Cooperative Control of Heterogeneous Connected Vehicle Platoons: An Adaptive Leader-Following Approach

Junyan Hu, *Student Member, IEEE*, Parijat Bhowmick, *Member, IEEE*, Farshad Arvin, *Member, IEEE*, Alexander Lanzon, *Senior Member, IEEE*, and Barry Lennox, *Senior Member, IEEE*

***Abstract*—Automatic cruise control of a platoon of multiple connected vehicles in an automated highway system has drawn significant attention of the control practitioners over the past two decades due to its ability to reduce traffic congestion problems, improve traffic throughput and enhance safety of highway traffic. This paper proposes a two-layer distributed control scheme to maintain the string stability of a heterogeneous and connected vehicle platoon moving in one dimension with constant spacing policy assuming constant velocity of the lead vehicle. A feedback linearization tool is applied first to transform the nonlinear vehicle dynamics into a linear heterogeneous state-space model and then a distributed adaptive control protocol has been designed to keep equal inter-vehicular spacing between any consecutive vehicles while maintaining a desired longitudinal velocity of the entire platoon. The proposed scheme utilizes only the neighbouring state information (i.e. relative distance, velocity and acceleration) and the leader is not required to communicate with each and every one of the following vehicles directly since the interaction topology of the vehicle platoon is designed to have a spanning tree rooted at the leader. Simulation results demonstrated the effectiveness of the proposed platoon control scheme. Moreover, the practical feasibility of the scheme was validated by hardware experiments with real robots.**

***Index Terms*—Intelligent transportation systems, multi-robot systems, robust/adaptive control of robotic systems.**

I. Introduction

The notion of Intelligent Highway Vehicle Systems (IHVSs) was introduced more than two decades ago and has attracted considerable attention of the transportation engineering communities to accommodate exponentially increasing number of highway traffic in today's life along with minimizing the risk of traffic accidents [1]. The key strategy to implement IHVSs is to employ automatic or semi-automatic cruise control mechanisms for highway vehicles and thereby reducing human intervention in driving the vehicles.

Fig. 1. A platoon of driverless vehicles in an automated highway system.

The fundamental issue in controlling a string (also called platoon) of connected vehicles is how to ensure string stability when the lead vehicle in a string suddenly accelerates or decelerates due to exogenous disturbances. String stability of automated vehicles guarantees that the spacing and velocity errors between the successive vehicles do not grow or amplify upstream along the string [2]. Efficacy of a platoon control scheme depends highly on the vehicular information available for feedback and on the spacing policy used. For instance, the acceleration feedback via Vehicle-to-Vehicle (V2V) communication (such as DSRC and VANET [3]) in addition to position and velocity feedback information enhances the relative string stability of a platoon [4]. Spacing policy also plays a major role in deciding a particular platoon control strategy. Constant spacing policy is quite popular in the literature since it ensures constant inter-vehicular spacing independent of the cruising velocity of the platoon and also offers high traffic capacity and easier implementation (as portrayed in Fig. 1).

A Cooperative Adaptive Cruise Control (CACC) technique was proposed in [5] for heterogeneous vehicle platoon along with practical validation results and also examined the string stability of the practical setup. The article [6] has introduced the concept of $\mathcal{L}_p$ string stability for connected vehicles and analysed string stability properties of cooperative adaptive cruise control strategies for vehicle platoons with experimental validation results. In [7], string stability of homogeneous and heterogeneous vehicle platoons with constant time headway spacing policy has been investigated considering time delays in sensors and actuators. An adaptive bidirectional platoon control scheme using a coupled sliding-mode control technique is proposed

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(Corresponding author: Parijat Bhowmick.)

The authors are with the Department of Electrical and Electronic Engineering, School of Engineering, University of Manchester, Manchester M13 9PL, U.K. (e-mail: junyan.hu@manchester.ac.uk; parijat.bhowmick@manchester.ac.uk; farshad.arvin@manchester.ac.uk; alexander.lanzon@manchester.ac.uk; barry.lennox@manchester.ac.uk).

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in [8] to ensure string stability of the platoon and to improve its performance. In [9], a distributed finite-time adaptive integral sliding-mode control framework is designed for a platoon of vehicles subjected to bounded exogenous disturbances. However, these studies have considered fixed interaction topology among the vehicles which reduces the flexibility and reliability of a platoon control scheme. Of late, in [10], a fully decentralized control scheme is developed including obstacle avoidance feature for a platoon of car-like vehicles equipped with on-board cameras to detect the obstacles. In the present context, the experimental work done in [11] is a major contribution in the area of cooperative driving in which a fleet of miniature robotic cars is developed for conducting platoon control activities.

With the advent of multi-agent consensus theory and formation control techniques [12]–[14], cooperative control of connected vehicles platoons has started emerging in the recent years [15]–[17]. The major objective of cooperative control of connected vehicle platoon is to maintain uniform velocity of all the vehicles in a platoon moving in one dimension on a straight flat road alongside ensuring a desired inter-vehicular spacing at the steady-state. Pioneering research works have been carried out in [15] and [16] to apply the consensus based cooperative control techniques in developing distributed control schemes for connected vehicle platoons moving in one dimension. While [15] have developed the control schemes for homogeneous vehicle platoons, [16] have extended their previous results for heterogeneous platoons. However, in [15] and [16], the proposed cooperative control protocols are not adaptive and the case of communication link failure have not been explored.

