

# Event-Triggered Control for String-Stable Vehicle Platooning

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**Abstract**—Cooperative adaptive cruise control (CACC) is a promising technology that is proven to enable the formation of vehicle platoons with small inter-vehicle distances, while avoiding amplifications of disturbances along the vehicle string. As such, CACC systems can potentially improve road safety, traffic throughput and fuel consumption due to the reduction in aerodynamic drag. Dedicated short range communication (DSRC) is a key ingredient in CACC systems to overcome the limitations of onboard sensors. However, wireless communication also involves inevitable network-induced imperfections, such as a limited communication bandwidth and time-varying transmission delays. Moreover, excessive utilization of communication resources jeopardizes the reliability of the DSRC channel. The latter might restrict the minimum time gap that can be realized safely. As a consequence, to harvest all the benefits of CACC, it is important to limit the communication to only the information that is actually required to establish a (string-)stable platoon over the wireless network and to avoid unnecessary transmissions. For this reason, an event-triggered control scheme and communication strategy is developed that takes into account the aforementioned network-induced imperfections and that aims to reduce the utilization of communication resources, while maintaining the desired closed-loop performance properties. The resulting  $\mathcal{L}_2$  string-stable control strategy is experimentally validated by means of a platoon of three passenger vehicles.

**Index Terms**—Cooperative adaptive cruise control (CACC), event-triggered control, string stability, networked control systems, vehicle-to-vehicle (V2V) communication, vehicle platoon.

## I. INTRODUCTION

### A. Cooperative Adaptive Cruise Control

THE main objective of Cooperative Adaptive Cruise Control (CACC) systems is to maintain a desired, not necessarily constant, (small) distance between vehicles, while ensuring that disturbances are attenuated throughout the string of vehicles. The latter property is also referred to as *string stability*, see, e.g., [6], [40], [46], and in essence constitutes an

$\mathcal{L}_p$ -stability property (for  $p \in [1, \infty)$ ) [52]. Vehicle platoons that are string unstable lead to so-called phantom traffic jams due to excessive braking. Hence, string stability forms an important property in enhancing traffic flow. Besides this string stability property, it is desired to realize a small inter-vehicle distance as it increases the road capacity and reduces fuel consumption. Well-known Adaptive Cruise Control (ACC) systems allow vehicles to maintain a desired distance or time gap, based on measurements of the distance and distance rate that are obtained via onboard sensors such as a radar and/or camera. Cooperative Adaptive Cruise Control systems (CACC) offer to enhance the behavior of traffic flow in terms of string stability while realizing small inter-vehicle distances commonly expressed in terms of the *time gap*, being the distance between a leader and a follower vehicle, divided by the follower vehicle speed (typically less than 1 second). A key ingredient to achieve these two (somehow conflicting) goals simultaneously, is the wireless vehicle-to-vehicle (V2V) communication via a Dedicated Short Range Communication (DSRC) channel. It is shown in, e.g., [34], [40], that CACC significantly improves the attenuation of disturbances along the vehicle with small time gaps compared to conventional ACC systems.

### B. Wireless Communication

The use of wireless communication also has drawbacks as it comes with inevitable network-induced imperfections caused by the digital nature of the communication network. To be more concrete, the communication is packet-based, where the rate at which these data-packages can be transmitted is limited and the communication channel is subject to communication delays. As shown in [10], [15], [16], [25], [29], [36], [37], [40], and [42], these communication imperfections can have a significant influence on the performance of CACC systems in the sense that if the communication delays are too large and/or the rate at which transmissions occur is too small, string stability and other performance properties for a given time gap might no longer be guaranteed. Hence, the number of transmissions in time should be sufficiently large and communication delays sufficiently small in order to obtain the desired platooning behavior. The latter is not trivial to realize as high communication rates degrade the reliability of the DSRC channel and increase the transmission delays as reported in [5], [27], and [30]. This, in turn, might put restrictions on the minimum time gap that can be achieved safely in dense traffic and might as a consequence impede the benefits of CACC with respect to traffic throughput and

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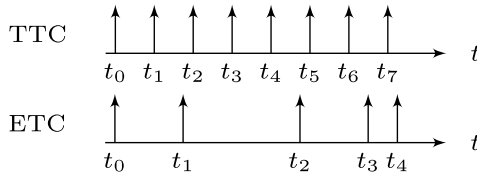


Fig. 1. Paradigm shift in digital control from time-triggered control (TTC) systems to event-triggered control (ETC) systems.

fuel consumption, see also [40]. Hence, in order to secure the reliability and the quality of the network (in terms of communication delays), it is of importance that only the information that is actually required to establish a string-stable platoon is being transmitted and that the transmission of unnecessary information is avoided. Despite the growing interest in vehicle platooning, only a few works available in the literature address this important topic, including [25], [28], [36], [37], [42].

### C. Time-Triggered Versus Event-Triggered Communication

In CACC systems, the desired acceleration is transmitted over the DSRC channel. Due to the packet-based nature of DSRC, these transmissions only occur at discrete instants in time, which we denote by  $t_k$ ,  $k \in \mathbb{N}$ , satisfying  $0 = t_0 < t_1 < t_2 < \dots$ . In traditional (digital) control setups, these transmission instants are scheduled in a time-triggered fashion, typically according to a fixed sampling rate as illustrated in Fig. 1. Since the scheduling of transmission instants is purely based on time and not on the actual status of the plant, time-triggered communication often leads to inefficient use of communication resources which, in the context of CACC, is not desirable as discussed before. Hence, it seems more natural to use resource-aware control methods that determine the transmission instants on the basis of output measurements to allow a better balance between communication efficiency and control performance. Such a resource-aware control method is offered by *event-triggered control* (ETC).

In ETC schemes, the transmission instants are determined on-line by means of a “smart” triggering condition that depends on, *e.g.*, output measurements of the system such that, as illustrated in Fig. 1, transmission are only scheduled when needed in order to guarantee stability, safety and performance properties. As a consequence, event-triggered control has the potential to offer a better balance between the utilization of communication resources and the control performance than time-triggered control. An event-triggering mechanism (ETM) takes, for instance, the form

$$t_0 = 0, t_{k+1} := \inf \{t \geq t_k \mid |\hat{y}(t) - y(t)| \geq \sigma |y(t)|\}, \quad (1)$$

where  $y$  denotes the output measurement (*e.g.*, the desired acceleration of the vehicle equipped with this ETM) and  $\hat{y}$  the most recently transmitted value of  $y$ , and where  $\sigma \in (0, 1)$ . Observe that if the most recent transmitted value of  $y$ ,  $\hat{y}$ , is relatively close to the actual value of  $y$ , no transmission takes place. On the other hand, if the difference between  $\hat{y}$  and  $y$ , also referred to as the *network-induced error*, is relatively large, a new transmission instant is generated. As such, transmissions only take place when there is a significant

change in the value of  $y$  with respect to the most recently transmitted value. See also [3], [4], [9], [22], [47] for some early approaches and [21] for a recent overview on event-triggered control systems.

One of the main challenges in the design of such an *event-triggering mechanism* (ETM) is to guarantee the desired control performance, *e.g.*, in terms of  $\mathcal{L}_2$ -stability, together with a *positive minimum inter-event time* (MIET) despite the presence of disturbances [7], [12]. Obviously, the latter two properties are essential in the context of event-triggered vehicle platooning. The control performance guarantee is needed to establish a string-stable platoon and a positive MIET is required to avoid Zeno-behavior (an infinite number of events in finite time) and to enable practical implementation of the ETC system. In addition, the resulting ETC systems should be robust to time-varying communication delays induced by the DSRC channel. Although many ETC methods are available in the literature, in [7] it is shown that many of the proposed ETMs, including the ETM described by (1), do not have a positive MIET that is robust with respect to the presence of external disturbances. To deal with the aforementioned issue, recent works on ETC either use *time regularization* [1], [17], [20], [22], [48]–[50], in the sense that the next event transmission can only occur after a specific “waiting time”  $\delta$  since the last transmission has elapsed, or *periodic event-triggered control* (PETC) [9], [20], [22], [24], [32], [38], in the sense that the triggering condition is checked at fixed periodic sampling time instants with sampling period  $h$ , such that the MIET is larger than or equal to  $\delta$  or  $h$ , respectively. However, none of these works provides an output-based event-triggered control method that leads to the combination of  $\mathcal{L}_2$ -stability and robustness guarantees with respect to time-varying delays, which is required in the context of CACC.

### D. Event-Triggered Cooperative Adaptive Cruise Control

In this paper, the recently developed framework presented in [12] is used to construct an event-triggered control strategy that does comply with the aforementioned criteria. To be more specific, the proposed ETC framework offers an ETC strategy for a class of nonlinear feedback systems and results in

- a finite  $\mathcal{L}_2$ -gain,
- a strictly positive lower bound on the inter-event times (which guarantees Zeno-freeness),
- robustness guarantees with respect to (time-varying) transmission delays.

Key to obtaining all these beneficial properties is the unique combination of *dynamic* event-triggering conditions and time regularization. As shown in the present paper, the design of this class of dynamic event-triggered controllers is systematic. The proposed control methodology is experimentally validated by means of a platoon of three passenger vehicles.

### E. Organization of the Paper

The remainder of this paper is organized as follows. After presenting some preliminaries and notational conventions in Section II, we introduce the platoon model and the control

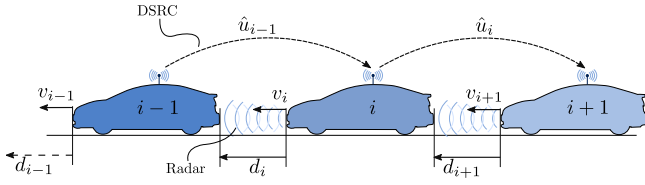


Fig. 2. Homogeneous vehicle platoon consisting of  $N$  vehicles equipped with radar and DSRC communication. The variable  $d_i$ ,  $i \in \tilde{N}$  represents the distance between vehicle  $i$  and its predecessor (vehicle  $i-1$ ),  $v_i$  the velocity of vehicle  $i$  and  $\hat{u}_i$  denotes the most recently transmitted measurement of the desired acceleration of vehicle  $i$ .

objectives in Section III leading to the problem statement. In Section IV, we shortly discuss the tuning of a CACC system for the network-free situation. In Section V, we discuss the imperfections induced by the wireless communication network and we provide a brief introduction on (*dynamic*) event-triggered control. Moreover, we adapt aforementioned the platoon model to include these network-induced errors. In Section VI this model is used to obtain design conditions for the proposed *dynamic* event-triggered strategy such that string stability is guaranteed under event-triggered communication.

Finally, we provide the concluding remarks in Section VIII.

## II. DEFINITIONS AND PRELIMINARIES

The following notational conventions will be used in this paper.  $\mathbb{N}$  denotes the set of all non-negative integers,  $\mathbb{N}_{>0}$  the set of all positive integers,  $\mathbb{R}$  the field of all real numbers and  $\mathbb{R}_{\geq 0}$  the set of all non-negative reals. For  $N \in \mathbb{N}$ , we write the set  $\{1, 2, \dots, N\}$  as  $\tilde{N}$ . By  $|\cdot|$  and  $\langle \cdot, \cdot \rangle$  we denote the Euclidean norm and the usual inner product of real vectors, respectively. For a matrix  $P \in \mathbb{R}^{n \times n}$ , we write  $P > 0$  ( $P \geq 0$ ) if  $P$  is symmetric and positive (semi-)definite, i.e.,  $x^\top P x > 0$  ( $x^\top P x \geq 0$ ) for all  $x \neq 0$ . Likewise, we write  $P < 0$  ( $P \leq 0$ ) if  $P$  is symmetric and negative (semi-)definite, i.e.,  $x^\top P x < 0$  ( $x^\top P x \leq 0$ ) for all  $x \neq 0$ . A function  $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to be of class  $\mathcal{K}$  if it is continuous, strictly increasing and  $\alpha(0) = 0$ . It is said to be of class  $\mathcal{K}_\infty$  if it is of class  $\mathcal{K}$  and it is unbounded. For a signal  $f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$ ,  $n \in \mathbb{N}_{>0}$  and  $t \in \mathbb{R}_{\geq 0}$ ,  $f(t^+)$  denotes the limit  $f(t^+) = \lim_{s \rightarrow t, s > t} f(s)$ , provided it exists.

