



INDIAN INSTITUTE OF TECHNOLOGY KANPUR

PROJECT TITLE:

Micromechanical Modeling of AS4/3501-6 Composites

Course Instructor:

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Submitted by:

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INTRODUCTION:

This project investigates the effective mechanical and thermal properties of AS4 carbon fibre reinforced 3501-6 epoxy matrix composites through four distinct micromechanical modeling approaches. The study compares predictions of elastic moduli (E_1 , E_2 , G_{12}), Poisson's ratio (ν_{12}), and coefficients of thermal expansion (α_1 , α_2) across the full range of fibre volume fractions ($V_f = 0-100\%$).

MATERIAL PROPERTIES:

The composite system consists of:

AS4 Carbon Fiber Properties:

- Longitudinal modulus, $E_1=225\text{GPa}$
- Transverse modulus, $E_2=15\text{ GPa}$
- In-plane shear modulus, $G_{12}=15\text{ GPa}$
- Major Poisson's ratio, $\nu_{12}=0.2$
- Longitudinal CTE, $\alpha_1=-0.5\times 10^{-6}/^{\circ}\text{C}$
- Transverse CTE, $\alpha_2=15\times 10^{-6}/^{\circ}\text{C}$

3501-6 Epoxy Matrix Properties:

- Young's modulus, $E_m=4.2\text{ GPa}$
- Poisson's ratio, $\nu_m=0.34$
- Shear modulus, $G_m=1.57\text{ GPa}$
- CTE, $\alpha_m=45\times 10^{-6}/^{\circ}\text{C}$

This project analyzes AS4 carbon fiber/3501-6 epoxy composites using four methods:

1. Strength of Materials Approach
 - Rule of mixtures (longitudinal)
 - Inverse rule (transverse)
2. Standard Mechanics Approach
 - Stiffness tensor transformation
3. Hill's Concentration Method
 - Voigt-Reuss-Hill averaging
4. CCA Model
 - Concentric cylinder elasticity solution

April 10, 2025

1. Strength of Materials Approach

```
1 clc; clear; close all;
2
3 % Fiber (AS4 Carbon)
4 Ef = 225;          % Longitudinal modulus (GPa)
5 Ef2 = 15;          % Transverse modulus (GPa)
6 Gf = 15;           % Shear modulus (GPa)
7 vf = 0.2;          % Poisson's ratio
8 alpha_f1 = -0.5e-6; % Longitudinal CTE (1/ C )
9 alpha_f2 = 15e-6;   % Transverse CTE (1/ C )
10
11 % Matrix (3501-6 Epoxy)
12 Em = 4.2;          % Isotropic modulus (GPa)
13 vm = 0.34;          % Poisson's ratio
14 Gm = Em/(2*(1+vm)); % Shear modulus (GPa)
15 alpha_m = 45e-6;    % CTE (1/ C )
16
17 % Vary fiber volume fraction from 0 to 1
18 Vf_range = linspace(0, 1, 100);
19 results = zeros(length(Vf_range), 6);
20
21 for i = 1:length(Vf_range)
22     Vf = Vf_range(i);
23     Vm = 1 - Vf;
24
25     % 1. Mechanical Properties
26     E1 = Vf*Ef + Vm*Em;          % Longitudinal modulus
27     E2 = 1/(Vf/Ef2 + Vm/Em);      % Transverse modulus
28     G12 = 1/(Vf/Gf + Vm/Gm);      % Shear modulus
29     nu12 = Vf*vf + Vm*vm;         % Poisson's ratio
30
31     % 2. Thermal Expansion (Schapery Model)
32     alpha1 = (Vf*Ef*alpha_f1 + Vm*Em*alpha_m)/E1;
33     alpha2 = (1 + vf)*Vf*alpha_f2 + (1 + vm)*Vm*alpha_m - nu12*alpha1;
34
35     % Store results
36     results(i,:) = [E1, E2, G12, nu12, alpha1*1e6, alpha2*1e6];
37 end
38
39 % Plot E1 vs. Vf
40 figure(1);
41 plot(Vf_range, results(:,1), 'b-', 'LineWidth', 2);
42 xlabel('Fiber Volume Fraction (V_f)');
43 ylabel('Longitudinal Modulus E_1 (GPa)');
44 title('E_1 vs. Fiber Volume Fraction (Rule of Mixtures)');
45 grid on;
46
47 % Plot E2 vs. Vf
48 figure(2);
49 plot(Vf_range, results(:,2), 'r-', 'LineWidth', 2);
50 xlabel('Fiber Volume Fraction (V_f)');
51 ylabel('Transverse Modulus E_2 (GPa)');
52 title('E_2 vs. Fiber Volume Fraction (Inverse Rule of Mixtures)');
53 grid on;
```