Motivated by the ongoing developments and existing challenges in the area of cooperative control of multiple connected vehicle platoon, in this paper, we consider the problem of designing a distributed control scheme for a platoon of heterogeneous connected vehicles moving in one dimension considering constant spacing policy and constant longitudinal velocity of the lead vehicle. Below, we summarize the contributions of this paper:

- A two-layer distributed and adaptive control architecture is developed for a heterogeneous platoon of bidirectionally connected vehicles to maintain same cruising velocity of all vehicles in the platoon and to keep equal inter-vehicular spacing between any two successive vehicles, which together preserve string stability of the platoon in presence of exogenous input applied to the lead vehicle.
- The proposed scheme takes into account the complete nonlinear vehicle dynamics during the controller design, hence, the heterogeneity of the vehicles due to different parameters are directly considered in this scheme.
- The proposed platoon control design being distributed and adaptive facilitates uninterrupted and reliable control of the platoon despite any bounded disturbance on the lead vehicle and also in the event of communication link failure.
- Effectiveness of the proposed method has been validated through affordable real robot experiments on a two-lane platoon of six connected vehicles, represented by autonomous, miniature Mona robots [18], in a real-time laboratory environment equipped with low-cost over-head camera tracking system.

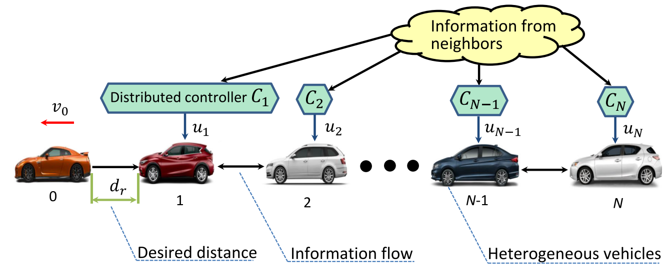


Fig. 2. Cooperative control architecture of a heterogeneous and connected vehicle platoon in which the vehicle labelled with 0 acts as the leader while the rest $1, 2, \ldots, N$ vehicles are considered as followers.

II. Technical Background and Problem Formulation

In this paper, we aim to establish the string stability of a heterogeneous vehicle platoon and thereby to ensure that all the vehicles in the platoon remain equidistant even when the lead vehicle is affected by exogenous input (e.g. sudden acceleration/deceleration). In order to achieve this, we developed a two-layer platoon control scheme which exploits the feedback linearization technique to first linearize the nonlinear vehicle dynamics and then applies a distributed cooperative control technique to develop a rectilinear static formation of the connected vehicles to keep them equidistant assuming that the leader vehicle has constant velocity.

A. Communication Topology

Suppose that the information links among following vehicles are bidirectional and there exists at least one directional link from the leader to followers. Consider a weighted and directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a nonempty set of nodes $\mathcal{V} = \{1, 2, \ldots, N\}$, a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and the associated adjacency matrix $\mathcal{A} = [\alpha_{ij}] \in \mathbb{R}^{N\times N}$. An edge rooted at the ith node and ended at the jth node is denoted by (i, j), which means information can flow from vehicle i to vehicle j. α_{ij} is the weight of edge (j, i) and $\alpha_{ij} > 0$ if $(j, i) \in \mathcal{E}$. Vehicle j is called a neighbour of vehicle i if $(j, i) \in \mathcal{E}$. Define the in-degree matrix as $D = \mathrm{diag}\{d_i\} \in \mathbb{R}^{N\times N}$ with $d_i = \sum_{j=1}^{N} \alpha_{ij}$. The Laplacian matrix $L \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $L = D - \mathcal{A}$. If the ith following vehicle observes the lead vehicle then an edge $(0, i)$ between them is said to exist with the pinning gain $g_i > 0$. We denote the pinning matrix as $G = \mathrm{diag}\{g_i\} \in \mathbb{R}^{N\times N}$. It is assumed that at least one follower is connected to the leader. In virtue of the bidirectional communication topology among the followers, all the eigenvalues of the matrix $L + G$ (denoted by λ_i for $1, 2, \ldots, N$) are real and positive [15].

B. Modelling of Heterogeneous Vehicles

We consider a platoon of connected vehicles with non-identical dynamics that forms a string running through dense traffic on a straight highway assuming there is no overtaking. In each platoon, the vehicle at the front is considered as the leader that moves at a constant velocity while rest of the downstream vehicles are considered as followers. A schematic diagram of the platoon is depicted in Fig. 2 where all the following vehicles try to maintain equal spacing between any two of them. In this paper, we will use the following notations: position, velocity and

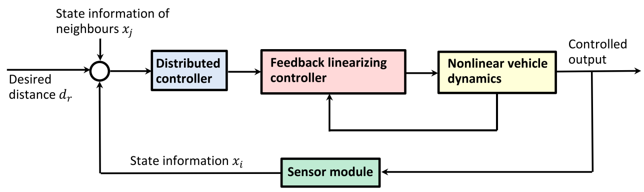


Fig. 3. A two-layer distributed platoon control scheme for connected vehicles.