## III. MODEL DESCRIPTION, CONTROL OBJECTIVES AND PROBLEM FORMULATION

In this section, we introduce a generic model of a homogeneous platoon consisting of  $N$  vehicles. Moreover, we provide formal definitions of *individual vehicle stability* and *string stability*, which are used to formalize the problem statement considered in this paper.

### A. Platooning Dynamics

In this paper, we consider a vehicle platoon consisting of  $N$  identical vehicles as illustrated in Fig. 2. The main objective of each vehicle  $\mathcal{V}_i$ ,  $i \in \tilde{N}$ , in the platoon is to maintain a desired distance  $d_{r,i}$  with respect to its predecessor (defined as vehicle  $i-1$ ). In this paper, we consider a constant time gap

policy in which the desired distance at time  $t \in \mathbb{R}_{\geq 0}$  is given by

$$d_{r,i}(t) = r_i + h v_i(t), \quad (2)$$

where  $v_i(t)$  denotes the velocity of vehicle  $i$  at time  $t$ ,  $r_i$  the standstill distance and where  $h \in \mathbb{R}_{>0}$  represents the desired constant time gap.

Let  $d_i(t)$  represent the actual distance between vehicle  $i$  and its preceding vehicle at time  $t \in \mathbb{R}_{\geq 0}$ , then we define the spacing error  $e_i(t)$ ,  $i \in \tilde{N} \setminus \{1\}$ , at time  $t \in \mathbb{R}_{\geq 0}$  as

$$e_i(t) := d_i(t) - d_{r,i}(t) = (q_{i-1}(t) - q_i(t) - L_i) - (r_i + h v_i(t)), \quad (3)$$

where  $q_i(t)$  denotes the (curvilinear) position of vehicle  $i$  at time  $t$ ,  $L$  the length of the vehicle  $i$ ,  $r$  the standstill distance and  $h$  the time gap as before. Since we consider a homogeneous vehicle platoon,  $h$  does not depend on the index  $i$ . The first vehicle in the platoon (corresponding to  $i = 1$ ) follows a *virtual reference vehicle* (corresponding to index  $i = 0$ ). As such, we defined  $e_1$  as the spacing error between the first vehicle and its virtual predecessor.

The dynamics of the virtual reference vehicle are given by

$$\mathcal{V}_0 : \begin{cases} \dot{v}_0(t) = a_0(t) \\ \dot{a}_0(t) = -\frac{1}{\tau_d} a_0(t) + \frac{1}{\tau_d} u_0(t), \end{cases} \quad (4)$$

where  $u_0(t)$  is the exogenous input of the vehicle platoon at time  $t$ . The plant dynamics of the  $i$ -th vehicle  $\mathcal{V}_i$ ,  $i \in \tilde{N}$ , are given by

$$\dot{e}_i(t) = v_{i-1}(t) - v_i(t) - h a_i(t) \quad (5a)$$

$$\mathcal{V}_i : \begin{cases} \dot{v}_i(t) = a_i(t) \\ \dot{a}_i(t) = -\frac{1}{\tau_d} a_i(t) + \frac{1}{\tau_d} u_i(t) \end{cases} \quad (5b)$$

$$\dot{u}_i(t) = -\frac{1}{h} u_i(t) + \frac{1}{h} \chi_i(t) \quad (5c)$$

where  $v_i(t)$  denotes the velocity at time  $t$ ,  $a_i(t)$  the acceleration at time  $t$ ,  $u_i(t)$  the desired acceleration at time  $t$ ,  $\chi_i(t)$  the control input at time  $t$  and  $\tau_d \in \mathbb{R}_{>0}$  the characteristic time constant of the vehicle drive-line of vehicle  $i$ . Note that (5a) is in correspondence with (3). Moreover, observe from (5b) that the vehicles are assumed to be acceleration-controlled since the dynamic behavior of the acceleration  $a_i(t)$ ,  $i \in \tilde{N}$ , is described by a first-order model. The latter can be realized by means of feedback linearization as described in [18], [43]. At last, observe from (5c) that the signal  $\chi_i(t)$  is filtered by a first order low-pass filter before it is fed into the vehicle drive line. This low-pass filter is used as a pre-compensator for the time-gap policy as in [40]. The latter is illustrated in Fig. 3, where  $H$  represents the spacing policy filter which is given by  $H(s) = h s + 1$ . Let us remark that this filter is such that the variable  $e_i(t)$  in Fig. 3 corresponds to the spacing error  $e_i(t)$ ,  $i \in \tilde{N}$ , as defined in (3). The transfer function  $H^{-1}$  represents the inverse of the spacing policy filter which corresponds to (5c). Observe that the control configuration is such that the asymptotic stability properties of the control-loop of vehicle  $i$  do not depend on  $h$  due to the cancellation



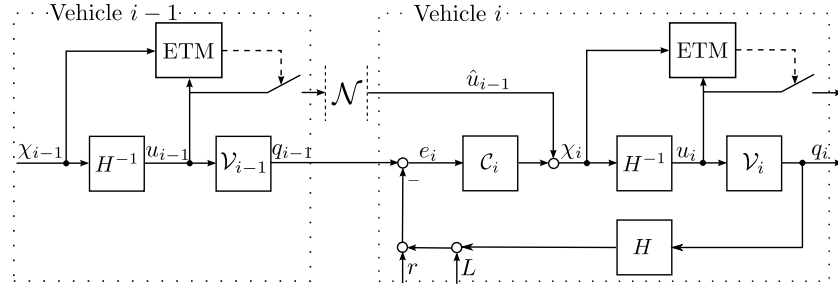


Fig. 3. Schematic representation of the proposed event-triggered CACC setup.

of the poles and zeros of  $H^{-1}$  and  $H$  in the dynamics of the closed-loop system. This is desirable as it enables the driver to set different time-gaps  $h$  without jeopardizing the individual vehicle stability which we formally define below in Section III-B.

Typically, a CACC scheme consists of a feedback controller  $C_i$ ,  $i \in \bar{N}$ , that depends on the spacing error and a feedforward component being the direct feedthrough of the predecessor's desired acceleration  $u_{i-1}(t)$  as illustrated in Fig. 2. The feedback controller often relies on measurements of a forward-looking radar whereas the feedforward component is obtained via Dedicated Short Range Communication (DSRC). In this paper, we consider a control law of the form

$$\chi_i(t) = \underbrace{k_p e_i(t) + k_d \dot{e}_i(t)}_{C_i} + \underbrace{\hat{u}_{i-1}(t)}_{\text{feedforward}}, \quad i \in \bar{N}, \quad (6)$$

where  $k_p$  and  $k_d$  are controller gains to be specified. Observe from (3) and (6) that a *one-vehicle look-ahead control strategy* is considered in the sense that the control law of vehicle  $i$  only depends on local information and information of its predecessor, vehicle  $i-1$ . The first two terms of (6) form the feedback controller  $C_i$  and  $\hat{u}_{i-1}(t)$  the feedforward of the predecessors desired acceleration  $u_{i-1}(t)$  that is sent over the DSRC channel as illustrated in Fig. 3. We employ the notation  $\hat{u}_{i-1}(t)$  to denote the most recently received information regarding  $u_{i-1}(t)$  by vehicle  $i$  at time  $t$ . Due to the packet based-nature of the communication channel and the presence of communication delays, we typically have that  $\hat{u}_{i-1}(t) \neq u_{i-1}(t)$  for  $t \in \mathbb{R}_{\geq 0}$ . Note that in case these communication delays are absent and the communication network is infinitely fast, it would hold that  $\hat{u}_{i-1}(t) = u_{i-1}(t)$  for all  $t \in \mathbb{R}_{\geq 0}$ . Observe from Fig. 3 that, as discussed in the introduction, the time instants at which  $u_i$  will be transmitted over the network are determined by means of an event-triggering mechanism (ETM).

*Remark 1: Besides the aforementioned time gap policy, also other spacing policies are reported in the literature such as a constant (velocity independent) spacing policy ( $h = 0$ ). However, as shown in [34], in one vehicle look-ahead platoons, i.e., platoons in which the vehicles can only obtain information of their predecessors, string stability properties can only be achieved when the spacing policy is velocity dependent. For this reason, we only focus on the constant time gap policy in this paper.*

*Remark 2: To streamline the exposition of the paper, we only consider a platoon of  $N$  vehicles whose dynamics are*

*given by (4), (5),  $i \in \bar{N}$ , and (6). However, as shown in [12], the event-triggered control framework and design methodology used in this paper can be used for a broad class of systems and thus can be applied to a much larger class of vehicle platoons that employ a one-vehicle look-ahead control strategy with possibly other vehicle, spacing policy and controller dynamics.*

### B. Problem Formulation

A well-designed CACC has to comply with two objectives: The first objective is vehicle following, i.e., regulating the spacing errors as given in (3) towards zero while realizing a small time-gap  $h$  (typically less than 1 second). This property is also referred to as *individual vehicle stability*. The second objective is to attenuate disturbances/shock waves along the vehicle platoons. The latter property is also referred to as *string stability* which, as mentioned in the introduction, forms an important property to enhance traffic flows and avoid so-called phantom traffic jams. To be more concrete, the platoon system consisting of  $N$  vehicles whose dynamics are given by (5),  $i \in \bar{N}$ , (4) and (6), should satisfy the following two control objectives.

- (i) *Individually vehicle stability:* For each vehicle  $i$ , it should hold that if  $v_{i-1}(t) = c$  with  $c$  some constant velocity, and  $\hat{u}_{i-1}(t) = 0$  for all  $t \in \mathbb{R}_{\geq 0}$ , then all corresponding solutions to (5),  $i \in \bar{N}$ , and (4) with the corresponding controller as in (6), satisfy

$$\lim_{t \rightarrow \infty} e_i(t) = 0. \quad (7)$$

- (ii) *String stability:* For any exogenous input  $u_0 \in \mathcal{L}_2$  and any  $x(0) \in \mathbb{R}^{(4N+3) \times (4N+3)}$  with  $x^\top = [x_0^\top \ x_1^\top \ \dots \ x_N^\top]^\top$  the lumped state vector, it should hold that for all  $i \in \bar{N}$ , there exists a  $\mathcal{K}_\infty$ -function  $\beta_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  such that the corresponding solution to (5),  $i \in \bar{N}$ , and (4) with the corresponding controller as in (6), satisfies

$$\|\chi_i\|_{\mathcal{L}_2} \leq \|u_0\|_{\mathcal{L}_2} + \beta_i(\|x(0)\|). \quad (8)$$

Let us remark that the string stability objective is closely related to the notion of  $\mathcal{L}_2$ -stability, see, e.g., [26]. In particular, if the system given by (5),  $i \in \bar{N}$ , (4) and (6) is string-stable, then the system is  $\mathcal{L}_2$ -stable with respect to exogenous input  $u_0 \in \mathcal{L}_2$  and every output  $\chi_i \in \mathcal{L}_2$ ,  $i \in \bar{N}$ , with an  $\mathcal{L}_2$ -gain less than or equal to one. Moreover, let us remark that the notion of string stability as presented above is similar to the weak string stability as presented in [37].

As already mentioned, Cooperative Adaptive Cruise Control (CACC) is a promising technology that is proven to be an effective method to achieve both individual vehicle stability and string stability while realizing small time-gaps. The wireless communication via DSRC forms an important ingredient to these conflicting goals. The use of wireless communication, however, also has drawbacks due to inevitable network-induced imperfections that result from the digital (packet-based) nature of the communication network. These imperfections include the presence of (time-varying) delays and a limited communication bandwidth. As already mentioned in the introduction, the aforementioned artifacts potentially degrade the performance of CACC systems in the sense that if the communication delays are too large and/or the time in between two consecutive transmission is too large, string stability for a given time-gap might no longer be guaranteed, see also [16], [25], [29], [36], [37], [42]. A second issue that arises when using wireless communication is that excessive use of the communication resources might lead to degradation of the reliability of the DSRC channel in terms of packet losses and transmission delays as reported in [5]. As such, it is important that unnecessary communication is avoided. Only relevant information should be transmitted over the DSRC channels. Given these facts and the objectives above, we can formulate the problem considered in this paper as follows.

