Group Assignment HT16

1 Introduction

The assignment is done in groups of three students. There are 3 hours of assistance (referred to as Lab in the course schedule) per week available, however, it is expected that the assignment tasks are solved on your own although the assistants are available for answering specific questions that you have prepared.

The assignment report shall be handed-in via e-mail, due on Monday October 31, along with the corresponding data files (models, simulation results etc.) sent to course assistant Fariba Rahimi (frahimi@kth.se) via e-mail.

Questions can be answered by any of the course assistants, either during the exercises or else preferably by e-mail. In case of short questions potentially solved by short answers, those will be given over e-mail as quickly as possible. More elaborate issues will be collected and responded to in person during the Lab sessions or otherwise on times/places as announced by the course assistants and/or communicated via the course homepage.

You can also use the KTH Social platform to post comments, questions and answers to/from your fellow students.

The goal with the assignment is for the students to develop a skill for how to approach and solve a mechatronics design problem using a model based approach. Based on a targeted product functionality such a skill involves (in short) conception of a system based on structured subsystems, selection of components, modeling and simulation of the system at various levels of detail, designing a control system and verification of system behavior.

1.1 Hint

Please read thoroughly the *whole* assignment before starting your work, and remember that

- ❖ Mastering the assignment also means that you possess skills valuable also when it comes to the final written exam, and most importantly in a future engineering career.

2 An electric nut-runner for tightening of bolted joints

2.1 System to be designed

Hand held electric nut runners are used in the manufacturing industry to aid assembly workers in doing precisely controlled bolt tightening, e.g. in the automotive industry. Such a nut runner is a good example of a quite advanced mechatronic system for which the characteristics of the device itself, of the bolted assembly and of the human operator must be considered and analysed together during the design process. In this assignment we will base modelling, design (some aspects) and analysis on the device depicted in Figure 1.

Figure 1. Cordless angle nut-runner - transducerized type Tensor ETV STB62-50-B10.

The basic elements of the Tensor ETV STB62-50-B10 nut runner are a basic cylinder shaped mechanical structure, a battery, a brushless DC-motor, a two-stage planetary gear, a torque transducer, a 90 degree spiral angle gear, an operator interface, an industrial radio communication interface (based on Bluetooth 2.0), a one-piece angle head for attachment to the bolted joint to be tightened, and finally an ergonomically designed handle as the physical interface to the operator.

2.2 Bolt tightening

Tightening of threaded bolt and nut assemblies requires control of both turning torque and angle of turn in order to reach the desired preload of the bolted assembly. The geometry of the assembly, the material characteristics of the involved components, and the friction between the parts (that move relative to each other during tightening) must be understood to control the tightening process.

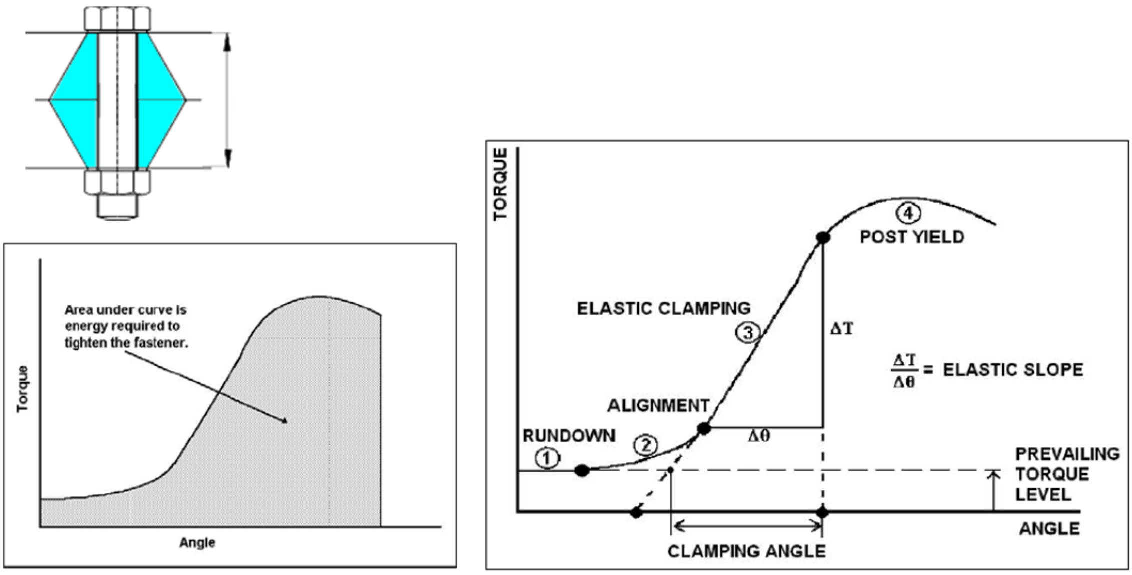


Figure 2. Turning torque versus turning angle during bolt tightening.

Tightening a bolt corresponds to an energy transfer process (figure 2 left) where the area under the torque-angle curve equals the energy consumed during the tightening process. The process of tightening a fastener involves:

- A rundown zone (1 in Figure 2) in which the nut is turned until the joint members start to align.
- An alignment or snugging zone (2) in which the mating surfaces of the joint are pressed into alignment to achieve a "snug" state.
- The third zone (3) corresponds to elastic deformation (stretching/compression) of the joint members where in principle there is a linear relationship between turning angle and elastic deformation.
- Finally the yield point of a joint member is reached and the following deformation is no longer elastic, but plastic.

In an advanced electrical nut-runner both the turning speed (or angle) and the turning torque is controlled.

