An Adaptive Switched Control Approach to Heterogeneous Platooning With Intervehicle Communication Losses

Youssef Abou Harfouch, Shuai Yuan, and Simone Baldi

***Abstract*—The advances in distributed intervehicle communication networks have stimulated a fruitful line of research in cooperative adaptive cruise control (CACC). In CACC, individual vehicles, grouped into platoons, must automatically adjust their own speed using on-board sensors and communication with the preceding vehicle so as to maintain a safe intervehicle distance. However, a crucial limitation of the state of the art of this control scheme is that the string stability of the platoon can be proven only when the vehicles in the platoon have identical driveline dynamics and perfect engine performance (homogeneous platoon), and possibly an ideal communication channel. This paper proposes a novel CACC strategy that overcomes the homogeneity assumption and that is able to adapt its action and achieve string stability even for uncertain heterogeneous platoons. Furthermore, in order to handle the inevitable communication losses, we formulate an extended average dwell-time framework and design an adaptive switched control strategy, which activates an augmented CACC or an augmented adaptive cruise control strategy depending on communication reliability. Stability is proven analytically and simulations are conducted to validate the theoretical analysis.**

***Index Terms*—Adaptive control, cooperative adaptive cruise control (CACC), heterogeneous platoon, networked control systems, switched control.**

I. Introduction

AUTOMATED driving is an active area of research striving to increase road safety, manage traffic congestion, and reduce vehicles' emissions by introducing automation into road traffic [1]. Platooning is an automated driving method in which vehicles are grouped into platoons, where the speed of each vehicle (except eventually the speed of the leading vehicle) is automatically adjusted so as to maintain a safe intervehicle distance [2]. The most celebrated technology to enable platooning is cooperative adaptive cruise control (CACC), an extension of adaptive cruise control (ACC) [3], where platooning is enabled by intervehicle communication in addition to on-board sensors.

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CACC systems have overcome ACC systems in view of their better string stability properties [4]: String stability implies those disturbances that are introduced into a traffic flow by braking and accelerating vehicles are not amplified in the upstream direction. In fact, while string stability in ACC strategies cannot be guaranteed for intervehicle time gaps smaller than 1 s [5], CACC was shown to guarantee string stability for time gaps significantly smaller than 1 s [6]. This directly leads to improved road throughput [7], reduced aerodynamic drag, and reduced fuel consumption [8] over ACC systems.

Despite this potential, state-of-the-art studies and demonstrations of CACC crucially rely on the assumption of vehicle-independent driveline dynamics (homogeneous platoon): Under this assumption, a one-vehicle look-ahead CACC was synthesized in [6], by using a performance-oriented approach to define string stability. An adaptive bidirectional platoon-control method was derived in [9], which utilized a coupled sliding mode controller to enhance the string stability characteristics of the bidirectional platoon topology. A longitudinal controller based on a constant spacing policy was developed in [10], showing that string stability can be achieved by broadcasting the leading vehicle's acceleration and velocity to all vehicles in the platoon. In [11], a linear controller was augmented by a model predictive control strategy to maintain the platoon's stability while integrating safety and physical constraints. In addition, for a platoon composed of identical agents with different controllers, Martinec *et al.* [12] assessed the performance and challenges, in terms of string stability, of unidirectional and asymmetric bidirectional control strategies.

Communication is an important ingredient of CACC systems: The work presented in [13] reviews the practical challenges of CACC and highlights the importance of robust wireless communication. From here, a series of studies aims at addressing the effect of nonideal communication on CACC performance: In order to account for network delays and packet losses caused by the wireless network, an H_∞ controller was synthesized in [14], guaranteeing string stability criteria and robustness for some small parametric uncertainty. Santini *et al.* [15] derived a controller that integrates intervehicle communication over different realistic network conditions, which models time delays, packet losses, and interference. Random packet dropouts were modeled as independent Bernoulli processes in [16], in order to derive a scheduling algorithm and design a controller for vehicular platoons with intervehicle network capacity limitation that guarantees string stability and zero steady-state spacing errors.

All of the aforementioned works rely on the crucial platoon's homogeneity assumption. However, in practice, having a homogeneous platoon is not feasible; there will always be some heterogeneity among the vehicles in the platoon (e.g., different driveline dynamics, parametric and networked-induced uncertainties). A study conducted in [17] assessed the causes for heterogeneity of vehicles in a platoon and their effects on string stability. A distributed adaptive sliding mode controller for a heterogeneous vehicle platoon was derived in [18] to guarantee string stability and adaptive compensation of disturbances based on constant spacing policy. While addressing heterogeneous platoons to some extent, the aforementioned work neglects the effect of wireless communication, as pointed out by Dey *et al.* [13].

The brief overview of the state of the art reveals the need to develop CACC with new functionalities that can handle platoons of heterogeneous vehicles, and guarantee string stability while adapting to changing conditions and unreliable communication. The main contribution of this paper is to address for the first time the problem of CACC for heterogeneous platoons with unreliable communication. The heterogeneity of the platoon is represented by different (and uncertain) time constants for the driveline dynamics and possibly different (and uncertain) engine performance coefficients. Using a model reference adaptive control (MRAC) augmentation method, we prove analytically the asymptotic convergence of the heterogeneous platoon to an appropriately defined string stable reference platoon. Furthermore, intervehicle communication losses, which are modeled via an extended average dwell-time framework, are handled by switching the control strategy of the vehicle at issue to a string stable ACC strategy with a different reference model. For this adaptive switching control scheme, stability with bounded state tracking error is proven under realistic switching conditions that match the packet error rate of the two most widely adopted vehicular wireless communication standards, namely, IEEE 802.11p/wireless access in vehicular environment (WAVE) and long-term evolution [19], [20].

This paper is organized as follows. In Section II, the system structure of a heterogeneous vehicle platoon with engine performance losses is presented. Section III presents an MRAC augmentation of a CACC strategy to stabilize the platoon. Moreover, Section IV presents an adaptive switched control strategy to stabilize the platoon in the heterogeneous scenario with engine performances losses while coping with intervehicle communication losses. Simulation results of the two controllers are presented in Section V along with some concluding remarks in Section VI.

Notation: The notation used in this paper is as follows: $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{N}^+$ represent the set of real numbers, natural numbers, and positive natural numbers, respectively. The notation $P = P^T > 0$ indicates a symmetric positive definite matrix P, where the superscript T represents the transpose of a matrix. The notation $\|\cdot\|$ represents the Euclidean norm. The identity matrix of dimension n is denoted by $I_{n\times n}$. The notation $\sup|\cdot|$ represents the least upper bound of a function.

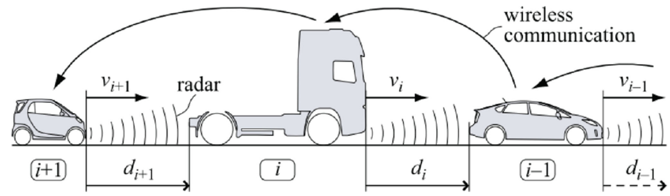

Fig. 1. CACC-equiped heterogeneous vehicle platoon [6].