*Problem 1:* Consider a homogeneous platoon consisting of  $N$  vehicles and whose dynamics are given by (4), (5),  $i \in \bar{N}$ , and (6). Propose a resource-aware (event-triggered) CACC system with guaranteed individual vehicle stability and string stability in the presence of time-varying communication delays (given upper bounds on the maximal delay values) and that significantly reduces the number of unnecessary transmissions compared to control methods that employ fixed transmission rates.

To address the problem stated above, we will adopt an emulation approach meaning that we first design the CACC system as in (6) such that the desired stability criteria are satisfied when the network-induced imperfections are ignored, i.e., when  $\hat{u}_{i-1}(t) = u_{i-1}(t)$ ,  $i \in \bar{N}$ , for all  $t \in \mathbb{R}_{\geq 0}$ . Secondly, we provide the design of the ETM. This triggering mechanism determines at which time instants a vehicle transmits a data package, containing its desired acceleration, to its successor. If the ETM is well-designed, the desired properties of the network-free system can still be preserved despite the presence of (event-based) packet-based communication and time-varying communication delays as we will show. In short, the proposed design procedure of a resource-aware controller consists of two steps: (i) the design of the CACC system for the network-free situation and (ii) the design of the ETM. One of the main advantages of this approach is that any control design method for continuous linear time-invariant systems can be used for the design of the CACC system in step (i), although here we focus on the setup as in (6). We will discuss the tuning of this control law in the next section.

#### IV. TUNING OF THE NETWORK-FREE CACC SYSTEM

The controller gains  $k_p$  and  $k_d$  as in (6) need to be tuned such that in absence of network-induced imperfections, i.e.,

when  $\hat{u}_{i-1}(t) = u_{i-1}(t)$  for all  $t \geq 0$ , each vehicle in the platoon is individually stable and that the platoon system itself is string-stable. As shown in [40], individual vehicle stability and string stability for the system given by (4), (5),  $i \in \bar{N}$ , and (6) are obtained for any  $h, k_d, k_p > 0$  for which  $k_d - k_p \tau_d > 0$  with  $\tau_d$  as in (5),  $i \in \bar{N}$ . It is important to notice that, in contrast to the string stability property, the individual vehicle stability is not affected by the network-induced imperfections and thus also not by the design of the ETMs.

In the remainder of the paper, we describe the effect of network-induced imperfections on the platoon dynamics given by (4), (5),  $i \in \bar{N}$ , and (6). Moreover, we will propose a triggering rule that aims to only communicate when necessary to achieve string stability.

#### V. WIRELESS AND EVENT-TRIGGERED COMMUNICATION

As mentioned before, typically we have that  $\hat{u}_{i-1}(t) \neq u_{i-1}(t)$ ,  $t \in \mathbb{R}_{\geq 0}$ , due to the presence of network-induced imperfections. In this section, we describe the effect of the network-induced imperfections more thoroughly by reformulating the platoon model given by (4), (5),  $i \in \bar{N}$ , and (6) in terms of network-induced errors defined by

$$e_{u_{i-1}}(t) := \hat{u}_{i-1}(t) - u_{i-1}(t), \quad i \in \bar{N}. \quad (9)$$

To do so, we first discuss the evolution of  $\hat{u}_i$  over time.

##### A. Wireless Communication

To model the packet-based communication and the presence of delays, note that at each transmission instant  $t_k^{i-1}$ ,  $k \in \mathbb{N}$ , a new measurement of  $u_{i-1}$  is collected and transmitted to vehicle  $i$ . After a communication delay of  $\Delta_k^{i-1}$ ,  $i \in \bar{N} \setminus \{1\}$ ,  $k \in \mathbb{N}$ , time units, the value of  $\hat{u}_{i-1}$  is updated according to

$$\hat{u}_{i-1}((t_k^{i-1} + \Delta_k^{i-1})^+) = u_{i-1}(t_k^{i-1}). \quad (10)$$

In between two update events, the value of  $\hat{u}_i$ ,  $i \in \bar{N}$ , is kept constant in a zero-order hold fashion (ZOH). As such, we adopt the following assumption.

*Assumption 1:* For all  $t \in (t_k^{i-1} + \Delta_k^{i-1}, t_{k+1}^{i-1} + \Delta_{k+1}^{i-1})$ , with  $i \in \bar{N} \setminus \{1\}$  and  $k \in \mathbb{N}$ , it holds that

$$\dot{\hat{u}}_{i-1}(t) = 0. \quad (11)$$

As in [12] and [23], we can distinguish two types of events, namely, events that correspond to time instants at which a vehicle  $i - 1$ , transmits a new measurement to its successor (referred to as *transmission events*), and events corresponding to time instants at which vehicle  $i$  receives a new measurement from its predecessor (referred to as *update events*).

*Remark 3:* In this paper, we assume that all transmissions are successful in the sense that no packet losses occur. In fact, this is justified as in the experiments we did not experience any losses of communication packets. However, by following [13], the event-triggered control scheme presented in the current paper can be extended such that a maximum number of successive packet dropouts (MANSD) can be tolerated. The latter

is possible for configurations with acknowledgment scheme, e.g., as in transmission control protocols (TCP), as well configurations without acknowledgment scheme, e.g., as in user datagram protocols (UDP).

In this paper, it is assumed that the communication delays  $\Delta_k^{i-1}$ ,  $i \in \bar{N} \setminus \{1\}$ ,  $k \in \mathbb{N}$ , are bounded from above by a (known) time-constant called the *maximum allowable delay* (MAD). In addition, we assume that before a vehicle transmits new information, the most recently transmitted information of that vehicle has been received by its succeeding vehicle. To be more specific, we adopt the following assumption.

**Assumption 2:** The transmission delays are bounded according to  $0 \leq \Delta_k^{i-1} \leq \tau_{mad} \leq t_{k+1}^{i-1} - t_k^{i-1}$  for all  $i \in \bar{N} \setminus \{1\}$  and  $k \in \mathbb{N}$ , where  $\tau_{mad}$  denotes the maximum allowable delay of transmissions sent by a vehicle in the platoon.

Observe that  $\tau_{mad}$  does not depend on index  $i$  as we consider a homogeneous vehicle string. Let us remark that Assumption 2 is indeed reasonable to make as in case of DSRC communication, the inter-transmission times are typically larger than the delays as shown in [30].

### B. Event-Based Communication

In this paper, we consider event-triggered control (ETC) schemes that determine the transmission instants by means of a triggering condition that depends on locally available output measurements, see, e.g., [2], [7], [11], [12], [14], [33], [41], [47], [51] and the references therein for more details on ETC. In this way, the communication resources are only used when necessary to maintain desired closed-loop behavior and the utilization of communication resources is reduced.

In particular, we consider *dynamic* event-generators for vehicle  $i \in \bar{N}$  of the form as proposed in [12] that schedule transmission instants according to

$$t_0^i = 0, t_{k+1}^i := \inf \left\{ t \geq t_k^i + \tau_{miet} \mid \eta_i(t) < 0 \right\}, \quad (12)$$

for  $k \in \mathbb{N}$  and where  $\eta_i(t)$  is the triggering variable at time  $t$  that evolves according to

$$\dot{\eta}_i(t) = \Psi_i(\chi_i(t), u_i(t), e_{u_i}(t), \tau_i(t)), \quad (13)$$

for all  $t \in \mathbb{R}_{\geq 0}$ , and where the function  $\Psi_i : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$  and the time-constant  $\tau_{miet} \in \mathbb{R}_{>0}$ , are to be properly designed as we will discuss in the next section. Observe that the time-constant  $\tau_{miet} \in \mathbb{R}_{>0}$  enforces a strictly positive lower-bound on the *minimum inter-event time* (MIET), i.e., the minimum time in between two consecutive transmissions. This strictly positive lower-bound is important to enable implementation of the ETC scheme in practice. Moreover, observe that Assumption 2 is satisfied when  $\tau_{miet} > \tau_{mad}$ , where  $\tau_{mad}$  typically follows from the timing specifications of the DSRC channel.

Given the control law in (6) with  $k_d, k_p > 0$  and  $k_d - k_p \tau_d > 0$  (which leads to individual vehicle stability as discussed before) and the ETM described by (12) and (13), the remaining part of Problem 1 can be reformulated as follows: provide design conditions for the time-constant  $\tau_{miet} \in \mathbb{R}_{>\tau_{mad}}$  and the function  $\Psi$  such that the system described by (4)-(6), and (9)-(13),  $i \in \bar{N}$ , is string-stable despite the

presence of time-varying delays (that are upperbounded by a given  $\tau_{mad}$ ) and such that the (average) time in between two consecutive transmission instants, also referred to as the inter-event times, is significantly larger than for time-triggered control schemes with a fixed transmission rate.

### C. State Space Formulation of Platoon Model

Since we are interested in string stability, it is of interest to evaluate the input-output behavior in terms of  $\mathcal{L}_2$ -gains with respect to  $\chi_{i-1}$  (as input) and  $\chi_i$  (as output). As already illustrated in Fig. 3 and as we will show later, this input-output behavior is not affected by other vehicles in the platoon. To describe this input-output behavior, let us define

$$\tilde{x}_i := [v_{i-1} \quad a_{i-1} \quad u_{i-1} \quad e_i \quad v_i \quad a_i \quad u_i]^\top, \quad (14)$$

for  $i \in \bar{N} \setminus \{1\}$ . Then the platoon dynamics in between the update events, as given in (4)-(6), can be formulated in terms of the following state-space model

$$\dot{\tilde{x}}_i(t) = A\tilde{x}_i(t) + B\chi_{i-1}(t) + E\hat{u}_{i-1}(t), \quad (15)$$

for  $t \in \mathbb{R}_{\geq 0} \setminus \{t_k^{i-1} + \Delta_{k+1}^{i-1}\}_{k \in \mathbb{N}}$ ,  $i \in \bar{N} \setminus \{1\}$ , and  $k \in \mathbb{N}$ , where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\tau_d} & \frac{1}{\tau_d} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{h} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -h & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_d} & \frac{1}{\tau_d} \\ \frac{k_d}{h} & 0 & 0 & \frac{k_p}{h} & -\frac{k_d}{h} & -k_d & -\frac{1}{h} \end{bmatrix} \quad (16)$$

$$B = \begin{bmatrix} 0 & 0 & \frac{1}{h} & 0 & 0 & 0 & 0 \end{bmatrix}^\top \quad (17)$$

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{h} \end{bmatrix}^\top, \quad (18)$$

and where  $\hat{u}_{i-1}$  evolves according to (10) and (11).

The input-output relation of  $u_0$  and  $\chi_1$  does not involve network-induced imperfections since the first vehicle follows a virtual reference vehicle, i.e.,  $\hat{u}_0(t) = u_0(t)$  for all  $t \in \mathbb{R}_{\geq 0}$ . Let us define  $\tilde{x}_1 := [v_0 \ a_0 \ e_1 \ v_1 \ a_1 \ u_1]^\top$ , then the input-output relation of  $u_0$  and  $\chi_1$  can be described by the following state space model

$$\dot{\tilde{x}}_1(t) = A_0\tilde{x}_1(t) + B_0u_0(t), \quad (19a)$$

$$\chi_1(t) = C_0\tilde{x}_1(t) + D_0u_0(t), \quad (19b)$$

for all  $t \in \mathbb{R}_{\geq 0}$ , where

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\tau_d} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -h & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_d} & \frac{1}{\tau_d} \\ \frac{k_d}{h} & 0 & \frac{k_p}{h} & -\frac{k_d}{h} & -k_d & -\frac{1}{h} \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ \frac{1}{h} \\ 0 \\ 0 \\ 0 \\ \frac{1}{h} \end{bmatrix}, \quad (20)$$

$$C_0 = [k_d \quad 0 \quad k_p \quad -k_d \quad -k_d h \quad 0], \quad D_0 = 1.$$



Let us remark that for all  $h, \tau_d, k_p, k_d \in \mathbb{R}_{>0}$ , a minimal realization of  $(A_0, B_0, C_0, D_0)$  is given by  $\chi_1(t) = u_0(t)$ ,  $t \in \mathbb{R}_{\geq 0}$ . In other words, for the zero initial condition, i.e., when  $\tilde{x}_1(0) = [0 \ 0 \ 0 \ 0 \ 0]^T$ , the solution to (19) satisfies  $\chi_1(t) = u_0(t)$ ,  $t \in \mathbb{R}_{\geq 0}$ . Given the latter, we find that (8) holds for  $i = 1$  and  $k_d, k_p > 0$  with  $k_d - k_p \tau_d > 0$  (which leads to individual vehicle stability as discussed in Section IV).