acceleration of the ith following vehicle are denoted by $p_i(t)$, $v_i(t)$, $a_i(t)$ for all $i \in \{1, 2, \ldots, N\}$, while those of the lead vehicle are denoted by $p_0(t)$, $v_0(t)$ and $a_0(t)$. In the remaining of the paper, explicit dependence on time $t \geq 0$ has been omitted. Now, the dynamics of each vehicle can be modelled, as shown in [2], by the following nonlinear input-affine differential equation

$$\dot{a}_i = f_i(v_i, a_i) + g_i(v_i) b_i \qquad \forall i \tag{1}$$

which encompasses the engine dynamics, brake system and aerodynamics drag. In (1), b_i is the input to the engine of the ith vehicle and the nonlinear functions $f_i(v_i, a_i)$ and $g_i(v_i)$ are given by

$$f_i(v_i, a_i) = -\frac{1}{\tau_i}\left(a_i + \frac{\sigma\phi_i c_{di}}{2m_i} v_i^2 + \frac{d_{mi}}{m_i}\right) - \frac{\sigma\phi_i c_{di}}{m_i} v_i a_i$$

$$\text{and } g_i(v_i) = \frac{1}{\tau_i m_i},$$

where σ denotes the specific mass of the air, ϕ_i is the cross-sectional area, c_{di} indicates the drag coefficient, m_i denotes the mass, d_{mi} represents the mechanical drag and τ_i symbolizes the engine time constant (also called the inertial time-lag). The group of terms $\frac{\sigma\phi_i c_{di}}{2m_i}$ models the air resistance [2].

Much of the literature on platoon control problems considers either double integrator dynamics or other simplified vehicle models [7], [13]. The controllers designed on such simplified models may not be sufficient to handle the complex nonlinear dynamics of real vehicles. On the contrary, in this paper, the complete nonlinear model of the car-like vehicles (1) has been adopted for controller design. Note that since most of the parameters mentioned in (1) are not identical for all vehicles, hence, the present work deals with control of heterogeneous vehicle platoon. Now, we would like to use the following feedback linearizing control law

$$b_i = u_i m_i + 0.5\sigma\phi_i c_{di} v_i^2 + d_{mi} + \tau_i \sigma\phi_i c_{di} v_i a_i \tag{2}$$

for all $i \in \{1, 2, \ldots, N\}$ to transform the nonlinear vehicle dynamics into a linearized model. Note that the local feedback linearizing control action operates as the inner-layer controller as shown in Fig. 3, does appropriate simplification of the complex vehicle dynamics and ensures asymptotic stability of any resulting zero dynamics. In (2), u_i is the new control input to be designed. Now, substituting (2) into (1), the longitudinal dynamics of each vehicle in the platoon is obtained as

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = a_i, \\ \tau_i \dot{a}_i + a_i = u_i \qquad \forall i, \end{cases} \tag{3}$$

which is then expressed in the standard state-space form as

$$\dot{x}_i = A_i x_i + B_i u_i \qquad \forall i, \tag{4}$$

where $x_i(t) = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix}$, $A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}$ and $B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}$. It can be readily verified that the pair (A_i, B_i) is controllable for all $i \in \mathcal{F}$. The leader vehicle runs with constant velocity under the steady-state condition (i.e., $u_0 = 0$). Note that even though the nonlinear vehicle dynamics (1) is linearized to (4) but it is still heterogeneous due to the presence of the parameter τ_i. This makes the present problem non-trivial and challenging.

C. Problem Formulation

Cooperative control of a platoon of $N + 1$ heterogeneous vehicles moving on a straight flat road is considered in this work, which includes a leader indexed by 0 and N followers indexed from 1 to N. Let $\mathcal{F} = \{1, 2, \ldots, N\}$ be the set of the followers. The control objective is to ensure all the following vehicles maintains the same speed as the leader while keeping a constant inter-vehicular spacing, thereby potentially improving road throughput. This policy can provide advantages such as high vehicle density and low energy consumption.

The heterogeneous connected vehicles shown in Fig. 2 are said to achieve the desired platoon control objectives with the lead vehicle having constant velocity if for any given bounded initial states

$$\begin{cases} \lim\limits_{t\to\infty} \|p_i(t) - p_j(t)\| = (i-j)d_r, \\ \lim\limits_{t\to\infty} \|v_i(t) - v_0(t)\| = 0 \quad \text{and} \\ \lim\limits_{t\to\infty} \|a_i(t) - a_0(t)\| = 0, \end{cases} \tag{5}$$

for all $i, j \in \mathcal{F}$, $j \neq i$. The platoon control activities including how to preserve string stability is formulated as a 'Leader-following' static formation of the connected vehicles in the platoon where the lead vehicle is considered as the root node. The desired static formation of the platoon is specified by the vector $d_i = [id_r, 0, 0]^T$ $\forall i \in \{1, 2, \ldots, N\}$ known to the ith following vehicle, where d_r is a pre-specified gap between any two consecutive vehicles in the platoon. In case of implementing constant spacing policy, the desired gap d_r remains constant for all i. Note that any acceleration/deceleration in a vehicle happening due to taking sharp turns, climbing uphill or going downhill can be considered as a bounded disturbance acting on the vehicle temporarily. String stability of a platoon ensures that the spacing error grows only during the period of disturbance but quickly decays to zero as soon as the disturbance (i.e. the acceleration/deceleration) disappears.

III. Control Protocol Design and Stability Analysis

This section develops distributed cooperative control scheme for a platoon of heterogeneous and connected vehicles, which is the main theoretical contribution of the paper. The control objectives are i) to keep equal and constant inter-vehicular spacing between successive vehicles, ii) maintain the velocities of all the following vehicles same as that of the leader and iii) to maintain zero acceleration for all the vehicles in the platoon. Motivated by the adaptive control techniques developed in [19]–[21], we propose a distributed control protocol given in Theorem 1 to satisfy the aforementioned objectives and thereby to maintain string stability of the platoon.

