ROYAL INSTITUTE OF TECHNOLOGY

Lab 1 tutorial Longitudinal Dynamics – Slip Control

SD2231 – Applied vehicle dynamics control

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1 Introduction

The following laboratory assignment is intended to give you knowledge and experience in the area of longitudinal control of rail and road vehicles.
More specifically, the focus will be on controlling the wheel behaviour during operation, and thus increasing the stability of the wheel as well as acceleration and braking performance of the vehicle.

2 Theory

2.1 Slip control

Traction and braking forces of ground vehicles (here passenger cars and trains) are transferred from the wheel through the contact patch onto the ground, which can be a road or a rail. Each wheel can be categorised into a driven or non-driven wheel. The driven wheel is connected to the motor of the vehicle and experiences its demanded tractive torque. If a torque is applied for acceleration or coasting at a certain speed, it leads to the effect that the wheel spins with a higher speed than the longitudinal speed of the vehicle. Thus, the driven wheel does not roll, but actually rotates faster than the corresponding longitudinal velocity of the vehicle. A non-driven wheel is free-rolling and rotates with the same speed as the vehicle. Brakes are usually found on every wheel. During braking the wheel rotates at a slower speed that the longitudinal speed of the vehicle.

This difference in speed between the wheel speed and longitudinal vehicle speed is described as the slip of the wheel. Depending on the driving condition, tractive slip s_t and braking slip s_b can be distinguished:

$$s_t = \frac{r\omega - v_x}{r\omega} \quad \text{and} \quad s_b = \frac{v_x - r\omega}{v_x} \tag{1}$$

where r = wheel radius, ω = rotational speed of the wheel and v_x = longitudinal speed of the vehicle. A qualitative slip curve of a rubber tyre including the maximum and dynamic friction coefficient (μ_{max} and μ_d) and the critical slip s_{crit} is shown in Figure 1. The maximum friction coefficient is reached at 10 – 20 % slip, depending on various tyre parameters (e.g. tyre pressure, rubber compound, thread, etc.), vehicle parameters (toe angle, camber angle, vehicle speed, etc.) and external factors (temperature, road surface, type of asphalt, etc.). Examples of slip curves are seen in Figure 2 for a road vehicle. The deviating characteristic of the brake slip for snow can be explained by snow piling up in front of the tyre like a wedge.

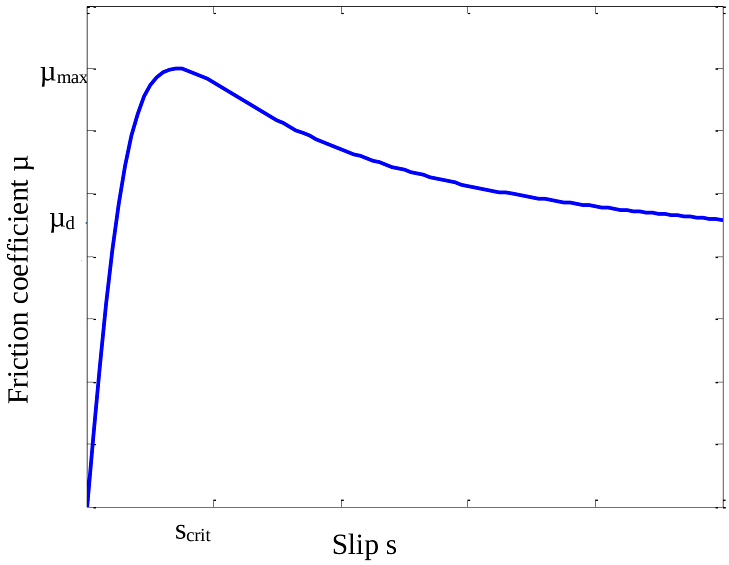

Figure 1. Qualitative slip curve of a typical rubber tyre of a passenger car.