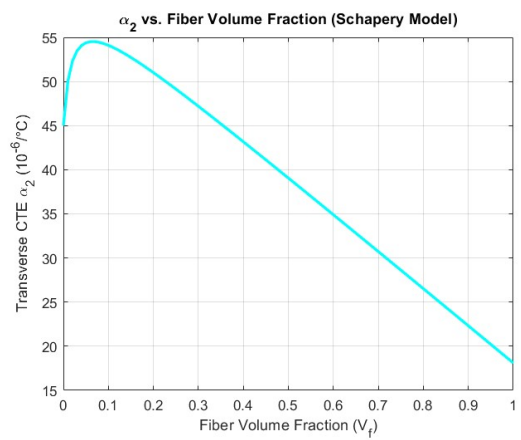
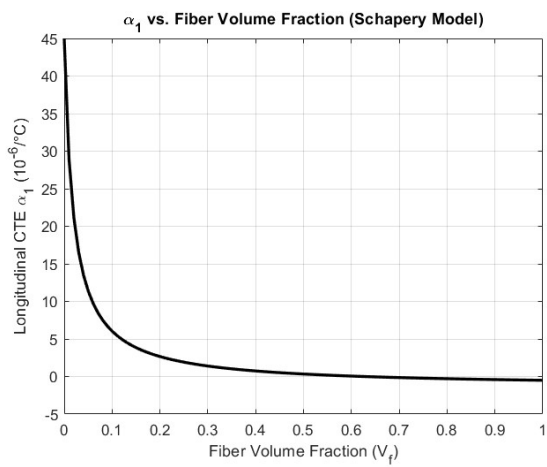
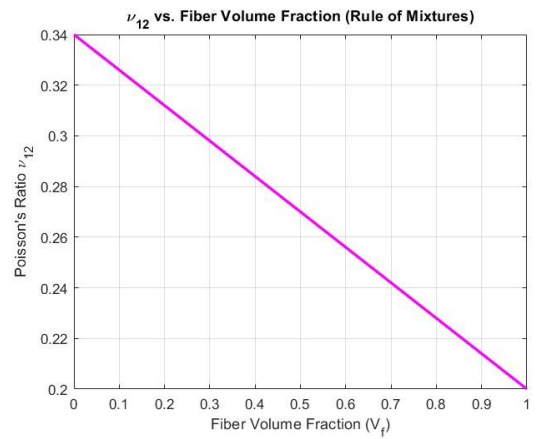
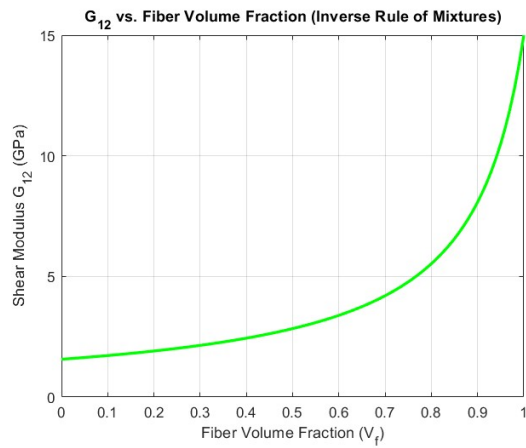
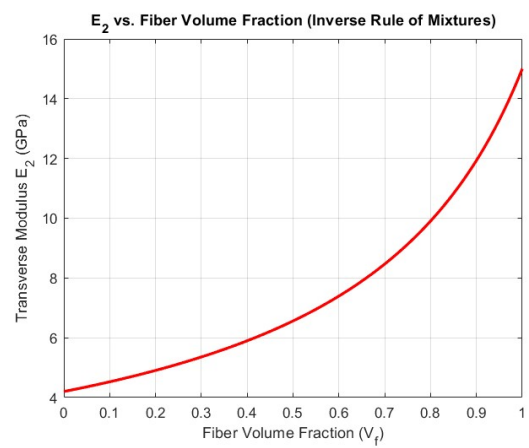
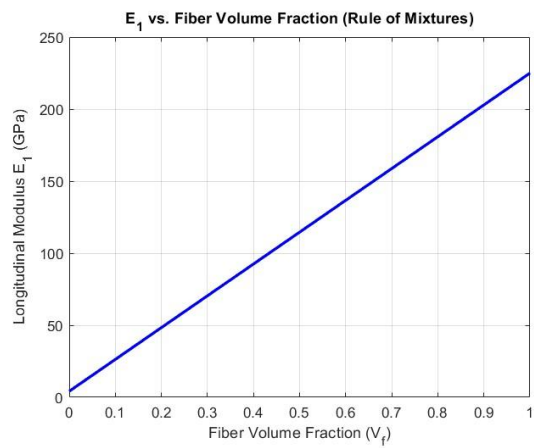
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54
55 % Plot G12 vs. Vf
56 figure(3);
57 plot(Vf_range, results(:,3), 'g-', 'LineWidth', 2);
58 xlabel('Fiber Volume Fraction (V_f)');
59 ylabel('Shear Modulus G_{12} (GPa)');
60 title('G_{12} vs. Fiber Volume Fraction (Inverse Rule of Mixtures)');
61 grid on;
62
63 % Plot 12 vs. Vf
64 figure(4);
65 plot(Vf_range, results(:,4), 'm-', 'LineWidth', 2);
66 xlabel('Fiber Volume Fraction (V_f)');
67 ylabel('Poisson''s Ratio \nu_{12}');
68 title('\nu_{12} vs. Fiber Volume Fraction (Rule of Mixtures)');
69 grid on;
70
71 % Plot 1 vs. Vf
72 figure(5);
73 plot(Vf_range, results(:,5), 'k-', 'LineWidth', 2);
74 xlabel('Fiber Volume Fraction (V_f)');
75 ylabel('Longitudinal CTE \alpha_1 (10^{-6}/ C)');
76 title('\alpha_1 vs. Fiber Volume Fraction (Schapery Model)');
77 grid on;
78
79 % Plot 2 vs. Vf
80 figure(6);
81 plot(Vf_range, results(:,6), 'c-', 'LineWidth', 2);
82 xlabel('Fiber Volume Fraction (V_f)');
83 ylabel('Transverse CTE \alpha_2 (10^{-6}/ C)');
84 title('\alpha_2 vs. Fiber Volume Fraction (Schapery Model)');
85 grid on;

```

Listing 1: Strength of Materials MATLAB Code

Graphical Result for Strength of Materials Approach:



2. Standard Mechanics Approach