II. System Structure

Consider a heterogeneous platoon with M vehicles. Fig. 1 shows the platoon where v_i represents the velocity (m/s) of vehicle i, and d_i is the distance (m) between vehicle i and its preceding vehicle $i-1$. This distance is measured using a radar mounted on the front bumper of each vehicle. Furthermore, each vehicle in the platoon can communicate with its preceding vehicle via wireless communication. The main goal of every vehicle in the platoon, except the leading vehicle, is to maintain a desired distance $d_{\mathrm{r},i}$ between itself and its preceding vehicle. Define the set $S_M = \{i \in \mathbb{N} |\ 1 \le i \le M\}$ with the index $i = 0$ reserved for the platoon's leader (leading vehicle). A constant time headway (CTH) spacing policy will be adopted to regulate the spacing between the vehicles [21]. The CTH is implemented by defining the desired distance as follows:

$$d_{\mathrm{r},i}(t) = r_i + h_i v_i(t), \quad i \in S_M \tag{1}$$

where r_i is the standstill distance (meters) and h_i the time headway (seconds) (or time gap). It is now possible to define the spacing error (meters) of the ith vehicle as follows:

$$\begin{aligned} e_i(t) &= d_i(t) - d_{\mathrm{r},i}(t) \\ &= (q_{i-1}(t) - q_i(t) - L_i) - (r_i + h_i v_i(t)) \end{aligned} \tag{2}$$

with q_i and L_i representing the rear-bumper position (m) and length (m) of vehicle i, respectively.

A desired behavior of the platoon is instantiated when the effect of disturbances (e.g., emergency braking) introduced along the platoon is attenuated as they propagate in the upstream direction [6]. Such behavior is denoted with the term string stability. A standard definition of string stability considered in this paper is given as follows:

Definition 1 (String Stability [6]): Let the acceleration of vehicle i be denoted with $a_i(t)$. Then, a platoon is considered string stable if

$$\sup_{\omega} |\Gamma_i(j\omega)| = \sup_{\omega} \left| \frac{a_i(j\omega)}{a_{i-1}(j\omega)} \right| \le 1, \quad 1 \le i \le M \tag{3}$$

where $a_i(s)$ is the Laplace transform of the acceleration $a_i(t)$ of vehicle i.

The control objective is to regulate e_i to zero $\forall i \in S_M$, while ensuring the string stability of the platoon. The following model is used to represent the dynamics of the vehicles:

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h_i \\ 0 & 0 & 1 \\ 0 & 0 & -\dfrac{1}{\tau_i} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} + \begin{pmatrix} 0 \\ 0 \\ \dfrac{\Lambda_i}{\tau_i} \end{pmatrix} u_i \tag{4}$$

where a_i and u_i are, respectively, the acceleration (m/s^2) and control input (m/s^2) of vehicle i. Moreover, τ_i represents each vehicle's unknown driveline time constant (s) and Λ_i represents the engine's performance: For the nominal performance, we have $\Lambda_i = 1$, while performance might decrease below 1 due to wear or wind gusts, or increase above 1 due to wind in the tail; Λ_i can also be affected by the slope of the road. Model (4) was proposed in [6] for the special case of $\Lambda_i = 1\ \forall i \in S_M$. The leading vehicle's model is defined as

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_0 \\ v_0 \\ a_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} u_0. \quad (5)$$

Note that under the assumption of a homogeneous platoon with perfect engine performance, we have $\tau_i = \tau_0$ and $\Lambda_i = 1\ \forall i \in S_M$. In this paper, we remove the homogeneous assumption by considering that $\forall i \in S_M$, τ_i can be represented as the sum of two terms as follows:

$$\tau_i = \tau_0 + \Delta\tau_i \quad (6)$$

where τ_0 is a known constant representing the driveline dynamics of the leading vehicle and $\Delta\tau_i$ is an unknown constant deviation of the driveline dynamics of vehicle i from τ_0. In fact, $\Delta\tau_i$ acts as an unknown parametric uncertainty. In addition, we remove the perfect engine performance assumption by considering Λ_i as an unknown input uncertainty. Substituting (6) into the third differential equation of (4), we obtain

$$\begin{aligned} \tau_i \dot{a}_i &= -a_i + \Lambda_i u_i \\ \dot{a}_i &= -\frac{1}{\tau_0} a_i + \frac{1}{\tau_0} \Lambda_i^* [u_i + \Omega_i^* \phi_i] \end{aligned} \quad (7)$$

where $\Lambda_i^* = \frac{\Lambda_i \tau_0}{\tau_i}$, $\Omega_i^* = -\frac{\Delta\tau_i}{\Lambda_i \tau_0}$, and $\phi_i = -a_i$.

Substituting (7) into (4), the vehicle model in a heterogeneous platoon with engine performance loss under spacing policy (1) can be defined as the uncertain linear-time invariant system as follows:

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h_i \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} \Lambda_i^* [u_i + \Omega_i^* \phi_i] \ \ \forall i \in S_M. \quad (8)$$

We can now formulate the control objective for the heterogeneous platoon as follows.

Problem 1 (Adaptive Heterogeneous Platooning): Design an adaptive control input $u_i(t)\ \forall i \in S_M$, such that the heterogeneous platoon described by (5) and (8) asymptotically tracks the behavior of a string stable platoon for any possible vehicles' parametric uncertainty under ideal communication between all consecutive vehicles.

III. Adaptive Heterogeneous Platooning

In order to design the control input, Section III-A presents string stable reference dynamics for the vehicles in the platoon, and Section III-B defines a stabilizing $u_i(t)$ through an MRAC augmentation approach.

A. CACC Reference Model

Under the baseline conditions of identical vehicles, perfect engine performance, and no communication losses between any consecutive vehicles, Ploeg *et al.* [6] derived, using a CACC strategy, a controller and a spacing policy, which proved to guarantee the string stability of the platoon. The time headway constant of the spacing policy (1) is set as $h_i = h^C$, $\forall i \in S_M$, where the superscript C indicates that communication is maintained between the vehicle and its preceding one. Moreover, the CACC baseline controller is defined as

$$h^C \dot{u}^C_{bl,i} = -u^C_{bl,i} + K^C_p e_i + K^C_d \dot{e}_i + u^C_{bl,i-1}, \ \ i \in S_M \quad (9)$$

where K^C_p and K^C_d are the design parameters of the controller. Without loss of generality here and in the following text, all initial conditions of controllers are set to zero, for example, $u^C_{bl,i}(0) = 0$, $\forall i \in S_M$. In addition, the leading vehicle control input is defined as

$$h_0 \dot{u}_0 = -u_0 + u_r \quad (10)$$

where u_r is the platoon's input representing the desired acceleration (m/s^2) of the leading vehicle, and h_0 a positive design parameter denoting the nominal time headway. The initial condition of (10) is $u_0(0) = 0$. The cooperative aspect of (9) resides in $u^C_{bl,i-1}$, which is received over the wireless communication link between vehicle i and $i-1$.