Theorem 1: Consider a platoon of heterogeneous connected vehicles with the feedback linearized dynamics given in (4). Define $\delta = \frac{\tau_0}{\max_{i\in\mathcal{F}} \tau_i}$, $\rho = \frac{\tau_0}{\min_{i\in\mathcal{F}} \tau_i}$ and select $\varphi \geq \frac{1}{2\delta \min_{i\in\mathcal{F}} \lambda_i}$. Then, the cooperative control objectives of the heterogeneous platoon are achieved when the following adaptive control protocol is applied to the ith follower vehicle for all $i \in \mathcal{F}$

$$\begin{cases} u_i = B_0^T \xi_i x_i + \varphi K \sum_{j=0}^{N} \alpha_{ij} \left((x_i - d_i) - (x_j - d_j) \right), \\ \dot{\xi}_i = \rho\, x_i^T B_0 K \sum_{j=0}^{N} \alpha_{ij} \left((x_i - d_i) - (x_j - d_j) \right), \end{cases} \tag{6}$$

with $K = -B_0^T P$ where $P > 0$ is a unique solution to the algebraic Riccati equation (ARE)

$$PA_0 + A_0^T P - PB_0 B_0^T P + \gamma I_3 = 0 \tag{7}$$

for a given $\gamma > 0$.

Proof: The closed-loop dynamics of the heterogeneous vehicle platoon given in (4) under the proposed distributed platoon control law (6) is obtained as

$$\dot{x}_i = \left(A_i + B_i B_0^T \xi_i\right) x_i + \varphi B_i K \sum_{j=0}^{N} \alpha_{ij} \left((x_i - d_i) - (x_j - d_j) \right). \tag{8}$$

For each connected vehicle, exploiting the controllable canonical structure of A_0, B_0, A_i and B_i for all i, it is always possible to find a constant vector ζ_i such that

$$A_0 = A_i + B_i B_0^T \zeta_i \qquad \forall i \in i \in \mathcal{F}. \tag{9}$$

Now, plugging (9) into (8), we get

$$\begin{aligned} \dot{x}_i &= A_0 x_i + B_i B_0^T (\xi_i - \zeta_i) x_i \\ &\quad + \varphi B_i K \sum_{j=0}^{N} \alpha_{ij} \left((x_i - d_i) - (x_j - d_j) \right). \end{aligned} \tag{10}$$

Define the formation tracking error $\varepsilon_i = x_i - d_i - x_0$ for each follower vehicle and let the vector $\varepsilon = [\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T]^T$. Since $A_0 d_i = 0$, $\dot{d}_i = [0\,0\,0]^T$ and noting that $\varepsilon_i - \varepsilon_j = (x_i - d_i) - (x_j - d_j)$ for any $i, j \in \mathcal{F}$, $\dot{\varepsilon}_i$ can be easily calculated by plugging (10)

$$\dot{\varepsilon}_i = A_0 \varepsilon_i + B_i B_0^T (\xi_i - \zeta_i) x_i + \varphi B_i K \sum_{j=0}^{N} \alpha_{ij} (\varepsilon_i - \varepsilon_j). \tag{11}$$

Then, (11) is expressed in the Kronecker product form as

$$\dot{\varepsilon} = (I \otimes A_0)\varepsilon + \varphi \mathrm{diag}\{B_i K\} \left((L+G) \otimes I \right) \varepsilon + \begin{bmatrix} B_1 B_0^T (\xi_1 - \zeta_1) x_1 \\ B_2 B_0^T (\xi_2 - \zeta_2) x_2 \\ \vdots \\ B_N B_0^T (\xi_N - \zeta_N) x_N \end{bmatrix}. \tag{12}$$

In this proof, the platoon control problem including the string stability has been cast as an asymptotic stability problem of the closed-loop formation tracking error dynamics $\dot{\varepsilon}$. To approach this problem via Lyapunov's method, we consider the following Lyapunov function candidate

$$V = \varepsilon^T \left((L+G) \otimes P \right) \varepsilon + \sum_{i=1}^{N} (\xi_i - \zeta_i)^2. \tag{13}$$

Note $V > 0$ since $L + G > 0$ and $P > 0$ for a given $\gamma > 0$. The time derivative of V is computed along the trajectories of (11) as