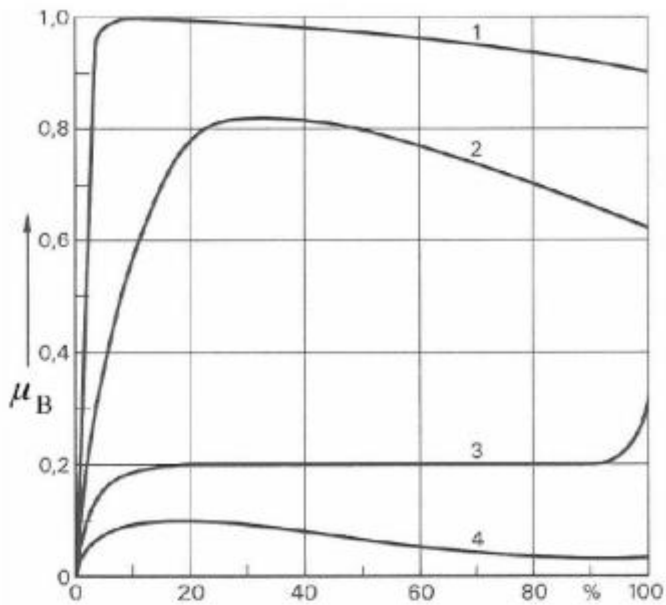

Figure 2. Typical brake slip curves of a rubber tyre for different road conditions: 1. Dry road, 2. Wet road, 3. Snowy road and 4. Icy road.

Rail vehicles have no rubber tyres, but a metal to metal contact patch. As explained before, slip is necessary for the force transfer of the wheel. The slip is created due to the massive weight of the rail vehicle. Both, the wheel and the rail expand and counteracts in different regions when the wheel is driven. Examples of slip curves are seen in Figure 3 for a rail vehicle.

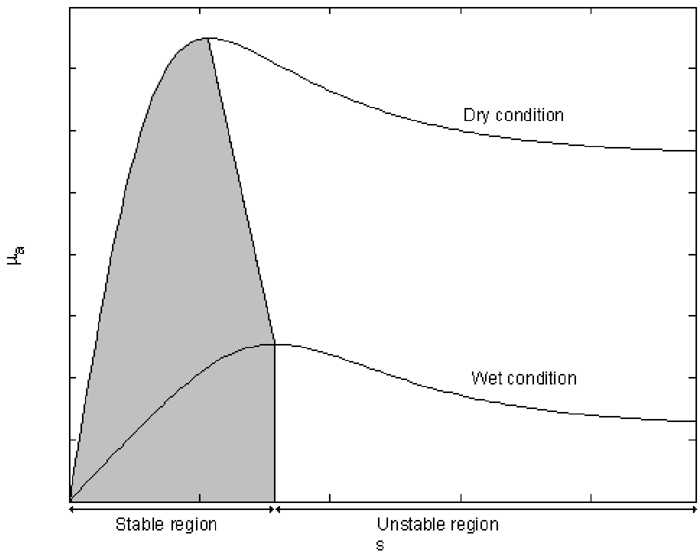

Figure 3. Brake slip curve of a rail vehicle for different rail conditions.

The aim of slip control is to keep the slip in a certain range during acceleration and braking in order to achieve the maximum traction and steer-ability of the vehicle. Special focus has to be given to braking manoeuvres.
If the braking torque on the wheel is too high, the wheel will lock and start sliding. Due to the dependency of longitudinal and lateral tyre forces (friction circle), a locked wheel will not be able to build up any lateral force. The longitudinal force utilises the dynamic friction coefficient at 100% brake slip, which is lower than the maximum friction coefficient, and therefore the stopping distance will also be longer. Thus, in order to prevent the wheel from locking, an anti-lock braking system (ABS) is used. It controls the relative slip of the wheels during braking to prevent wheel lock-up.
A typical case of a spinning wheel during acceleration is found on low friction ground, such as icy roads. The wheel starts spinning faster than the vehicle is moving forward, leading to a loss of the lateral tyre force and smaller longitudinal tyre force. A traction control system (TCS) is applied in order to utilize the available adhesion effectively and prevent high tractive slip values. Both systems help also to reduce the wear of a tyre or a wheel.

The following challenges for the slip control have to be considered:
• Detection of the slip on the wheel.
• Control of the slip within certain thresholds.

Anti-lock braking system
ABS was introduced in 1978 as the first active chassis system in passenger cars. The basic objective is to prevent the wheel from locking while braking by modulation of the brake pressure; i.e. braking just as much that the slip ranges in the area of the critical slip where a maximum force can be transferred, see Figure 4. Besides reaching a shorter stopping distance, the main advantage is the steer-ability during braking. Each wheel is controlled individually in order to utilise the available friction potentials at their optima.
A major practical challenge of the ABS is that wheel slip cannot be measured with inexpensive sensors on a passenger vehicle. Often the only measurements

available are those for the rotational speeds at each wheel. Longitudinal speed has to be determined within algorithms in order to calculate the actual slip values for each wheel.