We can now define the reference dynamics for (8) as the dynamics of system (8) with $\Omega_i^* = 0$, $\Lambda_i^* = 1$, and control input $u_{i,m} = u^C_{bl,i}$. The reference model can therefore be described by

$$\begin{pmatrix} \dot{e}_{i,m} \\ \dot{v}_{i,m} \\ \dot{a}_{i,m} \\ \dot{u}_{i,m} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & -h^C & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K^C_p}{h^C} & -\frac{K^C_d}{h^C} & -K^C_d & -\frac{1}{h^C} \end{pmatrix}}_{A^C_m} \underbrace{\begin{pmatrix} e_{i,m} \\ v_{i,m} \\ a_{i,m} \\ u_{i,m} \end{pmatrix}}_{x_{i,m}} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K^C_d}{h^C} & \frac{1}{h^C} \end{pmatrix}}_{B^C_w} \underbrace{\begin{pmatrix} v_{i-1} \\ u^C_{bl,i-1} \end{pmatrix}}_{w_i} \quad \forall i \in S_M \quad (11)$$

where $x_{i,m}$ and w_i are the vehicle i's reference state vector and exogenous input vector, respectively. Consequently, (11) is of the following form:

$$\dot{x}_{i,m} = A^C_m x_{i,m} + B^C_w w_i \ \ \forall i \in S_M. \quad (12)$$

Furthermore, using (10), the leading vehicle's model becomes

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \\ \dot{u}_0 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ 0 & 0 & 0 & -\frac{1}{h_0} \end{pmatrix}}_{A_r} \underbrace{\begin{pmatrix} e_0 \\ v_0 \\ a_0 \\ u_0 \end{pmatrix}}_{x_0} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{h_0} \end{pmatrix}}_{B_r} u_r. \tag{13}$$

Reference model (12) has been proven in [6] to be asymptotically stable around the equilibrium point

$$x_{i,m,\text{eq}} = \left(0\ \overline{v}_0\ 0\ 0\right)^T, \text{ for } x_0 = x_{i,m,\text{eq}} \text{ and } u_r = 0 \tag{14}$$

where $\overline{v}_0$ is a constant velocity, provided that the following Routh–Hurwitz conditions are satisfied:

$$h^C > 0,\quad K_p^C, K_d^C > 0,\quad K_d^C > \tau_0 K_p^C. \tag{15}$$

To assess the string stability of the reference platoon dynamics, it is found that

$$\Gamma_i(s) = \frac{1}{h^C s + 1} \quad \forall i \in S_M. \tag{16}$$

Therefore, we conclude that (16) satisfies the string stability condition (3) of Definition 1 for any $h^C > 0$ and, thus, the defined reference platoon dynamics (12) is string stable.

B. MRAC Augmentation of a Baseline Controller

In this section, reference model (12) will be used to design the control input $u_i(t)$ such that the uncertain platoon's dynamics described by (5) and (8) converge to string stable dynamics. With this scope in mind, we will augment a baseline controller with an adaptive term, using a similar architecture as proposed in [22]. To include the adaptive augmentation, the input $u_i(t)$ is split, $\forall i \in S_M$, into two different inputs

$$u_i(t) = u_{bl,i}(t) + u_{ad,i}(t) \tag{17}$$

where $u_{bl,i}$ and $u_{ad,i}$ are the baseline controller and the adaptive augmentation controller (to be constructed), respectively.

First, define the control input of the leading vehicle $u_0(t)$ as in (10). Moreover, define $u_{bl,i}(t) = u_{bl,i}^C(t)$. Substituting (17) into (8), we get the uncertain vehicle model

$$\dot{x}_i = A_m^C x_i + B_w^C w_i + B_u \Lambda_i^* \left[u_{ad,i} + \Theta_i^{*T} \Phi_i\right] \quad \forall i \in S_M \tag{18}$$

where $x_i = (\,e_i\ v_i\ a_i\ u_{bl,i}\,)^T$, and the matrices A_m^C and B_w^C are known and defined in (12), and $B_u = (\,0\ 0\ \frac{1}{\tau_0}\ 0\,)^T$.

The uncertain ideal parameter vector is defined as $\Theta_i^* = (\,K_{u,i}^*\ \Omega_i^*\,)^T$ where $K_{u,i}^* = 1 - \Lambda_i^{*-1}$. The regressor vector is defined as $\Phi_i = (\,u_{bl,i}\ \phi_i\,)^T$. Therefore, the heterogeneous platoon with engine performance loss and control input (17) can be defined as system (13)–(18).

Furthermore, taking (12) as the vehicle reference model, the adaptive control input is defined as

$$u_{ad,i} = -\Theta_i^T \Phi_i \tag{19}$$

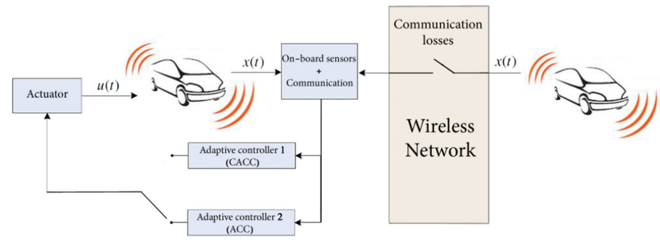

Fig. 2. Networked switched control system.

where Θ_i is the estimate of Θ_i^*. Define the state tracking error as

$$\tilde{x}_i = x_i - x_{i,m} \qquad \forall i \in S_M. \tag{20}$$

Replacing (19) into (18) and subtracting (12) results in the following state tracking error dynamics:

$$\dot{\tilde{x}}_i = A_m^C \tilde{x}_i - B_u \Lambda_i^* \tilde{\Theta}_i^T \Phi_i \tag{21}$$

where $\tilde{\Theta}_i = \Theta_i - \Theta_i^*$.

Since A_m^C is stable, there exists a unique symmetric positive define matrix $P_m = P_m^T > 0$ such that

$$(A_m^C)^T P_m + P_m A_m^C + Q_m = 0$$

where $Q_m = Q_m^T > 0$ is a designed matrix. Define the adaptive law

$$\dot{\Theta}_i = \Gamma_\Theta \Phi_i \tilde{x}_i^T P_m B_u \tag{22}$$

with $\Gamma_\Theta = \Gamma_\Theta^T > 0$ being the adaptive gain. Then, the following stability and convergence results can be stated.

Theorem 1: Consider the heterogeneous platoon model (8) with reference model (12). Then, the adaptive input (19) with adaptive law (22) makes the platoon's dynamics asymptotically converge to string stable dynamics. Consequently,

$$\lim_{t\to\infty} [x_i(t) - x_{i,m}(t)] = 0 \quad \forall i \in S_M$$

and

$$\lim_{t\to\infty} \|\Theta_i^T(t)\Phi_i(t)\| = 0 \quad \forall i \in S_M.$$

Proof: See Appendix A. ■

The results of Theorem 1 hold under the assumption of ideal continuous communication between the vehicles in the platoon. However, communication losses are always present in practice and coping with them is the subject of the next section.

IV. Adaptive Switched Heterogeneous Platooning

One way of handling the unavoidable communication losses is by switching between CACC and ACC depending on the network's state at each single communication link. This networked switched control system is outlined in Fig. 2. In this aim, an adaptive switched control method is presented for the scenario with joint heterogeneous dynamics and intervehicle communication losses. Note that ACC does not require intervehicle communication, but as a drawback it requires to increase the time gap in order to guarantee string stability [6]. So, the switched control system also takes into account that a different spacing policy might be active in the CACC case (indicated with h^C) and in the ACC case (indicated with h^L), where the superscript L stands for communication loss. The adaptive switched controller is based on a mode-dependent average dwell time (MDADT), which is used to characterize the network switching behavior as a consequence of communication losses.

Definition 2 (MDADT [23]): For a switched system with S subsystems, a switching signal $\sigma(\cdot)$, taking values in $\{1, 2, 3, \ldots, S\} = \mathcal{M}$, and for $s \geq t \geq 0$ and $k \in \mathcal{M}$, let $N_{\sigma k}(t, s)$ denote the number of times subsystem k is activated in the interval $[t, s)$, and let $T_k(t, s)$ be the total time subsystem k is active in the interval $[t, s)$. The switching signal $\sigma(\cdot)$ is said to have an MDADT τ_{ak} if there exist positive numbers N_{0k}, called mode-dependent chatter bounds, and τ_{ak} such that

$$N_{\sigma k}(t, s) \leq N_{0k} + \frac{T_k(t, s)}{\tau_{ak}} \quad \forall s \geq t \geq 0. \tag{23}$$

Furthermore, in the presence of switching losses, the following notion of stability must be introduced.

