Feedforward Strategies for Cooperative Adaptive Cruise Control in Heterogeneous Vehicle Strings

Ahmed M. H. Al-Jhayyish and Klaus Werner Schmidt

***Abstract*— String stability is an essential property to ensure that the fluctuations are attenuated along vehicle strings. This paper focuses on the fulfillment of string stability in the practical case of heterogeneous vehicle strings that comprise vehicles with different dynamic properties. Using the idea of predecessor following, acceleration feedforward, predicted acceleration feedforward, and input signal feedforward are considered as different possible feedforward strategies. For all strategies, the parameter ranges of predecessor vehicles that ensure string stability of a given vehicle are characterized, computed, and validated by simulation.**

***Index Terms*— Cooperative adaptive cruise control, string stability, heterogeneous vehicle strings, communication delay.**

I. Introduction

RECENT advances in automotive engineering aim at replacing human driver functionality [1], [2]. Cooperative Adaptive Cruise Control (CACC) automates the longitudinal vehicle motion to support safe vehicle following using distance measurements and state information communicated among vehicles [3]–[7]. In particular, CACC allows for smaller inter-vehicle spacings compared to the distance measurement-based automatic cruise control (ACC) [4].

CACC can be applied in different information flow topologies. The most commonly used topology is predecessor following [7] and [8], where each vehicle obtains data such as acceleration [4], [9]–[12] or input signal [6], [12]–[14] from its predecessor vehicle. A basic requirement when using CACC is to attenuate fluctuations along vehicle strings, which is captured by the formal condition of string stability. An extensive overview of different definitions for string stability is given in [13] and there are several methods for designing controllers so as to achieve string stability. For example, PD-controllers are used in [4], [13], and [14] and H_∞-control is found suitable in [6]. Reference [15] is based on model-predictive control and [16] and [17] use consensus control.

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Many research works on string stability consider the case of homogeneous vehicle strings with identical vehicles [6], [9]–[13], [15], [16], [18]–[21]. Nonetheless, heterogeneous vehicle strings, where vehicles have different dynamic properties as well as different communication delays, are encountered in practice. That is, string stability needs to be ensured for all combinations of potential following scenarios with different vehicles.

The research work on string stability of heterogeneous vehicle strings is limited. Reference [22] performs analysis and controller design in the leader-predecessor architecture. This work only considers three vehicle types, whereby the actual dynamics of the vehicles are not explicitly specified. The work in [4] establishes an empirical relationship between a fixed communication delay and the choice of controller parameters for string stability without considering the possibility of different communication delays. The dependency of string stability on actuator and communication delays is investigated in [14], assuming that the controller can be changed in real-time depending on the dynamics of the predecessor vehicle, which is difficult in practice. Reference [5] uses receding-horizon control to achieve string-stability for speed-change maneuvers of heterogeneous vehicle strings with limited length. The recent work in [17] provides a control algorithm for different information flow topologies and heterogeneous vehicles based on a distributed consensus assuming equal inter-vehicle gaps.

This paper performs a comprehensive analysis of strict L_2 string stability according to [13] for heterogeneous vehicle strings in the predecessor following topology. It is required that each vehicle satisfies string stability for any possible predecessor vehicle and associated communication delay. The first contribution of the paper is determining realistic vehicle parameter ranges from existing experimental studies. Next, feedforward strategies with communication (feedforward) of the acceleration signal, a predicted acceleration signal and the input signal by the predecessor vehicle are investigated. In this respect, the second contribution of the paper is proving that all considered feedforward strategies are inherently robust in the sense of ensuring strict L_2 string stability for predecessor vehicles in a certain parameter range that is characterized by a closed set. The third contribution of the paper is given by efficient algorithms for determining these parameter ranges. As a practical result, choosing a suitable controller for each vehicle, strict L_2 string stability for heterogeneous vehicle strings of arbitrary length is guaranteed as long as the vehicle parameters belong to the respective parameter ranges. The obtained results

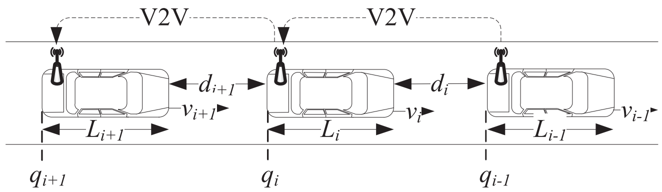

Fig. 1. Vehicle string with CACC.

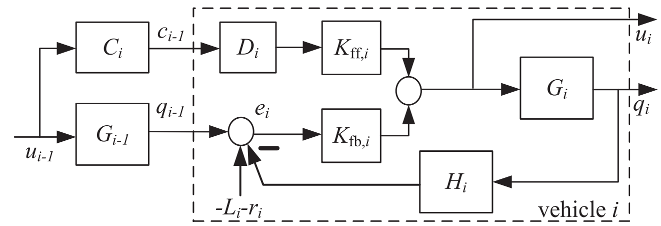

Fig. 2. Feedback loop for CACC.

are validated in a simulation environment with realistic vehicle parameters and wireless communication.

The remainder of the paper is organized as follows. Section II gives background information on CACC and string stability. In Section III, the string stability analysis for heterogeneous vehicle strings is motivated. The analysis results and algorithms are presented and discussed in Section IV and V for the case of input signal feedforward and acceleration feedforward/predicted acceleration feedforward, respectively. Section VI gives conclusions.

II. Preliminaries

A. Cooperative Adaptive Cruise Control (CACC)

CACC is introduced for safe vehicle following at small inter-vehicle spacings in order to increase the traffic throughput and to reduce fuel consumption [7], [8], [12], [23].

Using CACC, vehicles form *strings* as depicted in Fig. 1. For each vehicle i, we consider the *rear bumper position* q_i, the *length* L_i, the *velocity* v_i and the *inter-vehicle spacing* between vehicle i (front bumper) and $i-1$ (rear bumper)

$$d_i = q_{i-1} - q_i - L_i. \tag{1}$$

In practice, d_i can be measured by Radar or LIDAR in analogy to adaptive cruise control (ACC). Additionally, we assume that vehicle $i-1$ provides state information to vehicle i via vehicle-to-vehicle (V2V) communication using *predecessor following* (PF). Since PF only requires information exchange between adjacent vehicles, it is the most basic strategy [6], [7] that is most frequently used in the recent literature [8].

There are different policies for the desired vehicle spacing [7], [14]. We focus on the *constant-time-gap policy* with

$$d_{r,i} = r_i + h_i\, v_i \tag{2}$$

as the desired gap with the *headway time* h_i and the *desired spacing at standstill* r_i (for $v_i = 0$). We note that, using PF, other policies such as the constant gap policy can only guarantee marginal string stability even for homogeneous vehicle strings [6] and are hence not suitable for our study.

B. Control Architectures

Regarding the vehicle plant, we employ the linear model

$$G_i(s) = \frac{Q_i(s)}{U_i(s)} = \frac{e^{-\phi_i s}}{(1 + s\,\tau_i)\, s^2}, \tag{3}$$

that is frequently used in the recent literature [6], [12], [14]. τ_i is the time constant of the *driveline dynamics* and ϕ_i is the *actuator time delay* that can be different for each vehicle i. This model is obtained from a nonlinear model of the driveline dynamics based on feedback linearization and low-level control [24]–[26]. The low-level control loop ensures that τ_i is constant over a wide range of normal driving situations [4], [27] which are predominant for CACC [6].

Using the plant model in (3) and PF, we employ the control architecture in Fig. 2. Here, the spacing error of vehicle i is

$$e_i = d_i - d_{r,i} = q_{i-1} - q_i - L_i - r_i - h_i\, v_i \tag{4}$$

and (4) is realized by the transfer function $H_i(s) = 1 + h_i\, s$. $K_{\mathrm{fb},i}$ is a feedback controller and $K_{\mathrm{ff},i}$ is a feedforward filter that receives the signal c_i from vehicle $i-1$ via V2V communication. $D_i(s) = e^{-s\,\theta_i}$ represents a communication delay between vehicle $i-1$ and vehicle i. In agreement with the recent literature on PF, we investigate *input signal feedforward* (ISF) with $c_{i-1} = u_{i-1}$ and $C_i(s) = 1$ as well as *acceleration feedforward* (AF) with $c_{i-1} = a_{i-1}$ and $C_i(s) = \frac{e^{-s\,\phi_{i-1}}}{1+s\,\tau_{i-1}}$. In addition, we introduce the idea of *predicted acceleration feedforward* (PAF) with $c_{i-1}(t) = a_{i-1}(t + \phi_{i-1})$ and $C_i(s) = \frac{1}{1+s\,\tau_{i-1}}$.

