MATH 8050 HW1

Due Wednesday, August 31

Show all work. You may prepare either hand-written or typed solutions, but please make sure that they are complete and legible. Incomplete solutions or answers that cannot be read will be given no credit.

1. A large paint retailer has had numerous complaints from customers about underfilled paint cans. As a result, the retailer has begun inspecting incoming shipments of paint from suppliers. Shipments with underfill problems will be returned to the supplier. A recent shipment contained 2,440 gallon-size cans. The retailer randomly selected 50 cans and weighed each on a scale. The average weight of the 50 cans was recorded to estimate the average weight of all 2,440 cans. Properly filled cans weigh 10 pounds.

   (a) What is the population?
   (b) What is the sample?
   (c) What is the subject in this study?
   (d) What is the parameter of interest?
   (e) What is the statistic?

2. Eighteen measurements of the disbursement rate (in cm^3/s) of a chemical disbursement system are recorded and sorted:

6.50	6.77	6.91	7.38	7.64	7.74	7.90	7.91	8.21
8.26	8.30	8.31	8.42	8.53	8.55	9.04	9.33	9.36

   (a) Compute the sample mean and the sample variance.
   (b) Find the sample upper and lower quartiles.
   (c) Find the sample median.
   (d) Construct a boxplot of the data.
   (e) Find the 5^{th} and 95^{th} percentiles of the disbursement rates.

3. For a random variable X with pdf/pmf given by $f(x)$:

   (a) Show that $E(a + bX) = a + bE(X)$, where a, b are constants.
   (b) Show that $Var(a + bX) = b^2 Var(X)$.
   (c) Suppose we observe a random variable Y that depends on another observed value, x, through the relation
   $$Y = \beta_0 + \beta_1 x + \epsilon,$$
   where β_0, β_1, and x are constants, and $\epsilon \sim N(0, \sigma^2)$. Use parts (a) and (b) to find (i) $E(Y)$ and (ii) $Var(Y)$.

4. The average fat content of hot dogs is 18 grams. Suppose the standard deviation is 3 and that the fat content of all hot dogs is normally distributed.

   (a) What is the probability that the fat content of a randomly selected hot dog exceeds 19.2?

   (b) What is the probability that the average of 47 randomly selected hot dogs exceeds 19.2?

5. Eight hundred teenagers were sampled and the hours per week that each of them watches TV was recorded. The sample had a mean of 13.2 with a sample standard deviation of 1.6. Find and interpret a 95% confidence interval for the true mean weekly hours of TV watching for teenagers.

6. In order to test $H_0 : \mu = 4.5$ vs. $H_1 : \mu > 4.5$, a simple random sample of size $n = 13$ is obtained from a population that is known to be normally distributed with *population* standard deviation of 1.3.

   (a) The observed sample average is 4.9. Compute the appropriate test statistic and state its distribution under the null hypothesis.

   (b) Draw the appropriate distribution and shade the area that represents the p-value corresponding to the test statistic calculated in (a). Be sure to clearly label your picture, including the boundary of the shaded area and the parameter value(s) defining the distribution (i.e. give the degrees of freedom or the mean and variance of the distribution).

   (c) The p-value for this test is .1336. True or false: There is a 13.36% chance that the observed data came from a normal distribution with mean 4.5. Justify your answer.

   (d) State the result of this test using an $\alpha = .1$ significance level.

   (e) What is the probability of a Type I error?

   (f) What would it mean to commit a Type II error?

7. Below are listed some data I was provided a couple of years ago. I have listed here reported heights (in inches), separated by sex. Suppose we want to see if we have enough evidence to conclude that the average heights of males and females are significantly different. The data provided make up our sample.

Sex	Height				
Male	70	70	70	67	70
Female	64	66	62	62	69

   (a) Use this information to determine whether or not we have sufficient evidence to conclude that there is a true difference in the average heights of females and males. Assume equal variances in the heights of males and females (even though this is certainly questionable here!). Include the appropriate null and alternative hypotheses, the test statistic, the distribution of the test statistic under the null hypothesis, and calculate the p-value. Using a significance level of $\alpha = .1$, what would be the result of this test? State your conclusions in the context of the problem.

   (b) What is the probability of committing a Type I error in this problem?

   (c) The 90% confidence interval for $\mu_1 - \mu_2$ using these data is (2.0925, 7.5075). True or false - if we repeatedly sampled heights from males and females, approximately 90% of all our observed differences would fall between 2.0925 and 7.5075. Explain your choice.

8. The mean monthly expense for entertainment per household was obtained from data collected by telephone interviews of 1000 randomly selected households. The 95% confidence interval was computed as [$86.50, $161.50].

   (a) A magazine claims that, on average, households spend $120 per month on entertainment. Would we reject this claim at the $\alpha = .05$ level? Why or why not?

   (b) True or False: The true average monthly expense for entertainment has a 95% chance of being between 86.50 and 161.50. Justify your answer.