2.3 The operator

It is obvious that manually handling an electric nut-runner involves reaction forces/torques on the operator. It is also obvious that the characteristics of these reaction forces/torques are affected by the physical and control design of the nut-runner. From an ergonomic point of view it is therefore important to somehow take the operator into account while designing the nut-runner. Human dynamics can in situations like this often be modelled accurately enough using low-order linear models of the biomechanical system, see for example Lin et al (2003) which is available on-line as a full-text source via the KTH Library . The characteristics of such a model will then depend on the physical properties of the operator (strength, weight, etc.) and how (in which pose) the operator holds and operates the machine. Note that you must have access to the referenced paper in order to extract some model parameter values.

2.4 The system in some more detail

The overall system to be analysed and partly designed has the components, configurations, characteristics and parameter data as described in this section.

2.4.1 The nut-runner

A cordless angle nut-runner - transducerized type Tensor ETV STB62-50-B10 by AtlasCopco Tools is under study. The tool has dimensions according to Figure 3 below. In this assignment, the tool is oriented horizontally, whereas the bolt is oriented vertically.

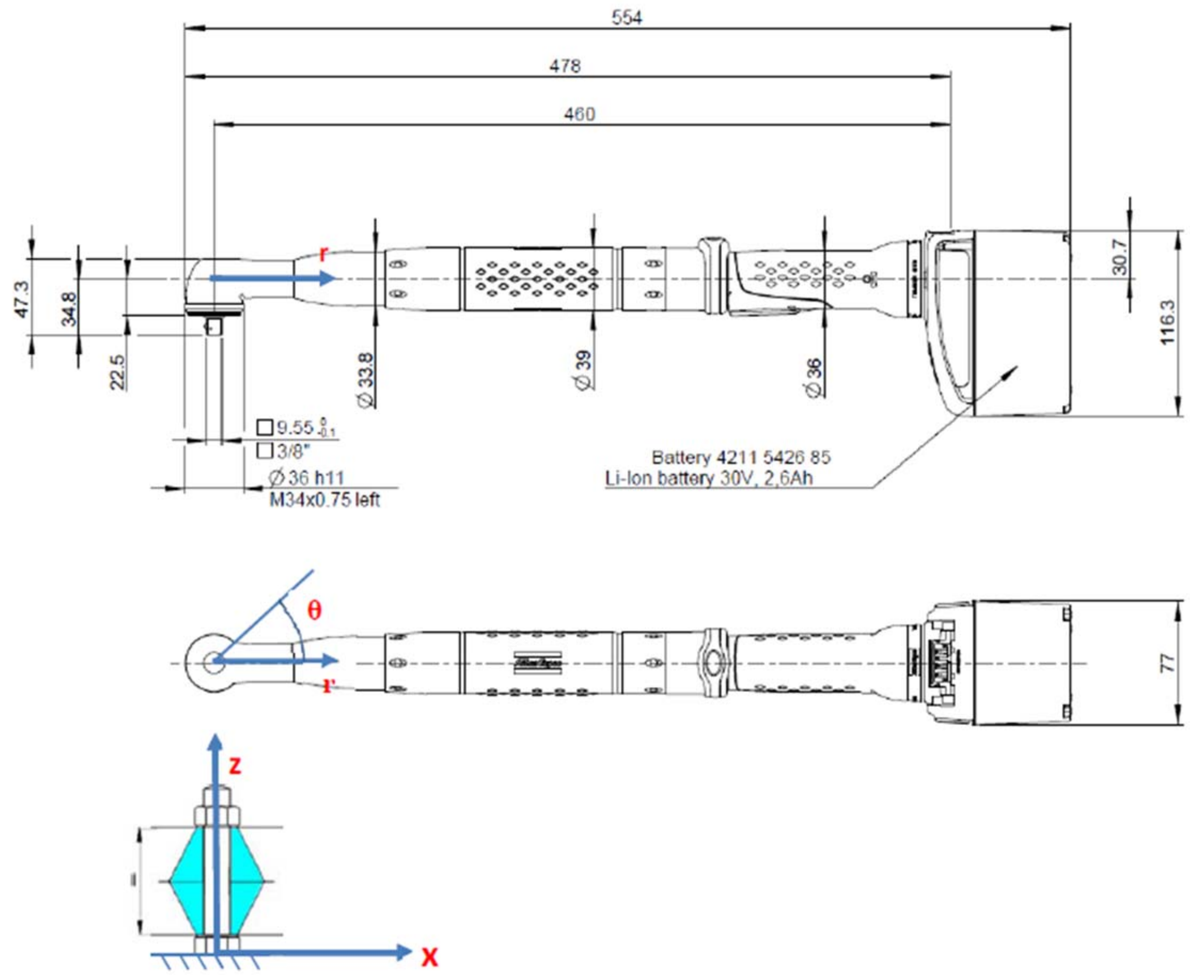


Figure 3. Dimensions of the nut-runner with recommended coordinate systems.

In addition to the figure 3 coordinates, the ***φ-coordinate*** corresponds to rotation (of motor and transmission parts) around the tool-fixed r-axis. The ***θ-coordinate*** corresponds to tool rotation around the z-axis of the inertial reference frame x-y-z. The ***α-coordinate*** corresponds to nut rotation around the same z-axis.

Some nut runner data needed for the assignment can be found in ref. [2 and 3] (Available on course home page). Note however that we apply a different motor than in the real case and that **we run this motor at 24 V. Motor data for the Faulhaber motor is found in ref. [4]**.

The **following assumptions and parameter values apply**:

- On the output shaft of the tool, a socket is attached to fit the nut to be turned. We assume this socket is stiffly connected and has a tight fit on the nut. In this way we can consider the tool chassis rotation θ to be in the horizontal plane only.
- The mass centres of the battery and of the tool itself (excluding the battery) are assumed to lie on the r-axis. The mass centre of the battery lies on the r-coordinate 498 mm. The mass centre of the tool itself (excluding the battery) lies on the r-coordinate 230 mm.
- The reaction force from the operator is assumed to act on the tool at r-coordinate 300 mm (in-between the two handles).