AF is for example employed in [4] and [9]–[12] and ISF is assumed in [6], [12], and [13]. PAF is first used in [14] as a special case of ISF where $C_i(s) = \dfrac{1}{1 + s\,\tau_{i-1}}$ is realized as a part of the filter transfer function $K_{\mathtt{ff},i}$ on vehicle i. We note that such implementation requires updating $K_{\mathtt{ff},i}$ depending on the predecessor vehicle time constant τ_{i-1} during runtime, which is generally undesired. Instead, we suggest to implement $C_i(s) = \dfrac{1}{1 + s\,\tau_{i-1}}$ on vehicle $i-1$ itself such that the communicated signal is $a_{i-1}(t + \phi_{i-1})$.

We finally note that most of the existing literature focuses on the case of homogeneous vehicles with identical vehicle models. The main subject of this paper is the case of heterogeneous driveline dynamics with $\tau_i \neq \tau_j$ and $\phi_i \neq \phi_j$ as well as different communication delays $\theta_i \neq \theta_j$ for $i \neq j$.

C. String Stability

The controller design for CACC requies to achieve a small spacing error in (4), while disturbances along vehicle strings must be attenuated in order to ensure driving safety, driving comfort and scalability [6], [8]. That is, small variations in signals such as velocity or acceleration of a vehicle i should not lead to increasing variations in the signals of its follower vehicles. Formally, we capture this property by the notion of *strict L_2 string stability* [6], [12], [13]. We write

$$Y_i(s) = P_i(s)\, U_1(s) \tag{5}$$

with the transfer function P_i between the leader input u_1 and the output signal y_i of vehicle i. Then, the transfer function between the output signals of successive vehicles is

$$\Gamma_{y_i}(s) = \frac{Y_i(s)}{Y_{i-1}(s)} = P_i(s)\, P_{i-1}^{-1}(s). \tag{6}$$

Strict L_2 string stability holds if and only if [12], [13]

$$||P_1(s)||_\infty < \infty \tag{7}$$

$$||\Gamma_{y_i}(s)||_\infty \leq 1, \quad \forall i. \tag{8}$$

($|| \bullet ||_\infty$ denotes the H_∞ norm). The advantage of strict L_2 string stability is its scalability when using PF: if (7) holds, it is only required to verify (7) for all values of i. Since $||\Gamma_{y_i}(s)||_\infty$ solely depends on vehicle i and its predecessor $i-1$ when using PF, we say that vehicle i is strictly L_2 string stable if (8) holds. An arbitrarily long vehicle string fulfills strict L_2 string stability if all of its vehicles are strictly L_2 string stable.

Using the physically meaningful output signal $y_i(t) = a_i(t)$ in the feedback loop in Fig. 2, we compute

$$\Gamma_{a_i}(s) = \frac{G_i}{G_{i-1}} \frac{C_i\, D_i\, K_{\text{ff},i} + G_{i-1}\, K_{\text{fb},i}}{(1 + H_i\, G_i\, K_{\text{fb},i})}. \tag{9}$$

We note that (9) with $C_i(s) = 1$ is used in [6] and [13], $C_i(s) = \frac{e^{-s\,\phi_{i-1}}}{1+s\,\tau_{i-1}}$ in [4] and [12] and $C_i(s) = \frac{1}{1+s\,\tau_{i-1}}$ in [14].

D. Controller Realization

The literature provides various controller realizations for guaranteeing string stability in the control architecture in Fig. 2 [4], [6], [12]–[14]. In this paper, we adopt the PD controller designs from [13] (ISF) and [4] (AF and PAF).

In the case of ISF, we use [13]

$$K_{\text{ff},i}(s) = \frac{1}{1 + h_i\, s} \quad \text{and} \quad K_{\text{fb},i}(s) = \frac{K_{\text{P},i} + K_{\text{D},i}\, s}{1 + h_i\, s} \tag{10}$$

That is, the PD-parameters $K_{\text{P},i}$ and $K_{\text{D},i}$ as well as the headway time h_i need to be selected in the design.

In the case of AF and PAF, we use [4]

$$K_{\text{ff},i}(s) = \frac{1 + s\,\tau_i}{1 + s\, h_i} \quad \text{and} \quad K_{\text{fb,i}}(s) = \omega_{K,i}\,(\omega_{K,i} + s). \tag{11}$$

Here, the parameters $\omega_{K,i}$ and h_i are needed for the design.

E. Bisection Method

In this paper, we use several bisection algorithms to approximate interval bounds. For ease of notation, we provide a generic formulation of such algorithm with a condition $P(\delta) = 0$ depending on the parameter δ. Here, it is assumed that there exists a value $\hat{\delta}$ such that for any $\delta_\text{l} < \hat{\delta}$ and $\delta_\text{u} > \hat{\delta}$, $\text{sign}(P(\delta_\text{l})) \neq \text{sign}(P(\delta_\text{u}))$ and it is desired to find $\hat{\delta}$. The generic procedure is presented in Algorithm 1, where δ_l^0 and δ_u^0 represent initial values for δ_l and δ_u, respectively. ϵ is a tolerance for the computation of $\hat{\delta}$. We write $\hat{\delta} = \texttt{Bisection}(P(\delta), \epsilon, \delta_\text{u}^0, \delta_\text{l}^0)$ when calling Algorithm 1.

Algorithm 1 Generic Bisection Algorithm

```
input  : P(δ), ϵ, δ_u^0, δ_l^0
output: δ̂
δ_u := δ_u^0, δ_l := δ_l^0
while δ_u − δ_l > ϵ do
    δ := (δ_l + δ_u) / 2
    if sign(P(δ)) = sign(P(δ_u)) then
        δ_u := δ
    else
        δ_l := δ
    end
end
δ̂ = (δ_l + δ_u) / 2
```

III. Motivation and Problem Statement

A. Parameter Combinations for Heterogeneous Vehicle Strings

In real life, different types of vehicles and communication delays are encountered and it is known from [6] that the system behavior is sensitive to variations in the vehicle model in (3) along the string. Nonetheless, the existing literature on CACC mostly focuses on homogeneous vehicles, whereby the specific parameters identified and used differ among such studies. We next summarize the observed parameter choices.

Reference [28] experimentally determines $\tau_i = 0.2$ and assumes that ϕ_i is bounded by 0.03 or 0.08. A low-level control loop which confirms small variations of τ_i between 0.19 and 0.23 is realized in [27]. Reference [3] confirms an overall time constant of the low-level control system in the order of $\tau_i = 0.5$ and an actuator delay between $\theta_i = 0.06$ and $\theta_i = 0.08$. Reference [4] identifies $\phi_i = 0.18$, $\theta_i = 0.06$ and $\tau_i \approx 0.38$ for an experimental vehicle and uses the time constants 0.1, 0.4, 0.5, 1.0 for heterogeneous vehicle simulations. References [6] and [29] determine $\tau_i = 0.1$, $\phi_i = 0.18$ or $\phi_i = 0.2$ and $\theta_i = 0.02$ and [11] finds $\tau_i = 0.8$, $\phi_i = 0.02$ and $\theta_i = 0.2$ after a system identification. Reference [10] obtains $\tau_i = 0.45$ and $\phi_i = 0.25$ from measurements. Neglecting the communication delay, [30] assumes $\tau_i = 0.1$, $\phi_i = 0.05$, [9] uses $\theta_i = 0.1$ and $\phi_i = 0.1$, [31] employs $\tau_i = 0.5$ and [14] assumes $\tau_i \leq 0.5$ and $\theta_i \leq 0.08$. Reference [32] argues that the information delay varies between $\theta_i = 0.10$ and $\theta_i = 0.33$, whereas $\tau_i = 0.05$ is assumed. References [16] and [33] use vehicles models with $\tau_i = 0$ and $\phi_i = 0$. Regarding the communication delay, [16] assumes that θ_i is bounded by 0.09 and [33] assumes that θ_i is between 0.1 and 0.4.

The respective parameter values are summarized in Table I. In this paper, we focus on parameter values from practical experiments as indicated by ”$\star$” in Table I. We consider that

$$0.1 \leq \tau_i \leq 0.8, \quad 0.02 \leq \phi_i \leq 0.25, \quad 0.02 \leq \theta_i \leq 0.2. \tag{12}$$

Respecting the previous discussion, the main aim of the paper is to investigate strict L_2 string stability according to (9). It holds that the model parameters τ_i, ϕ_i and the controller

TABLE I
PARAMETER VALUES FOR τ_i, ϕ_i AND θ_i

	τ	ϕ	θ
[29]⋆	0.2	0.03	—
[28]⋆	0.2	—	—
[3]⋆	0.5	0.07	—
[33]	0.05	—	0.1–0.33
[32]	0.5	—	—
[4]⋆	0.38, ≤ 1.0	0.18	0.06
[10]	—	0.1	0.1
[11]⋆	0.45	0.25	—
[12]⋆	0.8	0.02	0.2
[30], [6]⋆	0.1	0.18, 0.2	0.02
[31]	0.1	0.05	—
[15]	≤ 0.5	—	≤ 0.08
[34]	0	0	0.1–0.4
[17]	0	0	≤ 0.09

parameters $K_{\text{ff},i}$, $K_{\text{fb},i}$ and h_i of vehicle i are known by design. Differently, since any predecessor vehicle is possible, the parameters τ_{i-1}, ϕ_{i-1} of vehicle $i-1$ and the communication delay θ_i can assume any value in the ranges specified in (12).