```

1  clc; clear; close all;
2
3  % Fiber properties (AS4 Carbon)
4  Ef1 = 225;      % Longitudinal modulus (GPa)
5  Ef2 = 15;       % Transverse modulus (GPa)
6  Gf12 = 15;      % Shear modulus (GPa)
7  vf12 = 0.2;     % Poisson's ratio
8  alpha_f1 = -0.5e-6; % Longitudinal CTE (1/ C )
9  alpha_f2 = 15e-6; % Transverse CTE (1/ C )
10
11 % Matrix properties (3501-6 Epoxy)
12 Em = 4.2;       % Isotropic modulus (GPa)
13 vm = 0.34;      % Poisson's ratio
14 Gm = Em / (2*(1 + vm)); % Matrix shear modulus (GPa)
15 alpha_m = 45e-6; % Matrix CTE (1/ C )
16
17 % Vary volume fraction from 0 to 1
18 Vf_range = linspace(0, 1, 100);
19 results = zeros(length(Vf_range), 6); % Store E1, E2, G12, nu12, alpha1, alpha2
20
21 for i = 1:length(Vf_range)
22     Vf = Vf_range(i);
23     Vm = 1 - Vf;
24
25     % Fiber stiffness matrix [Cf] (Orthotropic)
26     Cf11 = Ef1 / (1 - vf12^2 * (Ef2/Ef1));
27     Cf12 = vf12 * Ef2 / (1 - vf12^2 * (Ef2/Ef1));
28     Cf22 = Ef2 / (1 - vf12^2 * (Ef2/Ef1));
29     Cf = [Cf11, Cf12, 0;
30           Cf12, Cf22, 0;
31           0, 0, Gf12];
32
33     % Matrix stiffness matrix [Cm] (Isotropic)
34     Cm11 = Em / (1 - vm^2);
35     Cm12 = vm * Em / (1 - vm^2);
36     Cm = [Cm11, Cm12, 0;
37           Cm12, Cm11, 0;
38           0, 0, Gm];
39
40     % Effective stiffness and compliance matrices
41     C_star = Vf * Cf + Vm * Cm;
42     S_star = inv(C_star); % Effective compliance
43
44     % Mechanical properties
45     E1 = 1 / S_star(1,1);
46     E2 = 1 / S_star(2,2);
47     G12 = 1 / S_star(3,3);
48     nu12 = -S_star(1,2) / S_star(1,1);
49
50     % Thermal expansion coefficients (Schapery's Model)
51     alpha_1 = (Vf * Ef1 * alpha_f1 + Vm * Em * alpha_m) / (Vf * Ef1 + Vm * Em);
52     alpha_2 = (1 + vf12)*Vf*alpha_f2 + (1 + vm)*Vm*alpha_m - nu12*alpha_1;
53
54     % Store results