$$\begin{aligned} \dot{V} =&\ \varepsilon^T \left((L+G) \otimes (PA_0 + A_0^T P) \right) \varepsilon + 2 \sum_{i=1}^{N} (\xi_i - \zeta_i) \dot{\xi}_i \\ &+ 2\varepsilon^T \left((L+G) \otimes I \right) \varphi \mathrm{diag}\{PB_i K\} \left((L+G) \otimes I \right) \varepsilon \\ &+ 2\varepsilon^T \left((L+G) \otimes P \right) \begin{bmatrix} B_1 B_0^T (\xi_1 - \zeta_1) x_1 \\ B_2 B_0^T (\xi_2 - \zeta_2) x_2 \\ \vdots \\ B_N B_0^T (\xi_N - \zeta_N) x_N \end{bmatrix} \\ =&\ \varepsilon^T \left((L+G) \otimes (PA_0 + A_0^T P) \right) \varepsilon \\ &+ 2\varepsilon^T \left((L+G) \otimes I \right) \varphi \mathrm{diag}\{PB_i K\} \left((L+G) \otimes I \right) \varepsilon \\ &+ 2 \sum_{i=1}^{N} \sum_{j=0}^{N} \alpha_{ij} (\varepsilon_i - \varepsilon_j)^T P B_i B_0^T (\xi_i - \zeta_i) x_i \\ &+ 2\rho \sum_{i=1}^{N} (\xi_i - \zeta_i) x_i^T B_0 K \sum_{j=0}^{N} \alpha_{ij} \left((x_i - d_i) - (x_j - d_j) \right) \\ =&\ \varepsilon^T \left((L+G) \otimes (PA_0 + A_0^T P) \right) \varepsilon \\ &+ 2\varepsilon^T \left((L+G) \otimes I \right) \varphi \mathrm{diag}\{PB_i K\} \left((L+G) \otimes I \right) \varepsilon \\ &+ 2 \sum_{i=1}^{N} \sum_{j=0}^{N} \alpha_{ij} (\varepsilon_i - \varepsilon_j)^T P (B_i B_0^T - \rho B_0 B_0^T)(\xi_i - \zeta_i) x_i. \end{aligned}$$

Now exploiting the structures of B_0, B_i and the parameter $\rho = \frac{\tau_0}{\min_{i\in\mathcal{F}} \tau_i}$, we can readily obtain the following relationship

$$B_i B_0^T - \rho B_0 B_0^T \leq 0. \tag{14}$$

Upon applying (14) into the last expression of $\dot{V}$, it gives

$$\begin{aligned} \dot{V} \leq&\ \varepsilon^T \left[(L+G) \otimes (PA_0 + A_0^T P) \right. \\ &\left. - 2\varphi \left((L+G) \otimes I \right) \mathrm{diag}\{PB_i B_0^T P\} \left((L+G) \otimes I \right) \right] \varepsilon \end{aligned}$$

which in turn implies

$$\begin{aligned} \dot{V} \leq&\ \varepsilon^T \left[\left((L+G) \otimes (PA_0 + A_0^T P) \right. \right. \\ &\left. - 2\varphi\delta\, (L+G)^2 \otimes PB_0 B_0^T P \right] \varepsilon, \end{aligned}$$

Algorithm 1: Procedure to Design the Control Law for Heterogeneous Vehicle Platoons.

1: **for** each vehicle $i \in \{1, \ldots, N\}$ **do**
2:   **initialize** p_i, v_i, a_i and ξ_i;
3:   set a bidirectional communication graph;
4:   set the desired distance d_r;
5:   **if** all the vehicles are connected with the leader as the root node **then**
6:     choose positive constants ρ and δ;
7:     select the positive parameter φ;
8:     compute controller parameter K by solving ARE (7) for a given γ;
9:     construct the control protocol u_i given in (6);
10:   **else**
11:    back to Step 3;
12:   **end if**
13: **end for**

since

$$B_i B_0^T - \delta B_0 B_0^T \geq 0 \qquad \forall i \in \mathcal{F} \tag{15}$$

due to a similar argument used to show (14). Now since $L + G > 0$, there always exists an unitary diagonalizing matrix $D \in \mathbb{R}^{N \times N}$ such that $\Lambda = D^T (L + G) D$ where $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$. Replacing $L + G$ in (15) by $D^T \Lambda D$, we have

$$\begin{aligned}\dot{V} \leq\ & \varepsilon^T (D^T \otimes I) \Bigg[\Lambda \otimes (P A_0 + A_0^T P) \\ & - \frac{1}{\min_{i \in \mathcal{F}} \lambda_i} \Lambda^2 \otimes P B_0 B_0^T P\Bigg] (D \otimes I) \varepsilon \\ \leq\ & \varepsilon^T (D^T \otimes I) \big[\Lambda \otimes (P A_0 + A_0^T P - P B_0 B_0^T P)\big] (D \otimes I) \varepsilon.\end{aligned} \tag{16}$$

Then, (16) implies $\dot{V} \leq 0$ using the ARE given in (6) for a given $\gamma > 0$ and $\dot{V} = 0$ only when $\varepsilon = 0$. Hence invoking LaSalle's invariance principle [22], asymptotic stability of the closed-loop tracking error dynamics ($\dot{\varepsilon}$) given in (11) can be guaranteed. Therefore, $\lim_{t \to \infty} \varepsilon(t) = 0$ which implies

$$\lim_{t \to \infty} x_i(t) - d_i - x_0(t) = 0 \qquad \forall i \in \mathcal{F}. \tag{17}$$

This completes the proof. ■

Remark 1: Different from the results shown in [4], [15] where only homogeneous vehicular platoons are considered (i.e., $A_i = A_0$, $B_i = B_0$, $\forall i \in \mathcal{F}$), the proposed control protocol in this work can deal with the heterogeneous dynamics of vehicles, which is more practical in real applications. Even though control of heterogeneous vehicle platoons is investigated in [8], it is required to know all the model information of the leader and followers before designing the control law, while in this work only the leader‘s model and the bound of the followers' models are needed, which largely reduces the computation complexity and thus provides more possibility in real hardware implementation.

Following the analysis presented above, the procedure to construct the control law u_i is given in Algorithm 1.