B. Region of Strict L_2 String Stability

In case of ISF, it holds that $C_i(s) = 1$ in Fig. 2. Using $D_i(s) = e^{-\theta_i s}$, we obtain from (9) and (3)

$$\Gamma_{a_i}(s) = e^{-\phi_i s}\frac{s^2(1+s\tau_{i-1})e^{-(\theta_i-\phi_{i-1})s}K_{\text{ff},i} + K_{\text{fb},i}}{(s^2(1+s\tau_i) + H_i K_{\text{fb},i} e^{-\phi_i s})}. \quad (13)$$

It first has to be noted that stability of Γ_{a_i} does not depend on the parameters τ_{i-1}, ϕ_{i-1} and θ_i, since these parameters only appear in the numerator of Γ_{a_i}. Regarding strict L_2 string stability, (8) needs to be fulfilled. Considering that $|e^{-j\omega\phi_i}| = 1$, (13) suggests that $||\Gamma_{a_i}(s)||_\infty$ only depends on τ_{i-1} and $\theta_i - \phi_{i-1}$ but not on the actual parameters θ_i and ϕ_{i-1}. Defining $\mu = \tau_{i-1}$ and $\eta = \theta_i - \phi_{i-1}$ with $\mu \in I_\mu := [0.1, 0.8]$ and $\eta \in I_\eta := [-0.23, 0.18]$ (see (12)), we introduce

$$\Gamma_{a_i}(s,\mu,\eta) = \frac{s^2\,(1+s\,\mu)\,e^{-\eta s}\,K_{\text{ff},i} + K_{\text{fb},i}}{(s^2\,(1+s\,\tau_i) + H_i\,K_{\text{fb},i}\,e^{-\phi_i s})}. \quad (14)$$

In principle, it is desired for each vehicle i to ensure strict L_2 string stability for all pairs $(\mu,\eta) \in I_\mu \times I_\eta$. In order to formalize this objective, let $\hat{\mu} \in I_\mu$ and $\hat{\eta} \in I_\eta$ be nominal values for μ and η such that $||\Gamma_{a_i}(s,\hat{\mu},\hat{\eta})||_\infty \leq 1$. We define the *region of strict string stability* for $\Gamma_{a_i}(s,\mu,\eta)$ and $(\hat{\mu},\hat{\eta})$.

Definition 1: Consider $\Gamma_{a_i}(s,\mu,\eta)$ in (14) with a given τ_i, ϕ_i, h_i. Assume that $K_i = \left[K_{\text{ff},i}\ K_{\text{fb},i}\right]$ is determined such that $||\Gamma_{a_i}(s,\hat{\mu},\hat{\eta})||_\infty \leq 1$ for $\hat{\mu} \in I_\mu$ and $\hat{\eta} \in I_\eta$. Then, the region of strict L_2 string stability (RSSS) for $\Gamma_{a_i}(s,\mu,\eta)$ and $(\hat{\mu},\hat{\eta})$ is defined as the largest connected set $A_{\hat{\mu},\hat{\eta}} \subseteq I_\mu \times I_\eta$ such that $(\hat{\mu},\hat{\eta}) \in A_{\hat{\mu},\hat{\eta}}$ and $||\Gamma_{a_i}(s,\mu,\eta)||_\infty \leq 1$ for all $(\mu,\eta) \in A_{\hat{\mu},\hat{\eta}}$.

In words, $A_{\hat{\mu},\hat{\eta}}$ is the largest possible connected set $A_{\hat{\mu},\hat{\eta}} \subseteq I_\mu \times I_\eta$ that contains $(\hat{\mu},\hat{\eta})$ and such that $||\Gamma_{a_i}(s,\mu,\eta)||_\infty \leq 1$ for all $(\mu,\eta) \in A_{\hat{\mu},\hat{\eta}}$. $A_{\hat{\mu},\hat{\eta}}$ characterizes all possible parameter combinations of vehicle $i-1$ around $(\hat{\mu},\hat{\eta})$ that ensure strict L_2 string stability of vehicle i. Accordingly, the first research problem is determining $A_{\hat{\mu},\hat{\eta}}$ and choosing K_i and h_i such that the RSSS contains all possible parameter combinations of vehicle $i-1$. Problem 1 is addressed in Section IV.

Problem 1: Consider $\Gamma_{a_i}(s,\mu,\eta)$ and $(\hat{\mu},\hat{\eta})$ as in Definition 1. Compute the set $A_{\hat{\mu},\hat{\eta}}$ and choose K_i and h_i such that $I_\mu \times I_\eta = A_{\hat{\mu},\hat{\eta}}$ for the controller realization in Section II-D.

C. Interval of Strict L_2 String Stability

In case of AF and PAF, it holds that $C_i(s) = \frac{e^{-s\,\lambda}}{1+s\,\tau_{i-1}}$ with $\lambda = \phi_{i-1}$ (AF) or $\lambda = 0$ (PAF) such that

$$\Gamma_{a_i}(s) = \frac{e^{-s\,\phi_i}}{1+s\,\tau_i}\,\frac{e^{-s\,(\theta_i+\lambda-\phi_{i-1})}\,K_{\text{ff},i} + \dfrac{1}{s^2}\,K_{\text{fb},i}}{(1+H_i\,G_i\,K_{\text{fb},i})}. \quad (15)$$

Again, stability of Γ_{a_i} is independent of vehicle $i-1$. In addition, since τ_{i-1} does not appear in Γ_{a_i}, strict L_2 string stability of Γ_{a_i} in (15) only depends on $\theta_i + \lambda - \phi_{i-1}$. We use

$$\Gamma_{a_i}(s,\nu) = \frac{1}{1+s\,\tau_i}\,\frac{e^{-s\,\nu}\,K_{\text{ff},i} + \dfrac{1}{s^2}\,K_{\text{fb},i}}{(1+H_i\,G_i\,K_{\text{fb},i})}, \quad (16)$$

with the parameter $\nu = \theta_i + \lambda - \phi_{i-1}$. ν characterizes the variation of $\theta_i \in I_\nu = [0.02, 0.2]$ and $\theta_i - \phi_{i-1} \in I_\nu = [-0.23, 0.18]$ for AF ($\lambda = \phi_{i-1}$) and PAF ($\lambda = 0$), respectively. Then, we analyze the robustness of a given controller $K_i = \left[K_{\text{ff},i}\ K_{\text{fb},i}\right]$ for vehicle i to variations in ν.

Definition 2: Consider $\Gamma_{a_i}(s,\nu)$ in (16) with a given τ_i, ϕ_i, h_i. Let $K_i = \left[K_{\text{ff},i}\ K_{\text{fb},i}\right]$ be such that $||\Gamma_{a_i}(s,\hat{\nu})||_\infty \leq 1$ for $\hat{\nu} \in I_\nu$. Then, the interval of strict string stability (ISSS) for $\Gamma_{a_i}(s,\nu)$ and $\hat{\nu}$ is defined as the largest interval $I_{\hat{\nu}} \subseteq I_\nu$ such that $\hat{\nu} \in I_{\hat{\nu}}$ and $||\Gamma_{a_i}(s,\nu)||_\infty \leq 1$ for all $\nu \in I_{\hat{\nu}}$.

In words, the interval $I_{\hat{\nu}}$ characterizes all possible values of θ_i around the nominal value $\hat{\nu}$ that ensure strict L_2 string stability of vehicle i. The related research problem studied in this paper is determining the set $I_{\hat{\nu}}$ and choosing K_i and h_i such that $I_\nu = I_{\hat{\nu}}$. Problem 2 is addressed in Section V.

Problem 2: Consider $\Gamma_{a_i}(s,\nu)$ and $\hat{\nu}$ as in Definition 2. Compute the interval $I_{\hat{\nu}}$ and choose K_i and h_i such that $I_\nu = I_{\hat{\nu}}$ for the controller realization in Section II-D.