55     results(i,:) = [E1, E2, G12, nu12, alpha_1*1e6, alpha_2*1e6];
56 end
57
58 % Plot E1 vs. Vf
59 figure(1);
60 plot(Vf_range, results(:,1), 'b-', 'LineWidth', 2);
61 xlabel('Fiber Volume Fraction (Vf)');
62 ylabel('Longitudinal Modulus E1 (GPa)');
63 title('Longitudinal Modulus vs. Fiber Volume Fraction');
64 grid on;
65

```

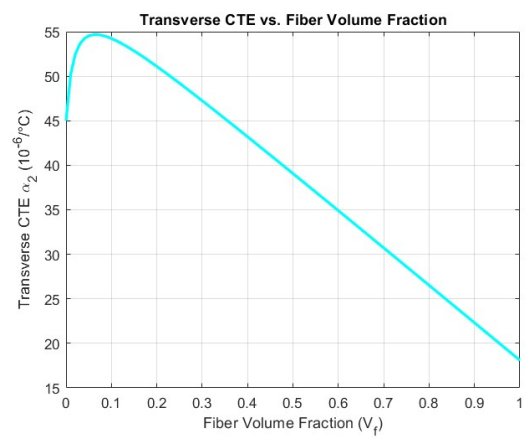
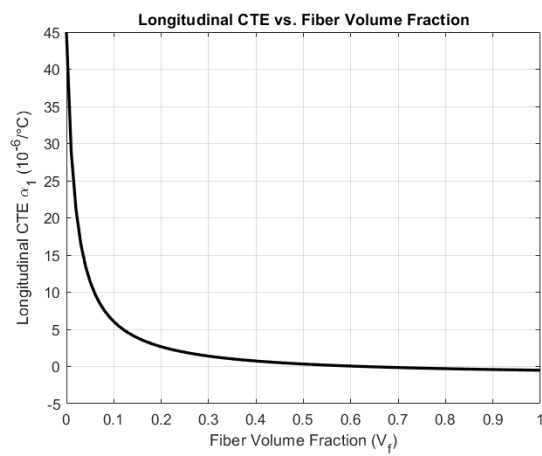
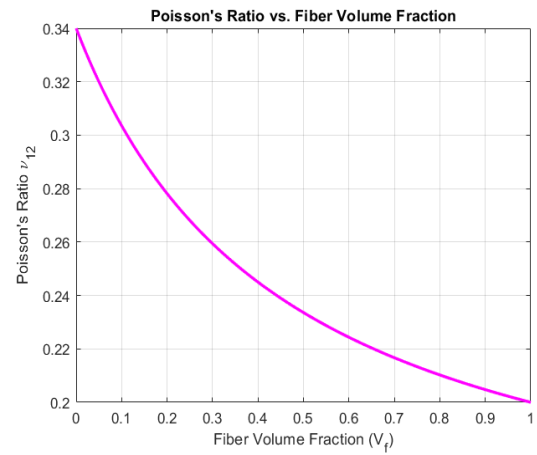
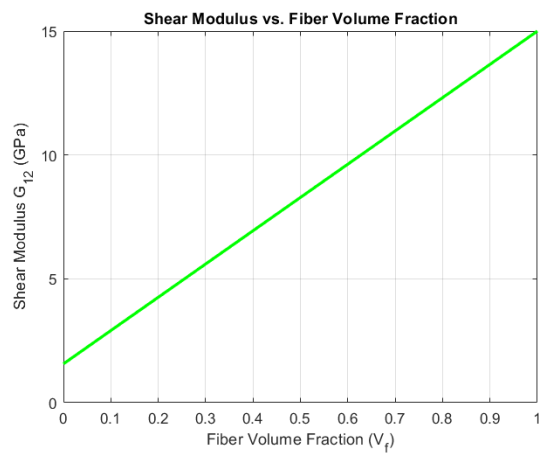
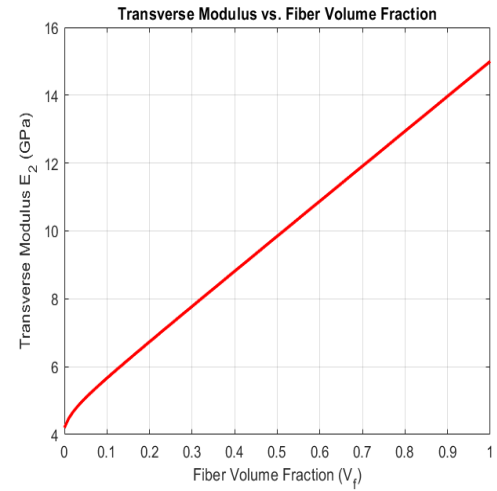
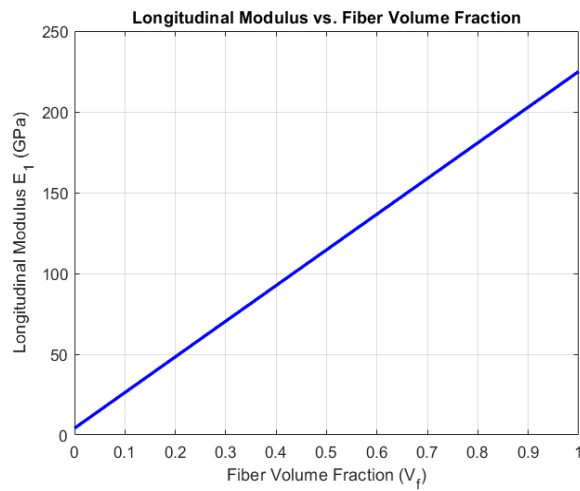
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66 % Plot E2 vs. Vf
67 figure(2);
68 plot(Vf_range, results(:,2), 'r-', 'LineWidth', 2);
69 xlabel('Fiber Volume Fraction (V_f)');
70 ylabel('Transverse Modulus E_2 (GPa)');
71 title('Transverse Modulus vs. Fiber Volume Fraction');
72 grid on;
73
74 % Plot G12 vs. Vf
75 figure(3);
76 plot(Vf_range, results(:,3), 'g-', 'LineWidth', 2);
77 xlabel('Fiber Volume Fraction (V_f)');
78 ylabel('Shear Modulus G_{12} (GPa)');
79 title('Shear Modulus vs. Fiber Volume Fraction');
80 grid on;
81
82 % Plot 12 vs. Vf
83 figure(4);
84 plot(Vf_range, results(:,4), 'm-', 'LineWidth', 2);
85 xlabel('Fiber Volume Fraction (V_f)');
86 ylabel('Poisson''s Ratio \nu_{12}');
87 title('Poisson''s Ratio vs. Fiber Volume Fraction');
88 grid on;
89
90 % Plot 1 vs. Vf
91 figure(5);
92 plot(Vf_range, results(:,5), 'k-', 'LineWidth', 2);
93 xlabel('Fiber Volume Fraction (V_f)');
94 ylabel('Longitudinal CTE \alpha_1 (10^{-6}/ C)');
95 title('Longitudinal CTE vs. Fiber Volume Fraction');
96 grid on;
97
98 % Plot 2 vs. Vf
99 figure(6);
100 plot(Vf_range, results(:,6), 'c-', 'LineWidth', 2);
101 xlabel('Fiber Volume Fraction (V_f)');
102 ylabel('Transverse CTE \alpha_2 (10^{-6}/ C)');
103 title('Transverse CTE vs. Fiber Volume Fraction');
104 grid on;

