Lab 3

Assignment

Applied Vehicle Dynamics Control

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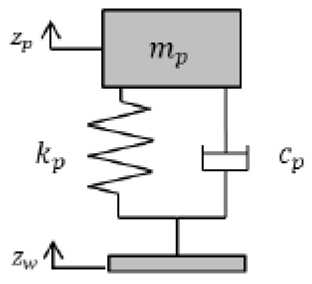
Task 1.1

Consider the single degree of freedom system shown in Figure 1 with the input disturbance $z_w(t)$. Write the equation of motion for mass m_p. Natural frequency and damping ratio of the system can be found by using the following relations:

$$\boldsymbol{w_n} = \sqrt{\frac{\boldsymbol{k_p}}{\boldsymbol{m_p}}}$$

$$\zeta = \frac{\boldsymbol{c_p}}{\boldsymbol{c_c}} = \frac{\boldsymbol{c_p}}{2\sqrt{\boldsymbol{k_p m_p}}}$$

Explain the physical interpretation of these two values. What is meant by under damped, critically damped and over damped systems?



Natural Frequency:
It is the frequency at which the system continues to oscillate in absence of any damping force. When there is no damping in the system, the systems continues to oscillate because of the linear restoring force.

Damping Ratio (ζ) :
The damping ratio explains how the oscillations in the system decay over time over a disturbance caused in the system. The damping factor is always positive or zero. When it is zero, the system oscillates indefinitely. The damping of any mechanical system can be described as being one of the following:

Figure 1 Single degree of freedom system. $zw(t)$ is the input displacement.

- Under damped Condition ($0<\zeta<1$)
  This system oscillates (at a reduced frequency than the un-damped case) with a exponentially decreasing amplitude. The system oscillations stop after some time. Though as per the control characteristics, there is no overshoot in the system.

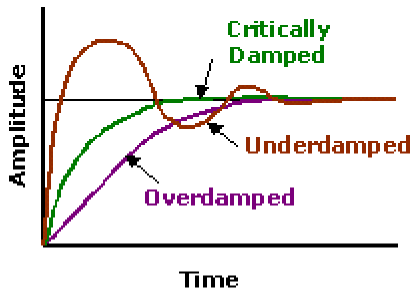


Figure 2 Types of Damping

- Over damped Condition ($\zeta<1$)

  The system exponentially decays to equilibrium without any oscillations. As per the control characteristics, there is an overshoot in the system and the settling time is more than that of a critically damped system

- Critically damped($\zeta = 1$)
  The system returns to equilibrium without any oscillations in least possible time without any overshoots.

Task 1.2

Calculate wn and ζ for the system in figure 1 Considering the following parameters: (assuming that zw is fixed).

mp=0.16 kg
cp=0.4 Ns/m
kp=6.32 N/m

$$\boldsymbol{w_n = \sqrt{\frac{k_p}{m_p}} = \sqrt{\frac{6.32}{0.16}} = 6.2849}$$
$$\boldsymbol{\zeta = \frac{c_p}{c_c} = \frac{c_p}{2\sqrt{k_p m_p}} = \frac{0.4}{2*\sqrt{6.32*0.16}} = 0.1989}$$

Task 1.3:

Now derive the transfer function from base disturbance $z_w(s)$ to the displacement $z_p(s)$ of the mass m_p ($\frac{z_p(s)}{z_w(s)}$). Write the transfer function in form of natural frequency w_n and damping ratio ζ. Plot the

Bode diagram of this transfer function and check the value of natural frequency on this plot. Compare the result with the bode diagram of the un-damped system i.e. c_p=0.

<u>Equation of Motion</u>

$m_p\ddot{z_p} + c_p(\dot{z_p} - \dot{z_w}) + k_p(z_p - z_w) = 0$

Transfer Function: $\frac{z_p(s)}{z_w(s)} = \frac{c_p s + k_p}{m_p s^2 + c_p s + k_p}$

Using equations of w_n and ζ, we can write transfer function in terms of them.

Transfer Function in terms of w_n *and* ζ: $\frac{z_p(s)}{z_w(s)} = \frac{2\zeta w_n s + w_n^2}{s^2 + 2w_n\zeta s + w_n^2}$

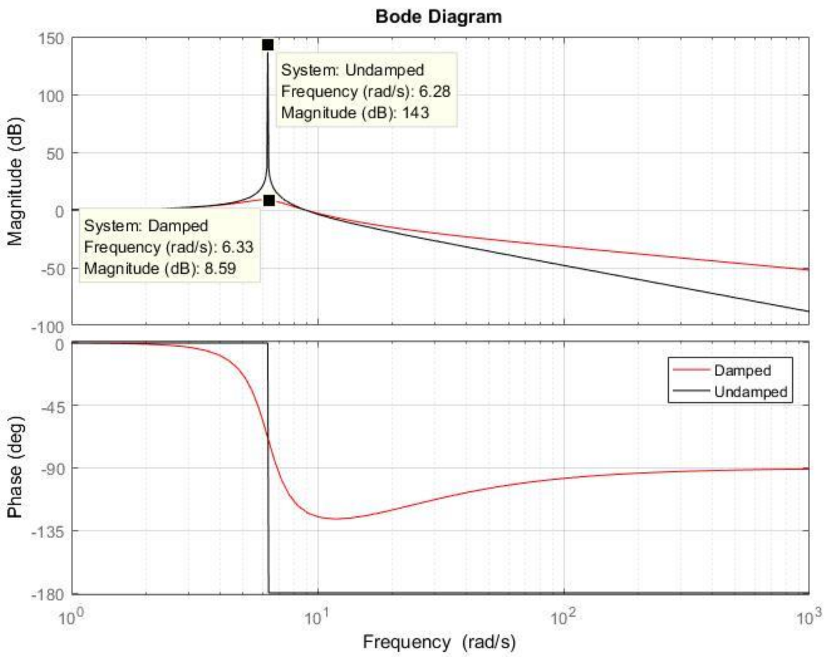


Figure 3 Bode Plot of Damped and Un-damped System

Looking at figure 3, we can clearly see that the amplitude of the un-damped system is very large as compared to the damped system.

Task 1.4:

Now study the response of this system to the different base excitations. For the step response, consider 3 cases where ζ<1, ζ=1 and ζ>1 (this means that you should vary the value of c_p to achieve the right range for the damping ratio). Discuss the differences between the three cases.

There are three different base excitations we have been given to work on:

a. Sinusoidal excitation with amplitude of 0.05m and an arbitrary frequency. (compare with frequency response function)

b. Step response with an amplitude of 0.05m.

c. Response to track excitation which is given in section 2.3 (Plot the PSD using the semilogy syntax)

(a) Sine Excitation:
For the sine excitation, we choose the signal as u = 0.05*sin(1*t). For a time period of 10 sec, we plot the transfer function against this input sine disturbance and get the following results as in figure 4. We can see the damped curve follows closely with the input curve.

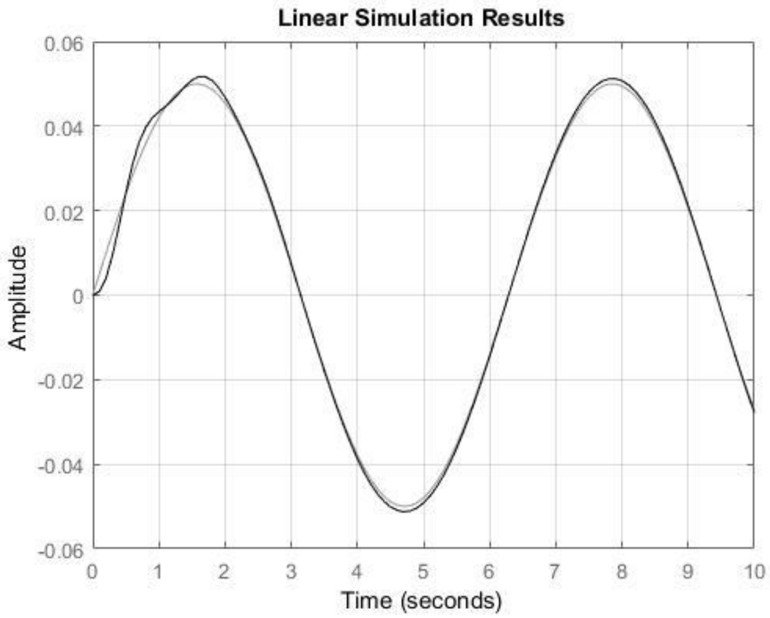


Figure 4 Sine excitation of a damped passive system

(b) Step Excitation :
For the step excitation, we chose the amplitude of 0.05 m and plotted the system response for natural frequency of the system taken as 1(critically damped system), less than one(Under damped system) and more than one (over damped system)

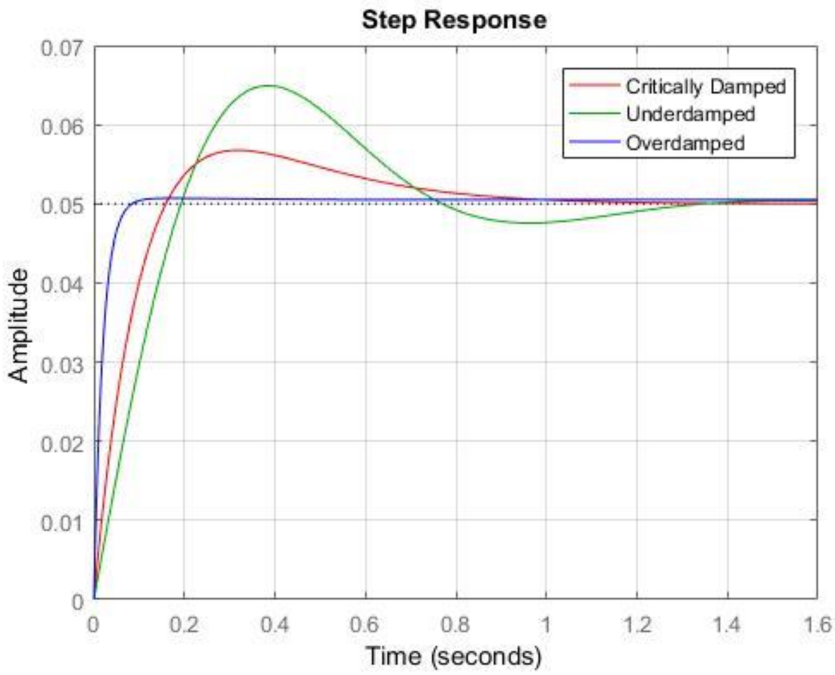


Figure 5 Step response of a damped passive system

Figure 5 shows step response of a damped passive system. We can see one peculiar thing about the graph. Because the numerator of transfer function is not constant, so we get an overshoot even when we choose the critically damped system. This is different from the theoretical graph of critical damping when no overshoot must be observed. This is because the present system contains zeros which overshoots the system. A system with no zeros will give a critically damped graphs similar to one studied in theory.

The difference between these cases are the control characteristics, overshoot is zero usually for a overdamped system, while settling time is the best for the critically damped system.

(c) Response to Track excitation:
The track irregularities can be described as Power Spectral Density Plots (PSD). PSD is calculated by multiplying frequency response function with PSD of track irregularities.

$$S_p(w)=|H|^2 * S_w(w)$$

We used the PSD of track irregularity of the example mentioned.

$$S_w(w) = \frac{4.028 * 10 - 7}{2.88 * 10 - 4 + 0.68\mathrm{w}^2 + \mathrm{w}^4}$$

Figure 6 shows a PSD plot drawn using the above S_w function. The frequency range taken is 0-25Hz.

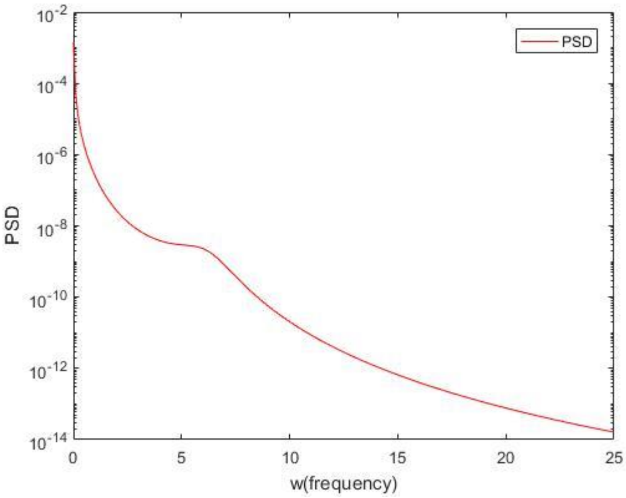


Figure 6 PSD for a damped passive system

Task 2.1

Derive equation of motion in time domain. Once you have found the equation of motion, use it to derive transfer function $\frac{zp(S)}{zw(S)}$.

In this task, the damped passive system has been replaced by a Proportional-Derivative controller. Equation of Motion for the system can be written as:

$$m_p\ddot{z_p} + k_p(z_p - z_w) + \mathrm{F(s)} = 0$$

The force calculated by this controller is

$$F(s){=}D(s){*}zp(s)$$

and controller $D(s)$ is

$$D(s){=}{-}(d_p{+}d_d s)$$

The actuator force in time domain will be

$$F(t){=}{-}d_p z_p(t){-}d_d\dot{z_p}(t)$$

The equation modifies to:

$$m_p\ddot{z_p} + k_p(z_p - z_w) - d_p z_p - d_d\,\dot{z_p} = 0$$

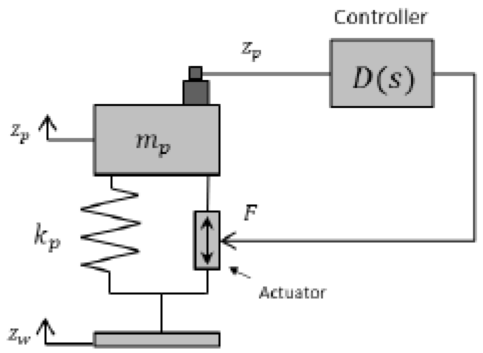


Figure 7 Single Mass System with active PD control

The Transfer Function can be written as:

$$\frac{z_p(s)}{z_w(s)} = \frac{k_p}{m_p s^2 + d_d s + (k_p - d_p)}$$

Task 2.2

After obtaining the transfer function, it is time to find the right values for d_p and d_d. Before searching for these two parameters, compare the denominators of the transfer function you just obtained with the one for damped passive system. They are both of second order. We can get almost the same response from both of the transfer functions, if coefficients of denominator are the same. You can use this technique to find d_p and d_d

The transfer function of the damped system is

$$\frac{z_p(s)}{z_w(s)} = \frac{c_p s + k_p}{m_p s^2 + c_p s + k_p}$$

And the transfer function of the Active PD control system is

$$\frac{z_p(s)}{z_w(s)} = \frac{k_p}{m_p s^2 + d_d s + (k_p - d_p)}$$

On comparing the denominators of the two transfer functions, we get the values:

$d_d = c_p = 0.4$

$d_p = 0.$

Now if we compare these two systems on a step response of amplitude 0.05m, we see almost similar response as in figure 8.