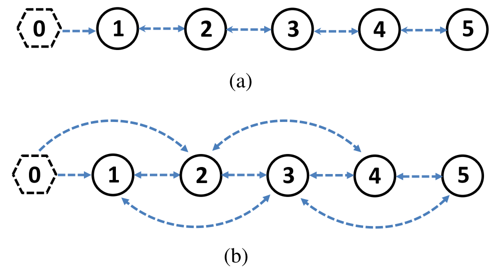


Fig. 4. Two communication topologies used in the simulation case study on the platoon control mission. (a) Topology 1: predecessor-following (PF) and (b) Topology 2: two-predecessor following (TPF).

TABLE I
VEHICLE PARAMETERS USED IN THE SIMULATION

Index	0	1	2	3	4	5
m_i (kg)	1753	1837	1942	1764	1029	1688
τ_i (s)	0.51	0.55	0.62	0.52	0.33	0.48

IV. Nonlinear Simulation Results

In this section, we provide Matlab/Simulink simulation results to show the usefulness and effectiveness of the proposed cooperative platoon control scheme. We consider a heterogeneous platoon of six connected vehicles, of which one is leader while the rest five are followers, interconnected via two different communication topologies as shown in Fig. 4(a) and 4(b). The parameters of each connected vehicle are arbitrarily selected and tabulated below in Table I. Note that in the simulation study, the actual nonlinear vehicle dynamics given in (1) has been considered.

The initial state of the leader is arbitrarily assigned to $p_0(0) = 200\,\mathrm{m}$, $v_0(0) = 8\,\mathrm{m/s}$, and $a_0(0) = 0\,\mathrm{m/s}^2$. Note that the acceleration or deceleration of the leader can be viewed as disturbances in a platoon. The initial position of the ith follower is given by $p_i(0) = (40 - 8i)$ m. In the simulation study, the desired inter-vehicular spacing is set to $d_r = 5$ m. Now based on the vehicle parameters given in Table I, we calculate the controller parameters as $\rho = 1.545$, $\delta = 0.823$ and $\varphi = 10$. Then, choosing $\gamma = 100$ and by solving the ARE (7), we get

$$P = \begin{bmatrix} 180.287 & 112.517 & 7.1 \\ 112.517 & 195.7535 & 12.8004 \\ 7.1 & 12.8004 & 7.2787 \end{bmatrix} > 0$$

which form the feedback gain matrix

$$K = -B_0^T P = -\begin{bmatrix} 10 & 18.0287 & 10.2517 \end{bmatrix}.$$

Two sets of simulation results are presented in Fig. 5(a) and 5(b) that correspond to Topologies 1 and 2, respectively, shown in Fig. 4(a) and 4(b). Each set of results contains position (p_i) and velocity (v_i) trajectories of the vehicles, the coupling weights (ξ_i) designed for them and the spacing error (ε_i) corresponding to each vehicle. For the first 10s of the simulation, the vehicles in the platoon move all together at the same cruising velocity $v_0(0) = 8$ m/s as that of the leader and hence, the inter-vehicular spacing between any two consecutive vehicles remains zero. Distance traversed by the vehicles during this phase increases at a constant rate and the coupling weights also remain constant at

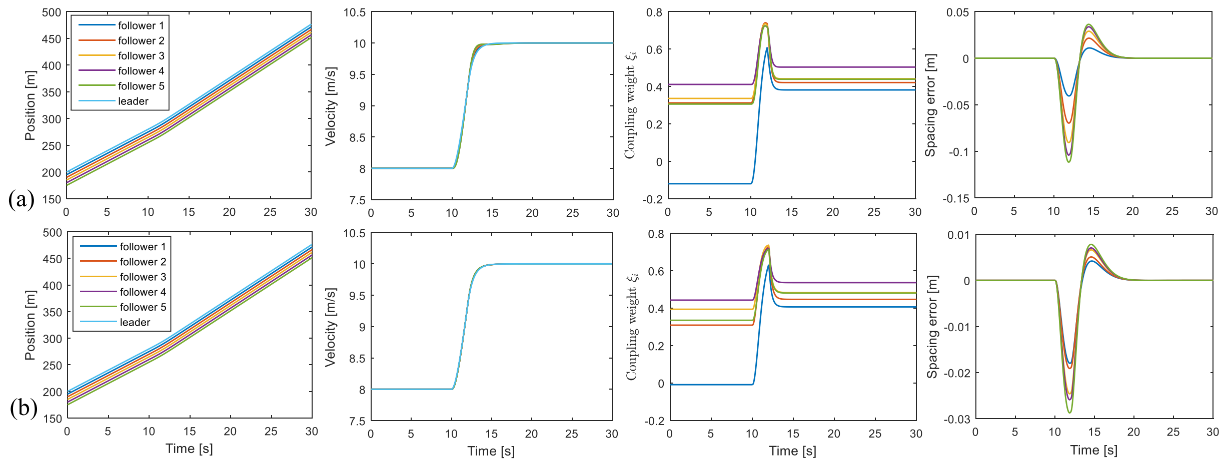


Fig. 5. Simulation results shows the performance achieved by the proposed two-layer cooperative control scheme when applied to a platoon of six connected and heterogeneous vehicles of which one acts as the leader while the rest five are followers. (a) Position $p_i(t)$, velocity $v_i(t)$, coupling weight $\xi_i(t)$ and spacing error $\varepsilon_i(t)$ for each of the vehicles in the platoon corresponding to Topology 1 and (b) $p_i(t)$, $v_i(t)$, $\xi_i(t)$, $\varepsilon_i(t)$ corresponding to Topology 2.