- The electromechanical actuation system of the tool is depicted in Figure 4 below.

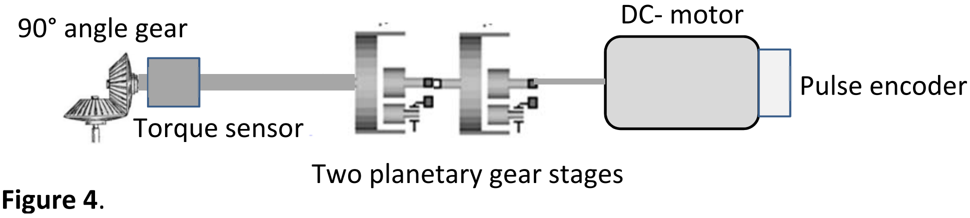


Figure 4.

- The transmission ratio of the 90 degree angle gear is equal to 1.

2.4.2 The bolted assembly

When tightening a bolted joint the torque is transferred from the tool (and operator) to the nut. The real tightening process is quite complex considering the non-linear friction conditions, the stretching and twisting of the bolt, as well as compression of the assembly members being fitted together.

In this assignment we will consider the nut running process as consisting of two phases; rundown and clamping (Figure 2). For the whole assignment, the ***bolt head is considered fixed to a reference frame***.

Run-down phase

In the run-down phase, the tool is to overcome only the friction and the inertia of the free-running nut. The reaction torque on the tool is hence very limited and therefore we ***assume that the tool remains in a fixed orientation*** although it will follow the nut downwards (no reaction forces on the operator although he/she is assumed to counteract gravitational forces of the tool). Further we assume that the ***friction conditions are linear.***

Clamping phase

In the clamping phase the screw-nut assembly is ***modelled as a stiffness*** (comparatively stiff, with damping). You can regard this stiffness as a spring/damper acting along the centre axis of the bolt. The very stiff "spring behaviour" corresponds to the stretching of the bolt during tightening. I.e., we disregard the compression elasticity of the assembly members being pressed together.

Initially when setting up the model equations we assume that the conditions are linear, but since the bolted joint is self-locking (i.e. rotation of the nut leads to longitudinal stretching of the bolt, but stretching of the bolt does not – due to friction – lead to nut rotation) the final behaviour is non-linear.

2.4.3 The operator model

When tightening a joint, a torque is applied. This torque must be counteracted somehow. In the nut-runner case the motion of the nut and the torque application is provided and controlled by the tool. However, the operator holding the tool must be able to counteract the applied torque. Hence in order to analyse the effects on the operator, an operator model must be applied. A principle figure of a linear operator model is depicted Figure 5 below. ***To get realistic model parameters of operator characteristics you should pick average values for the case of "a right-angle handle used on a horizontal surface" as presented by plots in Lin et al 2003*** (ref 1)**.**

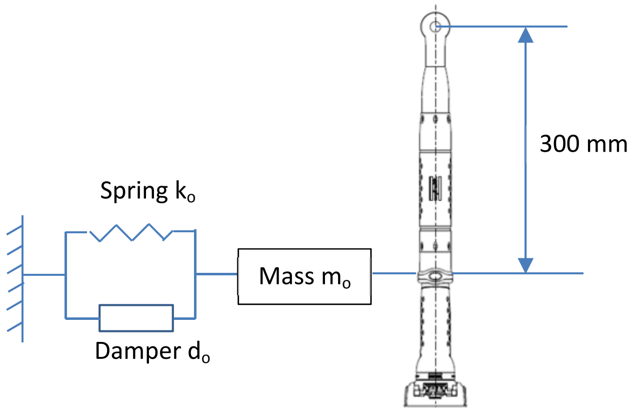


Figure 5. Operator model.

The operator is based on the following "ideas". The overall body of the operator is heavy compared to the tool, arms and hands of the operator. This is "visualized" by the fact that the spring/damper system is locked to a fixed frame to the left. The parameter values of the spring, damper and mass components should approximate part of the human arm/hand system mass as well as the stiffness and damping characteristics of the human neuromuscular system.

2.5 A first set of assignment tasks

The assignment tasks correspond to typically important engineering tasks that are performed during design of a mechatronic system, although in this case a conceptual design is already present.

The assignment is structured in an incremental way, going step by step from rather simple tasks to more complex issues. When solving the tasks it is important to ***adopt a structured approach simplifying reuse of early results in later stages of the process, and to document the work throughout*** (this will substantially facilitate creating a report of high quality). Simulation and analysis is to be done using Matlab and Simulink.

The first set of assignment tasks deals with modelling, analysis and the first stage of control design of the above system for the purpose of angular velocity control during the run-down phase (Figure 2). The main subsystems in terms potentially having large impact on the system behaviour are the ***DC-motor, the gearings and shafts, and the nut/bolt system.*** These main subsystems are modelled and their behaviour analysed such that the models can be verified to be reasonable and of suitable complexity (not unnecessarily detailed) for control design.

2.5.1 Assumptions

- Lumped parameters and linear models can be used. Linear models means in this case linear differential equations with constant coefficients.
- All mechanical interconnections except the human operator and the transmission within the tool are considered as infinitely stiff.
- The damping in the overall system comes from DC-motor resistance, the linear friction in the nut, the damping in the operator, and finally friction in the tool (also considered linear).

Additional parameters:

- Some parameters and system properties are found under heading 2.4 above.
- DC motor parameters can be found in the referenced data sheet.