IV. STRING STABILITY ANALYSIS USING ISF

A. Strict String Stability Analysis

We first investigate the dependency of $\Gamma_{a_i}(s,\mu,\eta)$ in (14) on μ. Proposition 1 shows that strict L_2 string stability for a vehicle i is preserved if the time constant μ of vehicle $i-1$ lies within an interval around a nominal time constant $\hat{\mu}$.

Proposition 1: Consider $\Gamma_{a_i}(s,\mu,\eta)$ in (14) for $\mu \geq 0$. Assume that τ_i, ϕ_i, $\hat{\mu}$ and η are given and let $K_i = \left[K_{\text{ff},i}\ K_{\text{fb},i}\right]$ be a controller such that $||\Gamma_{a_i}(s,\hat{\mu},\eta)||_\infty \leq 1$. Then, there is an interval $I(\tau_i,\phi_i,\eta,K_i) \ni \hat{\mu}$ such that $||\Gamma_{a_i}(s,\mu,\eta)||_\infty \leq 1$ for $\mu \in I(\tau_i,\phi_i,\eta,K_i)$ and $||\Gamma_{a_i}(s,\mu,\eta)||_\infty > 1$ otherwise.

Proof: We compute the absolute value of $\Gamma_{a_i}(j\omega, \mu, \eta)$:

$$\begin{aligned}\Gamma_{a_i}(s, \mu, \eta) &= \frac{s^2\, e^{-s\,\eta}\, K_{\mathrm{ff},i} + K_{\mathrm{fb}}}{H_i\,(s^2\,(1 + s\,\tau_i) + K_{\mathrm{fb},i}\, e^{\phi_i\, s})} \\ &\quad + \frac{s^3\, e^{-s\,\eta}\, K_{\mathrm{ff},i}}{H_i\,(s^2\,(1 + s\,\tau_i) + K_{\mathrm{fb},i}\, e^{\phi_i\, s})}\,\mu \\ &=: F_1(s) + F_2(s)\,\mu. \\ |\Gamma_{a_i}(j\omega, \mu, \eta)|^2 &= |F_1(j\omega)|^2 + |F_2(j\omega)|^2\,\mu^2 \\ &\quad + F_2(j\omega)F_1^\star(j\omega)\,\mu + F_1(j\omega)F_2^\star(j\omega)\,\mu.\end{aligned}$$

Hereby, $(\bullet)^\star$ denotes the conjugate complex. Further

$$\begin{aligned}\frac{\partial}{\partial\mu}\,|\Gamma_{a_i}(j\omega, \mu, \eta)|^2 &= F_2(j\omega)F_1^\star(j\omega) + F_1(j\omega)F_2^\star(j\omega) \\ &\quad + 2\,|F_2(j\omega)|^2\,\mu = 0 \\ \Rightarrow \mu_{\min}(\omega) &= -\frac{F_2(j\omega)F_1^\star(j\omega) + F_1(j\omega)F_2^\star(j\omega)}{2\,|F_2(j\omega)|^2}, \\ \frac{\partial^2}{\partial\mu^2}\,|\Gamma_{a_i}(j\omega, \mu, \eta)|^2 &= 2\,|F_2(j\omega)|^2 > 0.\end{aligned}$$

That is, for each value of ω, $|\Gamma_{a_i}(j\omega, \mu, \eta)|^2$ is a parabola in μ with a minimum at $\mu_{\min}(\omega)$. Noting that $|\Gamma_{a_i}(j\omega, \hat{\mu}, \eta)|^2 \le 1$ by design, it holds that $|\Gamma_{a_i}(j\omega, \mu_{\min}, \eta)|^2 \le 1$. Hence, there are two values $\mu_1(\omega) \le \mu_{\min}(\omega)$ and $\mu_{\mathrm{r}}(\omega) \ge \mu_{\min}(\omega)$ such that $|\Gamma_{a_i}(j\omega, \mu, \eta)|^2 = 1$, whereby $\mu_1(\omega) \le \hat{\mu}$ and $\mu_{\mathrm{r}}(\omega) \ge \hat{\mu}$. Evaluating $\mu_1(\omega)$ and $\mu_{\mathrm{u}}(\omega)$ for all frequencies, we define

$$\mu^{\min} = \max_{0 \le \omega < \infty} \{0, \mu_1(\omega)\} \tag{17}$$

$$\mu^{\max} = \min_{0 \le \omega < \infty} \{\mu_{\mathrm{r}}(\omega)\}. \tag{18}$$

Hereby, we respect in (17) that only non-negative values of μ are practically meaningful. We next show that $||\Gamma_{a_i}(s, \mu, \eta)||_\infty \le 1$ for $\mu^{\min} \le \mu \le \mu^{\max}$. Choose some μ such that $\mu^{\min} \le \mu \le \mu^{\max}$. Then, (17) and (18) imply that $|\Gamma_{a_i}(j\omega, \mu, \eta)|^2 \le 1$ for all values of ω. That is, $||\Gamma_{a_i}(s, \mu, \eta)||_\infty \le 1$ directly follows. Now assume that $\mu < \mu^{\min}$ or $\mu > \mu^{\max}$ and recall that $\mu \ge 0$ by definition. First, let $0 \le \mu < \mu^{\min}$. It follows from (17) that there exists an $\hat{\omega}$ such that $\mu < \mu_1(\hat{\omega})$. That is, $|\Gamma_{a_i}(j\hat{\omega}, \mu, \eta)|^2 > 1$, which implies that $||\Gamma_{a_i}(s, \mu, \eta)||_\infty > 1$. Similarly, it follows from (18) that there exists and $\hat{\omega}$ such that $\mu > \mu_{\mathrm{r}}(\hat{\omega})$ if $\mu > \mu^{\max}$. Again, $|\Gamma_{a_i}(j\hat{\omega}, \mu, \eta)|^2 > 1$ and hence $||\Gamma_{a_i}(s, \mu, \eta)||_\infty > 1$. Defining $I(\tau_i, \phi_i, \eta, K_i) = [\mu^{\min}, \mu^{\max}]$ completes the proof. □

Using Proposition 1, it is possible to characterize the RSSS $A_{\hat{\mu},\hat{\eta}}$ in Theorem 1. $A_{\hat{\mu},\hat{\eta}}$ contains all parameter combinations (μ, η) within some boundary around the nominal values $(\hat{\mu}, \hat{\eta})$.

Theorem 1: Assume that $K_{\mathrm{ff},i}$ is a strictly proper transfer function. Then, $A_{\hat{\mu},\hat{\eta}}$ is a non-empty, closed, complete set.

Proof: $A_{\hat{\mu},\hat{\eta}}$ is non-empty by assumption since $(\hat{\mu}, \hat{\eta}) \in A_{\hat{\mu},\hat{\eta}}$. We next argue that $||\Gamma_{a_i}(s, \mu, \eta)||_\infty$ is continuous in μ and η. Since $\Gamma_{a_i}(j\omega, \mu, \eta)$ is continuous in ω, μ, η,

$$||\Gamma_{a_i}(s, \mu, \eta)||_\infty = \sup_{\omega \in \mathbb{R}} |\Gamma_{a_i}(j\omega, \mu, \eta)|.$$

Using (14) and strict properness of $K_{\mathrm{ff},i}$, it follows that $|\Gamma_{a_i}(0, \mu, \eta)| = 1$ and $\lim_{\omega\to\infty} |\Gamma_{a_i}(j\omega, \mu, \eta)| = 0$. Hence, there is a compact interval $[0, \omega_{\max}]$ such that $|\Gamma_{a_i}(j\omega, \mu, \eta)|$ assumes its supremal value in $[0, \omega_{\max}]$ for each $(\mu, \eta) \in I_\mu \times I_\eta$. Since the supremum of a continuous multi-variable function over a compact interval is again continuous, also $||\Gamma_{a_i}(s, \mu, \eta)||_\infty = \sup_{\omega\in\mathbb{R}} |\Gamma_{a_i}(j\omega, \mu, \eta)| = \sup_{\omega\in[0,\omega_{\max}]} |\Gamma_{a_i}(j\omega, \mu, \eta)|$ is continuous. To show that $A_{\hat{\mu},\hat{\eta}}$ is closed, let $S_1 = \{(\mu, \eta) \in \mathbb{R} \times \mathbb{R} |\, ||\Gamma_{a_i}(s, \mu, \eta)||_\infty = 1\}$ be the 1-level set of $||\Gamma_{a_i}(s, \mu, \eta)||_\infty$. Noting that $||\Gamma_{a_i}(s, \mu, \eta)||_\infty \ge 1$ since $|\Gamma_{a_i}(0, \mu, \eta)| = 1$, we have that $S_1 = \{(\mu, \eta) \in \mathbb{R} \times \mathbb{R} |\, ||\Gamma_{a_i}(s, \mu, \eta)||_\infty \le 1\}$. Since $||\Gamma_{a_i}(s, \mu, \eta)||_\infty$ is continuous in μ and η, and any level set of a continuous function is closed, it follows that S_1 is closed. Hence, $A_{\hat{\mu},\hat{\eta}} = S_1 \cap I_\mu \times I_\eta$ is closed.