Definition 3 (Global Uniform Ultimate Boundedness [24]): A signal $\phi(t)$ is said to be globally uniformly ultimately bounded (GUUB) with ultimate bound if there exists a positive constant b, and for arbitrarily large $a \geq 0$, there is a time instant $T = T(a, b)$, where b and T are independent of t_0, such that

$$\|\phi(t_0)\| \leq a \Rightarrow \|\phi(t)\| \leq b \quad \forall\, t \geq t_0 + T. \tag{24}$$

By extension, we say that a system is GUUB when its trajectories are GUUB.

A. Mixed CACC-ACC Reference Model

In order to design the switched adaptive control input, we present in this section mixed CACC-ACC string stable dynamics, which serve as reference dynamics of the vehicles in the platoon. Let S_M^L be the subset of S_M containing the indices of the vehicles that lose communication with their preceding vehicle. In addition, let S_M^C be the subset of S_M containing the indices of the vehicles with maintained communication with their preceding vehicle. In the presence of intervehicle communication losses, reference dynamics (12) fail in general to guarantee the string stability of the platoon, since $u_{bl,i-1}^C$ is now no longer present for measurement $\forall i \in S_M^L$, and (3) might be violated. In this case, the time headway constant of the spacing policy (1) is set as $h_i = h^L, \forall i \in S_M^L$, with h^L to be determined in order to recover string stability. To do so, we define a new ACC baseline controller as follows:

$$h^L \dot{u}_{bl,i}^L = -u_{bl,i}^L + K_p^L e_i + K_d^L \dot{e}_i,\ \forall i \in S_M^L \tag{25}$$

where K_p^L and K_d^L are the design parameters of the controller, and $u_{bl,i}^C(0) = 0, \forall i \in S_M^L$, without loss of generality. Similar to the CACC case, the ACC reference model is defined as system (8) with $\Omega_i^* = 0$, $\Lambda_i^* = 1$, and control input $u_{i,m} = u_{bl,i}^L$. Thus, the reference model can be described by

$$\begin{pmatrix} \dot{e}_{i,m} \\ \dot{v}_{i,m} \\ \dot{a}_{i,m} \\ \dot{u}_{i,m} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & -h^L & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K_p^L}{h^L} & -\frac{K_d^L}{h^L} & -K_d^L & -\frac{1}{h^L} \end{pmatrix}}_{A_m^L} \begin{pmatrix} e_{i,m} \\ v_{i,m} \\ a_{i,m} \\ u_{i,m} \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d^L}{h^L} & 0 \end{pmatrix}}_{B_w^L} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{bl,i-1}^C \end{pmatrix}}_{w_i} \quad \forall i \in S_M^L \tag{26}$$

which is of the form

$$\dot{x}_{i,m} = A_m^L x_{i,m} + B_w^L w_i \quad \forall i \in S_M^L. \tag{27}$$

The asymptotic stability of the reference model (27) around equilibrium point (14) can be guaranteed by deriving conditions on K_p^L and K_d^L through the Routh–Hurwitz stability criteria. These conditions were found to be the same as (15). String stability of (27) can be additionally guaranteed by deriving sufficient conditions on the gains of controller (25) using condition (3) of Definition 1; when vehicle i is operating under ACC ($i \in S_M^L$), $\Gamma_i(s)$ is

$$\Gamma_i(s) = \frac{K_p^L + K_d^L s}{(\tau_0 s^3 + s^2 + K_d^L s + K_p^L)(h^L s + 1)} \quad \forall i \in S_M^L. \tag{28}$$

It gives

$$|\Gamma_i(j\omega)| = \frac{\sqrt{(K_d^L \omega)^2 + K_p^{L^2}}}{\sqrt{(h^L \omega)^2 + 1}\sqrt{(K_p^L - \omega^2)^2 + (K_d^L \omega - \tau_0 \omega^3)^2}}. \tag{29}$$

For a defined h^L, $\sup_w |\Gamma_i| \leq 1, \forall i \in S_M^L$, is verified by choosing K_p^L and K_d^L such that $\forall \omega > 0$,

$$\begin{aligned} &(h^L \tau_0)^2 \omega^6] + ((h^L)^2 - 2K_d^L \tau_0 (h^L)^2 + \tau_0^2)\omega^4 + (1 - 2K_p^L h^L \\ &+ (h^L K_d^L)^2 - 2K_d^L \tau_0)\omega^2 + ((h^L K_p^L)^2 - 2K_p^L) \geq 0. \end{aligned} \tag{30}$$

Thus, for a homogeneous platoon with no engine performance loss, when a communication link is lost, one can switch, for that link, from a string stable CACC strategy designed via (20) to a string stable ACC strategy designed via (28).

The resulting string stable mixed CACC-ACC reference dynamics can be described by

$$\dot{x}_0 = A_r x_0 + B_r u_r \tag{31}$$

$$\dot{x}_{i,m} = A_m^C x_{i,m} + B_w^C w_i \quad \forall i \in S_M^C \tag{32}$$

$$\dot{x}_{i,m} = A_m^L x_{i,m} + B_w^L w_i \quad \forall i \in S_M^L. \tag{33}$$

B. Formulation and Main Result for Platooning With Intervehicle Communication Losses

In this section, reference models (32) and (33) will be used to design the piecewise continuous control input $u_i(t)$ such that the uncertain platoon's dynamics described by (5) and (8) track with a bounded error string stable dynamics even in the presence of communication losses.

We define a new switched control input as

$$u_i(t) = u_{bl,i}(t) + u_{ad,i}(t) \qquad \forall i \in S_M \tag{34}$$

where

$$u_{bl,i}(t) = \begin{cases} u^C_{bl,i}, & \text{when communication is present} \\ u^L_{bl,i}, & \text{when communication is lost.} \end{cases} \tag{35}$$

In the presence of intervehicle communication losses, the following problem is defined.

Problem 2 (Adaptive Switched Heterogeneous Platooning): Design the adaptive laws for (34) and the switching parameters τ_{ak} and N_{0k} as in (23) such that for any MDADT switching signal satisfying (23) and in the presence of vehicles' parametric uncertainties, the heterogeneous platoon, described by (5) and (8), with communication losses tracks the behavior of a string stable platoon with GUUB error.

Remark 1: The reason for seeking GUUB stability (in place of asymptotic stability) is that asymptotic stability of switched systems with large uncertainties and average dwell time is a big open problem in control theory [25].

First, defining the control input of the leading vehicle $u_0(t)$ as in (10) results in a lead vehicle model as in (31). Then, substituting (17) into (8), the uncertain switched linear system vehicle model becomes, $\forall i \in S_M$ and $\sigma_i(t) \in \mathcal{M} := \{1, 2\}$, as

$$\dot{x}_i = A_{m,\sigma_i(t)} x_i + B_{w,\sigma_i(t)} w_i + B_u \Lambda^*_i \big[u_{ad,i} + \Theta^{*T}_i \Phi_i\big] \tag{36}$$

where $\sigma_i(\cdot)$ is the switching law of vehicle i (defined at the single link level), and $A_{m,\sigma_i(t)}$ and $B_{w,\sigma_i(t)}$ are time-variant matrices taking values, depending on the activated subsystem, as the known matrices $A_{m,k}$ and $B_{w,k}$, respectively, defined in (32) and (33), with $k \in \mathcal{M}$ representing the two subsystems in our system. In fact, subsystem $k = 1$ is activated by $\sigma_i(\cdot)$ when communication is maintained between vehicle i and its preceding one (when $i \in S^C_M$), and subsystem $k = 2$ is activated by $\sigma_i(\cdot)$ otherwise (when $i \in S^L_M$).

