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Cooperative Multiple Vehicle Trajectory Planning using MIQP

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***Abstract*—This paper considers the problem of cooperative trajectory planning for multiple, communicating, automated vehicles in general non-hazardous on-road scenarios. Cooperative behavior is introduced by optimizing a collective cost function for all automated vehicles. A novel approach based a Mixed-Integer Quadratic Programming is presented that guarantees to yield the globally optimal solution. Numerical experiments are provided to demonstrate the feasibility of our approach and the benefits compared to priority-based approaches and *non-cooperative* individual motion planning.**

I. Introduction

Due to its potential to increase traffic safety and energy efficiency research in automated driving has experienced tremendous progress over the last decades [1]. An important task for automated vehicles is the generation of collision-free trajectories which satisfy constraints arising from vehicle dynamics, road geometry and traffic rules while optimizing certain criteria such as comfort, energy efficiency and safety. Numerous approaches can be found in literature dedicated to single vehicle motion planning. An introduction and overview can be found here [2]–[4].

A promising category of methods is based on Mixed Integer Programming [5]–[7]. This formulation is well suited for the context of automated driving since logical constraints such as arising from traffic rules or the combinatorial nature of different maneuver choices [8] can be seamlessly integrated. Unlike gradient descent based optimization approaches, which are likely to get stuck in some local optimum, sophisticated algorithms are known that guarantee globally optimal solutions [6].

Considering multiple automated vehicles, the advances in V2X communication technology [9] enables them to communicate with one another, exchanging sensor data, or even coordinate cooperative maneuvers. In literature, this is referred to as cooperative driving or cooperative motion planning.

Current literature shows that a variety of promising approaches exists to exploit the potential of cooperative motion planning. Many of these, however, are dedicated to specific traffic scenarios e.g. solving the coordination problem at intersections [10]–[14], cooperative control for lane change and merge maneuvers [15] or maximizing throughput by quickly reaching a platooning state[16].

The focus of most work has been developing cooperative strategies for emergency situations [13], [17]–[19]. Rather than having a complex objective function to optimize the main goal here is to avoid or at least mitigate collisions. This is a particularly promising topic since connected automated vehicles are able to react faster and coordinate cooperative maneuvers on a level not possible for humans[18].

Less research has been done considering cooperative motion planning for general, non-hazardous on-road scenarios. Here cooperative trajectories can be generated with respect to various objectives e.g. passenger comfort, energy efficiency and traffic flow. A method dedicated to general on-road scenarios is presented in [20]. They exploit priority-based motion planning to orchestrate multiple vehicles, trying to minimize a collective cost function. A priority list is calculated to decompose the multi-vehicle into a series of consecutive single vehicle trajectory planning problem.

While priority-based motion planning decreases computational complexity by decreasing the configuration space, it also reduces the solution space. Even with an optimal priority assigned to the cooperative vehicles, optimality can not be guaranteed and in some cases, the simplified problem becomes infeasible even though a solution in the full configuration space exists[18].

The approach presented in this paper tackles a similar scenario as in [20]: general on-road traffic optimizing a collective cost function considering comfort, energy efficiency and travel time.

The major contribution of this paper is a novel Mixed-Integer-Quadratic-Programming (MIQP) based approach generating optimal trajectories for an ensemble of communicating automated vehicles. In contrast to existing approaches the method exploits the entire configuration space which enables the generation of cooperative trajectories that are not possible with, e.g., priority based approaches guaranteeing globally optimal solutions. Furthermore, the approach is applied to an overtaking scenario and compared to *non-cooperative* individual motion planning as well as to priority based approaches.

The remaining of this paper is structured as follows: In section II the problem of cooperative trajectory planning is stated. Section III presents the cooperative MIQP formulation to generate optimal trajectories for the ensemble of communicating automated vehicles. Simulation results comparing *non-cooperative* individual motion planning, priority-based motion planning and the proposed cooperative MIQP approach are presented in section IV. Section V concludes the paper and gives an outlook of future research.