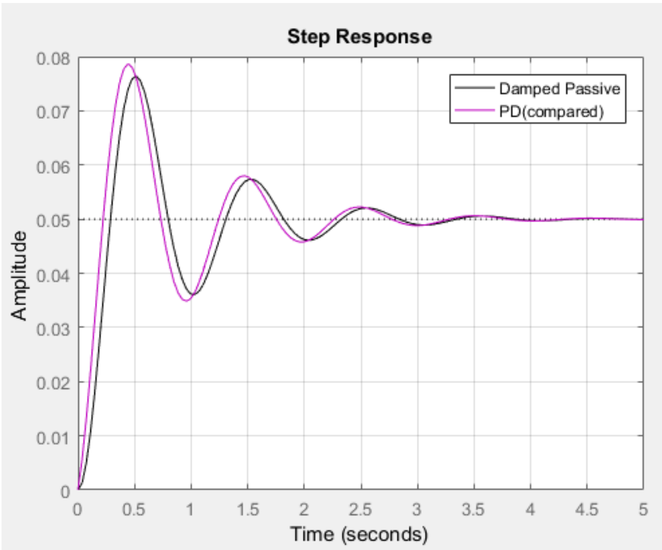


Figure 8 Step response of Damped passive system against a PD System using comparative values

Task 2.3

Once you have found these two para meters using the mentioned technique, you can further fine tune them to meet the response requirements to different base excitations. (keeping in mind that response to step input should be under critically damped and you should not change the un-damped natural frequency of the system.)

As we start tuning for d_d, we know that a regular road vehicle has ζ in the range of 0.3-0.5, so using that value, and using natural undamped frequency of the system,

$d_d = c_p = 2 * \zeta * w_n * m_p$

$d_d = 2 * 0.5* 6.2849 *0.16= 1.005$ (Real Case with PD)

For tuning d_d

Tuning values of d_p using step response function with an amplitude of 0.05 m, varying d_d between $0.1<d_d<2$, we get Figure 9

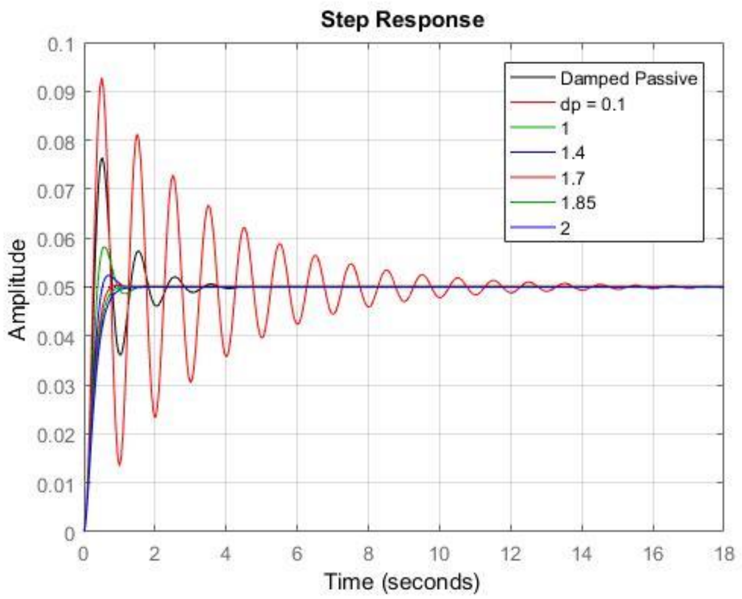


Figure 9 Tuning dd values using a step response

If we plot only the damped system, the PD system compared to damped system with values of d_d = 0.4 and d_p = 0, and the real case system for which d_d = 1.005, we get figure 10.

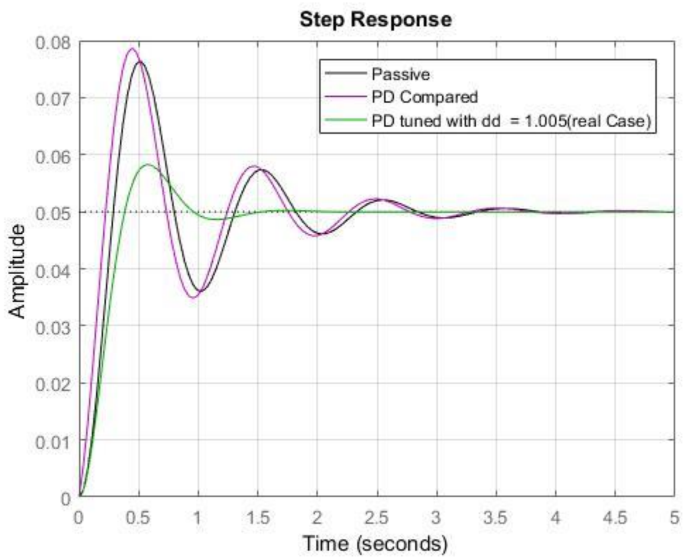


Figure 10 Step response for a Passive, PD Comparative to Passive and tuned PD controllers

Here we can see that choosing dd as 1.005 is a considerable judgement for a PD system with a little overshoot in the system. We can see clearly that the new response is better than the Passive response.

On tuning for dp values, we notice that for all values of dp other than zero, we get a steady state error. This is shown in figure 11. Because of this reason, we will keep the value of zero for k_p

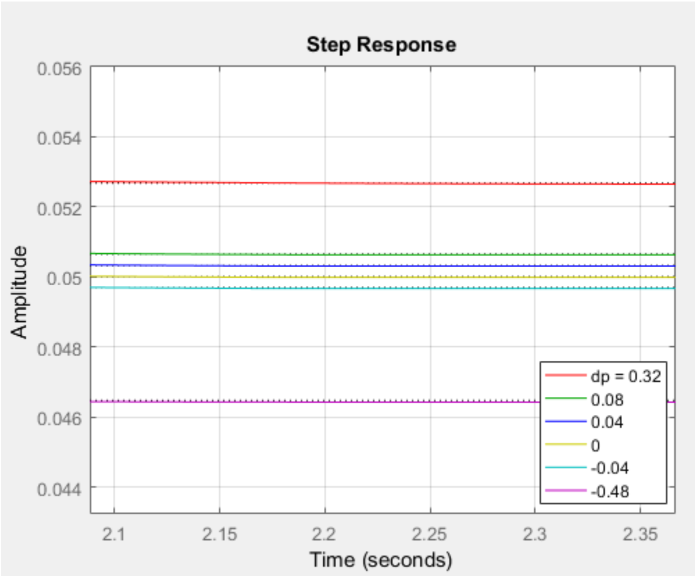


Figure 11 Step response (zoomed in) of PD response with tuned value of dp

Extra Task 1

Can you find the value of *dd* for which the response is critically damped? (Systems that are critically or over critically damped, show bumpy ride and are not very comfortable. On the other hand, systems with too little damping, damp out vibration very slowly which is not desirable either.)

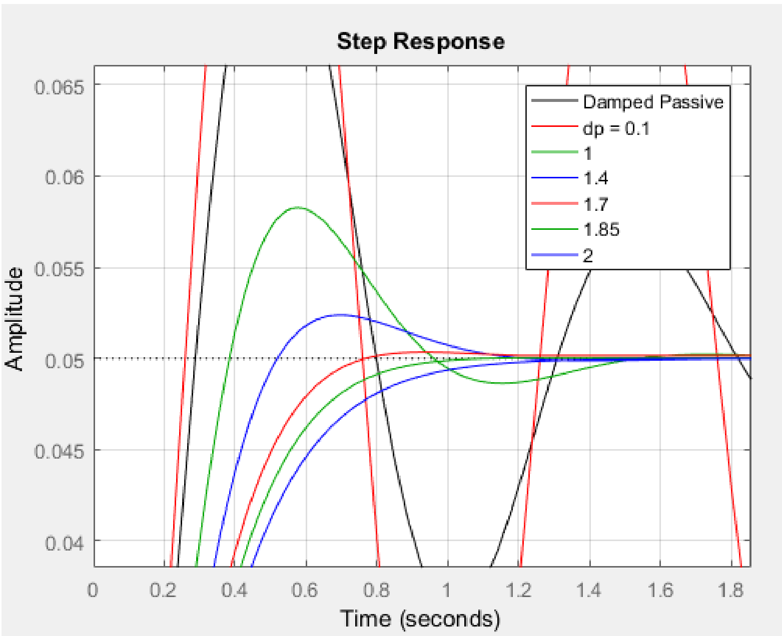


Figure 12 Step response for a PD controller with different values of dp

As we can see from figure 12, the value of dp for critical damping with minimum settling time is dp = 1.85. Values less than 1.85 exhibit overshoot(dp =1 is the ideal case with a little overshoot) and values less than 1.85 exhibit over damped system.

Value of d_p for critical damping :

$$d_p = 1.85$$

Let us see the response of a sine curve on this tuned PD controller: Figure 13 shows the sine response of the tuned and the critical PD controller.

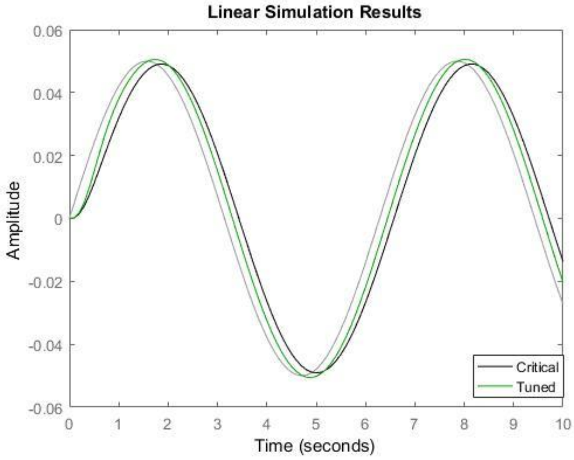


Figure 13 Sine response on a PD with tuned and critical values

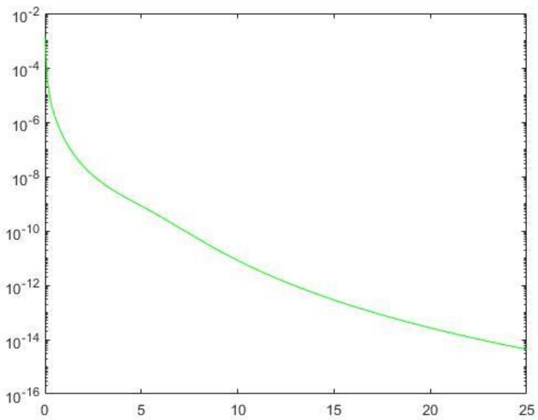


Figure 14 PSD for a tuned PID controller

PID Control

Task 3.1

First derive equation of motion in time domain and then find the transfer function $\frac{z_p(s)}{z_w(s)}$ like previous part. Then try to find the right parameters for the PID controller. Try tuning parameters until you meet the response requirements to different base excitations. (Keep in mind that h_p varies the natural frequency of the system and therefore it should have a small value.)

Equation of Motion for single mass system can be written as :
$m_p\ddot{z_p} + \ k_p(z_p - z_w)$ + F(s) = 0

where force of actuator can be written as :
F(s) = D(s) * z_p(s)

D(s) is the controller and for PID, it will have the following parameters:
D(s) = - $(h_p + h_d s + h_i/s)$

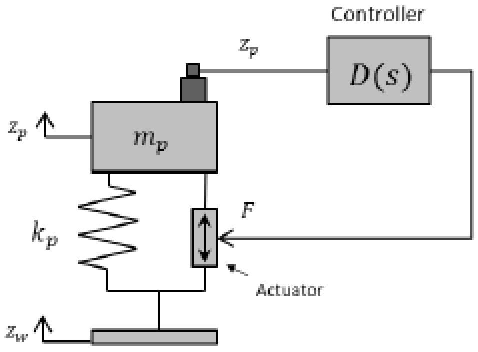


Figure 15 Single mass system with active PID Control

Using the value of D(s) and F(s) in Equation of motion of PID controller, the equation modifies to :

$$m_p\ddot{z_p} + \ k_p(z_p - z_w) = h_p + \dot{z_p}h_d + h_i \int z_p$$

Using this equation the transfer function can be written as:

$$\frac{z_p(s)}{z_w(s)} = \frac{k_p s}{m_p s^3 + h_d s^2 + (k_p + h_p)s + h_i}$$

We use this transfer function and use a step response with an amplitude of 0.05mm for tuning. When we start tuning this controller, we begin with values of h_p. When we vary the values of h_p from -1<h_p<1, we notice that at only values of hp = 0 there is no steady state error, as soon as the value goes up or down, the error rises accordingly. Hence, we will keep the value of hp = 1.

Moving on to the integral part, we see that no value of integral part for which there is a stable response except if we nullify the effect of integral controller and take hi = 0. For this reason, we proceed with hi = 0.

Moving on to the derivative part, the value of hd when varied from 0 to 4, the graph moves from a under damped response to an over damped response. Between values of hd = 1.6 to 1.85, gives the best response which is also close to the tuned PD controller response.