2.5.2 Your tasks

Task 1: Start the modeling with the voltage driven DC-motor only. The following models should be developed:
- Differential equations (1)
- Block diagram (2) (Simulink)
- State-space model (3) (Symbolically in Matlab + Simulink)
- Transfer function model (4) (Symbolically in Matlab + Simulink)

The modeling should first be done on a generic level without numerical parameter values. Implement models 2, 3 and 4 separately in Matlab/Simulink and define the model parameters in Matlab. Show the Simulink models and the Matlab parameter file in the report.

Task 2: Verify by simulation that the DC-motor model works as intended by applying a reasonable voltage step (considering motor specifications) on the input and studying the time-domain response in terms of current, angular acceleration and angular velocity. (From the data sheet you can also use for example stall torque, no-load speed, angular acceleration, and mechanical time constant to verify your model). **Make sure that that your time-domain plots are scaled such that the interesting behavior is captured clearly, and make sure that the curves are clearly visible. This principle should rule for your whole report.**

Convince yourself and ***make it clear in the report*** that all models are correct and give the same response (although all variables might not be as easy to access).

Task 3: Develop in Simulink an angular velocity PD controller for the DC-motor using ***the block-diagram model*** of the motor and **show the resulting block diagram model**. Assume that you can sense angular velocity and acceleration directly. Tune the controller to make a quick response without overshooting to an input reference step of 12 rad/s and make sure not to violate any voltage or current limitations of the motor. **Demonstrate the controller's performance by time domain plots and state your controller settings**.

Hint 1: Start with a low proportional gain and increase until you get some overshoot and oscillatory behavior. Then add derivative control to damp the system until a response without overshoot is achieved. In this first control task you can implement the P-controller as

$$u_c = k_p * (\omega_{ref} - \omega)$$

and the PD controller as

$$u_c = k_p * \left(\omega_{ref} - \omega\right) - k_d * \dot{\omega}$$

where k_p and k_d are the controller gains. As a tuning starting point for the derivative part you can calculate a controller gain to achieve the same damping factor as is given by the motor resistance (i.e. you see from the block diagram that motor resistance has the same principal damping effect as the derivative part of your controller).

Task 4: Analyze the pole locations of the motor model (without control). Draw conclusions **and describe them** for a well-motivated simplification of the model. **Derive the new model in terms of formalisms (1), (2), and (4) (see Task 1)**. Convince yourself that the models work properly **and demonstrate that in the report**.

Task 5: With the given specifications of the tool, **select a gear ratio for each of the two planetary gears** (while assuming they have equal gear ratios). **Motivate your choice**.
Hint: Electrical motors are typically associated with two different torque specifications – stall torque and continuous torque. Stall (zero speed) torque, which is the higher of the two, cannot be applied continuously due to over-heating of the motor. The continuous or rated (speed dependent) torque, on the contrary, can be applied for an extended period of time without risk. **Since – in our case – the nut-runner does not operate continuously we make the assumption that we can design for utilizing the stall torque**.

2.6 A second set of assignment tasks

2.6.1 Additional nut-runner information

In Figure 1 below, the main subsystems of the nutrunner are depicted.

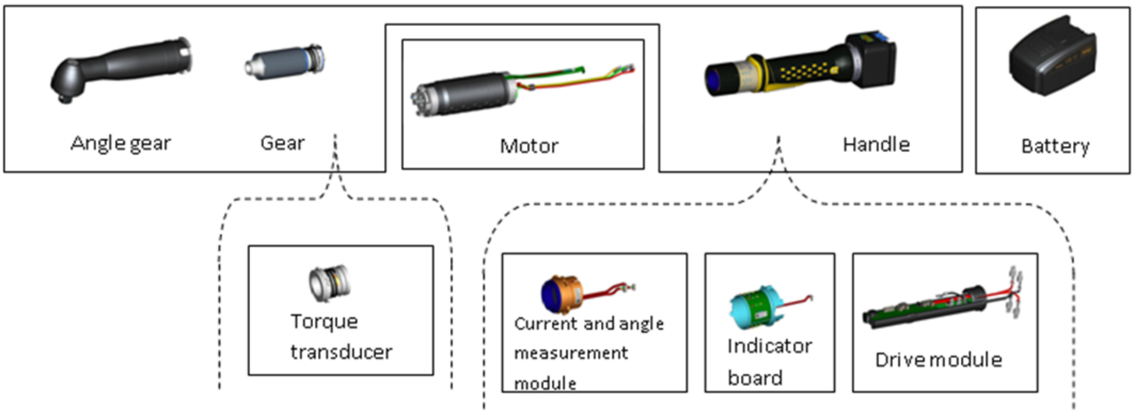


Figure 6. Nut-runner subsystems.

In Figure 7 below, the main parts of the transmission are depicted including some of the dimensions.