In essence, the dynamical system corresponding to the lumped state vector  $\tilde{x} = [\tilde{x}_1^T \ \tilde{x}_2^T \ \dots \ \tilde{x}_N^T]^T$  that can be obtained by means of (15)-(20),  $i \in \tilde{N}$ , constitutes an overlapping decomposition of the entire platoon model given by (4), (5),  $i \in \tilde{N}$ , and (6), see also [44]. The main advantages of having this decomposition is that we now only have to evaluate the input-output properties of these (overlapping) subsystems. In fact, by recalling that (8) holds for  $i = 1$ , we only need to evaluate the  $\mathcal{L}_2$ -gains of (15) with respect to  $\chi_{i-1}$  (as input) and  $\chi_i$  (as output) for  $i \in \tilde{N} \setminus \{1\}$ . Since we consider a homogeneous platoon, we only have to examine the  $\mathcal{L}_2$ -gain of a single subsystem. This leads to conditions that are computationally more tractable since the state-dimension of  $\tilde{x}_i$ ,  $i \in \tilde{N}$ , is in general much smaller than the state-dimension of  $x$ . More importantly, if these conditions are satisfied, the string stability property as given in (8) holds for any arbitrary platoon length  $N$ . Let us remark, however, that it is not trivial to extend this result to platoons with other communication topologies such as a two-vehicle look-ahead communication topology as described in, e.g., [39].

#### D. Hybrid System Formulation of Platoon Model

To facilitate the  $\mathcal{L}_2$ -gains with respect to  $\chi_{i-1}$  (as input) and  $\chi_i$  (as output), we reformulate the model given in (15) using the hybrid system formulation. To be more specific, consider hybrid systems  $\mathcal{H}$  of the form

$$\dot{\zeta} = F(\zeta, w), \quad \text{when } \zeta \in \mathcal{C}, \quad (21a)$$

$$\zeta^+ = G(\zeta), \quad \text{when } \zeta \in \mathcal{D}, \quad (21b)$$

where  $\zeta \in \mathbb{R}^n$  represents the state of the system,  $w \in \mathbb{R}^{n_w}$  an external input,  $F : \mathbb{R}^n \times \mathbb{R}^{n_w} \rightarrow \mathbb{R}^n$  describes the flow dynamics,  $G : \mathbb{R}^n \rightarrow \mathbb{R}^n$  the jump dynamics,  $\mathcal{C} \subset \mathbb{R}^n$  the flow set and  $\mathcal{D} \subset \mathbb{R}^n$  the jump set, see also [19]. A hybrid system with the data  $\mathcal{C}$ ,  $F$ ,  $\mathcal{D}$  and  $G$  as above is denoted by  $\mathcal{H} = (\mathcal{C}, F, \mathcal{D}, G)$  or, in short,  $\mathcal{H}$ . We now recall some definitions given in [19] regarding the solutions of such hybrid systems.

A *compact hybrid time domain* is a set  $\mathcal{E} = \bigcup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\} \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$  with  $J \in \mathbb{N}_{>0}$  and  $0 = t_0 \leq t_1 \leq \dots \leq t_J$ . A *hybrid time domain* is a set  $\mathcal{E} \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$  such that  $\mathcal{D} \cap ([0, T] \times \{0, \dots, J\})$  is a compact hybrid time domain for each  $(T, J) \in \mathcal{D}$ . A *hybrid signal* is a function defined on a hybrid time domain. A hybrid signal  $\zeta : \text{dom } \zeta \rightarrow \mathbb{R}^n$  is called a *hybrid arc* if  $\zeta(\cdot, j)$  is locally absolutely continuous for each  $j$ .

For the hybrid system  $\mathcal{H}$  given by the state space  $\mathbb{R}^n$ , the input space  $\mathbb{R}^{n_w}$  and the data  $(\mathcal{C}, F, \mathcal{D}, G)$ , a hybrid arc  $\zeta : \text{dom } \zeta \rightarrow \mathbb{R}^n$  and a hybrid input signal  $w : \text{dom } w \rightarrow \mathbb{R}^{n_w}$  is called a *solution pair*  $(\zeta, w)$  to  $\mathcal{H}$  if

- 1)  $\text{dom } \zeta = \text{dom } w$ ,
- 2) For all  $j \in \mathbb{N}$  and for almost all  $t$  such that  $(t, j) \in \text{dom } \zeta$ , we have  $\dot{\zeta}(t, j) \in \mathcal{C}$  and  $\dot{\zeta}(t, j) = F(\zeta(t, j), w(t, j))$ .
- 3) For all  $(t, j) \in \text{dom } \zeta$  such that  $(t, j+1) \in \text{dom } \zeta$ , we have  $\zeta(t, j) \in \mathcal{D}$  and  $\zeta(t, j+1) = G(\zeta(t, j))$ .

For the motivation and more details on these definitions, the interested reader is referred to [19]. We will often not mention  $\text{dom } \zeta$  explicitly, and understand that with each hybrid solution pair  $(\zeta, w)$  comes a hybrid time domain  $\text{dom } \zeta = \text{dom } w$ . A solution pair  $(\zeta, w)$  to system (21) is *nontrivial* if  $\text{dom } \zeta$  contains at least two points, *maximal* if there does not exist another solution  $\zeta'$  to  $\mathcal{H}$  such that  $\text{dom } \zeta$  is a proper subset of  $\text{dom } \zeta'$  and  $\zeta(t, j) = \zeta'(t, j)$  for all  $(t, j) \in \text{dom } \zeta$ , it is *complete* if its domain,  $\text{dom } \zeta$ , is unbounded, it is *Zeno* if it is complete and  $\sup_t \text{dom } \zeta < \infty$ , where  $\sup_t \text{dom } \zeta := \sup\{t \in \mathbb{R}_{\geq 0} : \exists j \in \mathbb{N} \text{ such that } (t, j) \in \text{dom } \zeta\}$ , and it is *t-complete* if  $\text{dom } \zeta$  is unbounded in the  $t$ -direction, i.e.,  $\sup_t \text{dom } \zeta = \infty$ .

In addition, we introduce the  $\mathcal{L}_2$ -norm of a function  $\zeta$  defined on a hybrid time domain  $\text{dom } \zeta = \bigcup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\}$  with  $J$  possibly  $\infty$  and/or  $t_J = \infty$  by

$$\|\zeta\|_{\mathcal{L}_2} = \sqrt{\sum_{j=0}^{J-1} \int_{t_j}^{t_{j+1}} |\zeta(t, j)|^2 dt} \quad (22)$$

provided the right-hand side is well-defined and finite. In case  $\|\zeta\|_{\mathcal{L}_2}$  is finite, we say that  $\zeta \in \mathcal{L}_2$ . Note that this definition is essentially identical to the usual  $\mathcal{L}_2$ -norm in case a function is defined on a subset of  $\mathbb{R}_{\geq 0}$ .

Moreover, we express the model in terms of the network-induced errors  $e_{u_{i-1}}$ ,  $i \in \tilde{N} \setminus \{1\}$ , as given in (9), which was also employed in [8], [12], [23], [35], and [41]. Before we reformulated the platoon model in terms of a hybrid system as in (21), we first discuss the dynamics of  $e_{u_{i-1}}$ ,  $i \in \tilde{N} \setminus \{1\}$ , as in (9) at update events. By recalling that at update events,  $\hat{u}_{i-1}$  is updated according to (10), we find that the dynamics of  $e_{u_{i-1}}$ ,  $i \in \tilde{N} \setminus \{1\}$ , at update events are given by

$$\begin{aligned} e_{u_{i-1}} &= ((t_k^{i-1} + \Delta_k^{i-1})^+ - \hat{u}_{i-1}((t_k^{i-1} + \Delta_k^{i-1})^+)) \\ &\quad - u_{i-1}(t_k^{i-1} + \Delta_k^{i-1}) \\ &= u_{i-1}(t_k^{i-1}) - \hat{u}_{i-1}(t_k^{i-1} + \Delta_k^{i-1}) + \hat{u}_{i-1}(t_k^{i-1} + \Delta_k^{i-1}) \\ &\quad - u_{i-1}(t_k^{i-1} + \Delta_k^{i-1}) \\ &\stackrel{(11)}{=} \underbrace{u_{i-1}(t_k^{i-1}) - \hat{u}_{i-1}(t_k^{i-1})}_{-e_{u_{i-1}}(t_k^{i-1})} \\ &\quad + \underbrace{\hat{u}_{i-1}(t_k^{i-1} + \Delta_k^{i-1}) - u_{i-1}(t_k^{i-1} + \Delta_k^{i-1})}_{e_{u_{i-1}}(t_k^{i-1} + \Delta_k^{i-1})} \\ &= -e_{u_{i-1}}(t_k^{i-1}) + e_{u_{i-1}}(t_k^{i-1} + \Delta_k^{i-1}). \end{aligned} \quad (23)$$

Let us highlight that for the third equality, we used Assumption 1.

To formulate the platoon dynamics in terms of jump and flow equations as in (21), we introduce the auxiliary variables  $\tau_i \in \mathbb{R}_{\geq 0}$ ,  $l_i \in \{0, 1\}$ ,  $s_i \in \mathbb{R}$ ,  $i \in \tilde{N}$ . The variable  $\tau_i$  constitutes a local timer that keeps track on the time elapsed

since the most recent transmission of vehicle  $i$ . The variable  $l_i$  indicates whether the next event at vehicle  $i$  is a transmission ( $l_i = 0$ ) or an update event ( $l_i = 1$ ). The variable  $s_i$  is used as a memory variable to store the value of  $-e_{u_i}$  at transmission instants  $t_k^i$ ,  $i \in \bar{N}$ ,  $k \in \mathbb{N}$ . Consider the state vector

$$\xi_i = (\tilde{x}_i, e_{u_{i-1}}, \tau_{i-1}, l_{i-1}, s_{i-1}, \eta_{i-1}) \in \mathbb{X}_i, i \in \bar{N} \setminus \{1\} \quad (24)$$

with  $\mathbb{X}_i := \mathbb{R}^7 \times \mathbb{R} \times \mathbb{R}_{\geq 0} \times \{0, 1\} \times \mathbb{R} \times \mathbb{R}_{\geq 0}$ . Based on (12) and Assumption 2, we find that the flow and jump sets are given by

$$\mathcal{C}_i := \{\xi_i \in \mathbb{X}_i \mid l_{i-1} = 0 \vee (0 \leq \tau_{i-1} \leq \tau_{mad} \wedge l_{i-1} = 1)\}, \quad (25)$$

and

$$\mathcal{D}_i := \{\xi_i \in \mathbb{X}_i \mid (\eta_{i-1} = 0 \wedge \tau_{i-1} \geq \tau_{miet}) \vee l_{i-1} = 1\}, \quad (26)$$

respectively. Observe that indeed, the system jumps in case the triggering condition given in (12) is violated. The flow dynamics are given by  $\dot{\xi}_i = F_i(\xi_i, \chi_{i-1})$  with the flow map  $F_i : \mathbb{X} \times \mathbb{R} \rightarrow \mathbb{X}$  defined as

$$F_i(\xi_i, \chi_{i-1}) := \left( f(\tilde{x}_i, e_{u_{i-1}}, \chi_{i-1}), g(\tilde{x}_i, \chi_{i-1}), 1, 0, 0, \Psi_{i-1}(\chi_{i-1}, u_{i-1}, e_{u_{i-1}}, \tau_{i-1}) \right). \quad (27)$$