On fine tuning, we obtain figure 16 where we see the PID controller response at hp = 0, hd = 1.3, hi = 0. The new response is a little underdamped as we want a vehicle response to be in real case.

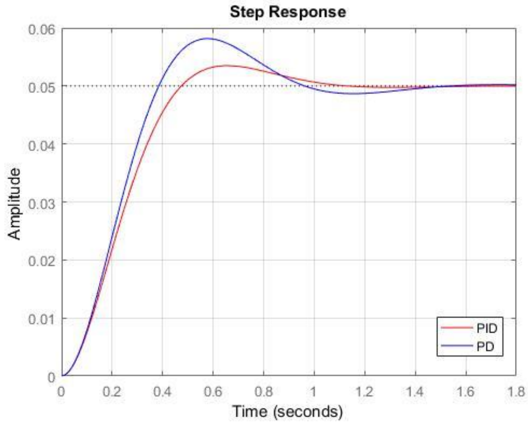


Figure 16 Step excitation for Tuned PID(Underdamped) response against PD response

Figure 16 shows that PID controller performs better than the PD controller even when we critically tune PD controller. The PID controller shows lesser settling time though all other parameters are the same for both of them.

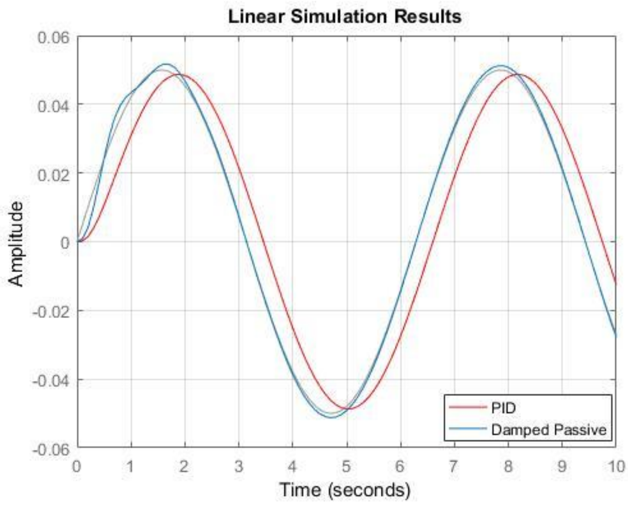


Figure 17 Sine response of PID control against a damped passive system

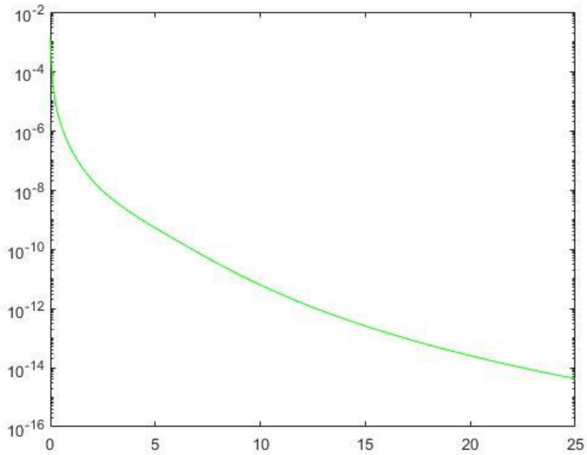


Figure 18 PSD of a tuned PID controller

Figure 17 and Figure 18 shows the sine excitation and the power spectral density of the tuned PID controller.

Task 3.2:

Explain how the integration parameter hi affects the step response?

When we vary hi from 0 to 5, we see that response worsens progressively. This lets us choose hi =0 for the PID control

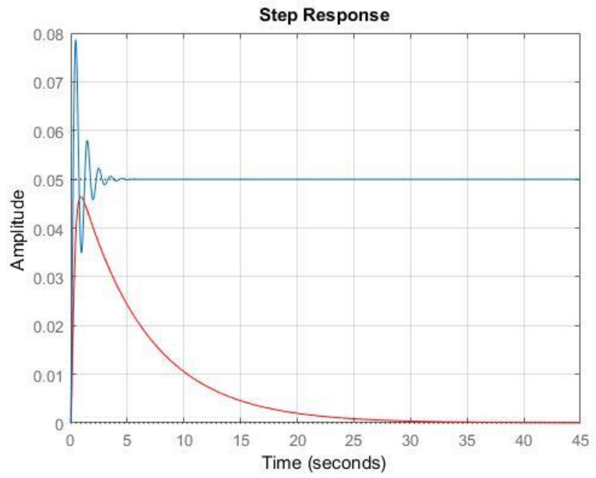


Figure 19 Tuning of integral controller with hi = 1

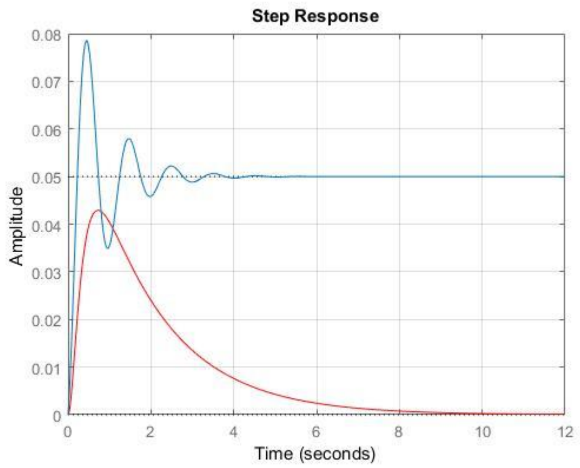


Figure 20 Tuning of integral controller with hi = 3

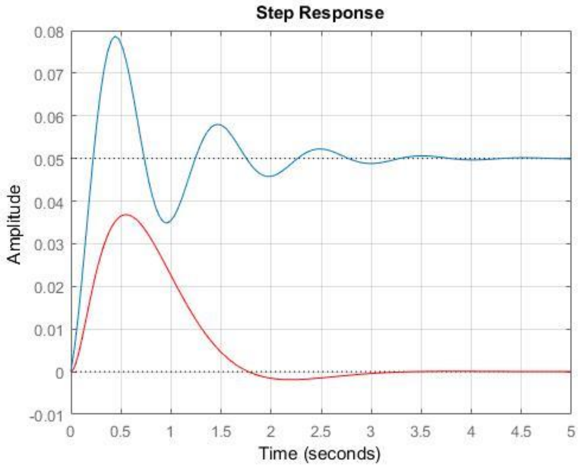


Figure 21 Tuning of integral controller with hi = 10

Task 4.1:

This is a simple and robust controller and the only parameter needs tuning is T. You can readily see that skyhook controller has exactly the same structure as a Derivative controller (Read part 2.1 to get help on Skyhook physical meaning). Derive the equation of motion and transfer function as previous parts. Try tuning the parameter T until you meet the response requirements to different base excitations.

The equation of motion for a skyhook controller can be written as :

$$m_p\ddot{z_p} + k_p(z_p - z_w) + F(s) = 0$$

Where force on actuator can be described as :

$$F(s) = D(s) * z_p(s)$$

And the controller D(s) can be

$$D(s) = -(T * s)$$

The equation modifies to:

$$m_p\ddot{z_p} + k_p(z_p - z_w) + T\,z_p = 0$$

and the transfer function can be written as :

$$\frac{z_p(s)}{z_w(s)} = \frac{k_p}{m_p + Ts + k_p}$$

When we start tuning the skyhook controller, there is only one parameter to tune, T. On manually changing values from 0.4 to 2, we obtain the following range of response as in figure 22.

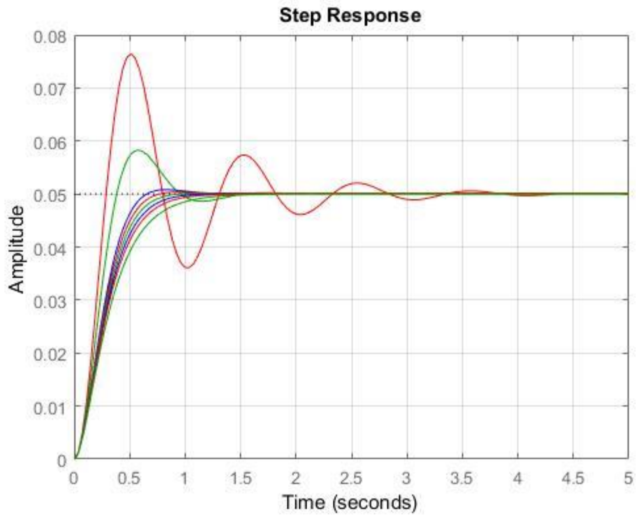


Figure 22 Tuning Skyhook with step excitation varying T from 0.04<T<2

On fine tuning T, we get the values T = 1 for a little under damped skyhook system and T = 1.9 for a critically damped skyhook system.

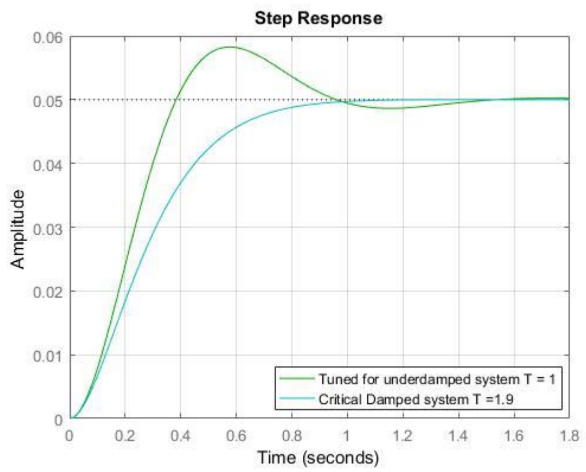


Figure 23 Skyhook controller for T =1.9 and T = 1

Figure 24 and Figure 25 shows sine excitation and PSD for a Skyhook controller with both values of T operating at critical and under damped systems.

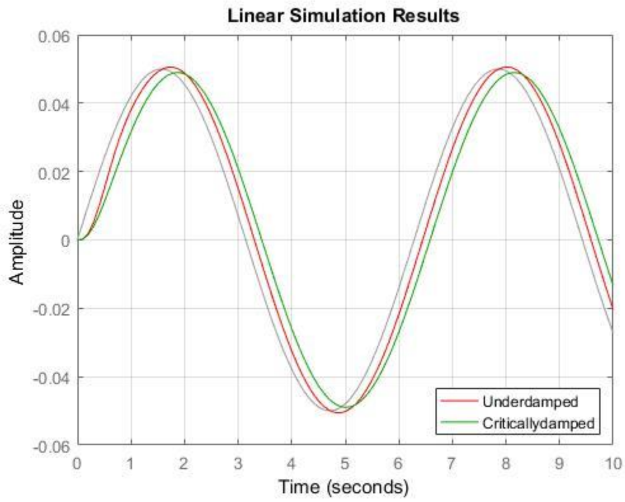


Figure 24 Sine excitation for a Skyhook controller

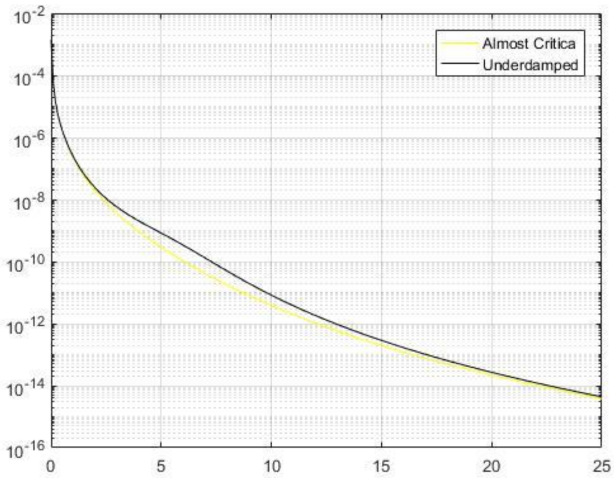


Figure 25 PSD for a Skyhook controller

Task 5.1:

Compare the amplitude of the transfer functions for the five studied systems (un-damped passive system, damped passive system, PD, PID and skyhook - A successful active control shows lower transfer function amplitude compared to ‘damped passive system’.

To compare the amplitude of the transfer functions for all the systems, we compare the Bode plots for all in Figure 26.

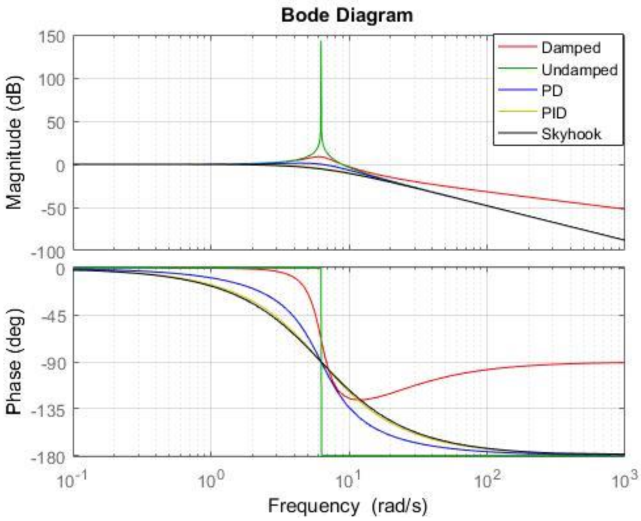


Figure 26 Bode Plot Comparison Chart

Task 5.2:

Compare also the responses to the 3 different base excitations for the damped system (for $\zeta<1$), PD, PID and Skyhook

Figure 27,28 and 29 shows the plot comparison for all the systems : Damped Passive, PD, PID, and Skyhook controllers. We can see in the step response in Figure 27 that PD and Skyhook's response almost overlaps. This is due to arbitrarily chosen under damping values for coefficients T, d_d and d_p.