Therefore, the heterogeneous platoon with engine performance loss under the control input $u_i(t) = u_{bl,i}(t) + u_{ad,i}(t)$ can be described by (31) and (36).

Furthermore, define the group of reference models representing the desired behavior of each subsystem as

$$\dot{x}_{m,i}(t) = A_{m,\sigma_i(t)} x_{m,i}(t) + B_{w,\sigma_i(t)} w_i(t) \; \forall i \in S_M, \sigma_i(t) \in \mathcal{M} \tag{37}$$

where $x_{m,i} = \big(e_{m,i} \; v_{m,i} \; a_{m,i} \; u_{i,m}\big)^T$. Note that (37) is of the form (32) for $\sigma_i(t) = 1$ (when $i \in S^C_M$) and (32) for $\sigma_i(t) = 2$ (when $i \in S^L_M$).

The adaptive control input is defined as

$$u_{ad,i}(t) = -\Theta^T_{i,\sigma_i(t)} \Phi_i \tag{38}$$

where $\Theta_{i,k}$ is the estimate of Θ^*_i of subsystem k. Moreover, the state tracking error is defined as in (20). Replacing (38) into (36) and subtracting (37), we obtain, $\forall i \in S_M$ and $\sigma_i(t) \in \mathcal{M} = \{1, 2\}$, the following state tracking error dynamics:

$$\dot{\tilde{x}}_i = A_{m,\sigma_i(t)} \tilde{x}_i - B_u \Lambda^*_i \tilde{\Theta}^T_{i,\sigma_i(t)} \Phi_i \tag{39}$$

where $\tilde{\Theta}_{i,k} = \Theta_{i,k} - \Theta^*_i$. Moreover, define (t_{k_l}, t_{k_l+1}) as the switch-in and switch-out instant pair of subsystem k, with $k \in \mathcal{M}$ and $l \in \mathbb{N}^+$.

Since $A_{m,k}$ is stable, there exist symmetric positive definite matrices $P_k = P^T_k > 0$ for every subsystem $k \in \{1, 2\}$ such that

$$A^T_{m,k} P_k + P_k A_{m,k} + \gamma_k P_k \leq 0.$$

Define $\overline{\lambda}_k$ and $\underline{\lambda}_k$ as the maximum and the minimum eigenvalue of P_k, respectively, and $\beta = \min_{k \in \mathcal{M}}\{\underline{\lambda}_k\}$. Furthermore, assume known upper and lower bounds for Θ^* such that $\Theta^* \in [\underline{\Theta}, \overline{\Theta}]$, and assume $\Lambda^*_i \geq 0$ with a known upper bound such that $0 \leq \Lambda^*_i \leq \overline{\Lambda}$.

Moreover, define the adaptive law for any $S_k = S^T_k > 0$ as

$$\dot{\Theta}^T_{i,k}(t) = S^T_k B^T_u P_k \tilde{x}_i(t) \Phi^T_i + F^T_{i,k}(t) \qquad \forall k \in \{1, 2\} \tag{40}$$

where $F_{i,k}(t)$ is a parameter projection term, defined in [26], that acts componentwise and guarantees the boundedness of the estimated parameters in $[\underline{\Theta}, \overline{\Theta}]$. In particular, $F_{i,k}$ is zero whenever the corresponding component of $\Theta_{i,k}$ is within the prescribed uncertainty bounds; otherwise, $F_{i,k}$ is set to guarantee that the corresponding time derivative of $\Theta_{i,k}$ is zero.

Furthermore, we define the switching law $\sigma_i(t)$ based on an MDADT strategy as follows:

$$\tau_{ak} > \frac{1+\zeta}{\gamma_k} \ln(\mu_k) \tag{41}$$

where $\zeta > 0$ is a user-defined positive constant, and $\mu_k, k \in \mathcal{M}$ defined as $\mu_1 = \frac{\overline{\lambda}_2}{\underline{\lambda}_1}$ and $\mu_2 = \frac{\overline{\lambda}_1}{\underline{\lambda}_2}$. The following stability and convergence results can be guaranteed by (40) and (41).

Theorem 2: Consider the heterogeneous platoon model (8) with reference models (32) and (33) in the CACC and ACC modes, respectively. Then, the adaptive input (38) with adaptive laws (40) makes the error dynamics (39) GUUB, provided that the switching between CACC and ACC satisfies the MDADT (41). Furthermore, the following state tracking error upper bound is derived:

$$\|\tilde{x}_i(t)\|^2 \leq \frac{1}{\beta} \exp\Big\{\sum_{k=1}^{2} N_{0k} \ln \mu_k\Big\} M \qquad \forall i \in S_M \tag{42}$$

where $M = \max_{k \in \mathcal{M}}\{\|\tilde{x}_i(t_0)\|^2 + c_1 + c_2, \kappa \frac{(1+\zeta)}{\zeta}(c_1 + c_2)\}$, $c_k = \mathrm{tr}\big[(\overline{\Theta} - \underline{\Theta}) S^{-1}_k (\overline{\Theta} - \underline{\Theta})^T \overline{\Lambda}\big] > 0$, and $\kappa = \max_{k \in \mathcal{M}}\{\mu_k\}$. Finally, the ultimate bound b on the norm of the state

TABLE I
PLATOON PARAMETERS, $M=5$

i	0	1	2	3	4	5
$\tau_i(s)$	0.1	0.5	0.7	0.3	0.7	0.9
Λ_i	-	0.5	0.7	0.75	0.7	0.7

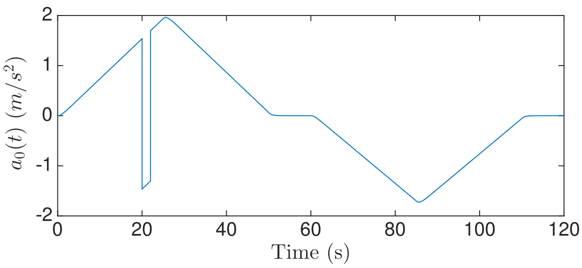

Fig. 3. Desired platoon acceleration $a_0(t)$.

tracking error is found to be

$$b \in \left[0, \sqrt{\exp\left(\sum_{k=1}^{2} N_{0k} \ln \mu_k\right) \frac{\kappa \overline{B}}{\beta}}\right] \tag{43}$$

with

$$\overline{B} = (c_1 + c_2)\frac{1+\zeta}{\zeta} > 0. \tag{44}$$

Proof: See Appendix B. ∎

Remark 2: The choice of ζ is based on the compromise between fast switching capabilities (41) (small ζ) and a small tracking error (42) upper bound (large ζ). Note that, as it is to be expected in any adaptive control setting [27], the error bounds are dependent on the size of the uncertainty set via c_1 and c_2.

Remark 3: Since reference models (37) were chosen to provide the desired string stable dynamics of the platoon under mixed network conditions as shown in Sections III-A and IV-A, then (40) and (41) guarantee that the heterogeneous platoon tracks, with a bounded tracking error, the behavior of a string stable platoon even in the presence of intervehicle communication losses.

Remark 4: The stability proof of Theorem 2 is based on two Lyapunov functions, one active when communication is present and one active when it is lost, cf., (47). Consequently, when communication is always maintained, only one Lyapunov function in (47) is active, from which we recover the asymptotic stability result as in Theorem 1.