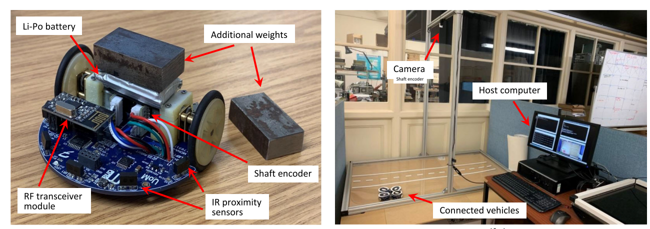
their initial values. Then, in order to disturb the constant velocity movement of the platoon, an exogenous input

$$w_0(t) = \begin{cases} 0, & 0\text{ s } \le t < 10\text{ s}; \\ 1, & 10\text{ s } \le t < 12\text{ s}; \\ 0, & t \ge 12\text{ s}; \end{cases} \tag{18}$$

is applied on the lead vehicle causing an acceleration in it followed by a steep rise in its velocity as revealed in the v_i vs. t graph of Fig. 5(a). This figure also illustrates that in response to the sudden increase in the velocity of the leader, the following vehicles also try to synchronize with the new stable velocity of the leader and after $t = 12$ s, all five followers attain the same velocity as that of the leader. This has been made possible by the influence of the proposed platoon control scheme. The graph ξ_i vs. t of Fig. 5(a) shows that the coupling weights adapt to new values to appropriately tune the controller gains to counteract the disturbance in the platoon. From the ε_i vs. t graph of Fig. 5(a), it is clear that during the transient period $10\text{ s} \le t < 20$s spacing error deviates from zero, exhibits bounded variation, then quickly decays to zero around $t = 20$s and after that, remains zero for all $t > 20$ s. This asserts that the proposed scheme has met the platoon control objectives and is able to withstand the perturbation (i.e. the sudden rise in the cruising velocity of the platoon) caused due to applying an acceleration input to the lead vehicle for a short span of time. Therefore the simulation results suggest that the proposed controller preserves string stability of the platoon. The same conclusion can also be drawn in the case of Fig. 5(b).

V. Experimental Validation

In order to establish the feasibility of the proposed two-layer distributed platoon control scheme, in this paper, laboratory-run hardware experiments involving mobile robots were performed. Below, we first briefly introduce the experimental setup and subsequently we will analyse the experimental results.



(a) (b)

Fig. 6. (a) Vehicle platform; (b) Experimental arena includes overhead camera tracking system, base station (i.e. the host PC), the aluminium framed glass platform and the mobile robots (Mona).

A. Experimental Setup

1) Vehicle Platform: In the experiment, we used autonomous mobile robots, Mona [18], that approximate the longitudinal dynamics of car-like vehicles. Mona is a low-cost, open-source, miniature robot which has been developed for swarm robotic applications. As shown in Fig. 6(a), Mona has two 6V low power DC gear-motors connected directly to its wheels having 32mm diameter and the robot has a circular chassis having 80mm diameter. The main processor of this robot is an AVR microcontroller equipped with an external clock of 16MHz frequency. Each robot is equipped with NRF24l01 wireless transceiver module for inter-robot communication. The robots are run and controlled by Arduino Mini/Pro microcontroller boards which are coded using the existing open-source software modules.

2) Experimental Platform: As shown in Fig. 6(b), the experimental platform includes a digital camera connected to a host PC which runs the tracking system, an arena (i.e., the glass frame), walls made of aluminium strut profile and the wooden floor. The position tracking system used in this experiment is an open-source multi-robotic localization system [23]. The tracking system can track the positions and orientations of the robots by identifying the unique circular tags attached on top of the robots. The position information is transmitted to the controller via the ROS communication framework. In this experiment, we considered two separate platoons in two separate lanes. Each of the two platoons consisted of three connected vehicles. The information

Fig. 7. Experimental results: course of motion of a two-lane platoon containing six connected vehicles, all represented by Mona robots, in the experimental arena. (a) Initial rest position for all vehicles at $t = 0$ s; (b) Lead vehicles has acquired the longitudinal motion from rest at $t > 0$; (c) Following vehicles gradually catch the velocity of the leaders and the desired inter-vehicular spacing is restored; and (d) At $t = 45$ s, the platoon (both the lanes) reaches the other end of the arena – mission accomplished.

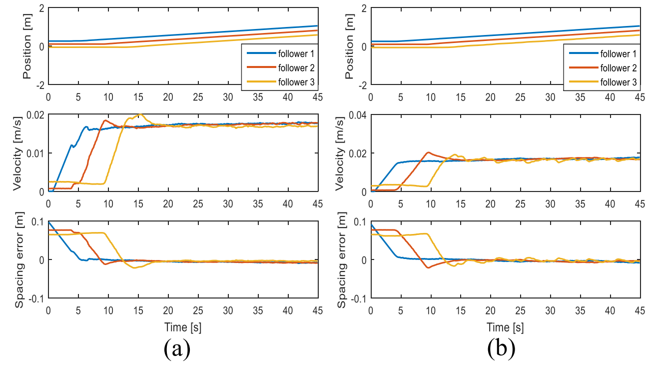


Fig. 8. Time evolvement of position $p(t)$, velocity $v(t)$ and spacing error $\varepsilon(t)$ of the leader and following vehicles during the experiment with real robots: (a) For the vehicles in Lane 1; (b) For the vehicles in Lane 2.