II. Problem statement

The general problem of motion planning for vehicles can be described as finding a collision-free trajectory from an initial state to a goal state. A classical approach to achieve this is by choosing control signals that minimize a cost function like (1) satisfying certain constraints e.g, arising from vehicle dynamics or obstacle avoidance for a planning horizon T.

$$\min_{\boldsymbol{u}} J = \min_{\boldsymbol{u}} \int_{t=0}^{T} j(\boldsymbol{x}(t), \boldsymbol{u}(t))dt \tag{1}$$

Function j represents the costs dependent on the state $\boldsymbol{x}$ and control input $\boldsymbol{u}$. When considering multiple vehicles this cost function has to be extended. To achieve cooperative behavior among vehicles, a common goal is needed [21], [22]. This can be achieved by introducing a collective cost function.

Consider a set of cooperative automated vehicles $\mathcal{V} = \{V^1, ..., V^N\}$ where $\boldsymbol{x}^n$ denotes the state and $\boldsymbol{u}^n$ the input vector of vehicle $V^n \in \mathcal{V}$ whose dynamics is described by:

$$\dot{\boldsymbol{x}}^n(t) = f(\boldsymbol{x}^n(t), \boldsymbol{u}^n(t)) \tag{2}$$

The goal of cooperative motion planning then can be stated as minimizing a collective cost function as followed

$$\min_{\boldsymbol{u}^n, \forall n \in N} J = \min_{\boldsymbol{u}^n, \forall n \in N} \int_{t=0}^{T} j(\{J^1, ..., J^N\}) + j_{inter}(\{\boldsymbol{x}^1, ..., \boldsymbol{x}^N\})dt \tag{3}$$

Where $j(\{J^1, ..., J^N\})$ is a function that combines the vehicle's individual costs, e.g. a weight sum, and $j_{inter}(\{\boldsymbol{x}^1, ..., \boldsymbol{x}^N\})$ a function that models costs introduced by interactions among cooperative vehicles. The generated cooperative motion plan has to satisfy several constraints such as arising from vehicles' dynamics (2), dynamic limits, road geometry, collision avoidance among each other and avoiding static and moving obstacles.

III. Cooperative Trajectory planning using MIQP

In this section, we present a novel MIQP formulation to generate globally optimal trajectories for an ensemble of co-operative communicating automated vehicles by optimizing a collective cost function like (3). The result is a sequence of control signals $\boldsymbol{u}^n(t)$ for each automated vehicle which can be tracked by a low-level controller.

The model used is similar to [5], [6], [20] and is based on receding-horizon Mixed-Integer Program. The road segment within a planning horizon T is assumed to be sufficiently straight so that the curvature can be neglected. This allows modeling longitudinal and lateral coordinates in a Cartesian representation. Furthermore, to solve this problem numerically, the planning horizon $t \in [0, ..T]$ is discretized into $K = T/\tau$ steps where τ denotes the duration of the discretization time step. The shape of a vehicle is approximated by a rectangle with length l and width w orientated along the x-axis. Obstacles and road boundaries are appropriately enlarged with the dimensions of the vehicle so that the vehicle can be treated as a single point.

A. Vehicle dynamics

Since the focus of this work is on general on-road driving rather than emergency situations where the vehicle is operated near physical limits, a simplified motion model is used. The longitudinal and lateral dynamics of each cooperative vehicle V^n is modeled as two triple integrators with state $\mathbf{x}^n \in \mathbb{R}^6$ and input $\mathbf{u}^n \in \mathbb{R}^2$ at the beginning of the time interval $t \in [k * \tau, (k+1) * \tau)$ given by

$$\mathbf{x}^n(k) = [p_x^n(k), v_x^n(k), a_x^n(k), p_y^n(k), v_y^n(k), a_y^n(k)]^T \tag{4}$$
$$\mathbf{u}^n(k) = [j_x^n(k), j_y^n(k)]^T \tag{5}$$

where $\square_x$ and $\square_y$ describe the longitudinal and lateral components of the position p, the speed v, the acceleration a and the jerk j in the inertial frame.