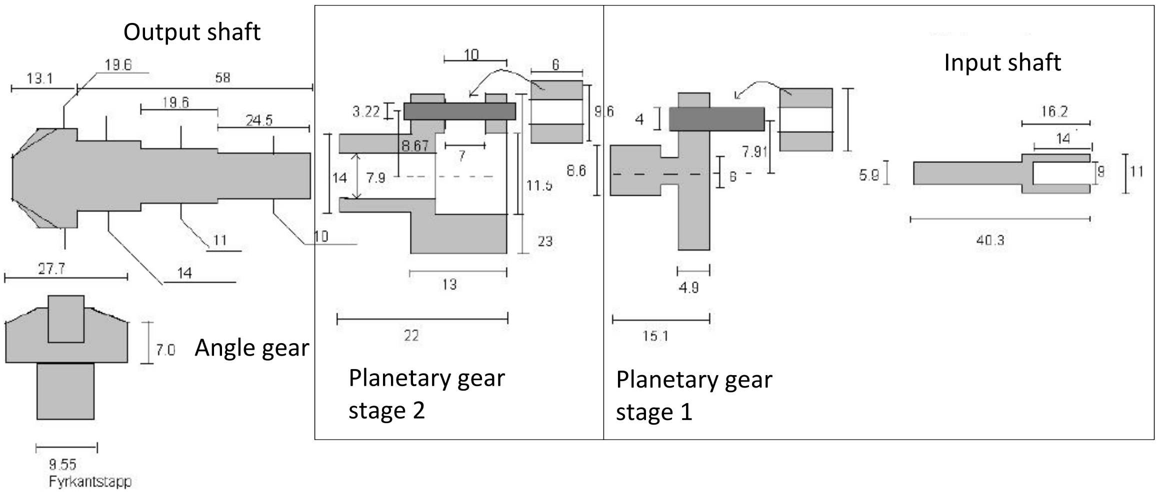


Figure 7. Nut-runner transmission

Assumptions:

- The angle gear, output and input shafts can be considered rigid (i.e. no flexibility)
- All components of the transmission are assumed to be made of steel.
- The flexibility in the transmission is assumed concentrated to the two planetary gear stages and equals $k_t = 739\ Nm/rad$ as ***estimated on the output (load) side of the planetary gear pair***.
- The transmission damping (friction) is assumed linear and equals $d_t = 1.5\ Nm \cdot s/rad$ as ***estimated on the output (load) side of the planetary gear pair***. The damping in the transmission should be considered as relative. See Janschek p 246. (This agrees with the Figure below).
- The damping in the screw/nut assembly (joint friction) is assumed linear and equals $d_j = 1.5\ Nm \cdot s/rad$. The damping in the nut/screw should be considered absolute (“sky hook” type). See Janschek p 246. (This agrees with the Figure below)
- The stiffness k_j (longitudinal) of the screw is calculated from screw dimensions and material properties. The screw is assumed not to undergo any torsional twist.

With these assumptions the transmission model can be reduced according to the figure below. ***Please notice*** that this system is linear and corresponds to a multiple mass oscillator according to the course text book (Janschek page 236-237).

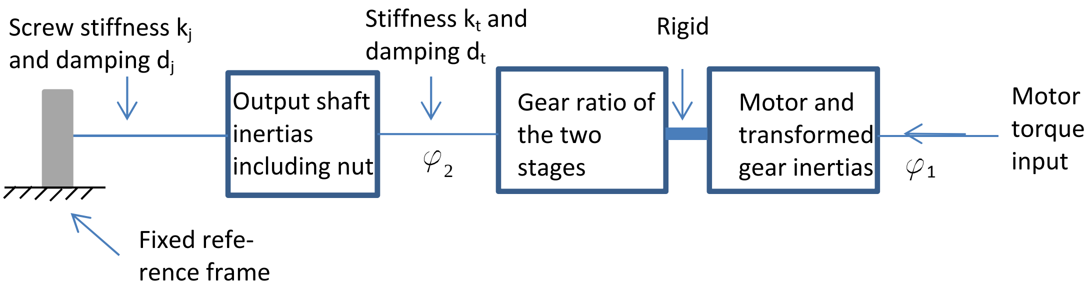
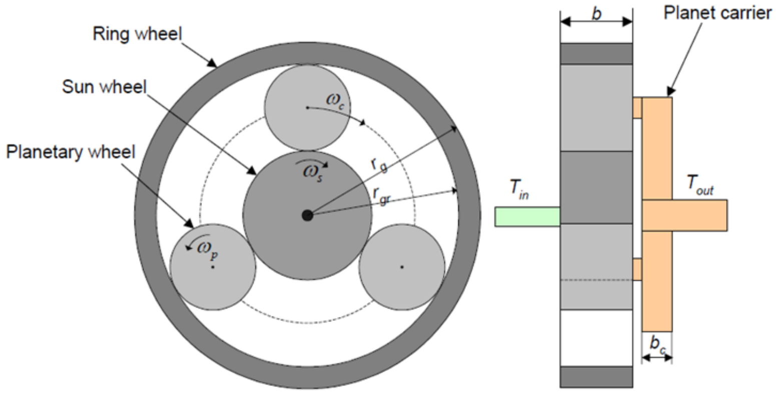


Figure 8. Idealized transmission model

During design work, the inertia of a planetary gear stage at the input (machine) side can be estimated [Roos, 2007, ref [5]] according to

$$J_{gs} = \frac{\rho_g \pi r_g^4}{32 C_{gr}^4} \frac{(9bn_s^2 + b_c n_s^2 - 36bn_s + 52b)}{(n_s - 1)^4}$$

Where $C_{gr} = r_g / r_{gr}$, ρ_g is material density, $n_s = n_{s1} = n_{s2}$ is gear ratio and the other parameters are according to Figure 9.



$$r_{g,1,2} = 0{,}0185$$
$$r_{gr,1,2} = 0{,}0163$$
$$b_{1,2} = 0{,}006$$
$$b_{c1} = 0{,}013$$
$$b_{c2} = 0{,}0049$$

Figure 9. Planetary gear notation and parameter values for this assignment.

The gear inertia of stage one as calculated from the above equation is referred to the motor axis **whereas the inertia of stage two must be recalculated via the transmission ratio of stage one**.

The inertia of the two output shafts should be estimated from Figure 7 by assuming an approximate average diameter and combined length of the two pieces (the gear ratio is one and all pieces are assumed rigid).

Task 6: Derive a model of the complete tool and nut rotation for the rundown phase, i.e. a 4th order model, using the simplified motor model that you have developed and the previously described assumptions. **Note**: During the run-down phase, the screw stiffness ***is not*** involved but friction damping in the nut/screw ***is*** involved. ***Use the state vector representation suggested in Janschek on page 237***, equation 4.37, i.e.:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix}$$

Where φ_1 and φ_2 are motor and transmission angles (1: motor side, 2: tool side of the final angle gear). **Note**: Since in this case the tool is considered fixed and the nut is considered free running the angle φ_2 equals the angle θ_n of the nut.