By substituting (9) in (15), we obtain that  $f(\tilde{x}_i, e_{u_{i-1}}, \chi_{i-1})$  is given by

$$f(\tilde{x}_i, e_{u_{i-1}}, \chi_{i-1}) := A_{11}\tilde{x}_i + A_{12}e_{u_{i-1}} + A_{13}\chi_{i-1}, \quad (28)$$

where

$$A_{11} = A + EC, \quad A_{12} = E, \quad A_{13} = B, \quad (29)$$

and where  $C := [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]$ , such that  $u_{i-1} = C\tilde{x}_i$ . The expression for  $g(\tilde{x}_i, \chi_{i-1})$  can be obtained from the fact that  $\dot{e}_{u_{i-1}} = -\dot{u}_{i-1}$  when  $\xi_i \in \mathcal{C}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , due to Assumption 1. As such,  $g(\tilde{x}_i, \chi_{i-1})$  is given by

$$g(\tilde{x}_i, \chi_{i-1}) := \frac{1}{h}C\tilde{x}_i - \frac{1}{h}\chi_{i-1}. \quad (30)$$

The jump dynamics are given by  $\xi_i^+ = G_i(\xi_i)$  with the jump map  $G_i : \mathbb{X} \rightarrow \mathbb{X}$  defined as

$$G_i(\xi_i) := (\tilde{x}_i, e_{u_{i-1}} + l_{i-1}s_{i-1}, l_{i-1}\tau_{i-1}, \\ -(1 - l_{i-1})e_{u_{i-1}}, 1 - l_{i-1}, \eta_{i-1}). \quad (31)$$

Observe that, when  $l_i = 0$ , the variable  $l_i$  jumps to the value 1, and, when  $l_i = 1$ ,  $l_i$  jumps to the value 0. The latter ensures that a transmission event can only be followed by an update event and vice versa. Hence, the flow and jump sets and maps comply with Assumption 2. Moreover, observe that the timer  $\tau_i$  is set to zero at transmission event (when  $l_i = 0$ ).

Observe that the jump map in (31) is defined such that  $s_{i-1}$ ,  $i \in \bar{N} \setminus \{1\}$ , is assigned the value of  $-e_{u_{i-1}}$  at a transmission event (when  $l_i = 0$ ). Given the latter and the fact that  $\dot{s}_{i-1} = 0$  for  $\xi_i \in \mathcal{C}_i$ , we can see that the jump dynamics of  $e_{u_{i-1}}$  defined

by (31) are in correspondence with (23). Note that the variable  $s_{i-1}$  is set to zero at update events (when  $l_i = 1$ ).

The input-output relation of  $\chi_{i-1}$  and  $\chi_i$  subject to network-induced errors is now described by the hybrid system  $\mathcal{H}_i = (\mathcal{C}_i, F_i, \mathcal{D}_i, G_i)$ ,  $i \in \bar{N} \setminus \{1\}$  with  $\mathcal{C}_i$ ,  $F_i$ ,  $\mathcal{D}_i$  and  $G_i$  as in (25), (27), (26) and (31), respectively, and where the output  $\chi_i$ ,  $i \in \bar{N} \setminus \{1\}$ , as in (6) is given by

$$\chi_i = C_z\tilde{x}_i + D_z e_{u_{i-1}}, \quad (32)$$

where

$$C_z = [k_d \ 0 \ 1 \ k_p \ -k_d \ -k_d h \ 0], \quad D_z = 1. \quad (33)$$

It is important to notice that this relation indeed only depends on (part of) the states of vehicle  $i - 1$  and the states of vehicle  $i$  and that it does not depend on states of other vehicles in the platoon.

Consider the following definition of  $\mathcal{L}_2$ -stability for the hybrid system  $\mathcal{H}_i$ ,  $i \in \bar{N} \setminus \{1\}$ .

*Definition 1:* The hybrid system  $\mathcal{H}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , is said to be  $\mathcal{L}_2$ -stable from input  $\chi_{i-1}$  to output  $\chi_i$  with an  $\mathcal{L}_2$ -gain less than or equal to  $\theta$ , if there exists a  $\mathcal{K}_\infty$ -function  $\beta$  such that for any exogenous input  $\chi_{i-1} \in \mathcal{L}_2$ , and any initial condition  $\xi_i(0, 0) \in \mathbb{X}_{0,i}$  with  $\mathbb{X}_{0,i} = \{\xi_i \in \mathbb{X} \mid l_{i-1} = 0\}$ , each corresponding maximal solution to  $\mathcal{H}_i$  is  $t$ -complete and satisfies

$$\|\chi_i\|_{\mathcal{L}_2} \leq \beta(\|(\tilde{x}_i(0, 0), e_{u_{i-1}}(0, 0), s_{i-1}(0, 0), \eta_{i-1}(0, 0))\|) \\ + \theta \|\chi_{i-1}\|_{\mathcal{L}_2}. \quad (34)$$

By means of this definition, we can now formalize Problem 1 as follows. Note that string stability is formulated in terms of an  $\mathcal{L}_2$ -gain being smaller than or equal to 1.

*Problem 2:* Determine the time constants  $\tau_{miet}$ ,  $\tau_{mad} \in \mathbb{R}_{>0}$  with  $\tau_{mad} \leq \tau_{miet}$ , the function  $\Psi_{i-1}$ ,  $i \in \bar{N} \setminus \{1\}$  (as in the event generator given by (12) and (13)), such that each system  $\mathcal{H}_i = (\mathcal{C}_i, F_i, \mathcal{D}_i, G_i)$ ,  $i \in \bar{N} \setminus \{1\}$  with  $\mathcal{C}_i$ ,  $F_i$ ,  $\mathcal{D}_i$  and  $G_i$  as in (25), (27), (26) and (31), respectively, is  $\mathcal{L}_2$ -stable from input  $\chi_{i-1}$  to output  $\chi_i$  with an  $\mathcal{L}_2$ -gain less than or equal to one, with a strictly positive  $\tau_{miet}$  to assure Zeno-freeness and with large (average) inter-event times  $t_{j+1}^{i-1} - t_j^{i-1}$ ,  $j \in \mathbb{N}$ .

By recalling that the individual vehicle stability is not affected by the ETM design and the fact that (8) holds for  $i = 1$ , we can conclude that solving Problem 2 is sufficient for solving the problem loosely stated at the end of Section III-B. In the next subsection, we present the design procedure for the time constants  $\tau_{mad}$  and  $\tau_{miet}$  and the function  $\Psi_{i-1}$ ,  $i \in \bar{N} \setminus \{1\}$ , such that the criteria mentioned in Problem 2 are satisfied.

## VI. ETM DESIGN WITH STRING STABILITY GUARANTEES

In the first part of this section, we specify the conditions for the design of  $\tau_{mad}$ ,  $\tau_{miet}$  and  $\Psi_i$  as in (13) based on the result in [11] and [12], and for obtaining string stability guarantees. Based on these conditions, we provide a systematic design procedure in the second part of this section, resulting in intuitive tradeoff curves between robustness in terms of  $\tau_{mad}$  and utilization of communication resources in terms of  $\tau_{miet}$ .



### A. Stability Analysis

Consider the following condition regarding the flow-dynamics of  $\tilde{x}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , as given in (27).

*Condition 1:* There exist constants  $\gamma$ ,  $\epsilon$ ,  $\varrho \in \mathbb{R}_{\geq 0}$  and  $\mu \in \mathbb{R}_{> 0}$  such that (35), as shown at the bottom of the next page, holds with  $A_{11}$ ,  $A_{12}$  and  $A_{13}$  as in (29)

Let us remark that Condition 1 in essence constitutes an  $\mathcal{L}_2$ -gain condition on the linear system given in (15), where  $\sqrt{1+\epsilon}$  is an  $\mathcal{L}_2$ -gain upper bound with respect to  $\chi_{i-1}$  and  $\chi_i$  and  $\gamma$  an  $\mathcal{L}_2$ -gain upper bound related to the influence of the transmission error  $e_{u_{i-1}}$  on the state  $\tilde{x}_i$ .

In addition, we consider the following condition regarding the time-constants  $\tau_{mad}$  and  $\tau_{miet}$ .

*Condition 2:* There exists a pair of time-constants  $(\tau_{mad}, \tau_{miet})$  such that

$$\gamma_1 \phi_1(\tau) \geq \gamma_0 \phi_0(\tau), \text{ for all } \tau \in [0, \tau_{mad}], \quad (36a)$$

$$\gamma_0 \phi_0(\tau_{miet}) \geq \lambda^2 \gamma_1 \phi_1(0) \quad (36b)$$

with  $\tau_{miet} \geq \tau_{mad}$ ,  $\lambda \in (0, 1)$  and the constants  $\gamma_0$  and  $\gamma_1$  given by

$$\gamma_0 := \gamma, \quad \gamma_1 := \frac{\gamma}{\lambda}, \quad (37)$$

and where  $\phi_l : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ ,  $l \in \{0, 1\}$ , satisfies

$$\frac{d}{d\tau} \phi_l = -\gamma_l (\phi_l^2 + 1) \quad (38)$$

with  $\phi_l(0) > 0$ .

At last, we consider that the function  $\Psi_{i-1} : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ ,  $i \in \bar{N} \setminus \{1\}$ , is of the form

$$\begin{aligned} \Psi_{i-1}(\chi_{i-1}, u_{i-1}, e_{u_{i-1}}, \tau_{i-1}) &= \varrho u_{i-1}^2 \\ &+ \omega(\tau_{i-1}) \left( \frac{1-\epsilon}{h^2} (\chi_{i-1} - u_{i-1})^2 - \bar{\gamma} e_{u_{i-1}}^2 \right), \end{aligned} \quad (39)$$

where

$$\omega(\tau_{i-1}) := \begin{cases} 0, & \text{for } \tau_{i-1} \leq \tau_{miet} \\ 1, & \text{for } \tau_{i-1} > \tau_{miet} \end{cases} \quad (40)$$

with

$$\bar{\gamma} = \gamma^2 \left( 1 + \frac{1}{\epsilon} \phi_0^2(\tau_{miet}) \right), \quad (41)$$

and where  $\epsilon \in (0, 1)$  and  $\varrho \in \mathbb{R}_{\geq 0}$  are tuning parameters.

Observe from (39) that  $\Psi_{i-1}(\chi_{i-1}, u_{i-1}, e_{u_{i-1}}, \tau_{i-1}) \geq 0$ ,  $i \in \bar{N} \setminus \{1\}$ , for  $\tau_{i-1} \leq \tau_{miet}$ . As such, the triggering variable  $\eta_i$  as in (13) does not decrease when  $\tau_{i-1} \leq \tau_{miet}$ . Moreover, observe from (39) that  $\Psi_{i-1}(\chi_{i-1}, u_{i-1}, e_{u_{i-1}}, \tau_{i-1}) \leq 0$ ,  $i \in \bar{N} \setminus \{1\}$  (and thus that the triggering variable  $\eta_{i-1}$  is decreasing) when the transmission error  $e_{u_{i-1}}$  is relatively large with respect to  $u_{i-1}^2 + \frac{1-\epsilon}{h^2} (\chi_{i-1} - u_{i-1})^2$ . Given the latter, we can conclude from (12) that a new transmission is scheduled if the

transmission error  $e_{u_{i-1}}$  is relatively large over time. Hence, in essence, this mechanism is similar to the ETM presented in the introduction, given by (1). The difference, however, is that for the ETM given by (12), (13) and (39), a robust positive MIET exists by design. Moreover, the ETM given by (12), (13) and (39) is a *dynamic* ETM as it employs the dynamic variable  $\eta_i$  to determine the transmission instants. The main motivation for using *dynamic* ETMs is that, in contrast to the commonly studied *static* ETMs (such as the ETM in (1)), the generated inter-event times do not converge to the enforced lower bound in presence of disturbances when the output is close to zero, as observed in [7] and [11], and typically lead to larger inter-event times.