Assuming piecewise constant input $\mathbf{u}_k^n$, the general vehicle dynamics (2) can be described by the linear, time-invariant, discrete state space equation:

$$\mathbf{x}_{k+1}^n = \begin{bmatrix} 1 & \tau & \frac{1}{2}\tau^2 & 0 & 0 & 0 \\ 0 & 1 & \tau & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \tau & \frac{1}{2}\tau^2 \\ 0 & 0 & 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k^n + \begin{bmatrix} \frac{1}{6}\tau^3 & 0 \\ \frac{1}{2}\tau^2 & 0 \\ \tau & 0 \\ 0 & \frac{1}{6}\tau^3 \\ 0 & \frac{1}{2}\tau^2 \\ 0 & \tau \end{bmatrix} \mathbf{u}_k^n \tag{6}$$

To enforce realistic dynamic limits the velocity v^n, the acceleration a^n, as well as the jerk j^n are bounded.

$$\mathbf{x}_k^n \in [\underline{\mathbf{x}}_k^n, \overline{\mathbf{x}}_k^n] \tag{7}$$
$$\mathbf{u}_k^n \in [\underline{\mathbf{u}}_k^n, \overline{\mathbf{u}}_k^n] \tag{8}$$

where $\underline{\square}$ specifies the minimum and $\overline{\square}$ the maximum value.

So far the motion in x and y-direction is fully decoupled which does not allow a realistic representation of the non-holonomic dynamics. By limiting the heading $\Theta \in [\underline{\Theta}, \overline{\Theta}]$, two additional constraints

$$\begin{aligned} v_y^n &\leq v_x^n \tan(\overline{\Theta}) \\ v_y^n &\geq v_x^n \tan(\underline{\Theta}) \end{aligned} \tag{9}$$

are introduced to ensure the dynamic feasibility of the generated trajectory [23].

Note that in contrast to previous work [5], [20], where first or second-order point-mass models have been used, we follow the approach of [6] and utilize a third-order description. This ensures a continuous acceleration profile of the generated trajectory and, as a result, a continuous yaw rate which a low-level controller can track smoothly.

B. Collision avoidance

1) Among cooperative vehicles: Consider any pair of cooperative vehicles V^n and $V^m \in \mathcal{V}$. Collision freeness can be ensured if for every time step k the vehicles are not occupying the same spatial area. This can be described by a set of logical *or* constraints.

$$\begin{aligned}
&(p_x^n <= p_x^m - l_{min}^{n,m}) \vee \\
&(p_x^n >= p_x^m + l_{min}^{n,m}) \vee \\
&(p_y^n <= p_y^m - w_{min}^{n,m}) \vee \\
&(p_y^n >= p_y^m + w_{min}^{n,m})
\end{aligned} \tag{10}$$

Hereby, $w_{min}^{n,m}$ is the minimum lateral and $l_{min}^{n,m}$ longitudinal distance necessary between the two vehicles. Note that additional safety margins, e.g., to cope with sensor noise, can be easily incorporated by increasing $w_{min}^{n,m}$ and $l_{min}^{n,m}$.

By applying the so-called Big-M method [24], [25], the logical constraints (10) can be transformed into a set of linear inequality constraints which is the standard semantic for mixed-integer programs. This is achieved by adding or subtracting an application-specific big positive constant M^{big} depending on the value of a binary variable δ_i. The binary variable is equal to 1 if a condition is active, 0 when it is inactive. The collision avoidance constraint at each time step k can then be written as:

$$p_{x,k}^n \leq p_{x,k}^m - l_{min}^{n,m} + (1 - \delta_{1,k}^{n,m})M^{big} \tag{11a}$$

$$p_{x,k}^n \geq p_{x,k}^m + l_{min}^{n,m} - (1 - \delta_{2,k}^{n,m})M^{big} \tag{11b}$$

$$p_{y,k}^n \leq p_{y,k}^m - d_{min}^{n,m} + (1 - \delta_{3,k}^{n,m})M^{big} \tag{11c}$$

$$p_{y,k}^n \geq p_{y,k}^m + d_{min}^{n,m} - (1 - \delta_{4,k}^{n,m})M^{big} \tag{11d}$$

$$\sum_{i=1}^{4} \delta_{i,k}^{n,m} \geq 1 \tag{11e}$$

The last constraint (11e) ensures that at least 1 original *or*-condition form (10) is satisfied.