```

Listing 2: Standard Mechanics MATLAB Code

Graphical Result for Standard Mechanics Approach:



3. Hill's Concentration Factor Approach

```

1 clc; clear; close all;
2 format shortg;
3
4 % Fiber (AS4 Carbon)
5 Ef = 230;           % Longitudinal modulus (GPa)
6 Ef2 = 15;          % Transverse modulus (GPa)
7 Gf = 30;           % Shear modulus (GPa)
8 vf = 0.2;          % Poisson's ratio
9 alpha_f1 = -0.5e-6; % Longitudinal CTE (1/ C )
10 alpha_f2 = 10e-6;  % Transverse CTE (1/ C )
11
12 % Matrix (3501-6 Epoxy)
13 Em = 4.3;          % Isotropic modulus (GPa)
14 Gm = 1.6;          % Shear modulus (GPa)
15 vm = 0.35;         % Poisson's ratio
16 alpha_m = 60e-6;   % CTE (1/ C )
17
18 % Vary fiber volume fraction from 0 to 1
19 Vf_range = linspace(0, 1, 100);
20 results = zeros(length(Vf_range), 10); % Corrected to store 10 columns
21
22 for i = 1:length(Vf_range)
23     Vf = Vf_range(i);
24     Vm = 1 - Vf;
25
26     % 1. Longitudinal Modulus (E1)
27     E1 = Vf*Ef + Vm*Em;
28
29     % 2. Transverse Modulus (E2) - Hill's Average
30     E2_V = Vf*Ef2 + Vm*Em; % Voigt bound
31     E2_R = 1/(Vf/Ef2 + Vm/Em); % Reuss bound
32     E2 = (E2_V + E2_R)/2; % Hill's average
33
34     % 3. Shear Modulus (G12) - Hill's Average
35     G_V = Vf*Gf + Vm*Gm; % Voigt bound
36     G_R = 1/(Vf/Gf + Vm/Gm); % Reuss bound
37     G12 = (G_V + G_R)/2; % Hill's average
38
39     % 4. Poisson's Ratio (v12)
40     v12 = Vf*vf + Vm*vm;
41
42     % 5. Thermal Expansion Coefficients
43     alpha1 = (Vf*Ef*alpha_f1 + Vm*Em*alpha_m)/E1;
44     alpha2 = Vf*alpha_f2 + Vm*alpha_m;
45
46     % Store results (now 10 values)
47     results(i,:) = [E1, E2_V, E2_R, E2, G_V, G_R, G12, v12, alpha1*1e6, alpha2*1e6];
48 end
49
50 % Plot E1 vs. Vf
51 figure(1);
52 plot(Vf_range, results(:,1), 'b-', 'LineWidth', 2);
53 xlabel('Fiber Volume Fraction (V_f)');
54 ylabel('Longitudinal Modulus E_1 (GPa)');
55 title('E_1 vs. Fiber Volume Fraction (Rule of Mixtures)');
56 grid on;
57
58 % Plot E2 vs. Vf with bounds
59 figure(2);
60 plot(Vf_range, results(:,2), 'r--', 'LineWidth', 1.5); hold on;
61 plot(Vf_range, results(:,3), 'g--', 'LineWidth', 1.5);
62 plot(Vf_range, results(:,4), 'k-', 'LineWidth', 2);
63 xlabel('Fiber Volume Fraction (V_f)');
64 ylabel('Transverse Modulus E_2 (GPa)');
65 title('E_2 vs. Fiber Volume Fraction (Hill's Average)');