The model simplifications according to the figure 8 above can be applied.

Derive the following

1. Differential equations symbolically
2. Transform the differential equations to a state-space model
3. Create the state space model in Matlab
4. Apply the state space model in Simulink and make sure to output all states.

5. Simulate the model by applying a voltage step of 1 V and convince yourself and the report reader that the model works properly.

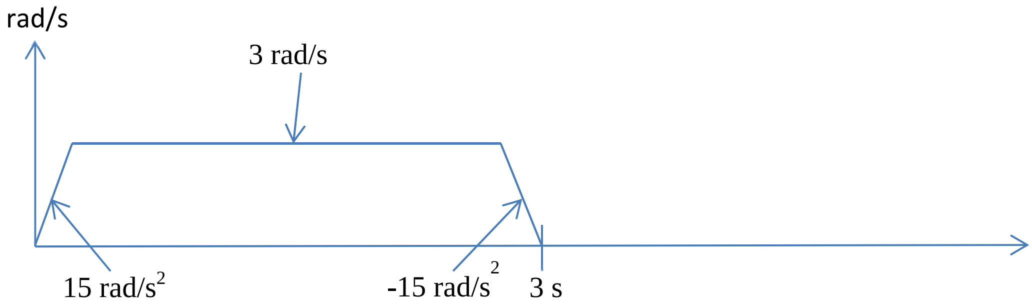
Show models 1-3 in the report. Show the simulation result (5) and convince yourself and the report reader that the model works properly by commenting on the below four hints.

Note: The systems to be modelled have a wide spread in natural frequencies. Such "stiff" systems can be hard to simulate and the choice of integration routine (solver in Matlab) can be critical. It is recommended to use *ode23t(mod.stiff/Trapezoidal).* Select method under Simulation/Configuration parameters in your Simulink model file window.

Hints:

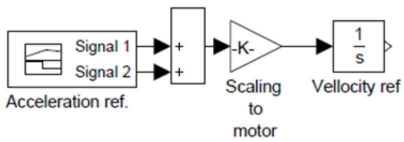
- Study the state variable traces and make sure that they are scaled correctly relative to each other (given the different gear ratios).
- The velocity saturates, but at which level, why, and is it correct?
- Estimate the mechanical time constant of the system and compare with that of the motor as specified in the motor data sheet. Explain the difference.
- If the internal damping would be low you would see some oscillatory behaviour. How many frequencies would you expect to see?

Task 7: Develop a PI angular velocity controller for the nut rundown phase by manual tuning. (You can for this case use the standard PID block in Simulink). The control target is to follow the below reference signal.



Try to make the control system behave nicely with smooth response, without overshooting and without steady state error.

For the reference velocity it is recommended to use the below construct (Model is on social as well: *Components.mdl*). The model sets constant angular velocity after a short acceleration phase. The scaling factor n2 (under K in the figure) is used to scale from 3 rad/s on the output shaft to the motor shaft.



Present clearly the control system response and your controller parameters. Present also the behaviour of the control voltage to the motor.

Task 8: This task deals with the bolt tightening case, which corresponds to an M8 screw of strength class 8.8, pitch 1.25 mm and with free length of 100 mm (thickness of joint members). For the bolt tightening process we assume the run-down and alignment phases are passed and that the elastic clamping phase starts at zero velocity.

Based on task 6 extend your model to include the screw/nut/joint subsystem and include the given data and the parameter data that you are supposed to calculate given the system dimensions and guidelines above. The extended model now corresponds to fig 8 above (including also the screw). Note: The translational stiffness of the screw can be translated into rotational stiffness as acting on the nut (via the pitch of the screw).

It is a good idea to use the sign (of matrix elements) and symmetry properties of the stiffness, damping and system matrices to check that the modeling is at least principally correct (Janschek p. 233, 236-237). Use the same generalized coordinates and states as before:

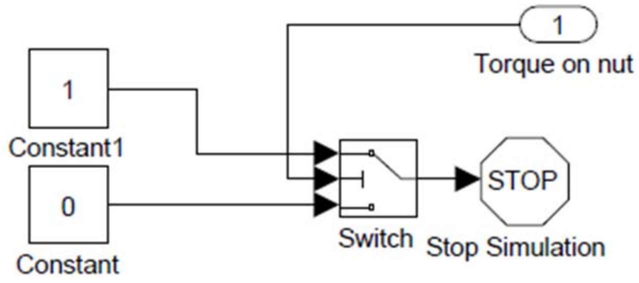
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix}$$

Where φ_1 and φ_2 are motor and transmission angles (1: motor side, 2: tool side of the final angle gear).

- Clearly present your state space model in symbolic form in your report.
- For a 24V input voltage to the motor how large is the torque on the nut? (You can calculate it by some extra blocks in Simulink using the angular twist between gear output and nut).
- How much is the corresponding turning angle of the nut?

Task 9: Develop an angular velocity controller for the elastic clamping phase ***considering the tool to be fixed***. The targeted tightening torque is 25 Nm and the targeted tightening speed is 3 rad/s. For the bolt tightening process we assume the run-down and alignment phases are passed and that the elastic clamping phase starts at zero velocity, and should end when the targeted torque is reached. Hence, ***you develop essentially a velocity controller and a torque limiter that switches the system off at the desired torque.***

To implement the controller switch-off when the targeted torque is reached you can use the below construct (which is also on KTH Social: *Components.mdl*). The input is the torque delivered at the nut as sensed by the ideal torque sensor (The placement of the torque sensor can be either before or after the gearing – it is up to you). In the switch block there is a threshold to set, which should correspond the targeted torque level. The stop simulation block activates when the input is non-zero.