2) Static or moving obstacle: Obstacle avoidance is modeled in the same way as shown above, again introducing four binary variables δ_i^o for each obstacle $o \in \mathcal{O}$, vehicle $V^n \in \mathcal{V}$ pair at every time step k. The shape of an obstacle o is approximated by a minimal bounding rectangle $[p_{x,k}^o - l^o,\ p_{x,k}^o + l^o] \times [p_{y,k}^o - w^o,\ p_{y,k}^o + w^o]$. More complex polygonal shapes are also possible, however, significantly increasing the complexity.

By modifying the position of the obstacle $(p_{x,k}^o, p_{y,k}^o)$ according to a predefined motion, this formulation also allows moving obstacles. In this paper non-cooperating vehicles, e.g. human-driven vehicles, are treated as such moving obstacles. The motion is assumed to be provided by a prediction module.

C. Collective cost function

To generate comfortable trajectories, a quadratic cost function to minimize acceleration, jerk as well as the deviation to a desired lateral position and longitudinal speed is chosen. The desired state is given by:

$$\mathbf{x}^{ref} = [p_x^{ref}, v_x^{ref}, a_x^{ref}, p_y^{ref}, v_y^{ref}, a_y^{ref}]^T \tag{12}$$

v_x^{ref} represents the desired speed, p_y^{ref} can be used to select a desired lateral offset e.g. for lane selection, a_x^{ref}, v_y^{ref} and a_y^{ref} are zero. The value of $p_x^{n,ref}$ is irrelevant since the corresponding weighting is zero.

Unlike in [5] where a one-norm is used the quadratic model has been chosen so increasing deviations from the desired state are penalized more than linearly. This should prevent a cooperative solution where one vehicle has an over proportionally high disadvantage while multiple others gain small advantages. E.g. 101 vehicles save 1 second each while crossing an intersection but therefore one vehicle has to wait unacceptable 100 seconds which would be a valid solution for a linear cost function that naively minimizes the over-all travel time. Using a weighted sum over all individual costs as the collective cost function, the cooperative trajectory planning problem for an ensemble of communicating automated vehicles $\mathcal{V}$ can now be written as:

$$\min_{\boldsymbol{u}^n, \forall n \in N} \sum_{n=1}^{N} w^n \left(\sum_{k=1}^{K} \|\mathbf{x}_k^n - \mathbf{x}_k^{n,ref}\|_Q + \sum_{k=0}^{K-1} \|\mathbf{u}_k^n\|_R \right) \tag{13a}$$

$$\textit{subject to } \forall n \in \mathcal{V}:$$

$$\textit{known inital state } \mathbf{x}_{k=0}^n = \mathbf{x}^n(0) \tag{13b}$$

$$\forall k \in [0, \dots, K-1]: \quad \mathbf{x}_{k+1}^n = A\mathbf{x}_k^n + B\mathbf{u}_k^n \tag{13c}$$