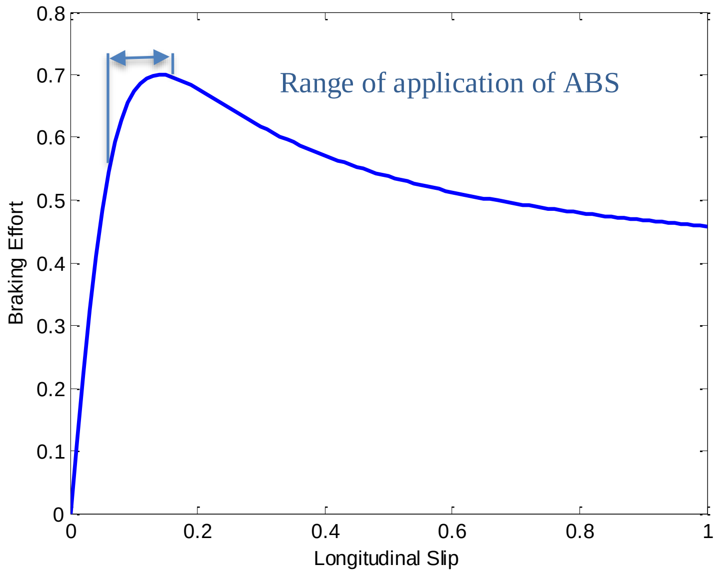

Figure 4. A typical slip curve of a braked wheel.

2.2 Control Theory

There are different control strategies which could be used for slip control. PID control, neural networks and fuzzy logic are some of the methods. Here the focus will be on the PID controller for controlling the slip.

2.2.1 PID Control

PID control is by far the most common way of using feedback in natural and man-made systems. It is a simple three term controller composed of a proportional, integral and derivative term where the transfer function of the most basic form of PID controller is;

$$C(s) = K_P + sK_D + \frac{K_I}{s} \tag{2}$$

Where K_P, K_I and K_D are proportional, integral and derivative gains respectively.

The controller could be implemented into system in different structure (output feedback, error feedback and so on). In Figure 5 the error feedback structure is demonstrated. Where "u" is control or input signal, "Y" is output or measured signal and "R" is reference or command signal.

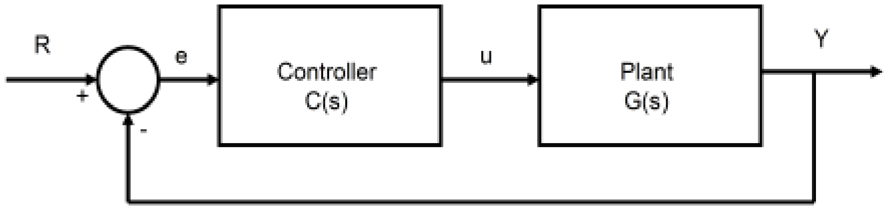

Figure 5. Sample of a closed loop control.

Generally in this context, the role of the PID-controller is to regulate the wheel slip and thereby the use of the adhesive force. A control method can be formulated by examining the adhesion characteristics. This can be done by choosing a reference slip and use this as a control signal. From the characteristics of the slip curve it is easy to observe that different slip implies different optimums of the adhesive force. The reason is that the adhesion coefficient differs between dry, wet and icy road, see Figure 2.

A PID controller takes control action based on past, present and prediction of future control errors and tries to minimize the error. However, in the absence of a feed forward term, the output never reaches the reference and hence we are left with non-zero steady state error.

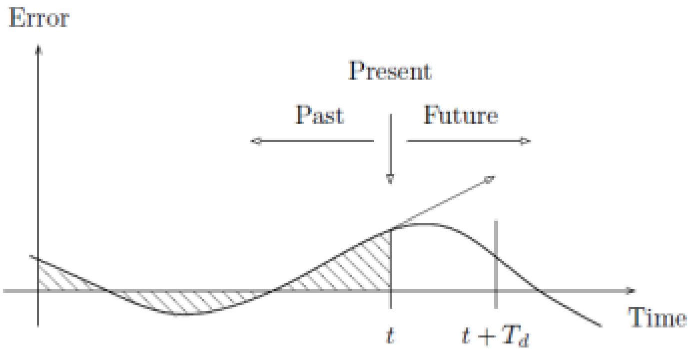

Figure 6. Formation of control signal, $\boldsymbol{u}$, from the error, $\boldsymbol{e}$.