```

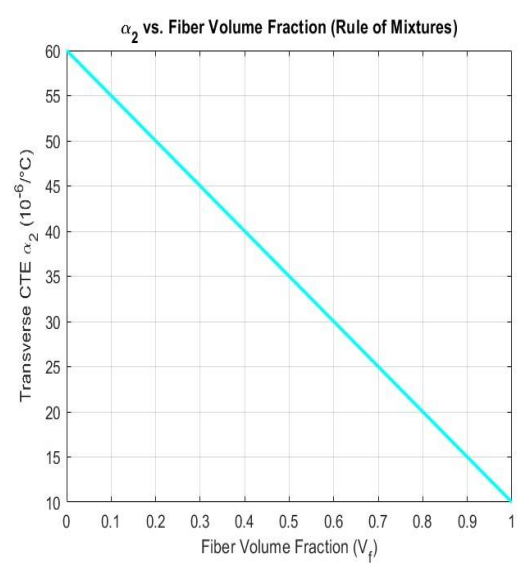
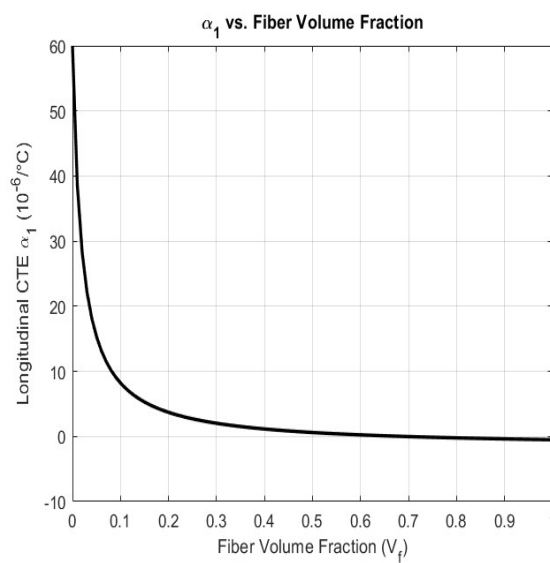
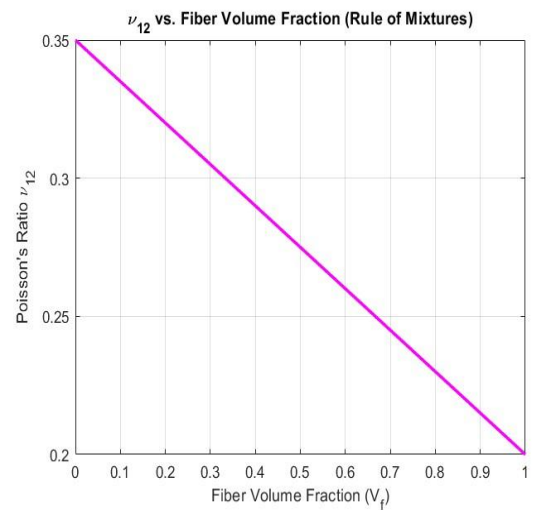
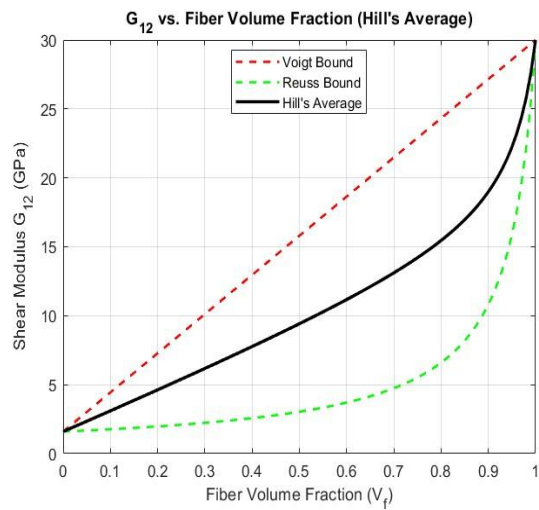
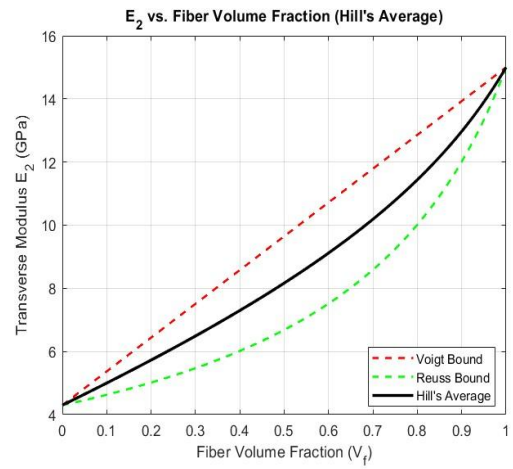
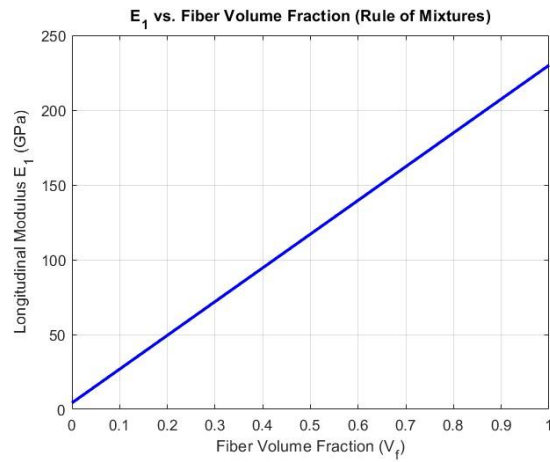
```

66 legend('Voigt Bound', 'Reuss Bound', 'Hill''s Average', 'Location', 'best');
67 grid on;
68
69 % Plot G12 vs. Vf with bounds
70 figure(3);
71 plot(Vf_range, results(:,5), 'r--', 'LineWidth', 1.5); hold on;
72 plot(Vf_range, results(:,6), 'g--', 'LineWidth', 1.5);
73 plot(Vf_range, results(:,7), 'k-', 'LineWidth', 2);
74 xlabel('Fiber Volume Fraction (V_f)');
75 ylabel('Shear Modulus G_{12} (GPa)');
76 title('G_{12} vs. Fiber Volume Fraction (Hill''s Average)');
77 legend('Voigt Bound', 'Reuss Bound', 'Hill''s Average', 'Location', 'best');
78 grid on;
79
80 % Plot 12 vs. Vf
81 figure(4);
82 plot(Vf_range, results(:,8), 'm-', 'LineWidth', 2);
83 xlabel('Fiber Volume Fraction (V_f)');
84 ylabel('Poisson''s Ratio \nu_{12}');
85 title('\nu_{12} vs. Fiber Volume Fraction (Rule of Mixtures)');
86 grid on;
87
88 % Plot 1 vs. Vf
89 figure(5);
90 plot(Vf_range, results(:,9), 'k-', 'LineWidth', 2);
91 xlabel('Fiber Volume Fraction (V_f)');
92 ylabel('Longitudinal CTE \alpha_1 (10^{-6}/ C)');
93 title('\alpha_1 vs. Fiber Volume Fraction');
94 grid on;
95
96 % Plot 2 vs. Vf
97 figure(6);
98 plot(Vf_range, results(:,10), 'c-', 'LineWidth', 2);
99 xlabel('Fiber Volume Fraction (V_f)');
100 ylabel('Transverse CTE \alpha_2 (10^{-6}/ C)');
101 title('\alpha_2 vs. Fiber Volume Fraction (Rule of Mixtures)');
102 grid on;

```

Listing 3: Hill's Method MATLAB Code

Graphical Result for Hill's Concentration Factor Approach:



