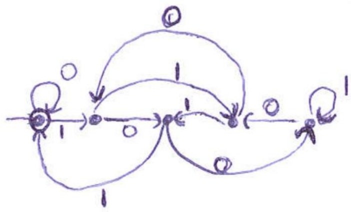


It consists of 4 states which are reprenting the ~~abs di~~ number of 0's – number of 1s upto that state.
Initially the difference is 0. ~~In case of~~
Two other accepting states are
1) The number of 0's – 1's is 1
2) " ———————————— –1

In case of the last state the value difference of the prefix becomes >1

220747

2. **(3 marks)** Let $\Sigma = \{0, 1\}$ and let $A = \{x \in \Sigma^* \mid x \text{ represents a multiple of five in binary}\}$. Note that leading zeros are permitted and ϵ represents the number 0. Construct a DFA M with at most 5 states such that $L(M) = A$. Give a brief and precise justification for your answer.



The 5 states represent the modulo of the current binary string with 5.

Intial state has modulo 0 which is the only accepting state

For any string in general arrival of a new 0 means that the modulo value is multiplied by 2. $\delta(q_e, 0) = (2a)$ modulo 5

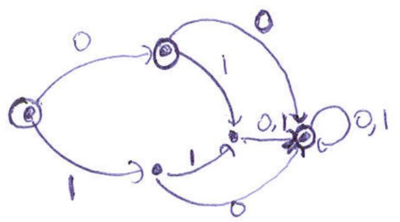
Arrival of 1 means, $\delta(a, 1) = (2a+1)$ modulo 5

~~With the~~

~~The~~ other ~~repres~~ states represent modulo 1,2,3,4

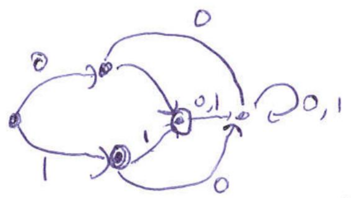
Roll No. 220747

3. **(5 marks)** Let $\Sigma = \{0, 1\}$ and let $A = \{1, 01, 11\}$. Construct a DFA M with at most 5 states such that $L(M) = \overline{A}$ (complement of A). Give a brief and precise justification for your answer.



The above DFA is achieved by first contructing the DFA for A. and then interchanging the set of accepted and rejected states.

For A, DFA is



. Hence reversing we obtain $L(M)$

220747

4. **(3 marks)** Let $\Sigma = \{0, 1\}$. Let M_1 be the automaton given in Figure 1 and M_2 be the automaton given in Figure 2.

Question. Is $L(M_1) \cap L(M_2) = \emptyset$? If "Yes", give a precise justification. If "No", give a string $x \in \Sigma^*$ such that $x \in L(M_1) \cap L(M_2)$ and explain.

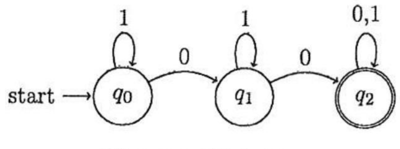


Figure 1: DFA M_1

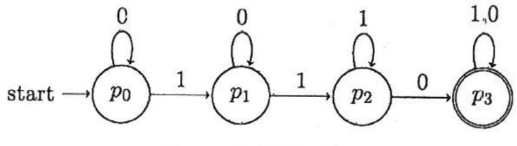


Figure 2: DFA M_2

No,

$x=1010$ ~~gets~~ is in language of both M_1 & M_2

the transition for 1010 is

M_1 $a_0 \xrightarrow{1} a_0 \xrightarrow{0} a_1 \xrightarrow{1} a_1 \xrightarrow{0} a_2$ Accepted

M_2 $P_0 \xrightarrow{1} P_1 \xrightarrow{0} P_1 \xrightarrow{1} P_2 \xrightarrow{0} P_3$ Accepted

M_1 has atleast 2 0's

M_2 has atleast 2 1's before 10

Roll no. 220747

5. **(5 marks)** Let $\Sigma = \{0, 1\}$. Let $M = (Q, \Sigma, \delta, s, F)$ be a DFA such that $\epsilon \notin L(M)$. Let $A_M = \{x \in \Sigma^* \mid$ there is no $y \in L(M)$ such that y is a prefix of $x\}$. Construct a DFA $M' = (Q', \Sigma, \delta', s', F')$ where $|Q'| \leq |Q| + 1$ such that $L(M') = A_M$. (Note: for a finite set X, we denote by $|X|$ the number of elements in X). Give a brief and precise justification for your answer.

$Q' = Q \cup \{Q_0\}$

$s' = s$

$$\delta'(q, a) = \begin{cases} \delta(q, a) & q \notin F, q \neq Q_0 \\ Q_0 & q \in F \\ Q_0 & q = Q_0 \end{cases}$$

$F' = \{x \mid x \in Q \text{ but } x \notin F\}$

The ~~A~~ state Q_0 has been artificially created to ~~p~~ depict that the achieved state already has a ~~ptd~~ pretix in M. Also the F states of M should not be accepted. The F states and Q_0 ~~in~~ in M' will not be accepted.

F' will consist of all states in Q not in F

Rough Work

$-1 \quad 01 \quad 11$