For the velocity controller design, you should **start with a P-controller, present and discuss the results (plots and controller setting), and then do the same for a PI-controller**.

Task 10: **Is the achieved tightening in good order** with respect to the strength properties of the screw? **How large is the tightening nut angle**? Consider that good screw tightening should be such that the screw tensile stress is close to the yield strength of the screw material.

If the tightening is not in good order (the force should be too high!), extend the model with coulomb friction, i.e. $Friction\ torque = \mu \cdot F_s. \cdot r_s$ between nut and screw and where μ is the friction coefficient, F_s the screw force and r_s the nominal radius of the thread. Typical friction coefficients can be in the range 0.1-0.2. **Check again that the achieved tightening is in good order, and if not reduce the torque target to a good level. How large is now the tightening angle**?

(Note that with Coulomb friction, the system is no longer linear, since friction force can only oppose motion. The modelling we do here still works in the linear domain because the nut motion is strictly in one direction. That is, we don't expect the friction to drive the nut, only oppose its motion.)

If needed, adjust your controller parameters.

Finally, show the results of your tightening process. How large is the deflection (twist) in the transmission, as seen on the load side and on the motor side respectively? Discuss the results.

Task 11: **When solving this task, please consult "Hints regarding task 12" on page 15-17 below**. Extend your fourth order model to a sixth order model by adding the operator model according to Figure 5, **and present the model**. Please note that now the angular motion of the nut (with respect to the fixed reference frame) is not equal to the angular motion of the output shaft (with respect to the tool) since the tool is no longer fixed. Again, ***it is a good idea*** to compare with Janschek p. 233, 236-237 and realize that model of Figure 4.8 (p 236), is complete enough to handle also this situation.

For the complete system model, the model of Figure 8 is to be extended with the model of Figure 5. Again ***note*** that the resulting model is still linear and agrees with the multiple mass oscillator according to the course text book (Janschek page 236-237). Use the following state vector definition:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{pmatrix}$$

Where φ_1 is tool rotation around the vertical axis (θ in figure 3) and φ_2 and φ_3 are motor and transmission angles (2: motor side, 3: tool side of the final angle gear).

Apply the same controller as in Task 10 and analyze the angular motion amplitude of the tool (discomfort to the operator). Check again the deflection (twist) in the transmission, is there a difference compared to Task 3? Present results and discuss them.

Task 12: How much electrical energy is consumed in one tightening? Assume the same amount is consumed during the combined run-down and alignment phase, how many screws can be tightened on one battery charge?

3 Reporting

There is **ONE** report per group.

The report is evaluated based on:

1. Correctness and coverage of the different assignment tasks.

2. Structure and understandability, i.e. the report should be easy to follow including the reasoning behind your solutions, results and conclusions. ***Be focused and precise rather that giving unnecessarily lengthy textual descriptions***.

3. Language and clarity.

As to the specific content of the report, ***please read carefully and stick rather closely to what is actually asked for under the different assignment tasks***.

Good luck!

4 References

[1] Lin JH, Radwin RG, Richard TG, A single-degree-of-freedom dynamic model predicts the range of human responses to impulsive forces produced by power hand tools. Journal of Biomechanics, 2003 Dec; 36(12):1845-52.
Available on-line via the KTH Library.

[2] Data sheet. Cordless angle nutrunner - transducerized type T e n s o r E T V S T B 6 2 - 5 0 - B 1 0.
Available on course homepage.

[3] Atlas Copco Tensor STB (Overview of tool and with some tools data).
Available on course homepage.

[4] Data sheet. Faulhaber brushless DC-motor 3268_BX4
Created from: fmcc.faulhaber.com/details/overview/PGR_4457_13822/PGR_13822_13814/en/

[5] Roos, Fredrik, Towards a methodology for integrated design of mechatronic servo systems, Doctoral thesis, KTH, 2007, Trita-MMK, ISSN 1400-1179; 2007:07. Accessible at: http://kth.diva-portal.org/smash/record.jsf?searchId=1&pid=diva2:12432

Hints regarding task 12

In task 12 you should extend the model with the human operator. This means that the model order will increase from 4 states to 6 states. The below modeling then applies given the assumption that the output inertia is concentrated at the nut.

Below is a complete principle description of the proposed model, which is then further elaborated in the following text.