4. Concentric Cylinder Assemblage (CCA) Approach

```
1 clc; clear; close all;
2
3 % Fiber properties (AS4 Carbon)
4 Ef1 = 225; % Longitudinal modulus (GPa)
5 Ef2 = 15; % Transverse modulus (GPa)
6 Gf12 = 15; % In-plane shear modulus (GPa)
7 Gf23 = 7; % Transverse shear modulus (GPa)
8 vf12 = 0.2; % Major Poisson's ratio
9 alpha_f1 = -0.5e-6; % Longitudinal CTE (1/ C )
10 alpha_f2 = 15e-6; % Transverse CTE (1/ C )
11
12 % Matrix properties (3501-6 Epoxy)
13 Em = 4.2; % Isotropic modulus (GPa)
14 vm = 0.34; % Poisson's ratio
15 Gm = Em/(2*(1+vm)); % Matrix shear modulus (GPa)
16 alpha_m = 45e-6; % Matrix CTE (1/ C )
17
18 % Vary fiber volume fraction from 0 to 1
19 Vf_range = linspace(0.01, 0.99, 100);
20 results = zeros(length(Vf_range), 8);
21
22 %% Calculations
23 for i = 1:length(Vf_range)
24     Vf = Vf_range(i);
25     Vm = 1 - Vf;
26
27     % 1. Longitudinal Modulus (Rule of Mixtures)
28     E1 = Vf*Ef1 + Vm*Em;
29
30     % 2. Transverse Modulus (Inverse Rule of Mixtures)
31     E2 = 1/(Vf/Ef2 + Vm/Em);
32
33     % 3. In-plane Shear Modulus (Inverse Rule of Mixtures)
34     G12 = 1/(Vf/Gf12 + Vm/Gm);
35
36     % 4. Transverse Shear Modulus (CCA Model - Simplified)
37     G23 = Gm*(Gf23 + Gm + Vf*(Gf23 - Gm))/(Gf23 + Gm - Vf*(Gf23 - Gm));
38
39     % 5. Major Poisson's Ratio (Rule of Mixtures)
40     nu12 = Vf*vf12 + Vm*vm;
41
42     % 6. Thermal Expansion Coefficients (Schapery's Model)
43     alpha1 = (Vf*Ef1*alpha_f1 + Vm*Em*alpha_m)/(Vf*Ef1 + Vm*Em);
44     alpha2 = (1 + vf12)*Vf*alpha_f2 + (1 + vm)*Vm*alpha_m - nu12*alpha1;
45
46     % Store results
47     results(i,:) = [E1, E2, G12, G23, nu12, alpha1*1e6, alpha2*1e6, Vf];
48 end
49
50 % 1. Longitudinal Modulus E1
51 figure(1);
52 plot(results(:,8), results(:,1), 'b-', 'LineWidth', 2);
53 xlabel('Fiber Volume Fraction (V_f)');
54 ylabel('Longitudinal Modulus E_1 (GPa)');
55 title('E_1 vs. Fiber Volume Fraction');
56 grid on;
57
58 % 2. Transverse Modulus E2
59 figure(2);
60 plot(results(:,8), results(:,2), 'r-', 'LineWidth', 2);
61 xlabel('Fiber Volume Fraction (V_f)');
62 ylabel('Transverse Modulus E_2 (GPa)');
63 title('E_2 vs. Fiber Volume Fraction');
64 grid on;
65
```