V. ILLUSTRATIVE EXAMPLE

To validate the different control strategies discussed earlier, we simulate in MATLAB/Simulink [28] a heterogeneous platoon of 5 + 1 vehicles (including vehicle 0) with vehicles' engine performance loss. The platoon's characteristics are shown in Table I, and are motivated by nominal values found and validated in the literature as in [6] and [29].

In order to test the string stability of the heterogeneous platoon, the desired platoon acceleration $a_0(t)$, shown in Fig. 3, represents a stop-and-go scenario that undergoes a sudden disturbance at $t = 20$ s.

We define three experiments to showcase the performance and results of controllers (17) and (34).

1) Experiment 1 (Perfect communication, no adaptation): Simulate the platoon under the control action of the CACC baseline controller (9) without adaptation.
2) Experiment 2 (Perfect communication, adaptation): Simulate the platoon under the control action of the augmented adaptive CACC controller (17).
3) Experiment 3 (Communication losses, adaptation): Simulate the platoon under the control action of the augmented adaptive switched controller (34) using TrueTime2.0 [30] to model a realistic wireless communication network (IEEE 802.11p/WAVE) with update frequency of 10 Hz.

In terms of spacing policies, ACC will operate with the standardized minimum time gap of $h^L = 1$ s. On the other hand, CACC was shown to guarantee string stability for any $h^C > 0$ provided (15) are verified. This motivates us to choose h^C as small as possible in order to guarantee maximum road throughput and fuel efficiency. However, Ploeg *et al.* [6] showed a compromise between a low value of h^C and the maximum allowed delay in the network for string stability. We choose a time gap of $h^C = 0.7$ s, which provides robustness toward delays up to 0.3 s. The baseline controllers' gains are chosen (for all experiments) as $K_p^C = 0.2$ and $K_d^C = 0.7$ for $u_{bl,i}^C$, and $K_p^L = 2.5$ and $K_d^L = 2.3$ for $u_{bl,i}^L$ in order to respect both string stability conditions (15) and (30). For all experiments, the control input of the lead vehicle is defined by setting $h_0 = 0.7$ s. In experiment 2, we designed the adaptive term (19) by setting $\Gamma_\Theta = 80 I_{2\times 2}$ and $Q_m = 5 I_{4\times 4}$.

Furthermore, in order to design the adaptive input (38) for experiment 3, we need to quantify the loss of communication between vehicles which is represented as the switching signals $\sigma_i(t)$. In fact, for a velocity range of approximately $[0, 50]$ (m/s) and intervehicle distance range of approximately $[0, 40]$(m), the packet error rate between consecutive vehicles was measured in practice to be around 1% [20]. Therefore, since our operating conditions, characterized by the desired platoon acceleration and the headway constants, fall inside the previously defined intervals, and since the total experiment duration is 120 s, the expected average time of loss of communication can be calculated as 1% of 120 s for one intervehicle communication network. This results in an average total communication loss time of 1.2 s between consecutive vehicles during the total operating time of 120 s. Accounting for single packet loss and consecutive packet loss, we define the switching signals of the five vehicles, shown in Fig. 5 (top), by the following MDADT characteristics $N_{01} = 2$, $N_{02} = 2$, $\tau_{a1} = 8.5$, and $\tau_{a2} = 0.7$, and a total communication loss time for one intervehicle communication link of 1.2 s.

Therefore, to keep the platoon stable when switching back and forth between control strategies, we need to design the adaptive term (38) such that the switching conditions for stability (41) are satisfied $\forall k \in \mathcal{M}$. In fact, by setting $\gamma_1 = 0.60$, $\gamma_2 = 1.00$, and $S_1 = S_2 = 100$, the following MDADT conditions are necessary to guarantee the overall stability of the switched system:

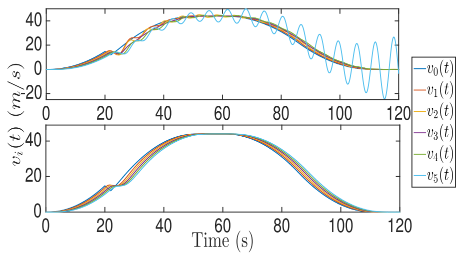

Fig. 4. Velocities of vehicles 0–5: $v_i(t)$, $i \in \{0, S_5\}$ in experiment 1 (top) and experiment 2 (bottom).

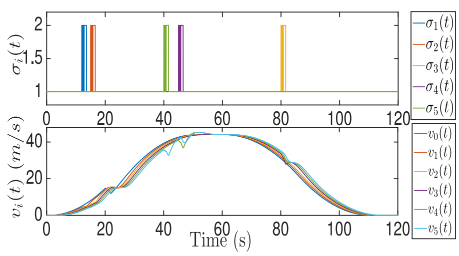

Fig. 5. Experiment 3: Switching signals $\sigma_i(t)$ of vehicles 1–5: $\sigma_i(t)$, $i \in S_5$ (top) and velocities of vehicles 0–5: $v_i(t)$, $i \in \{0, S_5\}$ (bottom).

$\tau_{a1} > 8.01$ and $\tau_{a2} > 0.66$. Therefore, since both conditions are satisfied by the switching signal's MDADT characteristics, then the switching controller in able to indeed guarantee the overall stability of the switched system.

From Fig. 4, it is clear that in Experiment 1, the CACC baseline controller (9), which guarantees the string stability of the platoon under the homogeneity and perfect engine assumptions, fails to maintain the platoon's stability when applied to the heterogeneous platoon. On the other hand, Fig. 4 also shows that, in Experiment 2, the augmented CACC controller (17), under the same platoon desired acceleration $a_0(t)$, succeeds in maintaining the string stability of the platoon even though the platoon is composed of unknown nonidentical vehicles that suffer from unknown engine performance loss.

Furthermore, Fig. 5 demonstrates the performance of the augmented adaptive switched controller (34) when communication loss is present in the platoon. We can see that controller (34) manages to maintain the string stability of the platoon while switching back and forth between control strategies to recover from the loss of communication throughout the platoon.

We can see from Fig. 5 that when a vehicle loses communication with its preceding one, it switches to a spacing policy characterized by a larger time gap. This is illustrated by the fact that the vehicle reduces its speed, for some time, in order to enlarge its intervehicle time gap and subsequently increases it speed again to match the platoon's speed. In turn, the following vehicles reduce their speeds in order to maintain their respective desired intervehicle spacing.

In terms of the norm of the state tracking error, Fig. 6 shows that when communication is always maintained, controller (17) regulates asymptotically the error to 0. Moreover, Experiment 3 shows that, under the action of controller (34), the platoon's dynamics track, with a bounded state tracking error, the

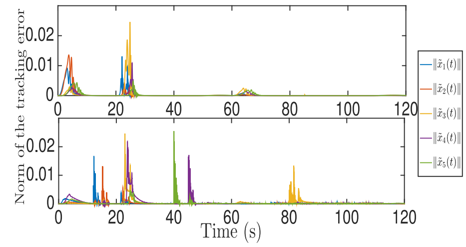

Fig. 6. Norm of the state tracking error of vehicles 1–5: $\|\tilde{x}_i(t)\|$, $i \in S_5$ in experiment 2 (top) and experiment 3 (bottom).

dynamics of a string stable platoon even when communication loss is present in the system.

VI. Conclusion

A novel adaptive switched control strategy to stabilize a platoon with nonidentical vehicle dynamics, engine performance losses, and communication losses has been considered. The proposed control scheme comprises a switched baseline controller (string stable under the homogeneous platoon with perfect engine performance assumption) augmented with a switched adaptive term (to compensate for heterogeneous dynamics and engine performance losses). The derivation of the string stable reference models and augmented switched controllers has been provided and their stability and string stability properties were analytically studied. When the switching respects a required MDADT, the closed-loop switched system is stable and signal boundedness is guaranteed. Numerical results have demonstrated the string stability of the heterogeneous platoon with engine performance losses under the designed control strategy.