By means of the Condition 1, Condition 2 and the definition of  $\Psi_{i-1}$ ,  $i \in \bar{N} \setminus \{1\}$ , as given in (39), we can now establish the following result.

*Theorem 1:* Consider the system  $\mathcal{H}_i = (\mathcal{C}_i, F_i, \mathcal{D}_i, G_i)$ ,  $i \in \bar{N} \setminus \{1\}$ , with  $\mathcal{C}_i$ ,  $F_i$ ,  $\mathcal{D}_i$  and  $G_i$  as in (25), (27), (26) and (31), respectively, and with  $\Psi_{i-1}$  given by (39) and suppose that Condition 1 with  $\epsilon = 0$  and Condition 2 hold. Then the system  $\mathcal{H}_i$  is  $\mathcal{L}_2$ -stable from input  $\chi_{i-1}$  to output  $\chi_i$  with an  $\mathcal{L}_2$ -gain less than or equal to one and thus the platoon system given by (5),  $i \in \bar{N}$ , (4) and (6) is string stable.

The proof is provided in the Appendix.

It is important to notice that Condition 1 and Condition 2 do not depend on index  $i$  due to the homogeneous nature of the vehicle string.

### B. ETM Design

The first step of the design procedure is to compute the constants  $\gamma$ ,  $\mu$  and the matrix  $P$ . The constants  $\gamma$ ,  $\mu$  and the matrix  $P$  can be obtained via a Linear Matrix Inequality (LMI) optimization problem in which  $\gamma$  is minimized subject to the LMI given in (35) where the constant  $\varrho$  is a tuning parameter and  $\epsilon$  is typically selected small. This LMI optimization problem can, for example, be solved using MATLAB with the YALMIP interface [31] and the SeDuMi solver [45]. Let us remark that if the system described by (4)-(6) is string-stable in the absence of network-induced imperfections, *i.e.*, when  $\hat{u}_{i-1}(t) = u_i(t)$  for all  $t \in \mathbb{R}_{\geq 0}$ , the LMI in (35) is always feasible for sufficiently large  $\gamma$  and sufficiently small  $\varrho$ .

As mentioned before, the constant  $\gamma$  is related the influence of the transmission error  $e_{u_{i-1}}$  on the state  $\tilde{x}_i$ . To be more specific, when  $\gamma$  is large, then the influence of transmission error  $e_{u_{i-1}}$  on the state  $\tilde{x}_i$  is also large and more transmissions might be needed in order to realize desirable closed-loop results.

The second step in the design procedure is to determine the  $(\tau_{mad}, \tau_{miet})$ -tradeoff curves. To do so, we solve (38) for  $\phi_0(0) = \frac{1}{\lambda}$  and  $\phi_1(0) \in (\frac{\phi_0 \gamma_0}{\gamma_1}, \frac{\phi_0 \gamma_0}{\gamma_1 \lambda^2}]$  with  $\lambda \in (0, 1)$ .

$$\begin{pmatrix} A_{11}^\top P + P A_{11} + \mu C_z^\top C_z + (\varrho + \frac{1}{h^2}) C^\top C & P A_{12} + \mu C_z^\top D_z & P A_{13} + \frac{1}{h^2} C^\top \\ A_{12}^\top P + \mu D_z^\top C_z & \mu D_z^\top D_z - \gamma^2 & 0 \\ A_{13}^\top P + \frac{1}{h^2} C & 0 & \frac{1}{h^2} - (1 + \epsilon) \mu \end{pmatrix} \leq 0, \quad P = P^\top \geq 0. \quad (35)$$

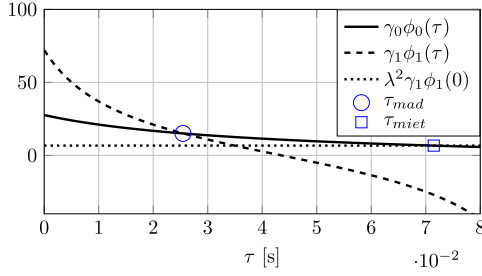


Fig. 4. Evolution of  $\gamma_l \phi_l(\tau)$ ,  $l \in \{0, 1\}$ , with  $\phi_l$  the solutions of (38) for  $\gamma = 8.442$ ,  $\lambda = 0.305$ ,  $\phi_0(0) = 3.279$  and  $\phi_1(0) = 8.557$ . The circle represents the point at which  $\gamma_0 \phi_0(\tau) = \gamma_1 \phi_1(\tau)$ , which corresponds to  $\tau_{mad} = 0.026$  and the rectangle represents the point at which  $\gamma_0 \phi_0(\tau) = \lambda^2 \gamma_1 \phi_1(0)$ , which corresponds to  $\tau_{miet} = 0.072$ .

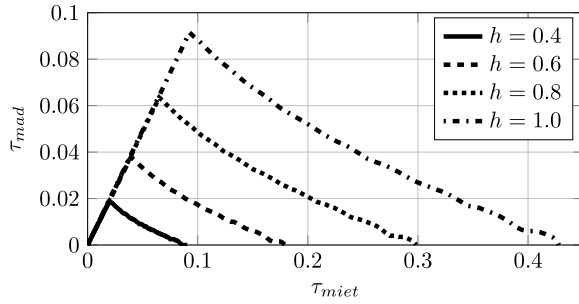


Fig. 5.  $(\tau_{mad}, \tau_{miet})$ -tradeoff curves corresponding to the platoon system described by (5),  $i \in \bar{N}$ , (4) and (6) with  $\tau_d = 0.1$ ,  $k_p = 0.2$ ,  $k_d = 0.7$ ,  $h \in \{0.4, 0.6, 0.8, 1.0\}$ ,  $\varrho = 0.04$  and  $\epsilon = 0.01$ .

The corresponding  $(\tau_{mad}, \tau_{miet})$ -combinations can be obtained by computing intersection of the functions  $\gamma_0 \phi_0$  and  $\gamma_1 \phi_1$  (which corresponds to  $\tau_{mad}$ ) and the intersection of  $\gamma_0 \phi_0$  and  $\lambda^2 \gamma_1 \phi_1(0)$  (which correspond to  $\tau_{miet}$ ), see also [23].

The latter procedure is also illustrated in Fig. 4 in which (38) was solved for  $\gamma = 8.442$ ,  $\lambda = 0.305$ ,  $\phi_0(0) = 3.279$  and  $\phi_1(0) = 8.557$ . As shown in Fig. 4, this results in  $(\tau_{mad}, \tau_{miet}) = (0.026, 0.072)$ .

By repeating the procedure described above for various values of  $\lambda$ , and  $\phi_1(0)$ , other  $(\tau_{mad}, \tau_{miet})$ -combinations can be obtained which are then used to establish the  $(\tau_{mad}, \tau_{miet})$ -tradeoff curve. This tradeoff curve provides an intuitive way to find appropriate values  $\lambda$ ,  $\phi_0(0)$  and thus  $\phi_0(\tau_{miet})$  which are part of the ETM given by (12), (13) and (39). As such, although the design procedure might seem difficult to carry out at first sight, the required conditions can be verified in a systematic manner and the procedure results in intuitive tradeoff curves. In Fig. 5, the  $(\tau_{mad}, \tau_{miet})$ -tradeoff curves are shown for the platoon system described by (5),  $i \in \bar{N}$ , (4) and (6) with  $\tau_d = 0.1$ ,  $k_p = 0.2$ ,  $k_d = 0.7$ ,  $h \in \{0.4, 0.6, 0.8, 1.0\}$ ,  $\varrho = 0.04$  and  $\epsilon = 0.01$ . The  $(\tau_{mad}, \tau_{miet})$ -combinations that are confined by the trade-off curves and the horizontal axis lead to a string-stable platoon.

As mentioned before, if the system is string-stable when network-induced errors are absent, i.e., when  $\hat{u}_{i-1}(t) = u_{i-1}(t)$  for all  $t \in \mathbb{R}_{\geq 0}$ , one can always find a matrix  $P$  and constants  $\gamma$ ,  $\epsilon$  and  $\varrho$  that satisfy (35). Consequently, after the constants  $\gamma$ ,  $\epsilon$  and  $\varrho$  are obtained, one can always find (possibly small) time-constants  $\tau_{miet}$  and  $\tau_{mad}$  such that string stability is preserved.

*Remark 4:* The LMI condition as given in (35) yields an  $\mathcal{L}_2$ -gain upper bound with respect to  $\chi_{i-1}$  and  $\chi_i$  of  $\sqrt{1 + \epsilon}$ . Hence, strictly speaking (and as indicated in Theorem 1), the vehicle platoon described by (4) and (5) is only guaranteed to be string-stable if  $\epsilon = 0$ . Let us remark, however, that the  $\mathcal{L}_2$ -gain is also lower-bounded by 1 due to the individual vehicle stability objective. As such, the LMI optimization problem stated in (35) is hard to solve from a numerical point of view. For this reason, we typically take  $\epsilon$  small ( $\epsilon \sim 10^{-2}$ ) but strictly positive to make the LMI computationally tractable. Let us remark that  $\epsilon$  is typically chosen such that its size is small with respect to the uncertainties introduced by the inevitable noise on radar and accelerometer measurements.

*Remark 5:* The variable  $\varrho$  constitutes a tuning parameter of the ETC system. To be more specific, from (39) we can see that the variable  $\varrho$  is part of the ETM given in (12) and (13), and therefore can have a significant influence on the average inter-event times generated by this ETM. Let us remark, however, that a bit of performance is sacrificed by choosing  $\varrho$  positive in the sense that it affects  $\epsilon$ .

*Remark 6:* Important to notice is that the triggering condition given by (12), (13) and (39) indeed only depends on locally available information since, under the assumption that no packet losses occur, the variables  $\chi_{i-1}$ ,  $u_{i-1}$ ,  $\hat{u}_{i-1}$  and  $\tau_{i-1}$ , are available at vehicle  $i - 1$ .

## VII. EXPERIMENTAL VALIDATION

To validate the proposed resource-aware control design and to demonstrate its technical feasibility, the event-triggered CACC strategy has been implemented on a platoon of three (almost) identical passenger vehicles. The Toyota Prius III Executive is selected as benchmark vehicle and is equipped with long-range radar, GPS and a communication module that uses the IEEE 802.11p-based ETSI ITS G5 standard for DSRC, see also Fig. 6. Let us remark that the same test-bed was used in [40]. A schematic overview of the vehicle architecture is provided in Fig. 7. Observe that the vehicle gateway provides access to the CAN-bus of the vehicle that is connected to the vehicle's actuators and sensors, which among others consists of the hybrid drive line, the accelerometer, the long-range radar measurements and the wheel speed encoders. Besides CAN-bus access, the vehicle gateway includes the low-level acceleration controller as discussed below (5) and safety-functionalities including a mechanism that allows the driver to overrule the system at any time.

The control design strategy is implemented on a real-time target, which provides reference commands to the vehicle gateway. Moreover, the real-time target, which runs MATLAB Simulink Real-Time applications at 100 Hz, is connected to the ITS G5 communication module and an HMI display that informs the driver about the target tracking functionalities, for example, it shows whether or not a wireless connection with the preceding vehicle is established. The characteristic time constant of the drive line is  $\tau_d = 0.1$ . As experimentally verified in [40], the first-order model used to describe the drive line which, neglecting the actuation delay, adequately describes the longitudinal dynamics. To include the actuation



Fig. 6. Benchmark platoon consisting of three passenger cars equipped with long-range radar and DSRC.