Regarding completeness, assume that $A_{\hat{\mu},\hat{\eta}}$ is not complete, denote its completion as $\overline{A}_{\hat{\mu},\hat{\eta}}$ and its boundary as $\delta A_{\hat{\mu},\hat{\eta}}$. Let $(\mu', \eta') \in \overline{A}_{\hat{\mu},\hat{\eta}} \setminus A_{\hat{\mu},\hat{\eta}}$. Since $A_{\hat{\mu},\hat{\eta}}$ is closed, $(\mu', \eta') \notin \delta A_{\hat{\mu},\hat{\eta}}$. In turn, there must be $\mu_1 < \mu'$ and $\mu_{\mathrm{u}} > \mu'$ such that $(\mu_1, \eta') \in \delta A_{\hat{\mu},\hat{\eta}}$ and $(\mu_{\mathrm{u}}, \eta') \in \delta A_{\hat{\mu},\hat{\eta}}$. However, this contradicts Proposition 1 since $\mu_1, \mu_{\mathrm{u}} \in I(\tau_i, \phi_i, \eta, K_i)$ but $||\Gamma_{a_i}(s, \mu', \eta')||_\infty > 1$ for $\mu' \in [\mu_1, \mu_{\mathrm{u}}] \subseteq I(\tau_i, \phi_i, \eta, K_i)$. □

The main outcome of Theorem 1 is the existence of a region $A_{\hat{\mu},\hat{\eta}}$ around the nominal parameter values $(\hat{\mu}, \hat{\eta})$ such that strict L_2 string stability is fulfilled for all parameter combinations in that region. In particular, a controller for vehicle i ensures strict L_2 string stability for all possible predecessor vehicles given by (12) if $A_{\hat{\mu},\hat{\eta}} = I_\mu \times I_\eta$.

Remark 1: We emphasize that Theorem 1 solely depends on the control architecture with ISF with $C_i(s) = 1$ in Fig. 2. That is, any controller realization with strictly proper $K_{\mathrm{ff},i}$ that achieves strict L_2 string stability for a given vehicle i and predecessor vehicle with time constant $\hat{\mu}$ provides robustness as identified in Theorem 1, whereby the size of $A_{\hat{\mu},\hat{\eta}}$ depends on K_i. The controller realization in (10) *fulfills this condition.*

B. Computation of the Region of Strict String Stability

We next develop an algorithm for computing $A_{\hat{\mu},\hat{\eta}}$. We use the bisection algorithm in Section II-E to compute the bounds of the interval $I(\tau_i, \phi_i, \eta, K_i) = [\mu^{\min}, \mu^{\max}]$ in Proposition 1. Regarding $\mu^{\max}$, we know that $||\Gamma_{a_i}(s, \mu, \eta)||_\infty \le 1$ if $\mu \le \mu^{\max}$ and $||\Gamma_{a_i}(s, \mu, \eta)||_\infty > 1$ if $\mu > \mu^{\max}$. We compute

$$\mu^{\max} = \mathtt{Bisection}(||\Gamma_{a_i}(s, \mu, \eta)||_\infty - 1, \epsilon, \mu_{\mathrm{u}}^{\max}, \mu_1^{\max}) \tag{19}$$

with the condition $||\Gamma_{a_i}(s, \mu, \eta)||_\infty - 1 = 0$ and a fixed value of η. Regarding the choice of $\mu_{\mathrm{u}}^{\max}$ and $\mu_1^{\max}$, we know that $||\Gamma_{a_i}(s, \hat{\mu}, \eta)||_\infty \le 1$ by design. Hence, $\mu^{\max} \ge \hat{\mu}$ and we can choose $\mu_1^{\max} = \hat{\mu}$. In addition, a large enough value for $\mu_{\mathrm{u}}^{\max}$ such that $||\Gamma_{a_i}(s, \mu_{\mathrm{u}}^{\max}, \eta)||_\infty > 1$ has to be selected.

Regarding $\mu^{\min}$, it can be directly concluded from Proposition 1 that $\mu^{\min} = 0$ if $||\Gamma_{a_i}(s, 0, \eta)||_\infty \le 1$. Otherwise,

$$\mu^{\min} = \mathtt{Bisection}(||\Gamma_{a_i}(s, \mu, \eta)||_\infty - 1, \epsilon, \mu_{\mathrm{u}}^{\min}, \mu_1^{\min}). \tag{20}$$

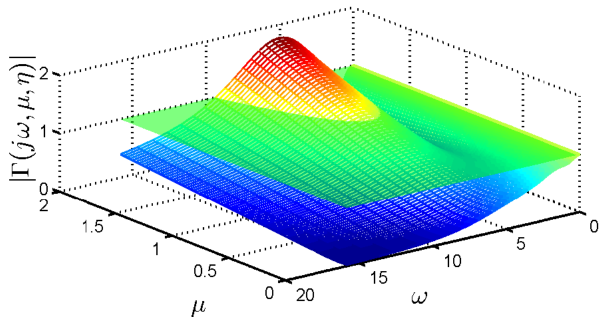

Fig. 3. $|\Gamma_{a_i}(j\omega, \mu, \hat{\eta})|$ for $\tau_i = 0.38$ and $\hat{\eta} = -0.12$.

Here, $\mu_1^{\min} = 0$ is suitable, since $||\Gamma_{a_i}(s, 0, \eta)||_\infty > 1$. Assuming that (20) is evaluated before (19) it is further possible to choose $\mu_{\mathrm{u}}^{\min} = \mu^{\max}$, since $||\Gamma_{a_i}(s, \mu^{\max}, \eta)||_\infty \leq 1$.

Using (19) and (20) it is now possible to compute the RSSS in Algorithm 2. To this end, the interval $I(\tau_i, \phi_i, \eta, K_i)$ with the bounds $\mu^{\min}$ and $\mu^{\max}$ is computed for values of $\eta \in I_\eta$ that are successively incremented in steps of $\Delta\eta$ within the maximum and minimum values of η.

Algorithm 2 Computation of $A_{\hat{\mu},\hat{\eta}}$

input : $\hat{\mu}$, $\hat{\eta}$, $\eta_{\max}$, $\eta_{\min}$, $\Delta\eta$; **output:** $A_{\hat{\mu},\hat{\eta}}$

```
η = η_min
A_{μ̂,η̂} = ∅
while η ≤ η_max do
    Compute I(τ_i, φ_i, η, K_i) for Γ_{a_i}(s, μ, η)
    A_{μ̂,η̂} = A_{μ̂,η̂} ∪ I(τ_i, φ_i, η, K_i)
    η = η + Δη
end
```

C. Evaluation for ISF

We next illustrate the obtained results. In the first evaluation, we consider Proposition 1. As an example, we select the parameters identified in [4] with $\tau_i = 0.38$, $\theta_i = 0.06$ and $\phi_i = 0.18$ and select a controller $K_i = \begin{bmatrix} K_{\mathrm{ff},i} & K_{\mathrm{fb},i} \end{bmatrix}$ as in (10) with $K_{\mathrm{P},i} = 2.9$, $K_{\mathrm{D},i} = 1.7$ and $h_i = 0.82$ that ensures strict L_2 string stability for $\hat{\mu} = 0.38$ and $\hat{\eta} = -0.12$. We determine the interval $I(\tau_i, \phi_i, \hat{\eta}, K_i) = [0, 1.25]$ using (20) and (19) with $\epsilon = 10^{-4}$. Fig. 3 shows $|\Gamma_{a_i}(j\omega, \mu, \hat{\eta})|$ for a relevant range of ω and μ. $|\Gamma_{a_i}(j\omega, \mu, \hat{\eta})|$ converges to zero for large values of ω and $|\Gamma_{a_i}(j\omega, \mu, \hat{\eta})| > 1$ for values of $\mu > 1.25$ and frequencies around $\omega = 5$ rad/sec. In addition, $|\Gamma_{a_i}(j\omega, \mu, \hat{\eta})|$ is a parabola for each value of ω such that indeed $\sup_\omega |\Gamma_{a_i}(j\omega, \mu, \hat{\eta})| \leq 1$ only for $\mu \in I(\tau_i, \phi_i, \hat{\eta}, K_i)$. We note that Fig. 3 only constitutes a representative example. We evaluated a large number of different parameter combinations with similar plots in agreement with Proposition 1.