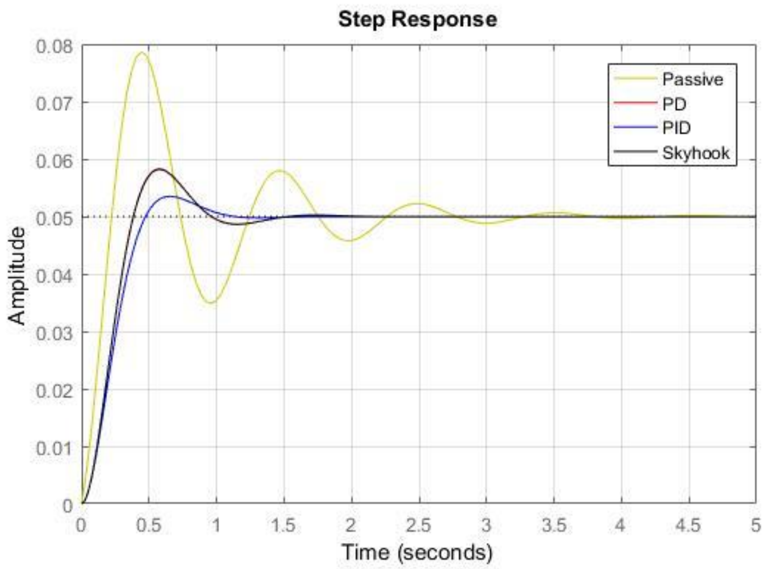


Figure 27 Step Response : All plots comparison

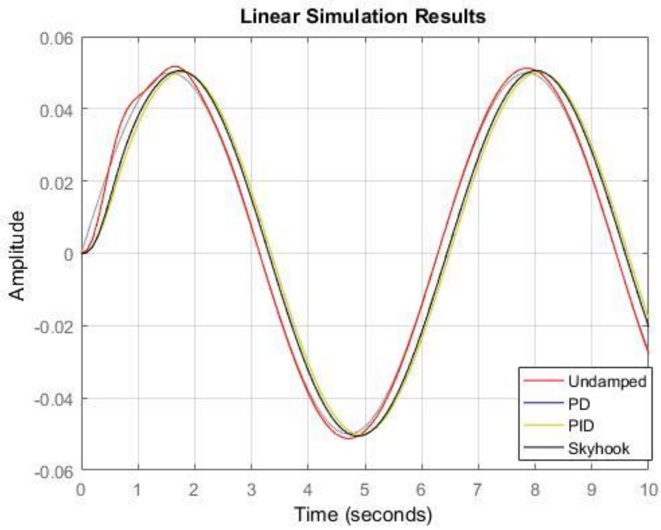


Figure 28 Sine Response : All plots comparison

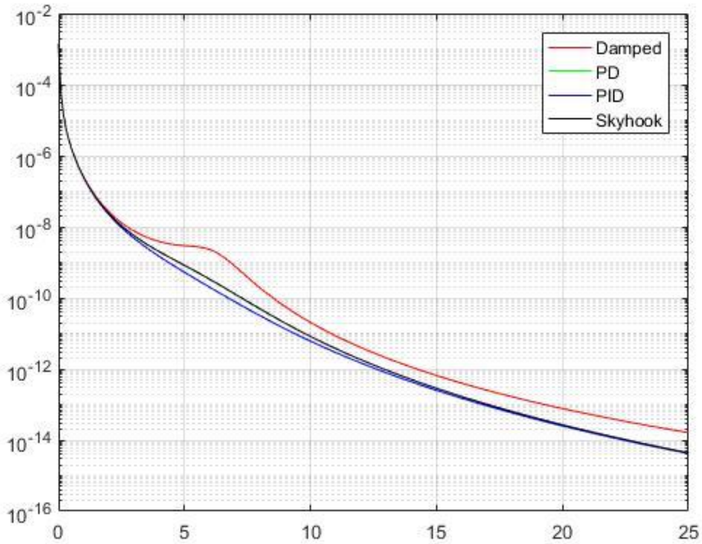


Figure 29 PSD Response: All plots comparison

Task 6.1:

In this part you should first derive the equation of motion for the two degrees of freedom system shown in Figure 9. z_w is the base excitation and is the input to the system. Once you have found the equations of motions, use them to derive the transfer function from the base excitation to the displacement of the secondary mass i.e. $z_s(s)=z_w(s)$

Hint: write the equation of motion for the two masses and then transform them to Laplace space. This will give you an algebraic equation with two equations and two unknown as function of 's'. Use Cramer's rule to find the relation $z_s(s)$ /$z_w(s)$

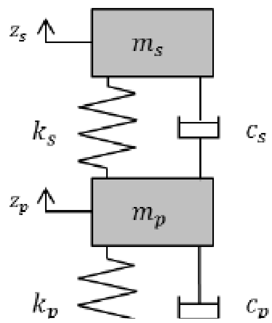


Figure 30 Two degrees of freedom with passive suspension

For the first mass, we can write the equation as :

$$m_s\ddot{z}_s^2 + k_s(z_s - z_p) + c_s(\dot{z}_s - \dot{z}_p) = 0$$

$$TF1 \;=\; \frac{z_s(s)}{z_p(s)} = \frac{c_s s + k_s}{m_s s^2 + c_s}$$

For the second mass,

$$m_p\ddot{z}_p^2 + k_p(z_p - z_w) + c_p(\dot{z}_p - \dot{z}_w) - k_s(z_s - z_p) - c_s(\dot{z}_s - \dot{z}_p) = 0$$

$$m_p z_p s^2 + k_p(z_p - z_w) + c_p s(z_p - z_w) - k_s(z_s - z_p) - c_s s(z_s - z_p) = 0$$

$$m_p z_p s^2 + k_p(z_p(s) - z_w(s)) + c_p s(z_p(s) - z_w(s)) - k_s(z_s(s) - z_p(s)) - c_s s(z_s(s) - z_p(s)) = 0$$

Using value of $z_p(s)$ and substituting it in the second mass equation above:

Final Transfer Function =

$$\frac{\mathrm{zs(s)}}{\mathrm{zw(s)}} = \frac{(s^2 c_p c_s + s(k_s c_p + k_p c_s) + k_p k_s)}{(s^4(m_p m_s) + s^3(c_p m_s + c_s m_p + c_s m_s) + s^2(k_p m_s + c_p c_s + k_s m_p + k_s m_s) + s(k_p c_s + c_p k_s) + k_p k_s);}$$

Task 6.2:

Study the Bode diagram of the derived transfer function and the response to the 3 excitations as mentioned in Section 3. Compare the amplitude of the bode diagram that you have obtained here with the bode diagram of section 3.1. What is the advantage of having two levels of suspension?

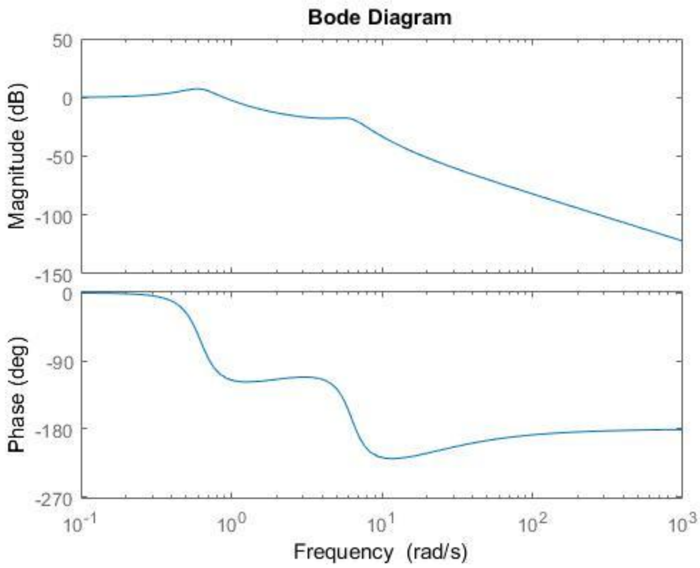


Figure 31 Bode plot for the 2 DOF System

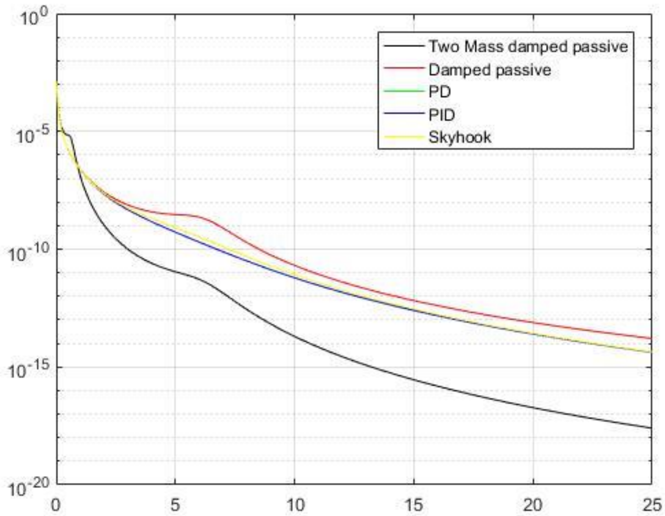


Figure 32 Bode plot of two mass as compared to one mas system

In figure 32, we are comparing the bode plot of the new two mass system with the older systems. We notice that there are two points of rising frequency owing to two masses.

The advantage of having two levels of suspension is that the overall damping of the system increases and both the masses' damping supplements each other. This, in return provides more isolation of z direction forces/vibrations from the road to the driver.

Response to different excitations:

Step Response:

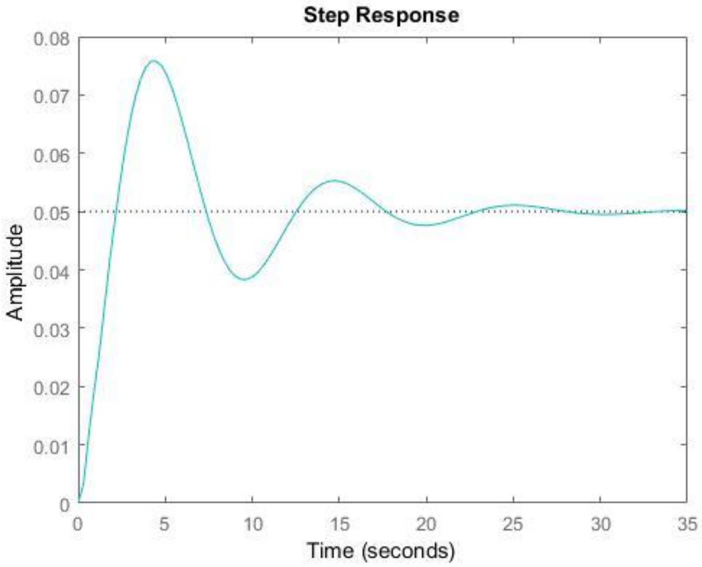


Figure 33 Sine excitation of a two mass damped passive system

In figure 33, we observe that the excitation is underdamped almost similar to that of a damped passive system but with more zeroes in the system which increases the settling time of the system.

Sine Response and PSD Plots

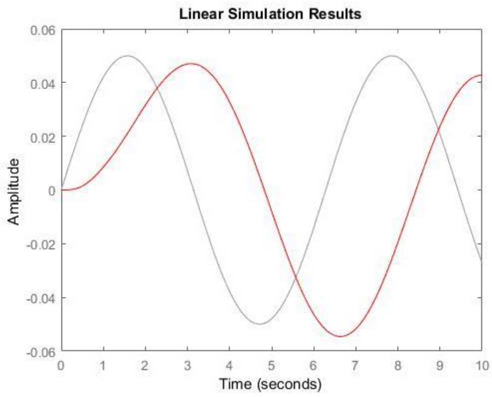


Figure 34 Sine excitation of a two mass damped passive system

We can clearly see as in Figure 34 that the curve is out of phase with the input frequency. The output curve is leading the system.

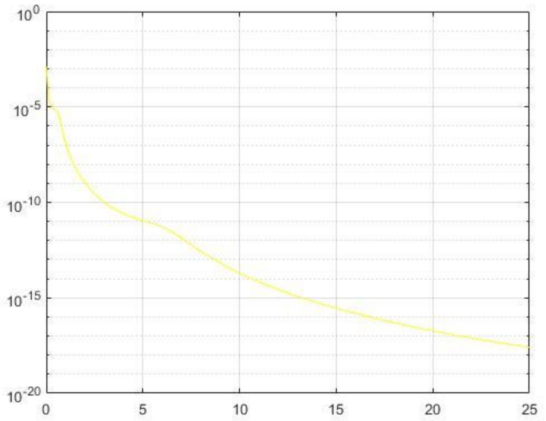


Figure 35 PSD plot of two mass damped passive system

Task 6.3

Start with deriving the **state space model** (work with state space in this section instead of transfer function!) of the system shown in Figure 10 considering that F, z_w and $\dot{z}_w$ are inputs to the system and z_s is the output and states can be selected to be as follow:

$$\mathbf{X = [z_s \, ; \dot{z}_s \, ; z_p ; \; \dot{z}_p];}$$

$$\mathbf{X} = \begin{bmatrix} z_s \\ \dot{z}_s \\ z_p \\ \dot{z}_p \end{bmatrix}$$

As we start deriving the state space, these are the A, B, C and D matrices that we get:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_s/m_s & 0 & k_s/m_s & 0 \\ 0 & 0 & 0 & 1 \\ k_s/m_p & 0 & -(k_s+k_p)/m_p & -c_p/m_p \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1/m_s & 0 & 0 \\ 0 & 0 & 0 \\ -1/m_p & k_p/m_p & c_p/m_p \end{bmatrix}$$

C = [1 0 0 0]

.D = 0

Task 6.4:

Once you have derived the state space model of the system it should be easy to implement the system in Simulink and then study the system for different base excitations as introduced in Section 3. Try controlling the vibration of the the mass m_s using a skyhook controller. The controller $D(s)$ for a skyhook controller is

$$D(s) = -T*s$$

Parameter T should be found so that the responses to excitations a. and b. introduced in Section 3 fulfil requirements mentioned in Section 3.1.