TABLE II
DESCRIPTION OF THE EXTERNAL LOAD ATTACHED TO THE ROBOTS AND THE GEAR RATIOS OF THE MOTORS

	Lane 1			Lane 2		
Index	1	2	3	1	2	3
Load (g)	0	150	300	0	150	300
Gear ratio	150:1	150:1	150:1	210:1	210:1	210:1

flow between each vehicle is shown in Fig. 7(a), which indicates some bidirectional and some unidirectional inter-distance exchanges. All vehicles were moving in one dimension from left to right direction on the glass board since only longitudinal motion of the vehicles was considered. Note that the mobile robots used in the experiment were not all identical. As mentioned in Table II, four among the six robots were burdened with unequal external weights attached in between the wheels and in addition to that, two different sets of micro-metal geared-motors having different gear ratios (i.e. 150:1 and 210:1) were used. Moreover, the robots were also functionally heterogeneous as the robots at the front of the platoon played the role of the leader vehicle while the rest were followers. Note that the over-head camera can be easily replaced in real-world implementation if the automated vehicles are equipped with on-board communication and positioning modules. For example, the road information (e.g., lane width, lane curvature) and the whereabouts of neighbouring vehicles (e.g., relative distance, relative speed) can be recorded by the Mobileye as used in [24].

B. Experimental Results

Fig. 7(a) shows the initial orientation of the platoon of six vehicles on the glass bed. The platoon is then brought into motion at $t > 0$ by applying an external input to the leader as seen in Fig. 7(b). Note here that the initial acceleration in any vehicle can be considered as a temporary bounded disturbance input since the platoon as a whole has been shown to be string-stable under the influence of the proposed control scheme. Similar theoretical viewpoint was established in [16]. The notion remains applicable when a vehicle decelerates or takes a turn. Due to communication delay, the follower vehicles cannot synchronize with the velocity of the leader instantaneously. As a result, at the beginning, spacing between the leader and the immediate follower becomes more than the desired spacing, but gradually the followers get synchronized with the leader and restore equal inter-vehicular spacing at the steady-state condition. Fig. 7(c) reflects this situation. Fig. 7(d) indicates that the platoon has reached its destination (i.e. the right end of the glass board) at $t = 45$ s satisfying all the platoon control objectives. Thus the mission is accomplished with the proposed platoon control scheme, which demonstrates the feasibility and effectiveness of the scheme in hardware experiment with real robots leading to its possible application in platoon control of real vehicles. Fig. 8(a) and 8(b) complement the experimental results shown in Fig. 7(a)-7(d) by providing the time variation of the positions $p_i(t)$ and velocities $v_i(t)$ of the vehicles and the associated spacing error $\varepsilon_i(t)$. We observe from Fig 8(a) that velocities of the followers in Lane 1 start rising in accordance with the leader from $t = 4$ s onwards, reach the leader around $t = 20$ s and remain synchronized until the end. The graph for the spacing error is also in agreement with this fact. It shows that the spacing error between any two vehicles decays to zero at the same time around $t = 20$ s and remains zero throughout the rest of experiment. The same conclusion can be drawn for the vehicles in Lane 2 as indicated via Fig. 8(b).

C. Discussions

In this paper, mobile robots (Mona) were used as an affordable, laboratory-run vehicle platform since real-world vehicles having dynamics governed by (1) can be locally feedback linearized into the longitudinal dynamics described by (3). The dynamics of the Mona robot including the geared-motors can also be approximated by (3) without loss of generality following [25]. Hence, it can be inferred that although the feasibility of the proposed distributed platoon control scheme was validated through small-scale mobile robots in a laboratory environment, it can also be applied to real-world vehicles. In that scenario, the local feedback linearizing controller will provide the appropriate simplification of the nonlinear vehicle dynamics while the cooperative control protocol (6) will be implemented on the linearized longitudinal dynamics to achieve the platoon control objectives.

In the future, more challenging and realistic situations (e.g. when the road is uneven or there is turning, when a vehicle at the middle of a platoon breaks down or starts malfunctioning due to hardware faults, when the vehicles move uphill or downhill roads, etc.) shall be taken into account by improving the proposed platoon control scheme using outer-loop optimization and artificial intelligence techniques. Furthermore, the proposed scheme could be extensively tested onto more sophisticated experimental platforms including full-sized real cars in a controlled environment.

VI. Conclusion

In this paper, we have introduced a two-layer decentralized and adaptive control scheme for heterogeneous connected vehicle platoon, which exploits first an input-output feedback linearization technique to linearize the nonlinear vehicle dynamics and then applies a distributed cooperative control law to serve the platoon control objectives. The proposed methodology ensures equal and constant inter-vehicular spacing between successive vehicles along with maintaining a desired cruising velocity of the platoon when the lead vehicle runs at constant velocity. Furthermore, the string stability of the platoon is preserved even when the lead vehicle is disturbed by exogenous disturbances (for example, due to acceleration/deceleration). It is investigated that the acceleration feedback, in addition to position and velocity feedback, improves the string stability and reliability of the proposed scheme. Simulation results shows satisfactory performance of the platoon control scheme in presence of leader's disturbance which is then further validated by real-time hardware experiments with mobile robots.

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