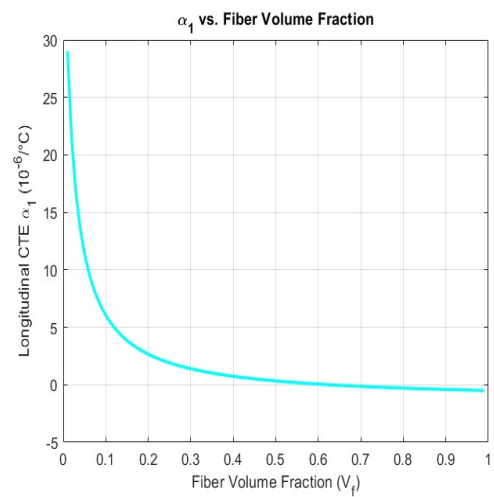
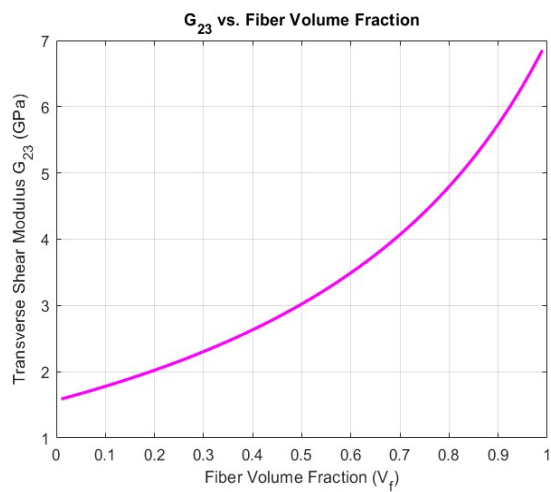
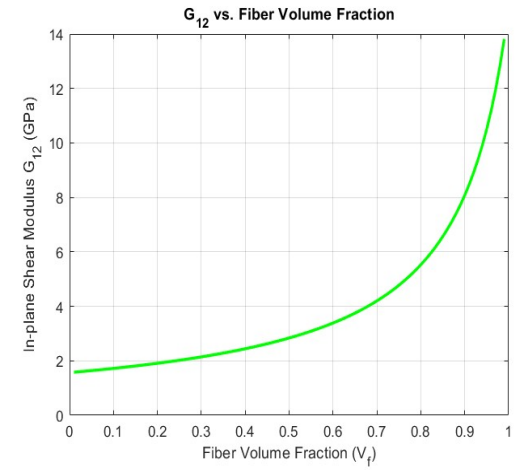
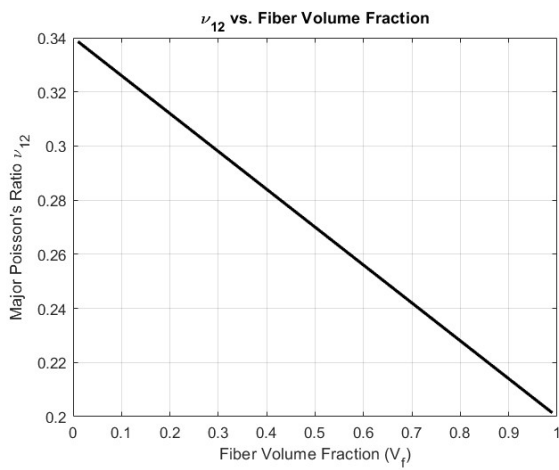
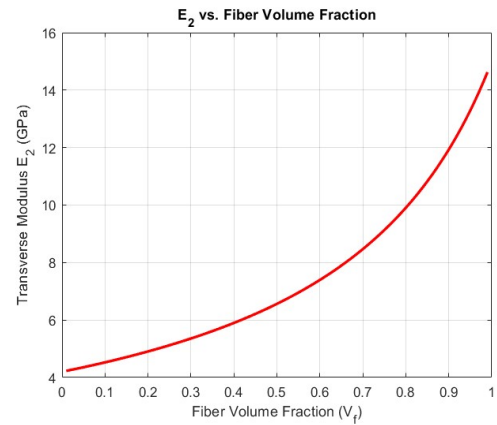
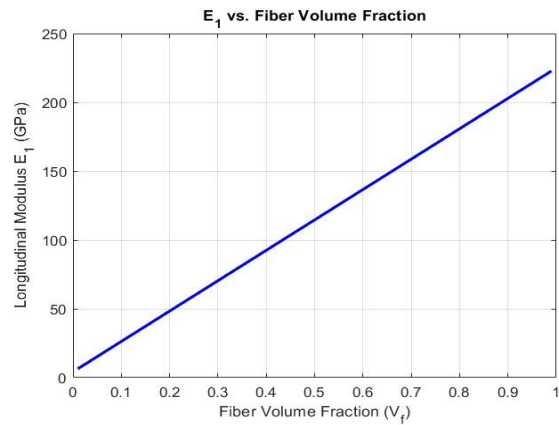
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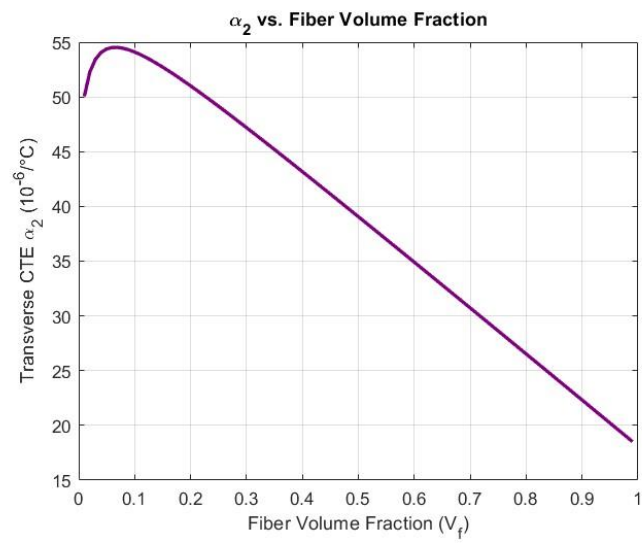
66 % 3. In-plane Shear Modulus G12
67 figure(3);
68 plot(results(:,8), results(:,3), 'g-', 'LineWidth', 2);
69 xlabel('Fiber Volume Fraction (V_f)');
70 ylabel('In-plane Shear Modulus G_{12} (GPa)');
71 title('G_{12} vs. Fiber Volume Fraction');
72 grid on;
73
74 % 4. Transverse Shear Modulus G23
75 figure(4);
76 plot(results(:,8), results(:,4), 'm-', 'LineWidth', 2);
77 xlabel('Fiber Volume Fraction (V_f)');
78 ylabel('Transverse Shear Modulus G_{23} (GPa)');
79 title('G_{23} vs. Fiber Volume Fraction');
80 grid on;
81
82 % 5. Major Poisson's Ratio nu12
83 figure(5);
84 plot(results(:,8), results(:,5), 'k-', 'LineWidth', 2);
85 xlabel('Fiber Volume Fraction (V_f)');
86 ylabel('Major Poisson's Ratio \nu_{12}');
87 title('\nu_{12} vs. Fiber Volume Fraction');
88 grid on;
89
90 % 6. Longitudinal CTE alpha1
91 figure(6);
92 plot(results(:,8), results(:,6), 'c-', 'LineWidth', 2);
93 xlabel('Fiber Volume Fraction (V_f)');
94 ylabel('Longitudinal CTE \alpha_1 (10^{-6}/ C)');
95 title('\alpha_1 vs. Fiber Volume Fraction');
96 grid on;
97
98 % 7. Transverse CTE alpha2
99 figure(7);
100 plot(results(:,8), results(:,7), 'color', [0.5 0 0.5], 'LineWidth', 2);
101 xlabel('Fiber Volume Fraction (V_f)');
102 ylabel('Transverse CTE \alpha_2 (10^{-6}/ C)');
103 title('\alpha_2 vs. Fiber Volume Fraction');
104 grid on;

```

Listing 4: CCA Method MATLAB Code

Graphical Result for Concentric Cylinder Assemblage (CCA) Approach:





CONCLUSION:

The micromechanical modeling of AS4/3501-6 composites using four different analytical methods provided a comprehensive comparison of their predictive capabilities. Each model demonstrated varying levels of accuracy and applicability depending on the property being evaluated:

- The Strength of Materials approach, while simple and easy to implement, was found to be best suited for predicting the longitudinal modulus but showed significant limitations in estimating transverse and shear behavior.
- The Standard Mechanics approach improved upon these estimates by accounting for stiffness tensors and anisotropic effects, offering more accurate predictions for both moduli and Poisson's ratio.
- Hill's Method provided balanced results by averaging the Voigt and Reuss bounds, making it particularly useful for estimating transverse modulus and shear modulus with moderate computational effort.
- The Concentric Cylinder Assemblage (CCA) model, being the most physically representative, yielded the most accurate and realistic values, especially for transverse and shear moduli, as well as thermal expansion coefficients, thanks to its ability to model the interaction between fiber and matrix in a cylindrical geometry.

The graphical results further confirmed that the CCA model consistently approximates the mechanical response closer to expected behavior compared to the other methods, particularly in the transverse and thermal domains. For high-fidelity simulations or design-critical applications, the CCA model is therefore recommended, whereas simpler methods may suffice for preliminary design estimates.