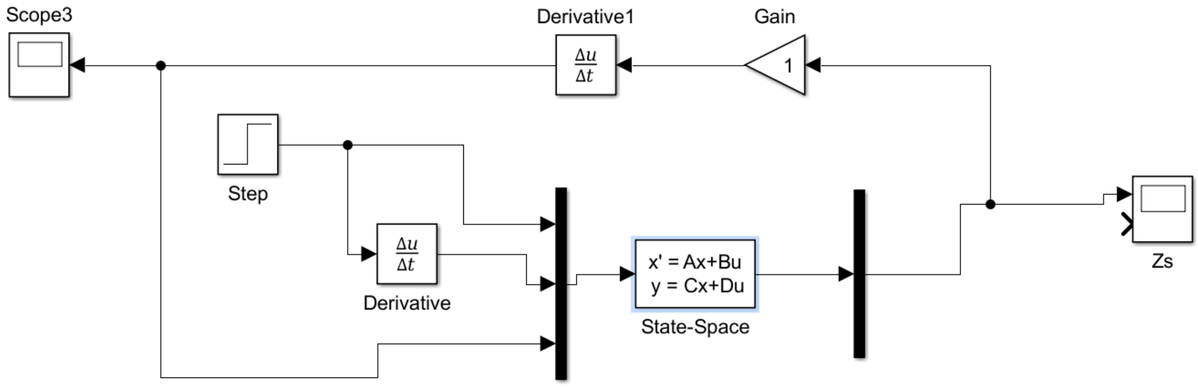


Figure 36 Simulink block for state space of a skyhook controller of a two mass damped passive system

Here is the Simulink block in figure 36
The input taken here is step response with an amplitude of 0.05m

On tuning with different values of T, to achieve a little under damped system, we notice varying T from 0.06-1.2 yields good response for the controller. We choose T = 0.1 for minimum settling time and less overshoot. The step response looks like Figure 37

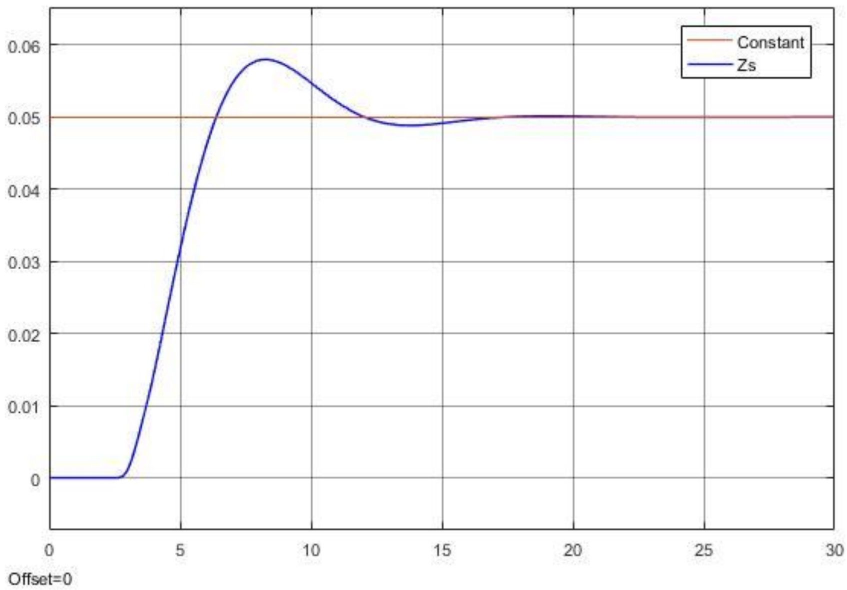


Figure 37 Step response for skyhook damping of a two DOF mass system

Also, the sine input with a frequency of 1 rad/sec and 0.05m amplitude is given to the same state space in the Simulink model and response as in figure 38 is observed with a value of T = 0.04. If we take the same value of T as in for step response, then the amplitude of the response is half of the input amplitude, which is not a good response. We can see that the input and output are out of phase and the output dampens the amplitude at a very slow pace.

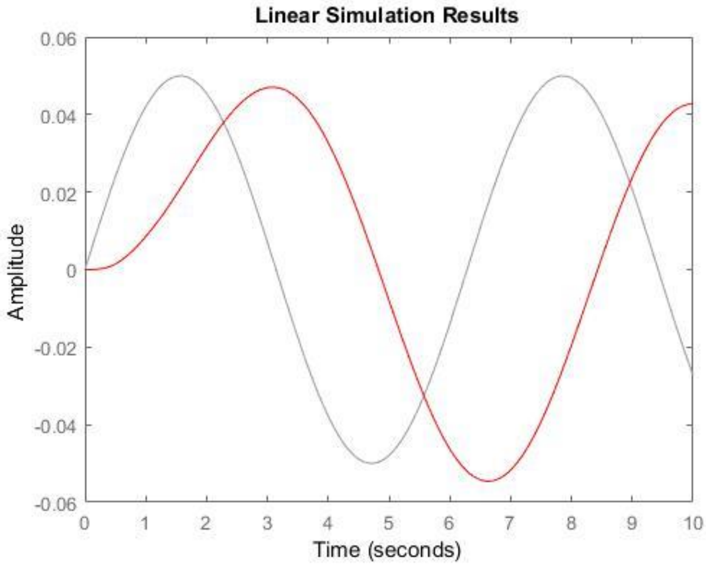


Figure 38 Sine response for skyhook controller of a 2 mass DOF model

Task 7.1:

Compare responses of the damped passive system and skyhook controller to excitations a. and b. (Section 3) and try to fulfil the response requirements in Section 3.1.

Figure 39 shows a comparison plot for step response of passively damped and a skyhook tuned system and respective control characteristics.

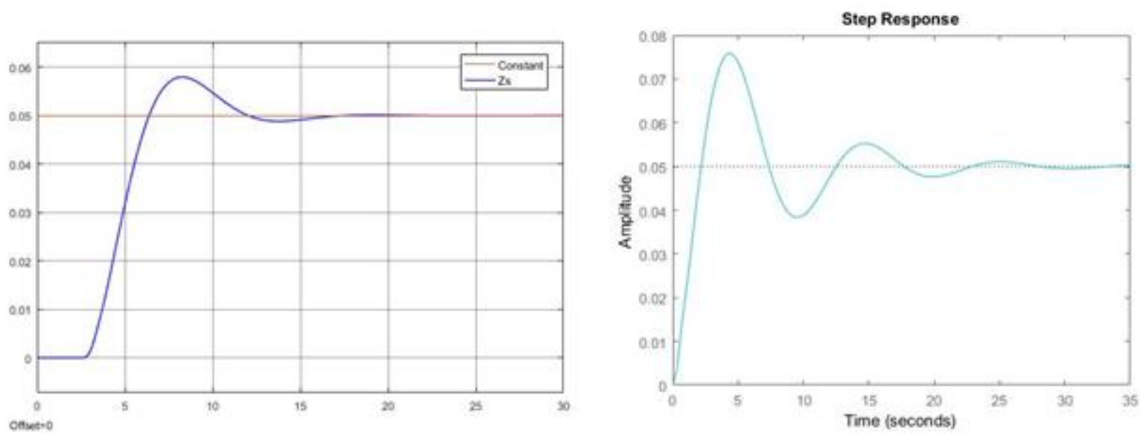


Figure 39 Comparison in 2 DOF system

We notice that skyhook controller dampens faster than the passive system with a less overshoot and less settling time as well. Passively damped system has a better rise time but it take a long time (20sec) to damped the whole motion. In an automobile, 20 seconds is a very large time to dampen the vibration, hence active control suing skyhook proves to be better than the passively damped system.

Control of Bounce and pitch for a simple vehicle model using skyhook and H

Task 8.1:

Derive both bounce and pitch equations of motion for the chassis shown in Figure 11 as function of parameters. Once you have found the equations of motion, try driving the state space model of the system where vectors of states, inputs and outputs should be as follow

The equation of motions were derived taking in reference the literature for Bounce & Pitch model of Ground Vehicle Dynamics course,

The equations of motion are as follows,

$$m\ddot{z} + (c_1 + c_2)\dot{z} + [(1-\lambda)c_2 - \lambda c_1]L\dot{\theta} + (k_1 + k_2)z + [(1-\lambda)k_2 - \lambda k_1]L\theta = F_z$$

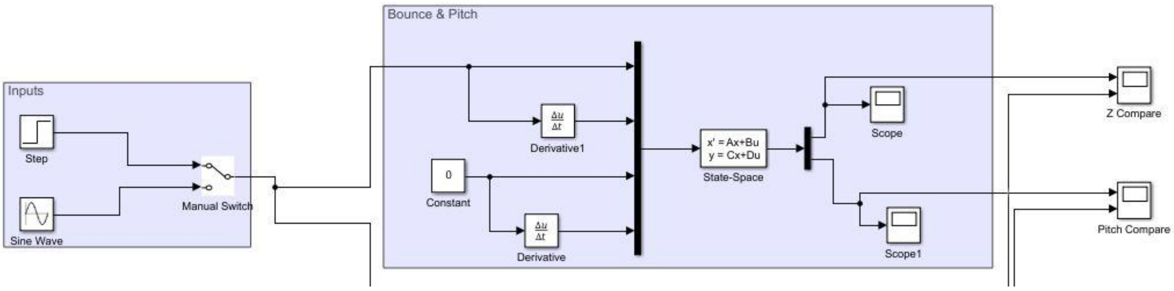
$$J\ddot{\theta} + [\lambda^2 c_1 + (1-\lambda)^2 c_2]L^2\dot{\theta} + [(1-\lambda)c_2 - \lambda c_1]L\dot{z} + \\ + [\lambda^2 k_1 + (1-\lambda)^2 k_2]L^2\theta + [(1-\lambda)k_2 - \lambda k_1]Lz = M_\theta$$

These equations can be derived into a State Space form as follows,

Task 8.2:

Once you have found the state space model, use Simulink to study the response of the model to the following excitations. Can you find the natural frequencies of the system?

The Figure below shows how the system is realised in Simulink,



The natural frequencies of the system can be found by calculating the Eigen of the Matrix A of the state space,

A =

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -(k1+k2)/_m & -(c1+c2)/_m & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.25(k1+k2)*L^2/_j & -0.25(c1+c2)*L^2/_j \end{bmatrix}$$

B =

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ {(k1)}/{m} & {(c1)}/{m} & {(k2)}/{m} & {(c1)}/{m} \\ 0 & 0 & 0 & 10 \\ (-k1*0.5*L)/j & (k2*0.5*L)/j & (c1*0.5*L)/j & (c2*0.5*L)/j \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

D = [0]

The following command was used to calculate the Eigen Frequencies,

F = eig(A) ;

Which gives the output as shown in the figure below,

```
F =

  -1.8182 + 7.1582i
  -1.8182 - 7.1582i
  -0.5143 + 3.8941i
  -0.5143 - 3.8941i
```

The above system was excited for three input conditions, one step input and two sinusoidal inputs of frequency 1 Hz & 8 Hz.

Excitation 1:

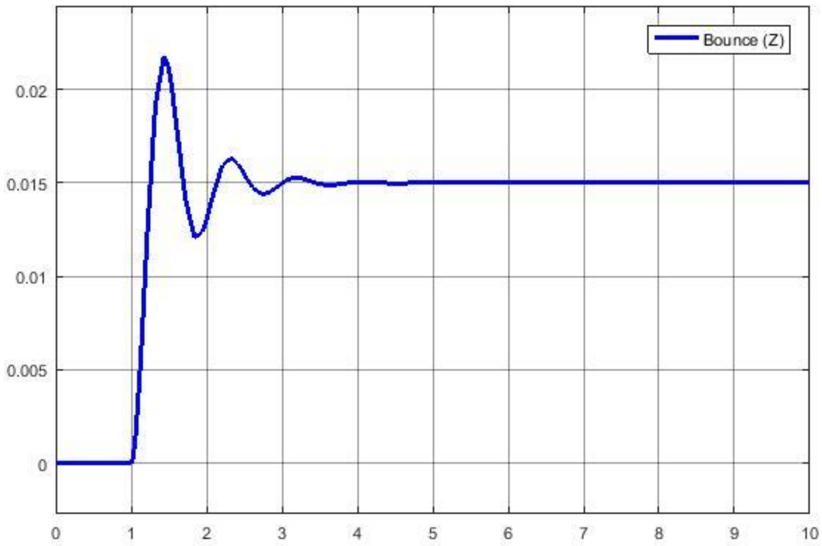
$$\begin{cases} z_{w1}(t) = 0.03 \ [m] & if\ t > 1, otherwise\ 0 \\ z_{w2}(t) = 0 & for\ -\infty < t < +\infty \end{cases}$$

Excitation 2:

$$\begin{cases} z_{w1}(t) = 0.01 * \sin(2\pi f t) & if\ t > 0, otherwise\ 0 \\ z_{w2}(t) = 0 & for\ -\infty < t < +\infty \end{cases}$$

where f is the frequency of excitation. Do the simulation for $f = 1\,Hz$ and $f = 8\,Hz$