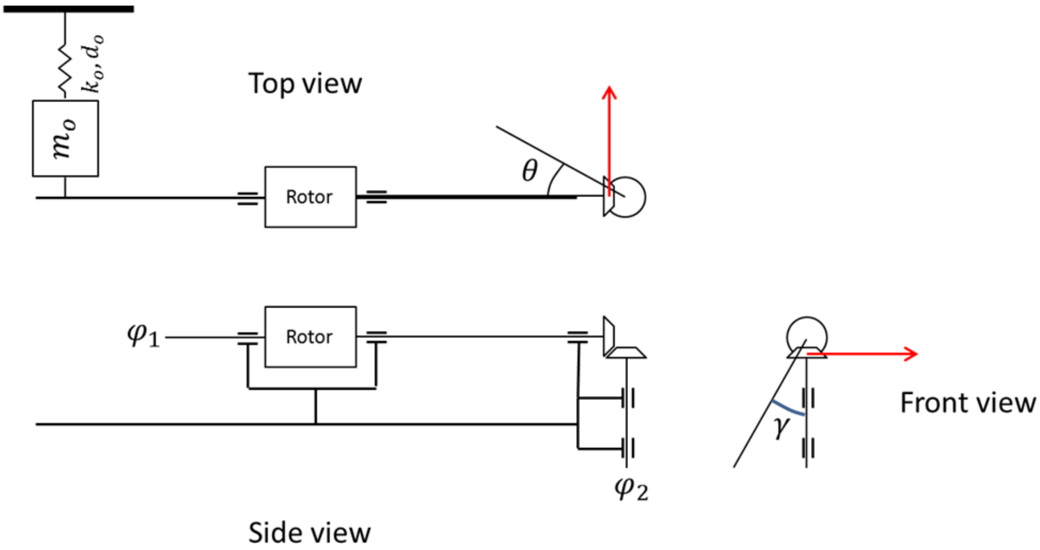


Figure 1. A principle sketch of the mechanical design

For clarity and simplicity the planetary gears are here initially assumed having ratio 1, the same goes for the final angle gearing. The coordinates (φ_2, θ) are defined in the inertial reference frame. The angle $\varphi_1$is defined in a frame fixed to the tool chassis.

1) Let's say we fix $\varphi_2 = 0$, then applying a torque on the motor will cause the motor output shaft to twist and the tool chassis to accelerate around θ. Hence, the motor is acting in a relative configuration (however between orthogonal generalized coordinates) in between tool chassis inertia and output shaft inertia. The torque due to the shaft twist is transferred to the output shaft via a gear tooth contact force as indicated in the Fig 1. In the θ direction, a reaction force is established causing the tool to turn, and the connection between this force and the operator goes via the rigid tool chassis. In the γ direction this force is balanced by our assumed holonomic constraint saying that the tool cannot rotate around this axis. Likewise, the rotational motor reaction torque onto the tool chassis is balanced by the same holonomic constraint.
2) Since φ_1 is defined relative to the tool chassis the back emf is a function of the derivative of this relative angle. Since this angle is not defined in an absolute frame it does not correspond directly to the absolutely defined output angle $\varphi_2$which essentially is what we want to control. Hence, the sensor feedback is relative and does not correspond directly to the controlled variable. We cannot cope with this problem in the controller since we do not

measure θ. However, the error will be rather small assuming that the operator keeps the tool rather fixed.

3) The above two points give us the following complete model, now including also the planetary gear.

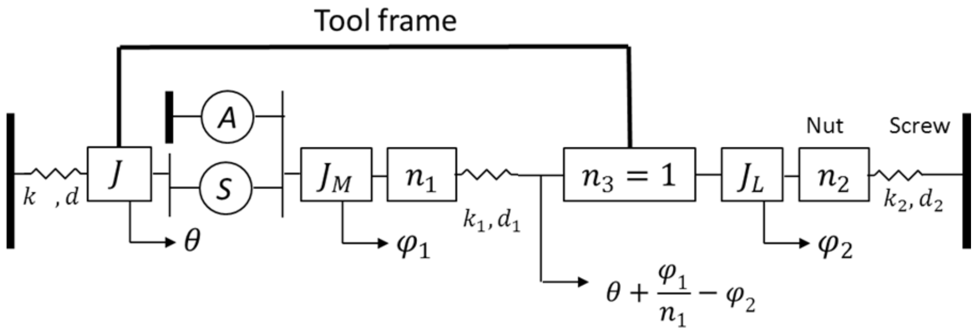
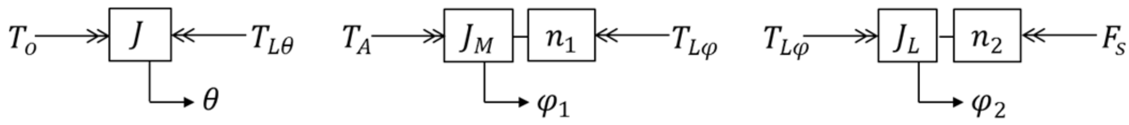


Figure 2. The more complete model

4) Model notation
   - In Fig 1, (m_o, k_o, d_o) corresponds to the operator model.
   - In Fig 2, the operator part of the model has been transformed to rotational motion and the operator mass adds to tool inertia (J_T around the z axis (battery+tool itself)) such that we get the new parameters (J, k, d). Note that when doing this transformation we are passing through the transmission n_0 in opposite direction compared to when we did this transformation at the motor side. n_o is the transmission ratio from operator translational motion *x* to tool rotation θ around the z axis.
   - Actuator A and sensor S are now active in a relative sense, i.e. not in relation to a fixed reference frame. This will affect your B and C matrices, i.e. how the motor affects the tool and what the sensor actually measures (the difference between to states), and this is what is to be fed back to the controller. See Janschek page 251-252.
   - The model part to the right of the motor indicates that the rotational deflection of the output shaft not only depends on motor angle but also on tool angle θ. This is explained by the tool frame. I.e. the nut (φ_2) can be turned either by turning the tool or by turning the motor. This now means that the deflection of the output shaft is
     - $\theta + \frac{\varphi_1}{n_1} - \varphi_2$

5) In Fig 2 the gear ration $n_3 = 1$ corresponds to the 90° output angle gear. Note here that the tool frame (and operator mass, both included in J) is rotating around a vertical axis. The torque (as a result of the motor actuation) acting on the tool frame is due to a reaction force due to the "red" force in the upper part of figure 1. Apparently, this torque has the same magnitude as the output torque acting on J_L, i.e. corresponding to the torque T_L due to the twist in the transmission (planetary gear). In the below figure the inertial components are separated and the forces/torques acting on them are indicated. Note the notation in the figure and that $|T_{L\theta}| = |T_{L\varphi}| = T_L$, i.e. the same torque magnitude but around different axes.


T_O
J
$T_{L\theta}$
θ
T_A
J_M
n_1
$T_{L\varphi}$
φ_1
$T_{L\varphi}$
J_L
n_2
F_s
φ_2