Appendix A
Proof of Theorem 1

Define a radially unbounded quadratic Lyapunov candidate function as

$$V_i(t) = \tilde{x}_i^T P_m \tilde{x}_i + \mathrm{tr}(\tilde{\Theta}_i \Gamma_\Theta^{-1} \tilde{\Theta}_i \Lambda_i^*).$$

Taking the time derivative of $V_i(t)$ and substituting the error dynamics into (21) results in

$$\dot{V}_i(t) = -\tilde{x}_i^T Q_m \tilde{x}_i - 2\tilde{x}_i^T P_m B_u \Lambda_i^* \tilde{\Theta}_i \Phi_i + 2\mathrm{tr}(\tilde{\Theta}_i \Gamma_\Theta^{-1} \dot{\Theta}_i \Lambda_i^*).$$

When calculating the time derivative, we have used the fact that the extra input from system $i-1$ in (11) to reference model i is canceled by the last term in (9). In such a way, we can proceed showing that this interconnection does not destroy stability. Using the identity $a^T b = \mathrm{tr}(ba^T)$ results in

$$\dot{V}_i(t) = -\tilde{x}_i^T Q_m \tilde{x}_i + 2\mathrm{tr}(\tilde{\Theta}_i^T \{\Gamma_\Theta^{-1} \dot{\Theta}_i - \Phi_i \tilde{x}_i^T P_m B_u\} \Lambda_i^*). \tag{45}$$

Choosing the adaptive law as in (22) reduces (45) to

$$\dot{V}_i(t) = -\tilde{x}_i^T Q_m \tilde{x}_i \leq 0 \tag{46}$$

which proves the uniform ultimate boundedness of $(\tilde{x}_i, \tilde{\Theta}_i)$. Furthermore, it can be concluded from (46) that $\tilde{x}_i \in L_2$. In addition, since $w_i(t)$ is bounded, then $x_{i,m} \in L_\infty$, and consequently, $x_i \in L_\infty$ and $u_{bl,i} \in L_\infty$. Moreover, since Θ_i^* is constant, then the estimated value is also bounded, $\tilde{\Theta}_i \in L_\infty$. Since $(x_i, u_{bl,i}) \in L_\infty$ and the components of the regressor vector Φ_i are locally Lipschitz continuous, then the regressor's components are bounded. Therefore, $u_i \in L_\infty$ and $\dot{x}_i \in L_\infty$. Hence, $\dot{\tilde{x}}_i \in L_\infty$, which implies that $\ddot{V}_i \in L_\infty$. Thus, $\dot{V}_i$ is a uniformly continuous function of time. In addition, since V_i has a lower bound, $\dot{V}_i \leq 0$, and $\dot{V}_i$ is uniformly continuous, then by Barbalat's Lemma, V_i tends to a limit, while its derivative tends to zero. Hence, the tracking error $\tilde{x}_i$ tends asymptotically to zero as $t \to \infty$. Furthermore, since V_i is radially unbounded, then $\tilde{x}_i$ globally asymptotically tends to zero as $t \to \infty$. This means that the tracking error dynamics are globally asymptotically stable. From (21), it can be deduced that $\ddot{\tilde{x}}_i \in L_\infty$, which indicates that $\dot{\tilde{x}}_i$ is uniformly continuous. Moreover, since $\tilde{x}_i \to 0$ as $t \to \infty$, then using Barbalat's lemma, $\lim_{t\to\infty} \|\dot{\tilde{x}}_i\| = 0$, which leads to

$$\lim_{t\to\infty} \|\tilde{\Theta}_i^T \Phi_i\| = 0 \qquad \forall i \in S_M.$$

This proves that for any bounded w_i, the closed-loop system globally asymptotically tracks the reference model as $t \to \infty$.

This completes the proof. ■

Appendix B
Proof of Theorem 2

The stability proof is based on two Lyapunov functions, one active when communication is present and one active when it is lost. An appropriate MDADT will be constructed in such a way that switching among the Lyapunov functions guarantees GUUB. Define the following Lyapunov function:

$$V_i(t) = \tilde{x}_i^T(t) P_{\sigma_i(t)} \tilde{x}_i(t) + \sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t) S_k^{-1} \tilde{\Theta}_{i,k}^T(t) \Lambda_i^*) \quad \forall i \in S_M. \tag{47}$$

Using the switched adaptive law (40), the derivative of $V_i(t)$ with respect to time between two consecutive discontinuities [i.e., $t \in [t_l, t_{l+1})$] is

$$\begin{aligned}\dot{V}_i(t) =& \tilde{x}_i^T(t)(A_{m\sigma_i(t_{l+1}^-)}^T P_{\sigma_i(t_{l+1}^-)} + P_{\sigma_i(t_{l+1}^-)} A_{m\sigma_i(t_{l+1}^-)})\tilde{x}_i(t)\\ &+ 2\mathrm{tr}[\tilde{\Theta}_{i,\sigma_i(t_{l+1}^-)} S_{\sigma_i(t_{l+1}^-)}^{-1} F_{i,\sigma_i(t_{l+1}^-)}^T \Lambda_i^*]\\ \leq& -\gamma_{\sigma_i(t_{l+1}^-)} \tilde{x}_i^T(t) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t)\\ &+ 2\mathrm{tr}[\tilde{\Theta}_{i,\sigma_i(t_{l+1}^-)} S_{\sigma_i(t_{l+1}^-)}^{-1} F_{i,\sigma_i(t_{l+1}^-)}^T \Lambda_i^*].\end{aligned}$$

In fact, the following two inequalities hold [26]:

$$\tilde{\Theta}_{i,\sigma_i(t_{l+1}^-)} S_{\sigma_i(t_{l+1}^-)}^{-1} F_{i,\sigma_i(t_{l+1}^-)}^T \Lambda_i^* \leq 0 \tag{48}$$

$$\sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t) S_k^{-1} \tilde{\Theta}_{i,k}^T(t) \Lambda_i^*) \leq c_1 + c_2$$

where $c_k = \mathrm{tr}\big[(\overline{\Theta} - \underline{\Theta}) S_k^{-1} (\overline{\Theta} - \underline{\Theta})^T \overline{\Lambda}\big]$ is a finite positive constant. This results in, for any $\zeta > 0$

$$\begin{aligned}\dot{V}_i(t) \leq& -\gamma_{\sigma_i(t_{l+1}^-)} \tilde{x}_i^T(t) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t)\\ &+ \gamma_{\sigma_i(t_{l+1}^-)}(c_1 + c_2) - \gamma_{\sigma_i(t_{l+1}^-)}(c_1 + c_2)\\ \leq& -\frac{\gamma_{\sigma_i(t_{l+1}^-)}}{1+\zeta} V_i(t) + \frac{\gamma_{\sigma_i(t_{l+1}^-)}}{1+\zeta}\left[(1+\zeta)(c_1+c_2) - \zeta V_i(t)\right].\end{aligned} \tag{49}$$

Let us define a finite positive constant $\overline{B}$ as in (44). Then, using (44) and (49), we can conclude that, between two consecutive discontinuities, $V_i(t)$ is

1) decreasing at an exponential rate when $V_i(t) > \overline{B}$ since $\dot{V}_i(t) \leq -\frac{\gamma_{\sigma_i(t_{l+1}^-)}}{1+\zeta} V_i(t)$;
2) non increasing when $V_i(t) \leq \overline{B}$ since $\dot{V}_i(t) \leq 0$.