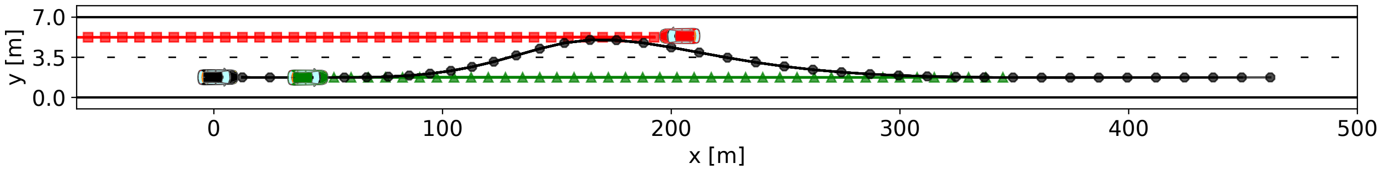
$$\forall k \in [0, \dots, K-1]: \quad \mathbf{u}_k^n \leq \overline{\mathbf{u}}_k^n, \quad \mathbf{u}_k^n \geq \underline{\mathbf{u}}_k^n \tag{13d}$$

$$\forall k \in [1, \dots, K]: \quad \mathbf{x}_k^n \leq \overline{\mathbf{x}}_k^n, \quad \mathbf{x}_k^n \geq \underline{\mathbf{x}}_k^n \tag{13e}$$

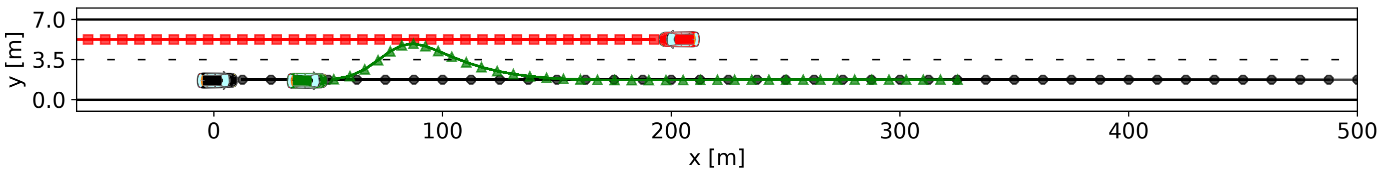
$$\forall k \in [1, \dots, K]: \quad v_y^n \leq v_x^n \tan(\overline{\Theta}), \quad v_y^n \geq v_x^n \tan(\underline{\Theta}) \tag{13f}$$

$$\begin{aligned}
&\forall k \in [1, \dots, K],\ \forall V^m \in \mathcal{V} | m \geq n: \\
&p_{x,k}^n \leq p_{x,k}^m - l_{min}^{n,m} + (1 - \delta_{1,k}^{n,m})M^{big} \\
&p_{x,k}^n \geq p_{x,k}^m + l_{min}^{n,m} - (1 - \delta_{2,k}^{n,m})M^{big} \\
&p_{y,k}^n \leq p_{y,k}^m - d_{min}^{n,m} + (1 - \delta_{3,k}^{n,m})M^{big} \\
&p_{y,k}^n \geq p_{y,k}^m + d_{min}^{n,m} - (1 - \delta_{4,k}^{n,m})M^{big} \\
&\sum_{i=1}^{4} \delta_{i,k}^{n,m} \geq 1
\end{aligned} \tag{13g}$$

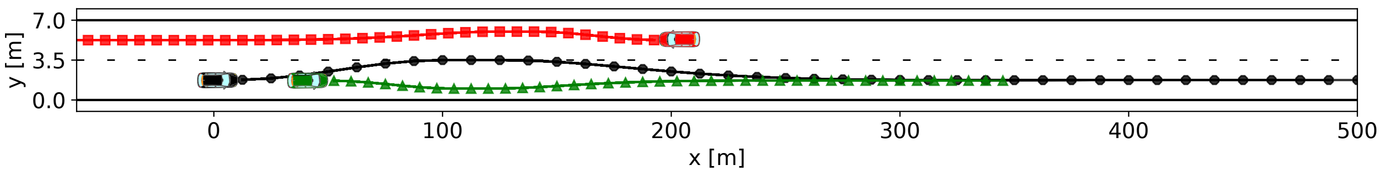
$$\begin{aligned}
&\forall k \in [1, \dots, K],\ \forall o \in \mathcal{O}: \\
&p_{x,k}^n \leq p_{x,k}^o - l_{min}^{n,m} + (1 - \delta_{1,k}^{n,o})M^{big} \\
&p_{x,k}^n \geq p_{x,k}^o + l_{min}^{n,m} - (1 - \delta_{2,k}^{n,o})M^{big} \\
&p_{y,k}^n \leq p_{y,k}^o - d_{min}^{n,m} + (1 - \delta_{3,k}^{n,o})M^{big} \\
&p_{y,k}^n \geq p_{y,k}^o + d_{min}^{n,m} - (1 - \delta_{4,k}^{n,o})M^{big} \\
&\sum_{i=1}^{4} \delta_{i,k}^{n,o} \geq 1
\end{aligned} \tag{13h}$$