The ideal version of the PID controller is given by the formula;

$$u(t) = k_p \cdot e(t) + k_i \cdot \int_0^t e(\tau) \cdot d\tau + k_d \frac{e(t)}{dt} \qquad (3)$$

Where u is the control signal and e is the control (tracking) error ($e = R - Y$), which is sent to the PID controller. The reference value (R) is also called the set-point. The control signal is thus a sum of three terms: a proportional term that is proportional to the error, an integral term that is proportional to the integral of the error, and a derivative term that is proportional to the derivative of the error.

The proportional part acts on the present value of the error, the integral represents an average of past errors and the derivative can be interpreted as a prediction of future errors based on linear extrapolation, Figure 6.

The controller can also be parameterized as:

$$u(t) = k_p \cdot \left(e(t) + \frac{1}{T_i} \cdot \int_0^t e(\tau) \cdot d\tau + T_d \frac{de(t)}{dt} \right) \qquad (4)$$

where T_i is the integral time constant and T_d is the derivative time constant.

2.2.2 Designing and tuning the PID controller

We are most interested in four major characteristics of the closed-loop step response, Figure 7. They are;

1. Rise Time (t_r): the time it takes for the plant output to rise beyond 90% of the desired level for the first time (0%-100% for underdamped second order systems, 5%-95% for critically damped and 10%-90% for over damped systems).
2. Overshoot: how much the peak level is higher than the steady state.
3. Settling Time (t_s): the time it takes for the system to converge to its steady state.
4. Steady-state error: the difference between the steady-state output and the desired output.

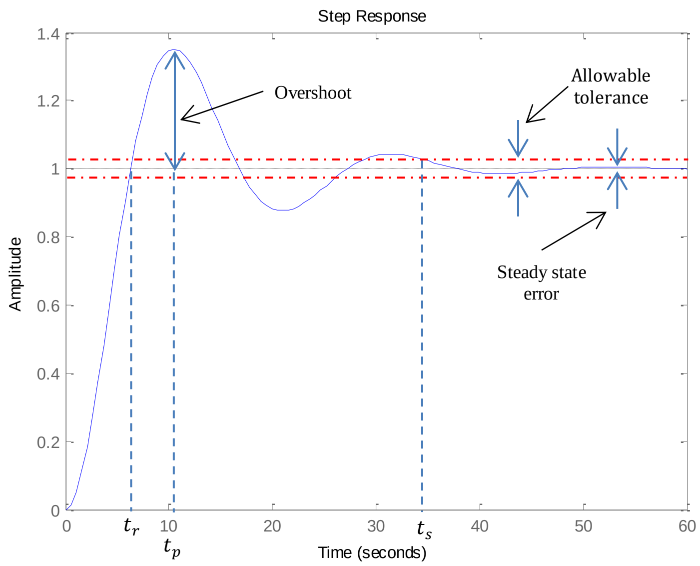

Figure 7. Characteristics of the closed-loop step response.

Generally, a proportional controller (K_P) will reduce the rise time but never eliminate the steady-state error. An integral control (K_I) will eliminate the steady-state error for a constant or step input, but it may make the transient response slower. A derivative control (K_D) increases the stability of the system, reducing the overshoot, and improving the transient response.

The effects of increasing each of the controller parameters K_P, K_I and K_D on system characteristics can be summarized in Table 1.

Table 1. Relation between controller parameters and system characteristics.

Response	Rise Time	Overshoot	Settling Time	S-S Error
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small Change	Decrease	Decrease	No Change

Note that these correlations might not be explicitly show the overall behaviour of the system after apply the individual parameter change, because K_P, K_I and K_D are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference for determining the values for K_P, K_I and K_D. Typical step response of a system affected by different controller is depicted in Figure 8.