The Bounce and Pitch results are,



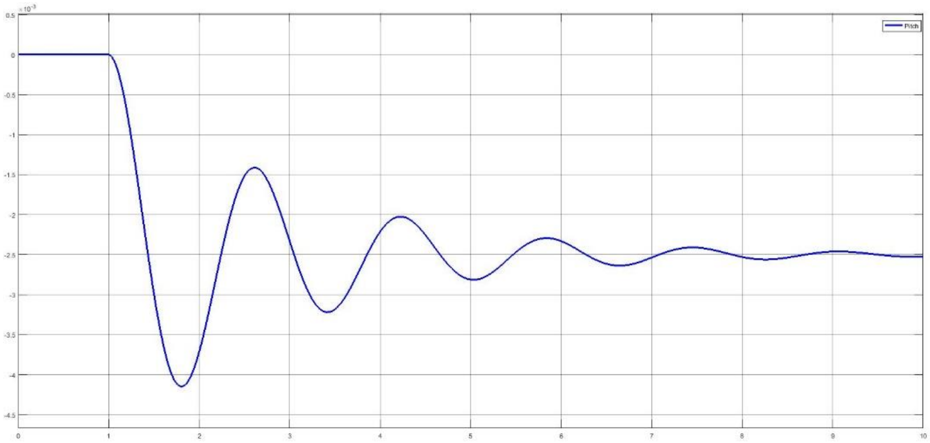


Figure 40 Bounce & Pitch for the passive system- Step Input

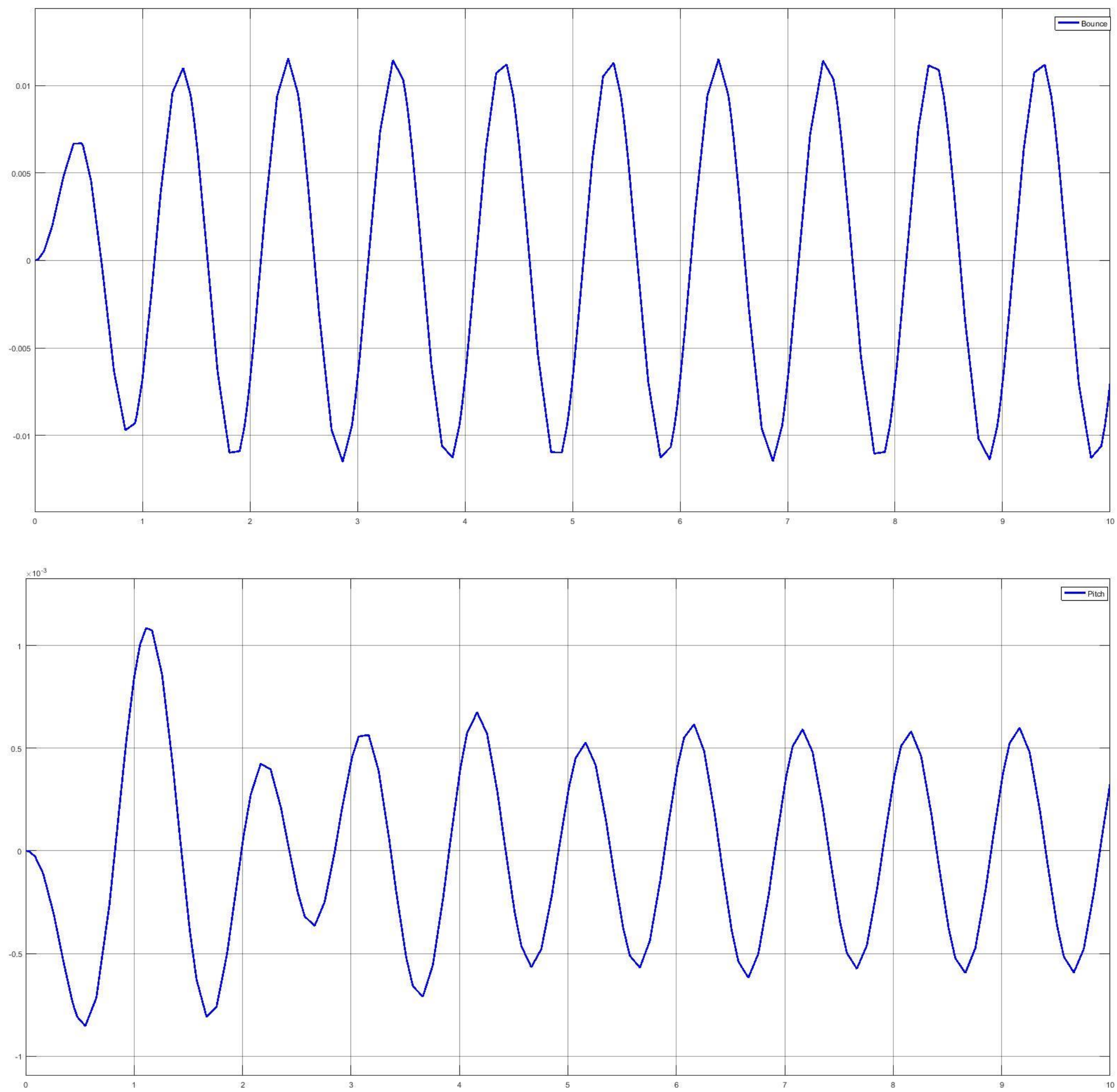


Figure 41 Bounce & Pitch for the passive system- Sinusoidal Input 1Hz

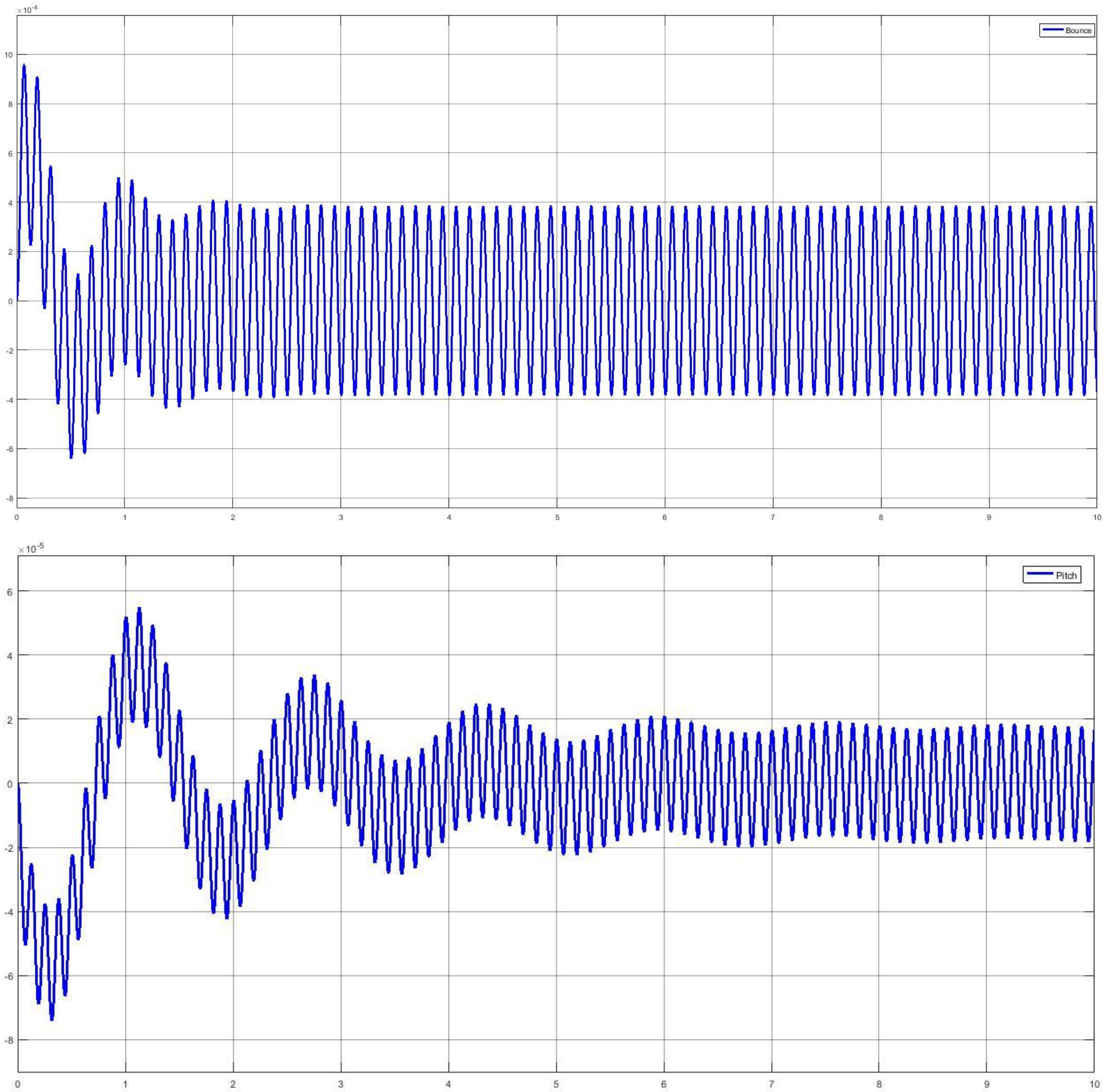
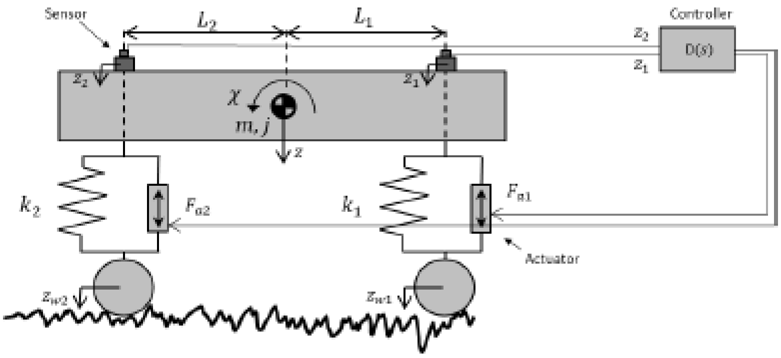


Figure 42 Bounce & Pitch for the passive system- Sinusoidal Input 8 Hz

Sky Hook Control

Task 9.1:

Derive equations of motion and state space model for sky hook system.



The equations of motion are same as for passive bounce & pitch model except the fact that the dampers are replaced by actuators and the actuator forces are fed as an input along with the road excitation.

The state space is derived from the equations of motion as shown,

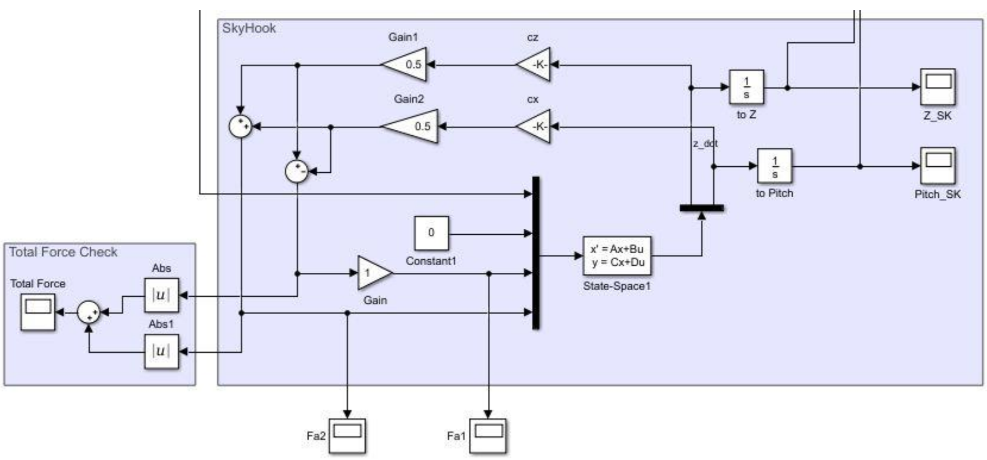
$$Ask = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2k/m & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & (-2kL^2)/j & 0 \end{bmatrix}$$

$$Bsk = \begin{bmatrix} 0 & 0 & 0 & 0 \\ k/m & k/m & -1/m & -1/m \\ 0 & 0 & 0 & 1 \\ -Lk/j & Lk/j & L/j & L/j \end{bmatrix}$$

$$Csk = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Dsk = [0]

The Sky Hook Model was implemented in Simulink as shown below,



Task 9.2: To complete your Simulink model you need to find out the relation between Fz,χ and $Fa1$,$Fa2$. Derive this relation and implement it in Simulink.

The force and torque which resist the bounce and pitch motion depend upon & which are sky hook damping parameters,

$$\text{Force} = F_z = -c_z * \dot{z} \quad \& \quad \text{Torque} = T_x = -c_x * \dot{x}$$

The force relation between the front and rear actuators can be derived by balancing the forces and moments about the centre of gravity.

$$F_{a1} = c_z * \dot{z} - c_x * \dot{x}$$

$$F_{a2} = c_z * \dot{z} + c_x * \dot{x}$$

The values of c_z & c_x were tune to match the response requirements.

Response requirements: (on displacement)

1. Response of the active suspension to sinusoidal excitation should show lower amplitude compared to passive damped system.
2. Step response of the active suspension should
   a. show under critically damped response (meaning that the response should oscillate before damping out)
   b. damp out faster than the 'damped passive system'
   c. have lower overshoot compared to 'damped passive system

The tuned values are,

$$c_z = 150000$$
$$c_x = 400000$$

Task 9.3: Report the tuned values of cz and $c\chi$ and compare the results (χ and z) with the passive case. In reality it is not possible to choose very high values for cz and $c\chi$, what is the limitation? (To be more realistic, you should try to limit the maximum absolute value of the force to 10 kN.)

The following figures show the comparison of the passive model with the sky hook model.