The next step is to assess the behavior of $V_i(t)$ at the discontinuous instants. We consider that subsystem $\sigma_i(t_{l+1}^-)$ is active when $t \in [t_l, t_{l+1})$ and subsystem $\sigma_i(t_{l+1})$ is active when $t \in [t_{l+1}, t_{l+2})$. Therefore, before switching, we have

$$V_i(t_{l+1}^-) = \tilde{x}_i^T(t_{l+1}^-) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t_{l+1}^-) + \sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t_{l+1}^-) S_k^{-1} \tilde{\Theta}_{i,k}^T(t_{l+1}^-) \Lambda_i^*) \tag{50}$$

and after switching, we have

$$V_i(t_{l+1}) = \tilde{x}_i^T(t_{l+1}) P_{\sigma_i(t_{l+1})} \tilde{x}_i(t_{l+1}) + \sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t_{l+1}) S_k^{-1} \tilde{\Theta}_{i,k}^T(t_{l+1}) \Lambda_i^*). \tag{51}$$

Since the tracking error $\tilde{x}_i(\cdot)$ and the parameter estimation error $\tilde{\Theta}_{i,k}(\cdot)$ are continuous, we have $\tilde{x}_i(t_{l+1}^-) = \tilde{x}_i(t_{l+1})$ and $\tilde{\Theta}_{i,k}(t_{l+1}^-) = \tilde{\Theta}_{i,k}(t_{l+1})$. Furthermore, we have the following properties:

1) $\tilde{x}_i^T(t) P_{\sigma_i(t_{l+1})} \tilde{x}_i(t) \leq \overline{\lambda}_{\sigma_i(t_{l+1})} \tilde{x}_i^T(t) \tilde{x}_i(t)$;
2) $\tilde{x}_i^T(t) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t) \geq \underline{\lambda}_{\sigma_i(t_{l+1}^-)} \tilde{x}_i^T(t) \tilde{x}_i(t)$;

where the first property is valid since we only have two subsystems and we know in advance to which subsystem we are switching to. Consequently, we get

$$\begin{aligned}V_i(t_{l+1}) - V_i(t_{l+1}^-) &= \tilde{x}_i^T(t)(P_{\sigma_i(t_{l+1})} - P_{\sigma_i(t_{l+1}^-)})\tilde{x}_i(t)\\ V_i(t_{l+1}) - V_i(t_{l+1}^-) &\leq \Big(\frac{\overline{\lambda}_{\sigma_i(t_{l+1})}}{\underline{\lambda}_{\sigma_i(t_{l+1}^-)}} - 1\Big) V_i(t_{l+1}^-)\\ V_i(t_{l+1}) &\leq \mu_{\sigma_i(t_{l+1})} V_i(t_{l+1}^-)\end{aligned} \tag{52}$$

where $\mu_{\sigma_i(t_{l+1})} = \overline{\lambda}_{\sigma_i(t_{l+1})} / \underline{\lambda}_{\sigma_i(t_{l+1}^-)}$. The next step is to analyze the overall behavior of $V_i(t)$. Considering the initial condition, we have two cases: $a)$ $V_i(t_0) > \overline{B}$ and $b)$ $V_i(t_0) \leq \overline{B}$.

Case a) $V_i(t_0) > \overline{B}$: Since $V_i(t)$ is decreasing at an exponential rate between two consecutive discontinuities, there exists a finite time instant $t_0 + T_1$ such that $V_i(t_0 + T_1) \leq \overline{B}$. Denote the number of intervals that subsystem k, $k \in \mathcal{M}$, is active by N_{1k}. Therefore, it follows from (49) and (52) that for

$t \in [t_0,\ t_0 + T_1)$,

$$\begin{aligned} V_i(t) &\leq \prod_{k=1}^{2} \mu_k^{N_{1k}} \exp\left\{-\sum_{k=1}^{2}\sum_{j=1}^{N_{1k}}(t_{k_j+1} - t_{k_j})\frac{\gamma_k}{1+\zeta}\right\} V_i(t_0) \\ &= \exp\left(\sum_{k=1}^{2} N_{1k}\ln\mu_k\right)\exp\left(-\sum_{k=1}^{2} T_k\frac{\gamma_k}{1+\zeta}\right)V_i(t_0) \\ &\leq \exp\left\{\sum_{k=1}^{2}\left[\left(N_{0k} + \frac{T_k}{\tau_{\mathrm{a}k}}\right)\ln\mu_k - T_k\frac{\gamma_k}{1+\zeta}\right]\right\}V_i(t_0) \\ &\leq \exp\left(\sum_{k=1}^{2} N_{0k}\ln\mu_k\right)\exp\left\{\sum_{k=1}^{2}\left(\frac{\ln\mu_k}{\tau_{\mathrm{a}k}} - \frac{\gamma_k}{1+\zeta}\right)T_k\right\}V_i(t_0) \end{aligned} \tag{53}$$

where T_k is the total time when subsystem k is active for $t \in t_0,\ t_0 + T_1)$. By substituting MDADT in (41)–(53), $V_i(t)$ can be attracted into the interval $[0, \overline{B}]$ with sufficiently big $T_1 > 0$. To study the value of $V_i(t_0 + T_1)$, we consider the special case: When $t = t_0 + T_1$, a switching is activated. Then, the interval $[0, \overline{B}]$ becomes $[0, \kappa\overline{B}]$, where the coefficient $\kappa := \max_{k\in\mathcal{M}} \mu_k$ is introduced by (52). Next, it is possible that $V_i(t)$ will diverge far away from the interval $[0, \kappa\overline{B}]$ due to fast switches when $t > t_0 + T_1$. By recursively performing the analysis above, we notice that it is possible that fast switches happen intermittently over the whole time horizon, which can only guarantee that the Lyapunov function enters and then exceeds the bound $\kappa\overline{B}$ intermittently over the whole time horizon. The worse scenario is that fast switches characterized by N_{0k} are initialized when the Lyapunov function exceeds the bound $\kappa\overline{B}$. This implies that only the following ultimate bound of the Lyapunov function can be guaranteed:

$$b_{\mathrm{V}} = \exp\left(\sum_{k=1}^{2} N_{0k}\ln\mu_k\right)\kappa\overline{B}. \tag{54}$$

Case b) $V_i(t_0) \leq \overline{B}$: The Lyapunov function is nondecreasing at the beginning, and it might exceed the bound $\overline{B}$. Therefore, with a similar analysis as in *case a)*, the same ultimate bound b_{V} of the Lyapunov function can be guaranteed as in (54). Hence, it can be concluded that the switched system (36) is GUUB according to (54). Furthermore, using (53), we can easily obtain an upper bound on $V_i(t)$ with a switching law based on MDADT (41) as follows $\forall t \geq t_0$:

$$V_i(t) \leq \exp\left(\sum_{k=1}^{2} N_{0k}\ln\mu_k\right)\max\left\{V_i(t_0),\ \kappa\overline{B}\right\}.$$

Since $V_i(t) \geq \beta\|\tilde{x}_i(t)\|^2$, it follows $\forall t \geq t_0$, the state tracking error upper bound is obtained as in (42). Moreover, using (54), an ultimate bound of the tracking error is obtained as in (43)

This completes the proof. ■

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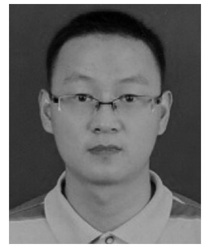

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