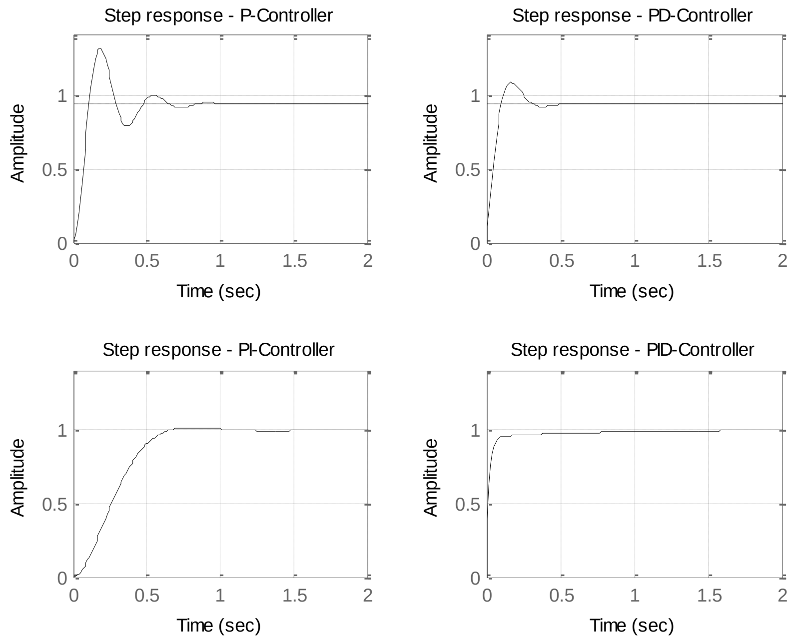

Figure 8. Step responses for different controllers.

Typical steps for designing a PID controller could be listed as following:

1. Obtain an open-loop response and determine what needs to be improved.
2. Determine what characteristics of the system need to be improved.
3. Use K_P to improve the rise time.

4. Use K_D to reduce the overshoot and settling time.
5. Use K_I to eliminate the steady-state error.
6. Adjust each of K_P, K_I and K_D until you obtain a desired overall response. You can always refer to the table shown earlier find out which controller controls what characteristics.

Finally, please keep in mind that you do not need to implement all three controllers (proportional, derivative, and integral) into a single system, if not necessary. For example, if a PI controller gives a good enough response, then you don't need to implement a derivative controller on the system. Keep the controller as simple as possible.

2.2.3 Estimating PID parameters

There are predefined and well-known methods for deciding the values of the PID controllers. However, the most popular methods were given by Ziegler and Nichols (1942), Astrom and Hagglund (1984), and more recently Zhuang and Atherton (1993) and Luyben and Eskinat (1994). Different tuning methods are due to different control objectives such as reference following and disturbance rejection, and different plants like first-order and second-order models.

Ziegler-Nichols is useful when there is no mathematical relation available for the system. This method is based on experiments and proposed rules for determining initial values of K_P, K_I and K_D based on the transient step response of a plant. It applies to plants whose unit-step response resembles an S-shaped curve with no overshoot where the system doesn't have any complex poles. This S-shaped curve is called the reaction curve, Figure 9.

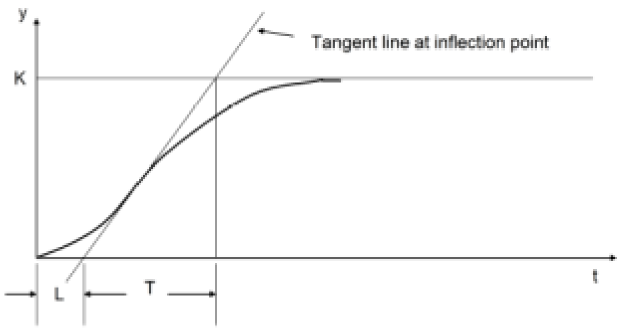

Figure 9. Reaction curve.

The S-shaped reaction curve can be characterized by two constants, delay time L and time constant T, which are determined by drawing a tangent line at the inflection point of the curve and finding the intersections of the tangent line with the time axis and the steady-state level line.
Using the parameters L and T, we can set the values of K_P, K_I and K_D according to the table below.

Table 2. Parameter identification using Ziegler-Nichols method

Controller	K_P	K_I	K_D
P	$\frac{T}{L}$	0	0
PI	$0.9\ \frac{T}{L}$	$0.27\ \frac{T}{L^2}$	0
PID	$1.2\ \frac{T}{L}$	$0.6\ \frac{T}{L^2}$	$0.6T$

These parameters will typically give a response with an overshoot about 25% and good settling time. We may then start fine-tuning the controller using the basic rules that relate each parameter to the response characteristics.

2.3 Frequency analysis

The frequency response method of controller design has certain advantages, especially in real-life situations such as modelling transfer functions from physical data.