Our second evaluation applies Algorithm 2 for determining the RSSS. To this end, we select different parameter combinations for τ_i, ϕ_i and θ_i within the specified bounds in Table I and experimentally determine $K_{\mathrm{P},i}$ and $K_{\mathrm{D},i}$ as well as h_i:

TABLE II
BOUNDS FOR μ DEPENDING ON η

	Case 1		Case 2		Case 3	
η	$\mu^{\min}$	$\mu^{\max}$	$\mu^{\min}$	$\mu^{\max}$	$\mu^{\min}$	$\mu^{\max}$
-0.23	0.0	0.91	0.0	1.30	0.0	3.03
-0.2	0.0	0.91	0.0	1.30	0.0	3.00
-0.16	0.0	0.91	0.0	1.29	0.0	2.95
-0.12	0.0	0.91	0.0	1.25	0.0	2.87
-0.08	0.0	0.91	0.0	1.21	0.0	2.77
-0.04	0.0	0.90	0.0	1.15	0.0	2.65
0.0	0.0	0.89	0.0	1.09	0.0	2.52
0.02	0.0	0.89	0.0	1.06	0.0	2.46
0.06	0.0	0.87	0.0	0.99	0.0	2,32
0.1	0.0	0.86	0.0	0.92	0.02	2.18
0.14	0.0	0.84	0.0	0.86	0.06	2.04
0.18	0.0	0.82	0.0	0.81	0.1	1.91

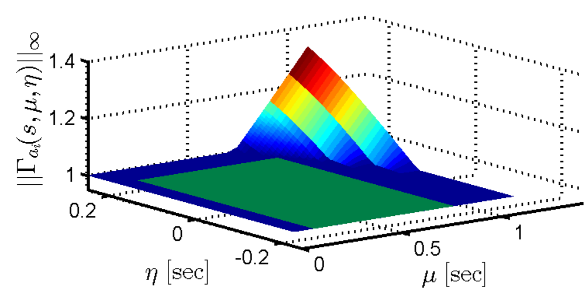

Fig. 4. $||\Gamma_{a_i}(j\omega, \mu, \eta)||_\infty$ for $\tau_i = 0.8$ and $\phi_i = 0.02$.

- Case 1: $\tau_i = 0.1$, $\phi_i = 0.2$, $\theta_i = 0.02$ as in [6] and $K_{\mathrm{P},i} = 1.39$, $K_{\mathrm{D},i} = 0.25$, $h_i = 1.0$,
- Case 2: $\tau_i = 0.38$, $\phi_i = 0.18$, $\theta_i = 0.06$ as in [4] and $K_{\mathrm{P},i} = 2.9$, $K_{\mathrm{D},i} = 1.7$, $h_i = 0.82$,
- Case 3: $\tau_i = 0.8$, $\phi_i = 0.02$, $\theta_i = 0.2$ as in [11] and $K_{\mathrm{P},i} = 3.2$, $K_{\mathrm{D},i} = 4.4$, $h_i = 0.6$.

Then, we evaluate the corresponding set $A_{\hat{\mu},\hat{\eta}}$ using Algorithm 2. The result of this evaluation is summarized in Table II.

It is readily observed that $I(\tau_i, \phi_i, \eta, K_i)$ for each value of η includes all possible vehicle time constants. That is, choosing a suitable K_i and h_i ensures $A_{\hat{\mu},\hat{\eta}} = I_\mu \times I_\eta$ such that strict L_2 string stability of the selected example vehicles is achieved for all predecessor vehicle parameters. Recalling that $\eta = \theta_i - \phi_{i-1}$, it can also be seen from Table II that the negative effect of large communication delays θ_i on $I(\tau_i, \phi_i, \hat{\eta}, K_i)$ is more significant than the effect of large actuator delays ϕ_{i-1}.

We further illustrate the obtained results by plotting $||\Gamma_{a_i}(j\omega, \mu, \eta)||_\infty$ for case 3. It can be seen that $A_{\hat{\mu},\hat{\eta}} = I_\mu \times I_\eta$, whereas $||\Gamma_{a_i}(j\omega, \mu, \eta)||_\infty > 1$ for most of the parameter combinations $(\mu, \eta) \notin I_\mu \times I_\eta$.

We next simulate a heterogeneous vehicle string with eight vehicles: the leader vehicle 1 has $\tau_1 = 0.1$ and $\phi_1 = 0.02$, vehicle 2, 3, 4, 5, 6, 7, 8 obey case 3, 2, 1, 1, 2, 3, 1 respectively. The leader performs sudden changes in the input signal in order to provide a signal that is difficult to follow as shown in Fig. 5 (d) (blue line). According to Table II, strict L_2 string stability is achieved for this heterogeneous vehicle string which is confirmed in Fig. 5. In particular, velocity and acceleration signals are attenuated along the vehicle string.

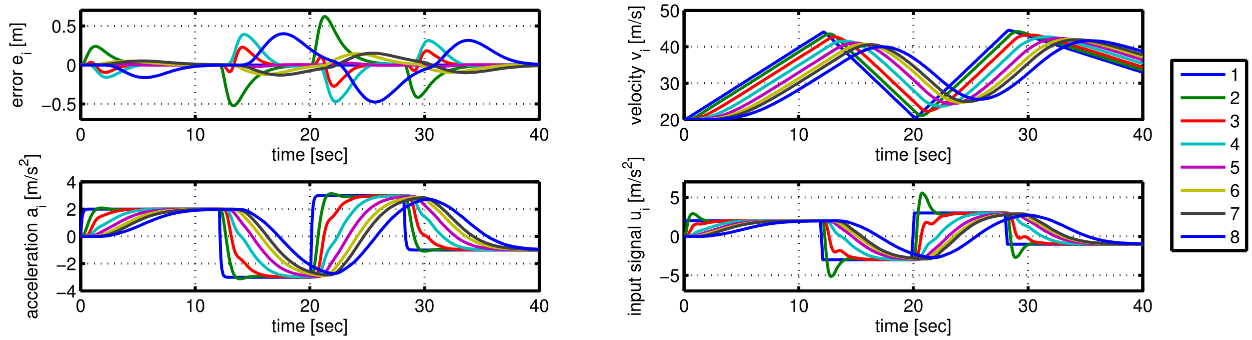

Fig. 5. Vehicle following for a vehicle string with 8 vehicles using ISF.

In addition, it is interesting to analyze the case of successive vehicles with different dynamics. For example, consider the input signal u_2 of the slower vehicle 2 with $\tau_2 = 0.8$ that follows a faster vehicle 1 with $\tau_1 = 0.1$ in Fig. 5. After each change of the input signal, a larger value of u_2 (compared to u_1) is required. The intuitive reason for this observation is the choice of the output signal $y_i = a_i$ in Section II-C. Vehicle 2 needs to apply larger input signal levels in order to adjust the actual acceleration a_2 to a_1. We further discuss the error signals in Fig. 5. Consider the faster vehicle 3 with $\tau_3 = 0.38$ that follows a slower vehicle 2 with $\tau_2 = 0.8$. Inspecting the figure, it turns out that the error signal of vehicle 3 is initially negative (decreased spacing). A possible explanation of this effect is that comparatively large input signal levels u_2 are communicated to the faster vehicle 3 which actually requires smaller input levels due to the faster dynamics. As a result, vehicle 3 initially accelerates more than necessary. The same effect can be seen for vehicle 4 and vehicle 8. Conversely, positive error signals (increased spacing) are initially observed when a slower vehicle follows a faster vehicle due to the relatively small input signal levels communicated by faster vehicles (for example vehicle 2 following vehicle 1).

In summary, the evaluation for ISF shows that strict L_2 string stability for each of the example vehicles with parameters τ_i, ϕ_i and θ_i can be obtained for predecessor vehicles within a complete set $A_{\hat{\mu},\hat{\eta}}$. The existence of this set is guaranteed by Theorem 1 and the boundary of the set is conveniently computed using Algorithm 2. Moreover, strict L_2 string stability for heterogeneous traffic with parameters in $I_\mu \times I_\eta$ identified from the literature can be guaranteed by a suitable choice of h_i when designing K_i for vehicle i.

V. String Stability Analysis Using AF and PAF

A. Strict String Stability Analysis

We first characterize the ISSS in Definition 2. It holds that this interval is a non-empty, closed set.

Theorem 2: Assume that $K_{\mathrm{ff},i}$ is proper. Then, $I_{\hat{\nu}}$ in Definition 2 is a non-empty, closed set.