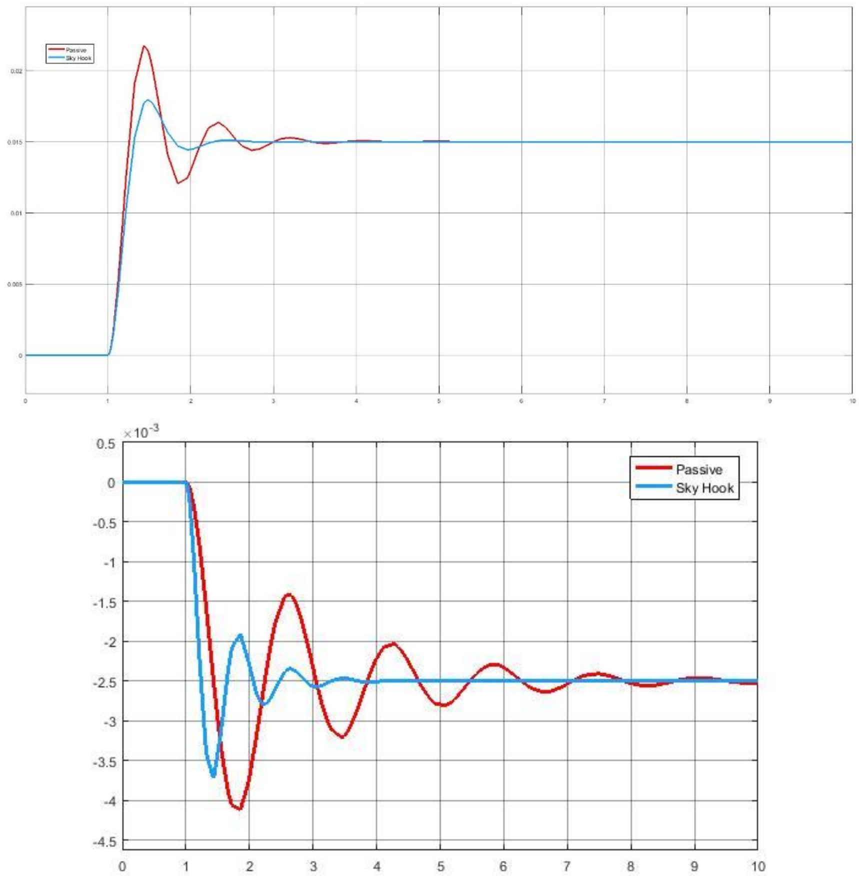
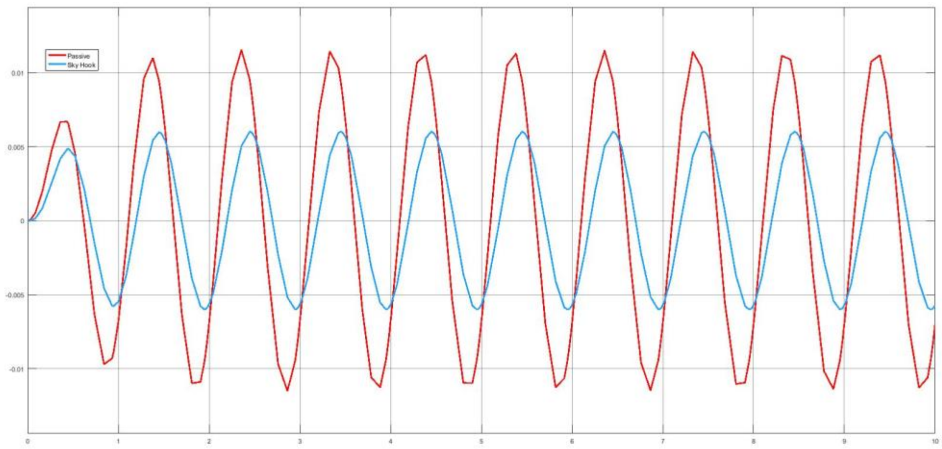


Figure 43 Comparison passive & sky hook system - Step Input



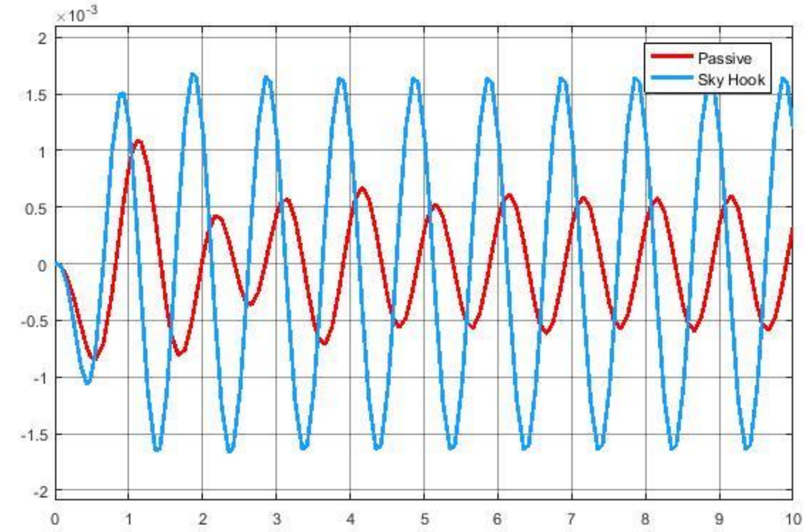


Figure 44 Comparison passive & sky hook system - Sinusoidal Input 1Hz

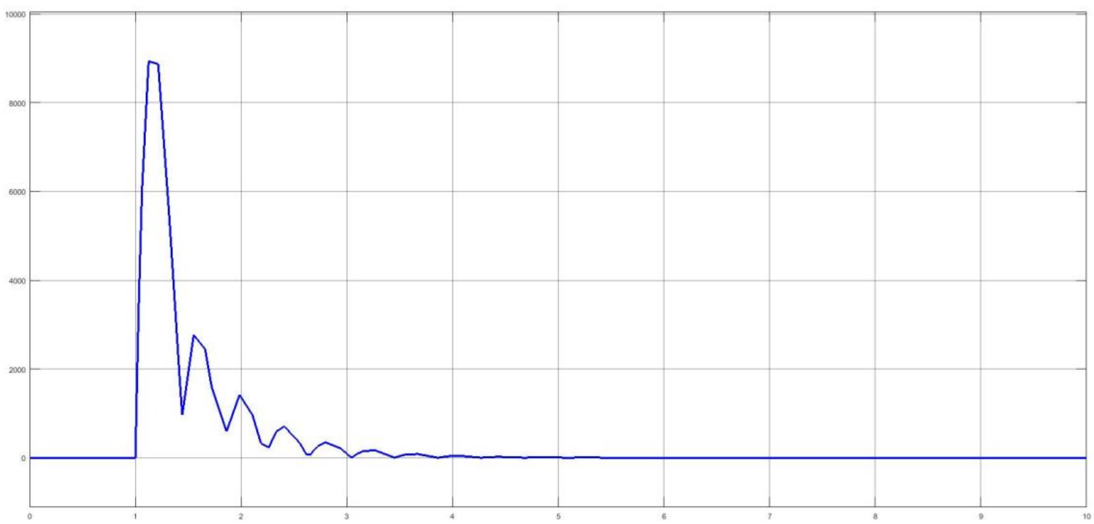


Figure 45 Total Force input to Actuators in Sky hook

Task 10.1: Calculate value of wnb.

$W\chi$ is designed in the same way as Wb, but is used to penalize the pitch motion. Try finding the frequency which should be penalized for pitch motion, $wn\chi$

The frequencies that must be penalized can be calculated as follows,

$$w_{nb} = \sqrt{\frac{2k}{m}} = 7.38\, rad/s$$

$$w_{nx} = \sqrt{\frac{2kL^2}{j}} = 7.8558\, rad/s$$

The Simulink model is realised as follows for the H infinity controller

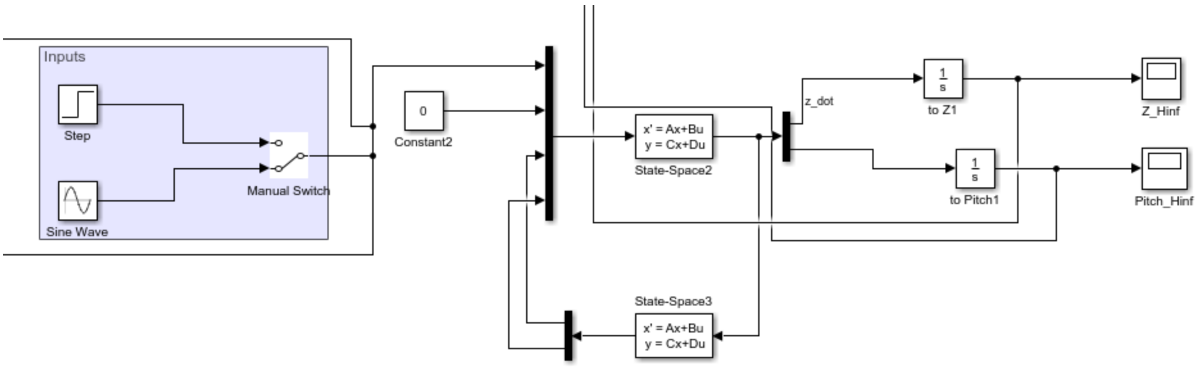


Figure 46 H Infinity Controller

Task 10.2:

Once a suitable controller is found study the system response to excitations defined in Section 5.1 and try to fulfil the response requirements in Section 5.2 (To be more realistic, you should try to limit the maximum absolute value of the force to 10 kN). Furthermore plot the magnitude of all the weighting functions.