2.3.1 Nyquist stability criterion

This is a unique method for determining stability of a closed loop system. A closed loop system is stable if all of the closed loop poles are in the left half of the s-plane. That's a very basic fact about a system. Using this technique it is possible to get information about closed loop pole location by plotting open loop frequency response data.

In order to understand the Nyquist stability criterion, having a perception of what a Nyquist plot is necessary. A Nyquist plot is a plot of the magnitude of the amplitude gain and phase shift of a system for sinusoidal inputs. Furthermore it is a map of the $j\omega$-axis in the s-plane using a transfer function, G(s), as the mapping function.

The Nyquist plot allows us to predict the stability and performance of a closed-loop system by observing its open-loop behaviour. The Nyquist criterion can be used for design purposes regardless of open-loop stability (remember that the Bode design methods assume that the system is stable in open-loop and when we try to determine the close loop stability of an unstable open loop plant with Bode, we might possibly come with wrong answers). Therefore, we use this criterion to determine closed-loop stability when the Bode plots display confusing information.

3 Assignment

3.1 Simulation setup

The simulation model in this assignment is based on Matlab/Simulink. You will use two different models to get to know the characteristics of the controller and implement a control strategy.
First, a small PID-controller introduction file gives you the possibility to investigate the different influences of each part of the controller (proportional, integral, derivative). The system that shall be analysed is a single mass system with a spring and a damper mounted in parallel, see Figure 10. The wheels are frictionless and positive x-direction is given. Please find further explanations in the comments of the file "PID_intro.m".

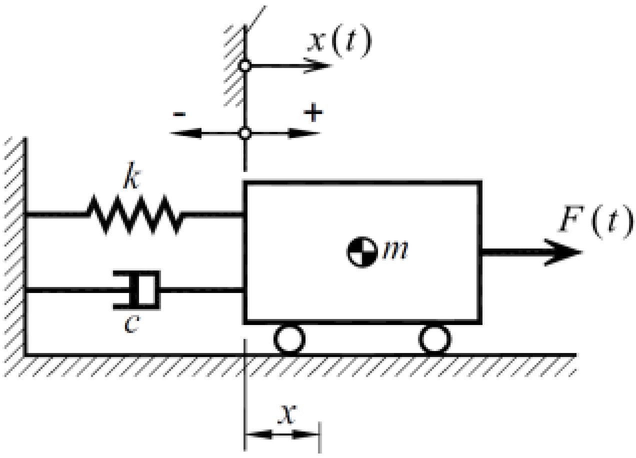

Figure 10 Single spring-mass-damper system.

The second model is the actual quarter car/train simulation model you will work on. It consists of an m-file initialising all parameters including the slip curves. Adjustments to the parameters are done in this file. The vehicle model is implemented in Simulink. The controller block (orange) is the subsystem that you will work on. Besides the report, you will hand in this controller block, so we can check each of the tasks. It is prepared in a way that you will have to make changes to this block only.
The driver model is a simple speed-dependent model that provides values between 0 and 1 for the throttle signal and 0 to -1 for the brake pedal signal. The delivered signal will always accelerate to a certain speed Dri.v_lim, hold this speed for the time Dri.t_const and decelerate the vehicle. The values between 0 and |1| can be set by changing the throttle and braking gain Dri.Ka and Dri.Kb.
A sensor block is given with the available signals in accordance to a real vehicle. Please use these signals only.
The actuator models represent a combustion engine and friction brakes with integrated time delays. The actuator torques are combined on the axle and

transferred to the wheel, which includes wheel dynamics with a given inertia and the tyre model that is based on a Magic Formula tyre model. The longitudinal dynamics represent the vehicle. Its acceleration and speed is determined here. Furthermore, a vertical dynamics model is implemented, yet deactivated. It will be used in the final task.

3.2 Tasks to do and questions to answer

In this section you will find information about the tasks that you should do in this laboratory exercise and also the questions that you have to answer in the report. Please read through this section and the following section 3.3 before you start working with your assignment.

Important notes:

* while writing the report, write the tasks in the order you have been asked in the handout and clearly mention the task and subtask number (Task 1.a, Task 2.c, etc) when you are giving explanations to a certain graph, question etc.

** The task which include plots or table, put them exactly after the relate task and not at the end of the document.

*** All numerical results should be arranged in tables in their respective section but not spread over the text.

Task 1. Determine control parameters.