Proof: $I_{\hat{\nu}}$ is non-empty since $\hat{\nu} \in I_{\hat{\nu}}$. It further holds that $||\Gamma_{a_i}(s,\nu)||_\infty$ is a continuous function in ν. Consider that

$$||\Gamma_{a_i}(s,\nu)||_\infty = \sup_{\omega\in\mathbb{R}} |\Gamma_{a_i}(j\omega,\nu)|$$

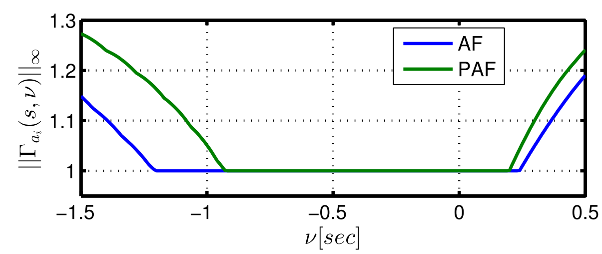

Fig. 6. Case 2: $||\Gamma_{a_i}(s,\nu)||_\infty$ for AF and PAF.

since $\Gamma_{a_i}(j\omega,\nu)$ is a continuous function in ω, ν. In addition, it is clear from (16) that $|\Gamma_{a_i}(0,\nu)| = 1$ and $\lim_{\omega\to\infty} |\Gamma_{a_i}(j\omega,\nu)| = 0$ considering that $K_{\mathrm{ff},i}$ is proper. Hence, there is a compact interval $[0,\omega_{\max}]$ such that $\Gamma_{a_i}(j\omega,\nu)$ assumes its supremal value in $[0,\omega_{\max}]$ for each $\hat{\nu} \in I_\nu$. Since the supremum of a continuous multi-variable function over a compact interval is again continuous, also $||\Gamma_{a_i}(s,\nu)||_\infty = \sup_{\omega\in\mathbb{R}} |\Gamma_{a_i}(j\omega,\nu)| = \sup_{\omega\in[0,\omega_{\max}]} |\Gamma_{a_i}(j\omega,\nu)|$ is continuous.

We define $S_1 = \{\hat{\nu} \in \mathbb{R} |\, ||\Gamma_{a_i}(s,\nu)||_\infty = 1\}$ as the 1-level set of $||\Gamma_{a_i}(s,\nu)||_\infty$. It holds that $S_1 = \{\nu \in \mathbb{R} |\, ||\Gamma_{a_i}(s,\nu)||_\infty \leq 1\}$ because $|\Gamma_{a_i}(0,\nu)| = 1$. Since $||\Gamma_{a_i}(s,\nu)||_\infty$ is continuous in ν, and any level set of a continuous function is closed, it follows that S_1 is closed. Hence, $I_{\hat{\nu}} = S_1 \cap I_\nu$ is closed. □

Theorem 2 shows the existence of an interval $I_{\hat{\nu}}$ around the nominal parameter $\hat{\nu}$ such that strict L_2 string stability is fulfilled for all parameter values in that interval. Consequently, a controller for vehicle i ensures strict L_2 string stability for all possible predecessor vehicles given by (12) if $I_{\hat{\nu}} = I_\nu$.

Remark 2: Note that the controller realization for AF and PAF in (11) *fulfills the condition in Theorem 2.*

Regarding the computation of $I_{\hat{\nu}}$, Theorem 2 states that $I_{\hat{\nu}}$ is a non-empty closed set. That is, writing $I_{\hat{\nu}} = [\nu^{\min}, \nu^{\max}]$, it holds that either $\nu^{\min} = -\infty$ or there is an interval $I_{\nu^{\min}}$ with $\nu^{\min} \in I_{\nu^{\min}}$ and

- $||\Gamma_{a_i}(s,\nu)||_\infty > 1$ for all $\nu \in I_{\nu^{\min}}$ with $\nu < \nu^{\min}$
- $||\Gamma_{a_i}(s,\nu)||_\infty \leq 1$ for all $\nu \in I_{\nu^{\min}}$ with $\nu \geq \nu^{\min}$.

Choosing $\nu_{\mathrm{l}}^{\min} \in I_{\nu^{\min}}$ with $\nu_{\mathrm{l}}^{\min} < \nu^{\min}$ and $\nu_{\mathrm{u}}^{\min} \in I_{\nu^{\min}}$ with $\nu_{\mathrm{u}}^{\min} \geq \nu^{\min}$,

$$\nu^{\min} = \mathtt{Bisect}(||\Gamma_{a_i}(s,\nu)||_\infty - 1, \epsilon, \nu_{\mathrm{u}}^{\min}, \nu_{\mathrm{l}}^{\min}) \quad (21)$$

precisely determines $\nu^{\min}$. Likewise, for $\nu^{\max}$, either $\nu_{\max} = \infty$ or there exists an interval $I_{\nu^{\max}}$ with $\nu^{\max} \in I_{\nu^{\max}}$

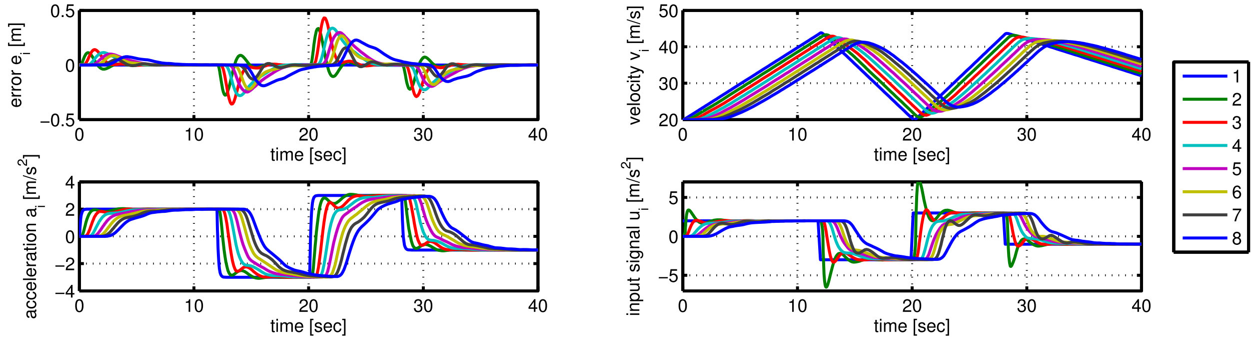

Fig. 7. Vehicle following for a vehicle string with 8 vehicles using AF.

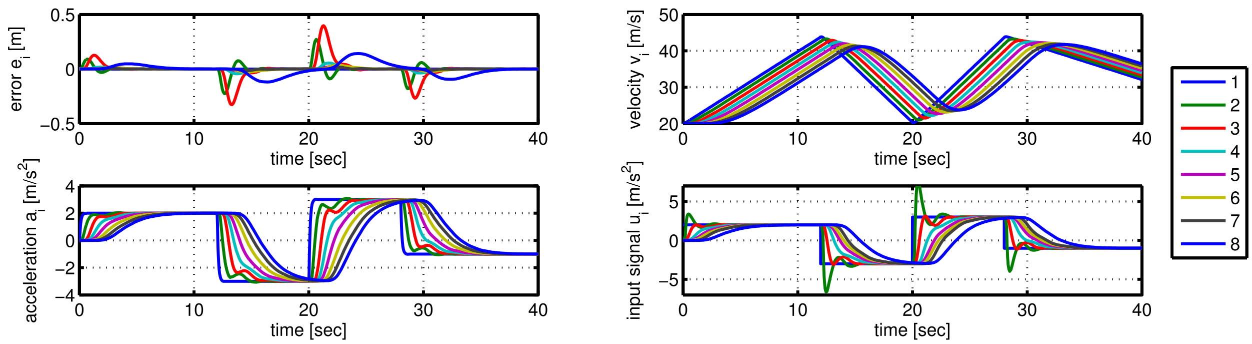

Fig. 8. Vehicle following for a vehicle string with 8 vehicles using PAF.

such that

- $||\Gamma_{a_i}(s,\nu)||_\infty > 1$ for all $\nu \in I_{\nu^{\max}}$ with $\nu > \nu^{\max}$,
- $||\Gamma_{a_i}(s,\nu)||_\infty \leq 1$ for all $\nu \in I_{\nu^{\max}}$ with $\nu \leq \nu^{\max}$.

That is, choosing $\nu_{\mathrm{l}}^{\max} \in I_{\nu^{\max}}$ with $\nu_{\mathrm{l}}^{\max} \leq \nu^{\max}$ and $\nu_{\mathrm{u}}^{\max} \in I_{\nu^{\max}}$ with $\nu_{\mathrm{u}}^{\max} > \nu^{\max}$,

$$\nu^{\max} = \mathtt{Bisect}(||\Gamma_{a_i}(s,\nu)||_\infty - 1, \epsilon, \nu_{\mathrm{u}}^{\max}, \nu_{\mathrm{l}}^{\max}) \quad (22)$$

precisely determines $\nu^{\max}$.

