MATH 8050 HW 2
Solution

① R code on Blackboard

② $Q(\beta_0, \beta_1) = \Sigma(y_i - \beta_0 - \beta_1 x_i)^2$

$$\frac{\partial Q}{\partial \beta_0} = -2\Sigma(y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\Rightarrow \Sigma y_i - n\beta_0 - \beta_1 \Sigma x_i = 0$$

$$\Rightarrow n\beta_0 = \Sigma y_i - \beta_1 \Sigma x_i$$

$$\Rightarrow \beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$\frac{\partial Q}{\partial \beta_1} = -2\Sigma(y_i - \beta_0 - \beta_1 x_i)x_i = 0$$

$$\Rightarrow \Sigma x_i y_i - \beta_0 \Sigma x_i - \beta_1 \Sigma x_i^2 = 0$$

$$\Rightarrow \Sigma x_i y_i - (\bar{y} - \beta_1 \bar{x}) n\bar{x} - \beta_1 \Sigma x_i^2 = 0$$

$$\Rightarrow \Sigma x_i y_i - n\bar{x}\bar{y} = \beta_1(\Sigma x_i^2 - n\bar{x}^2)$$

$$\Rightarrow \hat{\beta}_1 = \frac{\Sigma x_i y_i - n\bar{x}\bar{y}}{\Sigma x_i^2 - n\bar{x}^2} = \frac{S_{xy}}{S_{xx}}$$

$$\Rightarrow \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

(3) This is incorrect. The error term ε_i is random (and unobserved), which makes Y random also. Since we assume that $E(\varepsilon_i) = 0$, we have

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$
$$\Rightarrow E(Y_i) = \beta_0 + \beta_1 x_i$$

There is no error term in the expectation.

(4)

(a) Yes. The 95% C.I. about $\hat{\beta}_1$ does not contain zero, so if we were to test $H_0: \beta_1 = 0$ vs. $H_1: \beta_1 \neq 0$ at the $1 - .95 = .05$ level, we would reject H_0. This means we have significant evidence to suggest that Y is linearly related to x.

(b)

Since the firm (hopefully) does not have any marketing districts of population size 0, this value is outside the relevant range of the data. The intercept has no useful stand-alone meaning here. Also, since the C.I. contains zero, we have no reason to think $\beta_0 \neq 0$.

(5)

The negative slope estimate probably occurred due to simple chance variability. The large p-value tells us that we have no evidence to suggest that $\beta_1 \neq 0$. Thus, it is more accurate to say that we have no reason, based on the observed data, to believe advertising expenditures and sales are related at all.