(a) Individual motion planning



(b) Priority based motion planning



(c) Cooperative MIQP motion planning

Figure 1: Resulting trajectories of the three different cooperative motion planning approaches for an overtaking scenario: individual, *non-cooperative* (a), priority based (b) and cooperative MIQP (c) motion planning. V^1 is colored *black*, V^2 is the *green* car and V^3 the *red* car driving in negative x-direction

Where Q and R are semi-positive definite and positive definite weighting matrices respectively penalizing the deviation of the desired state and any control effort. If $\mathbf{x}^{n,ref}$ is independent or linearly dependent of other variables this yields a MIQP for which efficient solvers exist that guarantee globally optimal solutions [26]. As already mentioned in [6] if needed more complex non-quadratic cost functions or nonlinear vehicle dynamics can be used, however, leading to a Mixed Integer Nonlinear Program, which is much harder to solve.

IV. Example Scenarios

In this section the introduced MIQP formulation for cooperative trajectory planning is applied to demonstrate the feasibility in general on-road scenarios. A challenging overtaking scenario on a rural road with oncoming traffic is presented. Furthermore, the result is compared to those of a priority-based approach and to *non-cooperative* individual motion planning. The priority-based results are obtained by calculating the outcome of all possible priority permutations and taking the one with lowest overall costs. This way the result is independent of a specific priority selection heuristics and represents the best achievable result.

For the simulation the weighting w^n of each cooperative vehicle $V^n \in \mathcal{V}$ in the collective cost function (13) is set to 1. As a result, every vehicle is considered equally thus enabling the ensemble to reach the highest level of cooperative behavior according to [22]. The trajectory planning was performed for a time horizon of $T = 20s$ with a time step duration of $\tau = 0.5s$. The vehicles initially drive with their desired velocity and have a width of $w = 2m$. The width of a lane is $3.5m$, other parameters are listed in the table below.

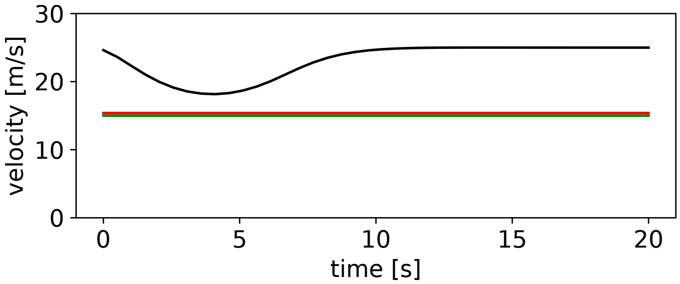
$\overline{\mathbf{x}} = [free, 30m/s, 3m/s^2, 6m, 2m/s, 2m/s^2]^T$
$\underline{\mathbf{x}} = [0, 0, -4m/s^2, 1m, -2m/s, -2m/s^2]^T$
$\overline{\mathbf{u}} = [3m/s^3, 2m/s^3]^T$, $\underline{\mathbf{u}} = -\overline{\mathbf{u}}$, $\overline{\Theta} = 0.4rad$, $\underline{\Theta} = -\overline{\Theta}$
$diag(Q) = \{0, 1, 2, 1, 2, 4\}$, $diag(R) = \{4, 4\}$