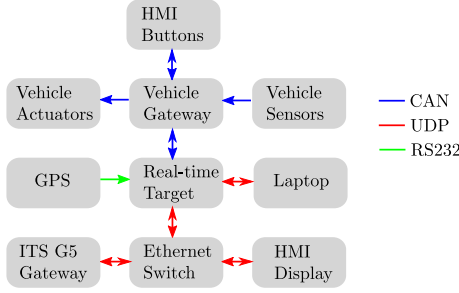


Fig. 7. Schematic overview of the vehicle architecture.

delay of 0.2 seconds in the model, we use a 12<sup>th</sup>-order Padé approximation to obtain a model including actuation delays that is used for the ETM design. For the experiments, we choose the controller gains as in (6) as  $k_p = 0.2$  and  $k_d = 0.7$ , the desired following distance as in (2) with  $r = 2.5$  meters and  $h = 0.6$  seconds and the ETM parameters as in (39) and (35) as  $\varrho = 0.04$ ,  $\varepsilon = 0.5$  and  $\epsilon = 0.01$ , which yield  $(\tau_{mad}, \tau_{miet})$ -tradeoff curves as depicted in Fig. 5. Based on these tradeoff curves, we choose  $(\tau_{mad}, \tau_{miet}) = (0.026, 0.072)$ , which corresponds to  $\gamma = 8.442$ ,  $\lambda = 0.305$ ,  $\phi_0(0) = 3.279$  and  $\phi_1(0) = 8.557$  and coincides with the situation illustrated in Fig. 4.

To enable the implementation of the ETM as described by (12), (13) and (39) on a digital real-time platform, we use exact discretization in order to obtain a triggering condition that is only verified as discrete-time instants. Let us remark that in this discrete implementation a triggering event is scheduled before it actually violates (12) in order to compensate for the sampling effect of the real-time target. Moreover, as the signal-to-noise ratio is large when  $u_i$  is close to zero, it is desirable to avoid transmissions due to this measurement noise when  $u_i$  is close to zero. As such, we employ, next to the ETM as given in (12) and (13), a constant threshold in the sense that no transmissions are being issued when  $|u_i| \leq 0.05$ .

To evaluate the proposed ETC method in terms of performance (in this case the spacing error) and utilization of communication resources (in this case the inter-event times), the experimental results are compared with the results obtained using a time-triggered control (TTC) scheme, in which the transmission instants are determined according to fixed transmission rate of  $1/T_s = 25$  Hz, as benchmark. The experimental results are shown in Fig. 8 and Fig. 9. To be more specific, in Fig. 8, the inter-event times generated by the first and second car of the platoon are shown. Moreover, to indicate the predictability of the proposed ETC scheme, also the simulation results are included. Observe from Fig. 8 and Fig. 9 that the inter-event times generated by the ETM

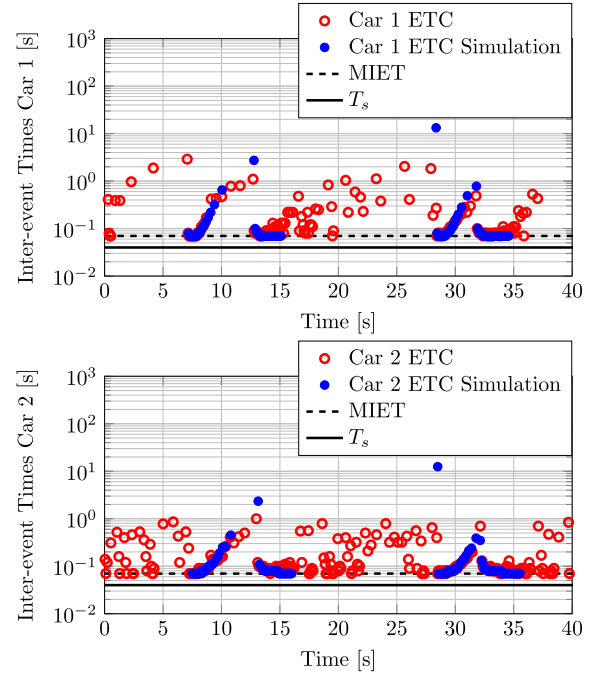


Fig. 8. Inter-event times generated by the ETM as given in (12), (13) and (39) resulting from the experiments (indicated in red) and from simulation (indicated in blue). The horizontal lines represent the minimum inter-event times  $\tau_{miet}$  and the fixed transmission period  $T_s = 0.04$  seconds as used in the benchmark TTC scheme.

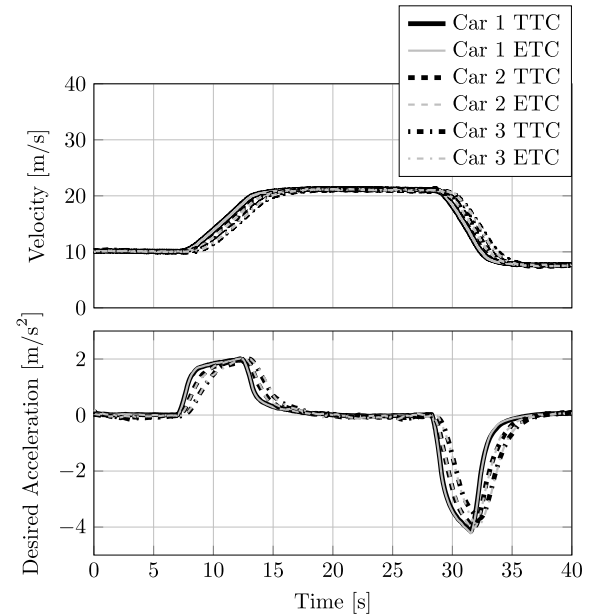


Fig. 9. Time response of the platoon of 3 passenger vehicles. In the top plot, the velocity  $v_i(t)$ ,  $i \in \{1, 2, 3\}$ , at time  $t$ , of each vehicle is displayed. The bottom plot shows the desired acceleration  $\chi_i(t)$ ,  $i \in \{1, 2, 3\}$  at time  $t$ , for all three vehicles.

are only small in case there is a significant change in the desired acceleration and otherwise, significantly larger than the enforced lower-bound  $\tau_{miet}$  or the communication period  $T_s$  of the TTC scheme (showing that “communication is only used when really needed”). In fact, the average inter-event times generated by the ETM in vehicle 1 and in vehicle 2, are  $\tau_{avg}^1 = 0.24$  and  $\tau_{avg}^2 = 0.16$ , respectively, which is clearly larger than  $\tau_{miet}$  and  $T_s$ .



In Fig. 9, the time-response of the velocity and desired acceleration for both the proposed ETC and the benchmark TTC strategy and all three vehicles are plotted. Observe that the responses corresponding to the ETC scheme look similar to the responses corresponding to the TTC scheme despite the significant reduction in communication achieved by the ETC scheme. The maximum absolute value of the spacing error (as defined in (3)) is approximately 0.8 m for both the TTC and the ETC implementation showing that the performance of the ETC implementation is similar to the performance of the TTC scheme. Summarizing, the experimental results illustrate the potential benefits of event-triggered communication for CACC systems, namely, having significantly larger inter-event times while realizing similar control performance in comparison with conventional time-triggered control methods.

### VIII. CONCLUSION

Dedicated Short Range Communication (DSRC) is a key ingredient in Cooperative Adaptive Cruise Control (CACC) systems to overcome the physical limitations of onboard sensors and enables to form string-stable platoons with small inter-vehicle distances. However, excessive utilization of communication resources can have a negative impact on the reliability of the DSRC channel and the size of the transmission delays. For this reason, a resource-aware CACC control strategy was proposed in this paper, which aims to reduce the utilization of communication resources in comparison with conventional time-triggered control methods while preserving the individual vehicle stability and string-stability guarantees. In addition, robustness with respect to time-varying delays and the presence of a strictly positive lower-bound on the minimum inter-event times (to avoid Zeno-behavior) is guaranteed by design. The proposed resource-aware control method relies on a recently developed ETC strategy that exploits the unique combination of *dynamic* event-triggered control and time-regularization (“waiting times”). Moreover, a systematic design procedure for this ETC strategy was provided that results in intuitive tradeoff curves between robustness in terms of the maximum allowable delay (MAD) and utilization of communication resources in terms of the minimum inter-event time (MIET). The proposed resource-aware CACC strategy was experimentally validated by means of a platoon of three passenger vehicles employing a time-gap of only  $h = 0.6$  seconds. The experimental results clearly demonstrated the potential benefits of event-triggered control, namely, having significantly larger inter-event times while realizing string stability and similar control performance in comparison with conventional time-triggered control methods. This result might be one of the important steps in order to realise the implementation of CACC on a large scale without introducing congestion of V2V communication network and/or the need to increase the time-gaps in the platoon.

### APPENDIX

To analyze the  $\mathcal{L}_2$ -gain property of the hybrid system  $\mathcal{H}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , we aim to find a positive semidefinite storage

function  $S_i$  that satisfies

$$\dot{S}_i \leq (1 + \epsilon)|\chi_{i-1}|^2 - |\chi_i|^2, \quad (42)$$

when  $\xi_i \in \mathcal{C}_i$  and

$$S_i(\xi_i^+) - S_i(\xi_i) \leq 0, \quad (43)$$

when  $\xi_i \in \mathcal{D}_i$ ,  $i \in \bar{N} \setminus \{1\}$ . We consider the following candidate storage function for the hybrid system  $\mathcal{H}_i$

$$U_i(\xi_i) = V_i(\tilde{x}_i) + \eta_{i-1} + \gamma_{l_{i-1}} \tilde{\phi}_{l_{i-1}}(\tau_{i-1}) W_i^2(e_{u_{i-1}}, s_{i-1}, l_{i-1}), \quad (44)$$

where

$$V_i(\tilde{x}_i) = \tilde{x}_i^\top P \tilde{x}_i \quad (45)$$

with  $P$  as in (35) and where  $\tilde{\phi}_{l_{i-1}}$  is given by

$$\tilde{\phi}_{l_{i-1}}(\tau) := \begin{cases} \phi_{l_{i-1}}(\tau) & \text{when } \tau \leq \tau_{miet} \\ \phi_0(\tau_{miet}) & \text{when } \tau > \tau_{miet} \end{cases} \quad (46)$$

for  $l_{i-1} \in \{0, 1\}$  and  $\tau \in \mathbb{R}_{\geq 0}$  with  $\phi_{l_{i-1}}$  satisfying (38). The function  $W_i$  is defined as

$$W_i(e_{u_{i-1}}, s_{i-1}, l_{i-1}) := \max\{\lambda^{l_{i-1}} |e_{u_{i-1}}|, |e_{u_{i-1}} + s_{i-1}|\}, \quad (47)$$

Observe that the functions  $V_i$ ,  $W_i$  are semi-positive definite. Moreover, observe that  $\tilde{\phi}_{l_{i-1}}(\tau) > 0$ ,  $l_{i-1} \in \{0, 1\}$ , for all  $\tau \in \mathbb{R}_{\geq 0}$  due to (36)-(38). Since, per definition of  $\mathbb{X}_i$ ,  $\eta_i(t, j) \geq 0$  for all  $(t, j) \in \text{dom } \xi_i$ , the candidate storage function  $U_i$  is positive semidefinite. Hence, indeed, the function  $U_i$  constitutes a valid candidate storage function.

Before we evaluate the behavior of  $U_i$ , we first consider the flow dynamics of the functions  $\tilde{\phi}_{l_{i-1}}$ ,  $l_{i-1} \in \{0, 1\}$ ,  $W_i$ ,  $V_i$  and  $\eta_i$ .

Consider the following lemma.

*Lemma 1:* For each solution  $\xi_i$  to  $\mathcal{H}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , with  $\xi_i(0, 0) \in \mathbb{X}_{0,i}$  and  $\chi_{i-1} \in \mathcal{L}_2$ ,  $\tau_{i-1}(t, j) \geq \tau_{miet}$ , for some  $(t, j) \in \text{dom } \xi_i$  implies that  $l_{i-1}(t, j) = 0$ .