B. Evaluation of (Predicted) Acceleration Feedforward

We first illustrate the result in Theorem 2 for AF and PAF using the test cases in Section IV-C. For a fair comparison with the ISF strategy, the controller parameter $\omega_{\mathrm{K},i}$ and the headway time h_i for each case are chosen such that the dominant poles of the closed loop are in close proximity to the dominant poles of the respective case using ISF in Section IV-C. Furthermore, the ISSS is computed using (21) and (22) (see Table III).

Table III indicates that it is possible in all cases to determine $K_{\mathrm{fb},i}$ and h_i such that strict L_2 string stability is ensured for the full parameter range of the communication delay $\nu \in [0, 0.2]$ for AF and $\nu \in [-0.23, 0.18]$ for PAF. Again, we emphasize that strict L_2 string stability only depends on the communication delay when using AF or PAF. In addition, it has to be noted that the headway times for AF and PAF are smaller compared to the case of ISF in Section IV-C. Moreover, a slight decrease of the headway time is possible when using PAF instead of AF. This fact is also illustrated in Fig. 6 that shows the ISSS for case 2 (AF and PAF). A slightly tighter ISSS can be obtained when using PAF.

We next simulate the same heterogeneous vehicle string with eight vehicles as in Section IV-C with the same input signal u_1. Strict L_2 string stability is achieved using AF and PAF which is confirmed in Fig. 7 and 8.

TABLE III
CONTROLLER PARAMETERS AND ISSS FOR DIFFERENT CASES

	AF			PAF		
	$\omega_{\mathrm{k},i}$	h_i	I_ν	$\omega_{\mathrm{k},i}$	h_i	I_ν
Case1	1.32	0.66	[-2.245,0.222]	1.5	0.6	[-1.952,0.192]
Case2	1.65	0.7	[-1.205,0.239]	1.9	0.67	[-0.928,0.195]
Case3	2.5	0.62	[-0.767,0.223]	2.8	0.6	[-0.695,0.216]

When looking at vehicles with different dynamic properties, the cases of AF and PAF exhibit similar behavior. For example, input signals of slow vehicles i (e.g. vehicle 2) assume larger values when following a faster vehicle and positive error signals are observed when applying positive input signals to the leader vehicle 1, since the communicated acceleration signal levels are independent of the predecessor vehicle dynamics. It only can be noted that PAF achieves smaller error signals compared to AF especially in cases where slower vehicles follow faster vehicles (for example vehicle 5, 6, 7).

C. Validation Using a Realistic Simulation Environment

In order to further validate the obtained results, we use the Plexe simulation environment for automated vehicle-following systems [34]. Plexe allows the coupled simulation of the V2V communication via the IEEE 802.11p protocol and realistic vehicle dynamics including detailed engine models, gearbox status and friction forces as described in [17]. In this paper, we use three vehicle types: Alfa Romeo 147 1.6 Twin Spark (V1), Audi R8 (V3) and Bugatti Veyron (V3) that are available in Plexe. Hereby, the vehicle parameters are adjusted such that

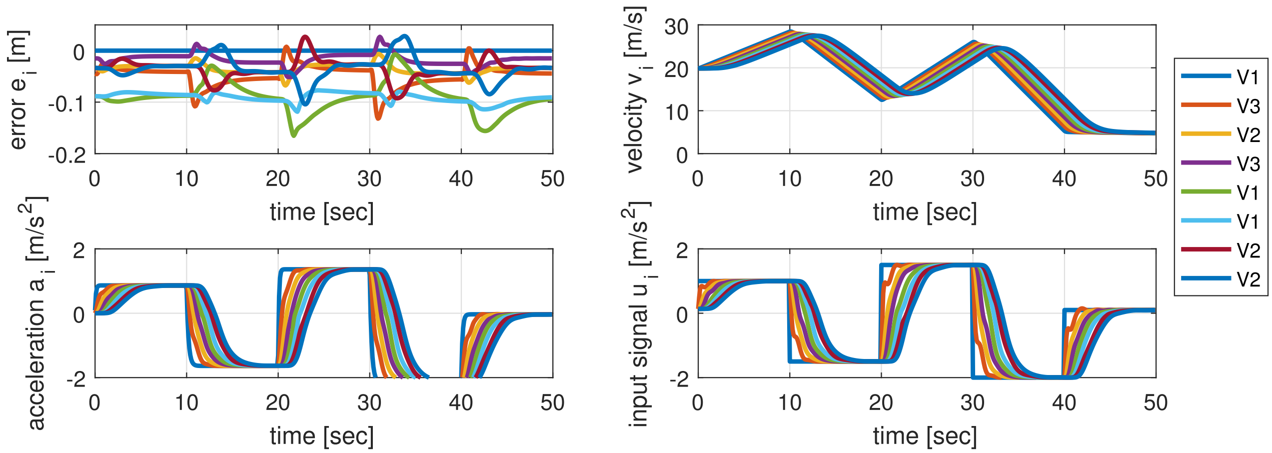

Fig. 9. Vehicle following simulation in Plexe for a vehicle string with 8 vehicles using PAF.

the time constants of the respective vehicles are in the order of $\tau_i = 0.1$ for V1, $\tau_i = 0.2$ for V2 and $\tau_i = 0.3$ for V3.

In addition to the already available ISF strategy, we implemented AF and PAF in Plexe and confirmed string stability in detailed simulations with the different strategies. In the scope of this paper, we focus on a representative case with the PAF strategy and a vehicle string with 8 vehicles. Choosing a headway time of $h_i = 0.5$ we determine controllers with the parameters $\omega_{k,i} = 1.5$ (V1), $\omega_{k,i} = 2$ (V2) and $\omega_{k,i} = 3$ (V3) that ensure string stability for the set $I_v = [-0.23, 0.18]$. In the simulation, delays of the transmitted data in the order of 100 ms are observed. An example simulation run with speed-up/braking maneuvers is shown in Fig. 9. It is readily observed that the follower vehicles safely track the leader, attenuating the variations in the acceleration signal. In particular, the distance error remains below 0.2 m. It is interesting to note that none of the follower vehicles requests a large input signal since the time constants of V1, V2 and V3 are close.

D. Comparison and Discussion

We finally discuss main features of the different control architectures. First, our evaluation indicates that strict L_2 string stability of the example vehicles i can be achieved for the full parameter range of predecessor vehicles by a suitable choice of the controller K_i and the headway time h_i for all feedforward strategies. Hereby, a PD-type feedback controller as in Section II-D is employed in this paper for simplicity. Nonetheless, other controller designs that fulfill the properties in Theorem 1 and 2 could be used as well.

Regarding the different feedforward strategies, we make the following observations. When using PD controllers, our experiments suggest that strict L_2 string stability can be achieved with a smaller headway time when using AF and PAF. This is essential when establishing tight vehicle following in order to increase the traffic capacity [35], [36]. On the other hand, ISF requires slightly smaller input signal levels which is relevant in case of input signal limitations. It also has to be highlighted that large values of the communication delay have a particularly negative effect on strict L_2 string stability for all feedforward strategies.

In summary, our results show that ISF, AF and PAF are all suitable for strict L_2 string stability in the identified parameter range of predecessor vehicles. It has to be noted that this range can be adjusted by changing the controller K_i and the headway time h_i. A detailed study of the controller design for strict L_2 string stability in heterogeneous vehicle strings based on the results of this paper is the subject of future work.

VI. Conclusions

Cooperative adaptive cruise control supports safe vehicle following in vehicle strings at tight inter-vehicle spacings. In this context, it is desired to fulfill the property of strict string stability such that fluctuations are attenuated along a vehicle string. This paper focuses on the string stability analysis for the practical case of heterogeneous vehicle strings. Hereby, the common case of predecessor following is considered since its realization and analysis scales to vehicle strings of arbitrary length. Different strategies with acceleration feedforward (AF), predicted acceleration feedforward (PAF) and input signal feedforward (ISF) are taken into account. It is shown that all feedforward strategies are robust in the sense that strict string stability for a certain vehicle can be achieved for arbitrary predecessor vehicles within a certain parameter range. The paper further provides algorithms that efficiently determine the respective parameter range. It is noted that this parameter range can be adapted to include different vehicle parameters by an appropriate controller design and choice of the headway time between vehicles.

It has to be emphasized that this paper analyzes strict string stability for heterogeneous vehicle strings using predecessor following and does not focus on the controller design. In future work, we will extend our study by controller design methods for time-delay systems such as [37] and [38] and to other information flow topologies such as leader-predecessor following in order to achieve strict string stability for heterogeneous vehicle strings and small headway times.

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