Task 1.a: Derive the equation of motion for the single mass system in Figure 10 and build the transfer function $T(s) = \frac{X(s)}{F(s)}$. Implement the transfer function T(s) in the m-file *PID_intro*. The m-file will plot the step response of the transfer function without an active controller. Sliders for each control parameter as well as for the mass, spring stiffness and damping coefficient will help you to adjust the step response.

Task 1.b: Use the model to find the control parameters that correspond to Figure 8 for a P-, PD-, PI- and PID-controller. You can use Table **1** to estimate the effectiveness of each controller parameter rather than a pure trial and error method.

> Note: please be aware that achieving a relatively similar behavior is enough!

Extra task 1.c: *Change mass, spring stiffness and damping coefficient to typical values of a quarter car model of a passenger car. Find the control parameters for the four controllers mentioned above. Note that the slider boundaries (min and max value) might need to be adjusted in the code.*

Extra task 1.d: *Analyse the stability of the open loop system and closed-loop systems using the frequency analysis method for all the controllers mentioned above, a total of five analyses.*

Task 2. Integrate feed-through control

Task 2.a: The current model will not run when pressing the simulate button. The controller block is empty and needs to be filled. The first task is the implementation of a feed-through control that translates the driver signals into actuator signals.

Task 2.b: Simulate both vehicles for the 3 given friction levels of the passenger car and 1 given friction level for the train. Change the friction levels and the vehicle type in the initial file accordingly and minimise the time to accelerate from 0 km/h to 90 km/h. Compare it to an acceleration run where the wheel speed is 10% higher than the optimum wheel speed which you got from acceleration minimisation. How much time do you gain or lose relatively? (Attach the graphs of both stable and unstable cases and present your results in a clear table and reflect and discuss the outcome.)

Task 2.c: Minimise the stopping distance while braking to standstill (standstill can be considered when $v_x$crosses zero the first time). Compare it to the stopping distance with a locked wheel. How many meters do you gain or lose? (Attach the graphs of both stable and unstable cases and present your results in a clear table and reflect and discuss the outcome.)

Task 3. Design PID controller

Task 3.a: Implement the tractive and braking slip in the control block and plot the slip over time.

Task 3.b: Add activation logics for the controllers (TCS and ABS). Note that it shall work for all driver inputs (throttle and brake pedal) as soon as traction threshold is reached.

Task 3.c: Design your own P-, PD-, PI-, PID-controller and compare /discuss the results for the different slip curves and vehicle types.

Task 3.d: Discuss each system and compare them based on system characteristics such as settling time, rise time, overshoot and steady state error. (for each of the controllers; P-, PD-, PI-, PID-controller)

Task 3.e: Is that necessary to use all type of controllers to follow the required response? Motivate your answer.

Task 4. PID controller optimisation

Task 4.a: Introduce the best controller that has the fastest response, least overshoot and smallest steady state error as well the best acceleration time and stopping distance. Assume that system is underdamped.

Task 4.b: Is it possible to eliminate the steady state error while maintain the previous mentioned properties?

Extra task 4.c: *Estimate the utilized friction value during the manoeuvre with the given sensor signals.*

Extra task 4.d: *Optimizing the given controller for uneven road/rail. Activate uneven road/rail disturbance in the vertical dynamics model.*

Task 5. Introducing disturbance to the system

- ***Extra task 5:*** *Introduce a disturbance such as an icy spot on a wet road into system for both acceleration and braking maneouvres. How does the disturbance change the system behaviour and what has to be adopted. Propose an optimized controller which accounts for this disturbance.*

Competition!

The winner should reach the **maximum acceleration** and **minimum braking distance** while keeping **minimum steady-state error** which can be realised through a **low cost controller**.

4 Examination

For details on the grading process of this course please consult the SD2231_Grading_Criteria.pdf document on Bilda.

4.1 Report writing

A good report includes exactly the information which is needed for the reader to understand the results and nothing more. Start working with the report from the start by writing down each of the steps you have done including the approach, gathered information and the rationale of each of the steps i.e. why you decided to go in a certain direction and why that is better than another way. This will help you in the course and it will be easier to finalize the report in time.

Note: arrange your report hierarchy exactly based on the tasks arrangement in this hand-out.

Include the following files when submitting your report:

- Your finalized report as *pdf* or *docx*
- A zip file including all your organized Matlab and Simulink files, teachers have to be able to run your files after downloading them from Bilda.

Do not forget to answer the questions in this hand-out!

You should use the template on Bilda for your final report.

4.2 People that will support this laboratory assignment

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