*Proof of Lemma 1:* Per definition of  $\mathbb{X}_{0,i}$ ,  $i \in \bar{N} \setminus \{1\}$ , we have that  $l_{i-1}(0, 0) = 0$ . Hence,  $\tau_{i-1}(0, 0) = \tau_{miet}$  implies that  $l_{i-1}(0, 0) = 0$ . Observe from (25) that when  $l_{i-1}(t, j) = 1$ , for some  $(t, j) \in \text{dom } \xi_i$ , the system is only allowed to flow if  $\tau_{i-1}(t, j) \leq \tau_{mad}$ . Given the latter and the fact that  $\tau_{mad} \leq \tau_{miet}$ , due to Assumption 2, it follows from (27) and (31) that  $l_{i-1}(t, j) = 0$ ,  $i \in \bar{N} \setminus \{1\}$ ,  $(t, j) \in \text{dom } \xi_i$ , when  $\tau_{i-1}(t, j) = \tau_{miet}$ .  $\square$

Given Lemma 1, we have that  $\tilde{\phi}_{l_{i-1}}(\tau_{miet}) = \phi_0(\tau_{miet})$ . By combining the latter fact with (38) and (46), we obtain that

$$\dot{\tilde{\phi}}_{l_{i-1}} = -(1 - \omega(\tau_{i-1}))\gamma_{l_{i-1}}(\tilde{\phi}_{l_{i-1}}(\tau_{i-1})^2 + 1). \quad (48)$$

By recalling (30), we obtain from (47) that for  $\xi_i \in \mathcal{C}_i$

$$\dot{W}_i \leq |\dot{e}_{u_{i-1}}| \leq \frac{1}{h} |\chi_{i-1} - u_{i-1}|. \quad (49)$$

From (27), we obtain that for  $\xi_i \in \mathcal{C}_i$

$$\begin{aligned} \dot{V}_i &= \tilde{x}_i^\top \left( A_{11}^\top P + P A_{11} \right) \tilde{x}_i + 2e_{u_{i-1}}^\top A_{12} \tilde{x}_i + 2\chi_{i-1}^\top A_{13} \tilde{x}_i \\ &\stackrel{(35)}{\leq} -\varrho u_{i-1}^2 - \frac{1}{h^2} (\chi_{i-1} - u_{i-1})^2 \\ &\quad + \mu((1+\epsilon)|\chi_{i-1}|^2 - |\chi_i|^2) + \gamma^2 |e_{u_{i-1}}|^2 \\ &\stackrel{(47)}{\leq} -\varrho u_{i-1}^2 - \frac{1}{h^2} (\chi_{i-1} - u_{i-1})^2 \\ &\quad + \mu((1+\epsilon)|\chi_{i-1}|^2 - |\chi_i|^2) + \gamma_{l_{i-1}}^2 W_i^2, \end{aligned} \quad (50)$$

where, for the sake of compactness, we omitted the arguments of  $W_i$ . By recalling Assumption 2 and using the fact that  $\tau_{mad} \leq \tau_{miet}$ , we can deduce from (47) that

$$\omega(\tau_{i-1}) \gamma_{l_{i-1}}^2 \left( 1 + \frac{1}{\varepsilon} \tilde{\phi}_{l_{i-1}}^2(\tau_{i-1}) \right) W_i^2 = \omega(\tau_{i-1}) \bar{\gamma}^2 |e_{u_{i-1}}|^2. \quad (51)$$

By means of the latter, we can deduce from (13) and (39) that for  $\xi_i \in \mathcal{C}_i$

$$\begin{aligned} \dot{\eta}_{i-1} &= \varrho u_{i-1}^2 + \omega(\tau_{i-1}) \\ &\quad \times \left( \frac{1-\varepsilon}{h^2} (\chi_{i-1} - u_{i-1})^2 - \gamma_{l_{i-1}}^2 \left( 1 + \frac{1}{\varepsilon} \tilde{\phi}_{l_{i-1}}^2(\tau_{i-1}) \right) W_i^2 \right). \end{aligned} \quad (52)$$

By means of (48)-(52), we can now obtain that

$$\begin{aligned} \dot{U}_i &\leq \dot{V}_i + 2\gamma_{l_{i-1}} \tilde{\phi}_{l_{i-1}} W_i \dot{W}_i \\ &\quad - (1 - \omega(\tau_{i-1})) \gamma_{l_{i-1}} \tilde{\phi}_{l_{i-1}} W_i^2 + \dot{\eta}_{i-1} \\ &\leq -\varrho u_{i-1}^2 - \frac{1}{h^2} (\chi_{i-1} - u_{i-1})^2 \\ &\quad + \mu((1+\epsilon)|\chi_{i-1}|^2 - |\chi_i|^2) + \gamma_{l_{i-1}}^2 W_i^2 \\ &\quad + 2\gamma_{l_{i-1}} \tilde{\phi}_{l_{i-1}} W_i \frac{1}{h} |\chi_{i-1} - u_{i-1}| \\ &\quad - (1 - \omega(\tau_{i-1})) \gamma_{l_{i-1}}^2 (\tilde{\phi}_{l_{i-1}}^2 + 1) W_i^2 \\ &\quad + \varrho u_{i-1}^2 + \omega(\tau_{i-1}) \left( \frac{1-\varepsilon}{h^2} (\chi_{i-1} - u_{i-1})^2 \right. \\ &\quad \left. - \gamma_{l_{i-1}}^2 \left( 1 + \frac{1}{\varepsilon} \tilde{\phi}_{l_{i-1}}^2 \right) W_i^2 \right) \\ &\leq -\frac{1}{h^2} (\chi_{i-1} - u_{i-1})^2 + 2\gamma_{l_{i-1}} \tilde{\phi}_{l_{i-1}} W_i \frac{1}{h} |\chi_{i-1} - u_{i-1}| \\ &\quad - \gamma_{l_{i-1}}^2 \tilde{\phi}_{l_{i-1}}^2 W_i^2 + \mu((1+\epsilon)|\chi_{i-1}|^2 - |\chi_i|^2) + \omega(\tau_{i-1}) \\ &\quad \times \left( \frac{1-\varepsilon}{h^2} (\chi_{i-1} - u_{i-1})^2 + \gamma_{l_{i-1}}^2 \left( 1 - \frac{1}{\varepsilon} \right) \tilde{\phi}_{l_{i-1}}^2 W_i^2 \right) \end{aligned} \quad (53)$$

for  $\xi_i \in \mathcal{C}_i$ . Now by using completion of the squares and the fact that for some constants  $a, b \in \mathbb{R}$  and  $\varepsilon > 0$ , it holds that  $2ab \leq (1/\varepsilon)a^2 + \varepsilon b^2$ , we obtain that

$$\begin{aligned} \dot{U}_i &\leq (\omega(\tau_{i-1}) - 1)(|\chi_{i-1} - u_{i-1}| - \gamma_{l_{i-1}} \tilde{\phi}_{l_{i-1}} W_i)^2 \\ &\quad + \mu((1+\epsilon)|\chi_{i-1}|^2 - |\chi_i|^2) \\ &\leq \mu((1+\epsilon)|\chi_{i-1}|^2 - |\chi_i|^2). \end{aligned} \quad (54)$$

At transmissions events, *i.e.*, when  $\xi_i \in \mathcal{D}_i$  with  $l_{i-1} = 0$ ,  $\tau_{i-1} \geq \tau_{miet}$  and the system jumps according to  $\xi_i^+ = G(\xi_i)$ ,

we have that

$$\begin{aligned} U_i(\xi_i^+) - U_i(\xi_i) &= \gamma_1 \tilde{\phi}_1(0) W_i^2(e_{u_{i-1}}, -e_{u_{i-1}}, 1) \\ &\quad - \gamma_0 \tilde{\phi}_0(\tau_{i-1}) W_i^2(e_{u_{i-1}}, s_{i-1}, 0) \\ &\stackrel{(38)}{\leq} (\gamma_1 \phi_1(0) \lambda^2 - \gamma_0 \phi_0(\tau_{miet})) |e_{u_{i-1}}|^2. \end{aligned} \quad (55)$$

At update events, *i.e.*, when  $\xi_i \in \mathcal{D}_i$  with  $l_{i-1} = 1$  (and thus  $0 \leq \tau_{i-1} \leq \tau_{mad}$ ) and the system jumps according to  $\xi_i^+ = G(\xi_i)$ , we have that

$$\begin{aligned} U_i(\xi_i^+) - U_i(\xi_i) &= \gamma_0 \tilde{\phi}_0(\tau_{i-1}) W_i^2(s_{i-1} + e_{u_{i-1}}, 0, 0) \\ &\quad - \gamma_1 \tilde{\phi}_1(\tau_{i-1}) W_i^2(e_{u_{i-1}}, s_{i-1}, 1) \\ &\stackrel{(38)}{\leq} (\gamma_0 \phi_0(\tau_{i-1}) - \gamma_1 \phi_1(\tau_{i-1})) |e_{u_{i-1}} + s_{i-1}|^2. \end{aligned} \quad (56)$$

By recalling (36) and (46), we obtain that

$$U_i(\xi_i^+) - U_i(\xi_i) \leq 0, \quad (57)$$

for all  $\xi_i \in \mathcal{D}_i$ . From (54)-(57), we can conclude that the storage function  $S_i = U_i/\mu$  satisfies (42) and (43).

At last, we need to show that all maximal solutions of the hybrid system  $\mathcal{H}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , are  $t$ -complete. To do so, we verify the conditions provided in [19, Proposition 6.10]. First of all, observe from (31) that  $G_i(\mathcal{D}_i) \subset \mathcal{C}_i \cup \mathcal{D}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , since for all  $\xi_i \in G_i(\mathcal{D}_i)$ , it holds that  $\tau_{i-1}^+ \geq 0$ ,  $\eta_{i-1}^+ \geq 0$ . Next, we show that for any point  $p \in \mathcal{C}_i \setminus \mathcal{D}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , there exists a neighborhood  $U$  of  $p$  such that, it holds for every  $q \in U \cap \mathcal{C}_i$  and every  $w \in \mathbb{R}$  that  $F_i(q, w) \in T_{\mathcal{C}_i}(q)$ , where  $T_{\mathcal{C}_i}(q)$  denotes the tangent cone<sup>1</sup> to  $\mathcal{C}_i$  at  $q$ . Observe that for each point  $p \in \mathcal{C}_i$  (recall that  $p = (\tilde{x}_i, e_{u_{i-1}}, \tau_{i-1}, l_{i-1}, s_{i-1}, \eta_{i-1})$ ),  $T_{\mathcal{C}_i}(p) = \mathbb{R}^7 \times \mathbb{R} \times T_{\mathbb{R}_{\geq 0}}(\tau_{i-1}) \times \{0\} \times \{0\} \times T_{\mathbb{R}_{\geq 0}}(\eta_{i-1})$ . From (25) and (31), we obtain that  $\mathcal{C}_i/\mathcal{D}_i = \{\xi_i \in \mathbb{X}_i : (l_{i-1} = 0 \wedge \tau_{i-1} < \tau_{miet}) \vee \eta_{i-1} > 0\}$ . Given the facts that according to (27) and (39),  $\dot{\tau}_{i-1} = 1$  and that  $\dot{\eta}_{i-1} > 0$  when  $\tau_{i-1} < \tau_{miet}$  and  $\eta_{i-1} = 0$ , for all  $i \in \bar{N} \setminus \{1\}$ , it indeed follows that for any  $p \in \mathcal{C}_i \setminus \mathcal{D}_i$  there exists a neighborhood  $S$  of  $\xi$  such that, it holds for every  $q \in S \cap \mathcal{C}_i$  and every  $w \in \mathbb{R}$  that  $F_i(q, w) \in T_{\mathcal{C}_i}(q)$ . Since finite escape times are excluded due to (54), we can conclude that all maximal solutions of the hybrid system  $\mathcal{H}_i$ ,  $i \in \bar{N} \setminus \{1\}$ , with  $\xi_i(0, 0) \in \mathbb{X}_{0,i}$  and  $\chi_{i-1} \in \mathcal{L}_2$ , are  $t$ -complete, which completes the proof.

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<sup>1</sup>The tangent cone to a set  $S \subset \mathbb{R}^n$  at a point  $x \in \mathbb{R}^n$ , denoted  $T_S(x)$ , is the set of all vectors  $\omega \in \mathbb{R}^n$  for which there exist  $x_i \in S$ ,  $y_i > 0$  with  $x_i \rightarrow x$ ,  $y_i \rightarrow 0$  as  $i \rightarrow \infty$  such that  $\omega = \lim_{i \rightarrow \infty} (x_i - x)/y_i$  (see Definition 5.12 in [19]).

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