The Values of kb and kchi were tuned to match the response requirements,

Kb = 7750

Kchi = 30000

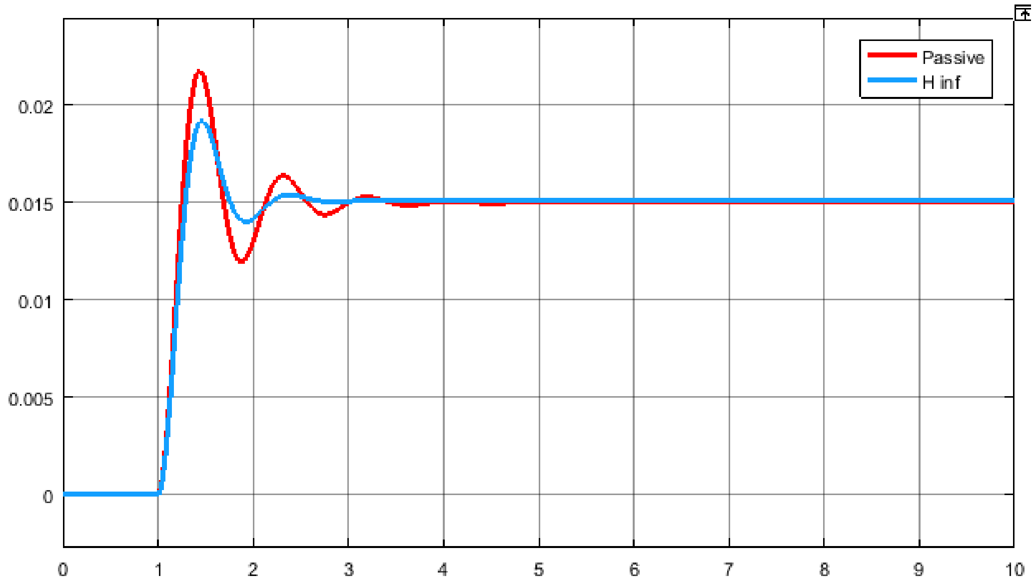


Figure 47 Comparison of Bounce passive & Hinf system - Step Input

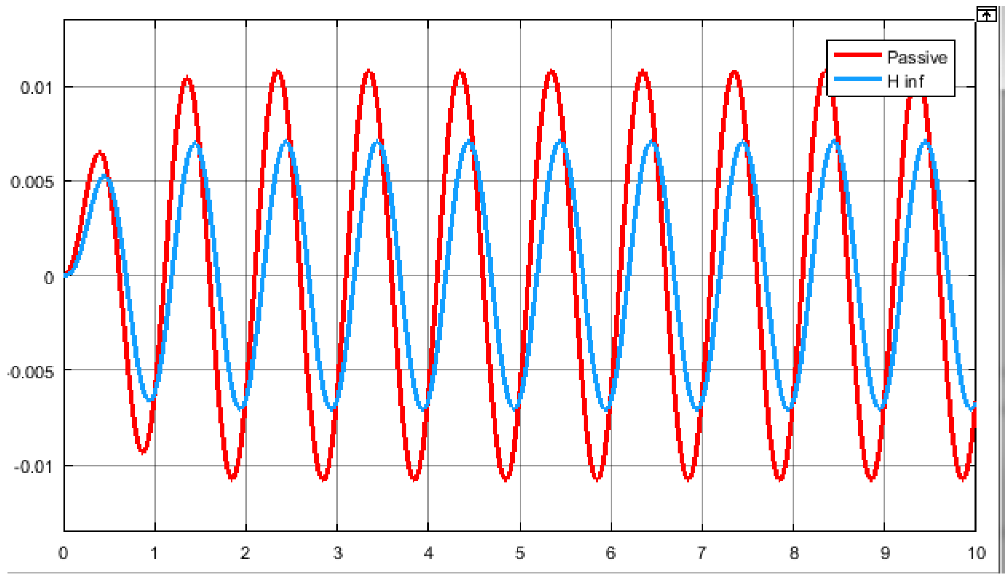


Figure 48 Comparison of Bounce passive & Hinf system - Sine Input 1Hz

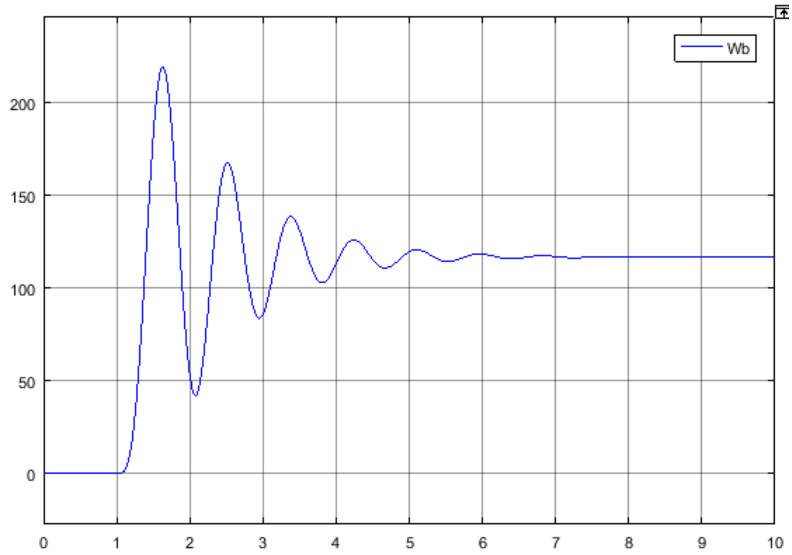


Figure 49 Weighing Function Wb

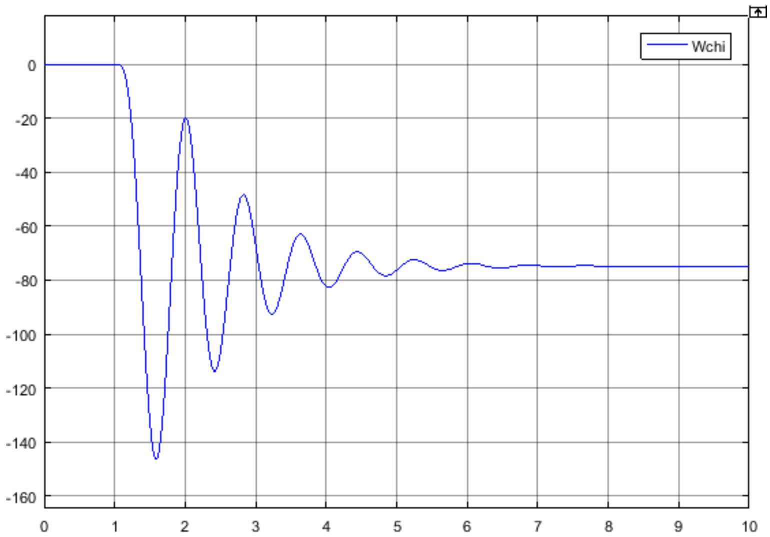


Figure 50 Weighing Function Wchi

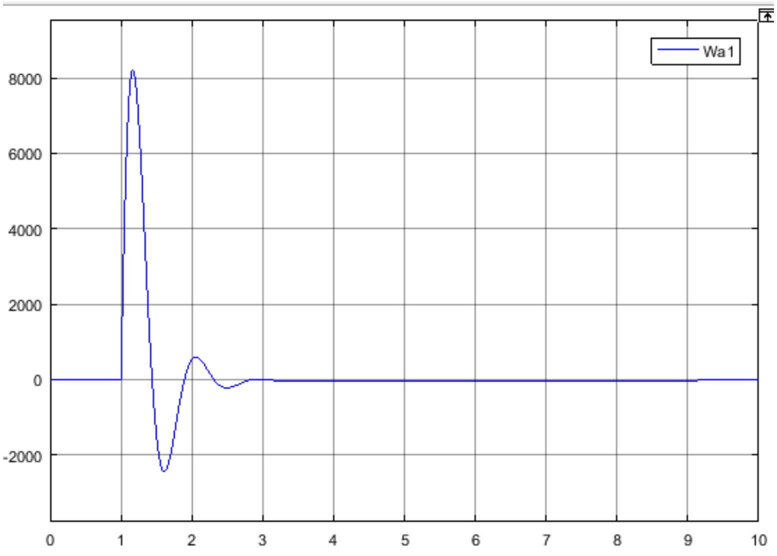


Figure 51 Weighing Function Wa1

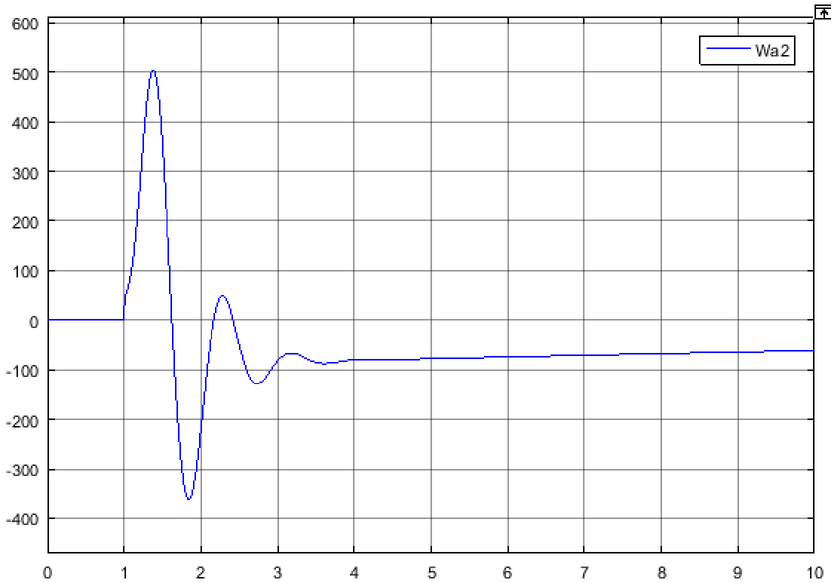


Figure 52 Weighing Function Wa2

Figure 53 Total force H inf system (N)

Task 10.3: Study the **step response** of the model with the original controller and vary the parameters of the system within 15% tolerance once at a time. Is the system still stable? Is absolute value of force still below 10 kN? (Parameters you may consider are mass-inertia, damping and stiffness.)

Increasing the mass & stiffness by 15% makes the system more stable. Whereas increasing the damping makes no difference as for H infinity controller the dampers are replaced by actuators.

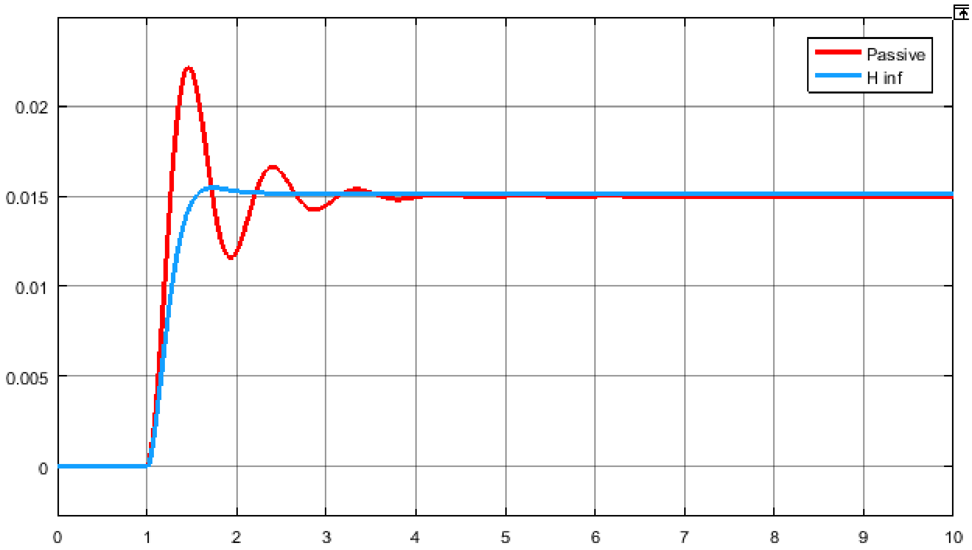


Figure 54 Bounce when mass is increased by 15 %

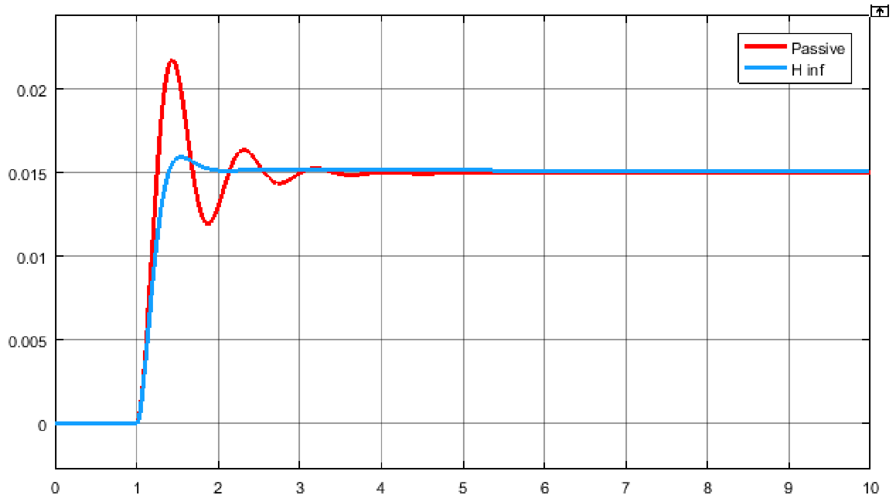


Figure 55 Bounce when Stiffness in increased by 15%

Task 11.1: Compare responses of the passive system, skyhook and $H\infty$ to excitations defined in Section 5.1 and try to fulfil the response requirements in Section 5.2

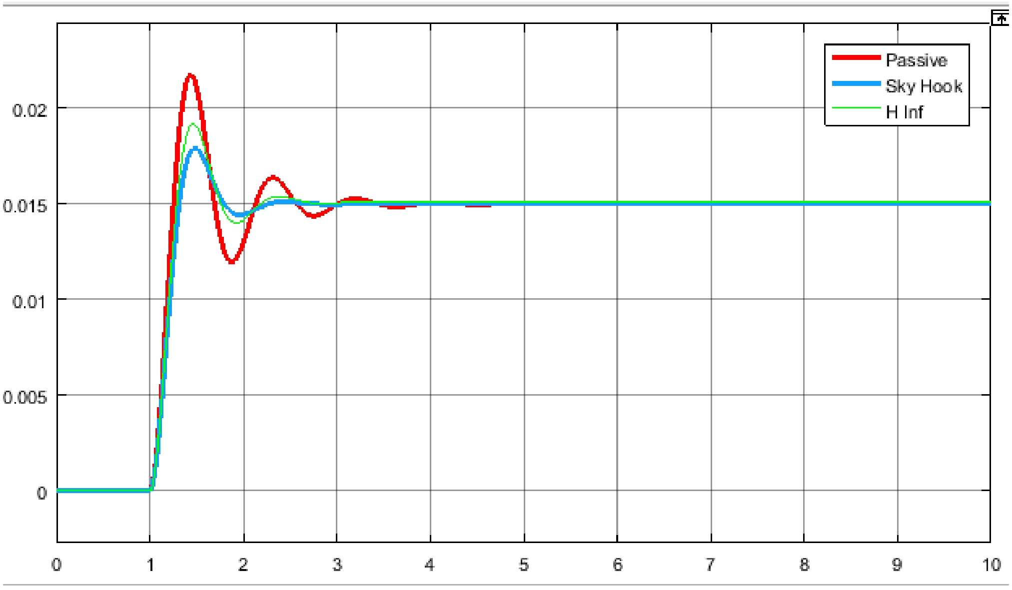


Figure 56 Bounce - Passive, Sky Hook & H Infinity controller - Step Input

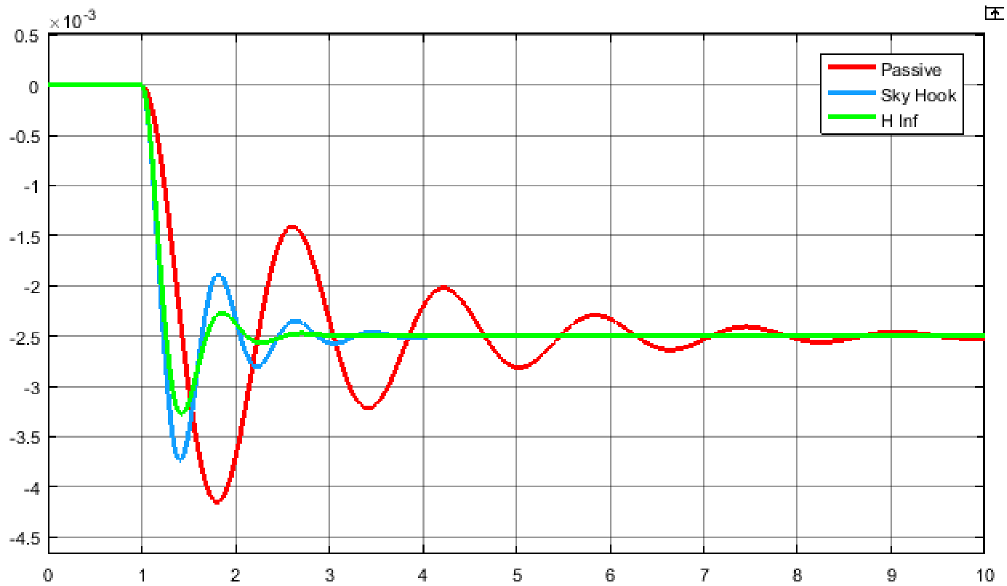


Figure 57 Pitch - Passive, Sky Hook & H Infinity controller - Step Input

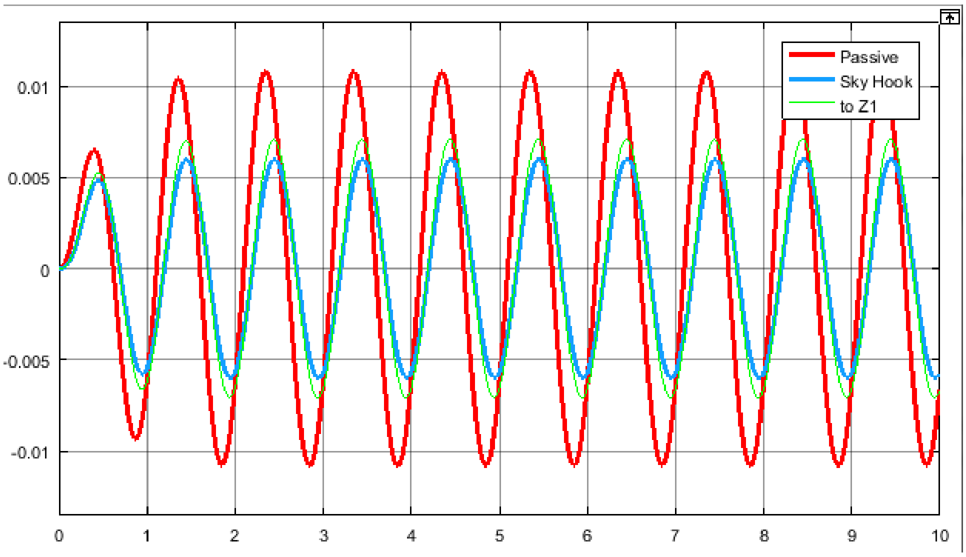


Figure 58 Bounce - Passive, Sky Hook & H Infinity controller - Sin Input 1Hz

When Compared the Sky Hook & H infinity controllers are better than the passive damped system where as the Sky Hook & H Infinity didn't have much difference between them with sky hook out performing H infinity at times.

But H Infinity give more control (tuneable) parameters (weights) whereas in Sky Hook we can only tune one gain. So for more complex systems H Infinity would serve better than Sky Hook