Table I: Parameters used for the numerical experiment

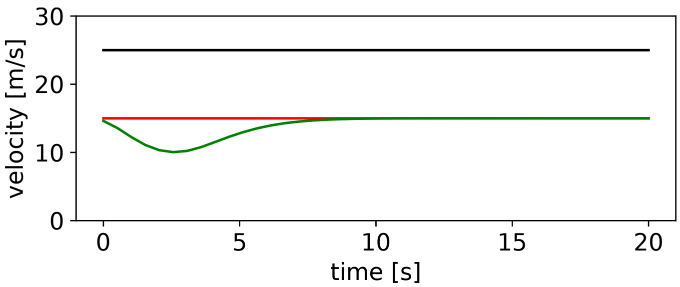
An overtaking scenario on a two-lane rural road is considered as shown in figure 1. Vehicle V^1 with a desired speed of $v_x^{1,des} = 25m/s$ is approaching a slower driving vehicle V^2, $v_x^{2,des} = 15$. Overtaking is potentially dangerous due to the oncoming vehicle V^3, $v_x^{3,des} = 15m/s$, on the adjacent lane. In case of individual motion planning, two options remain for V^1, either strong acceleration to overtake before passing V^3, which might not be possible due to a lack of engine power, or brake and overtake after passing V^3.

Figure **??** shows the resulting trajectories of individual, *non-cooperative* motion planning. In this case each vehicle generates a trajectory by minimizing an individual cost function not considering the costs of others. V^2 and V^3 therefore follow their desired velocities as shown in **??**. V^1 however, has to decelerate sharply and wait until V^3 has passed to finally overtake V^2 and continue driving with its desired speed. Decelerating and staying behind V^2 with significantly lower speed causes high costs for V^1. Today's traffic can for the most part be described as such planning.

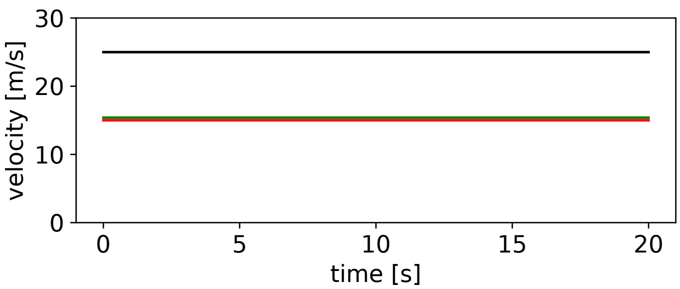
The priority assignment which generates trajectories with the lowest collective costs is $V^1 > V^3 > V^2$. Since V^2 has the lowest priority, it plans its motion last, adapting to the plans of V^1, V^3. The priority-based approach yields slightly lower



(a) Individual motion planning



(b) Priority based motion planning



(c) Cooperative MIQP motion planning

Figure 2: Speed profile of the three different cooperative motion planning approaches.

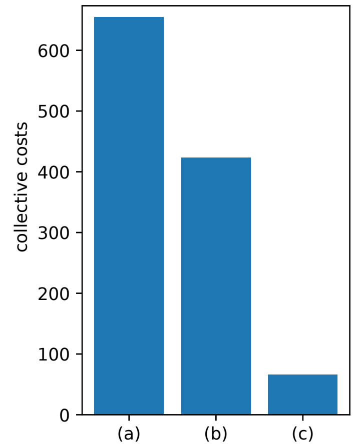


Figure 3: Comparison of the collective cost (a) individual, *non-cooperative* costs: 654.47 (b) priority based costs: 423.75 (c) cooperative MIQP costs: 65.88.

collective costs than individual motion planning, mainly because big deviations form the desired speed of any vehicle can be mitigated. As mentioned before, the major drawback of the priority-based approach is, that the vehicle with the highest priority doesn't adapt to any other vehicle's behavior even if small own disadvantages would allow others to greatly improve their trajectory.

By exploding the entire configuration space the proposed cooperative MIQP approach yields the globally optimal solution, which in the overtaking scenario is an ad-hoc creation of a third lane see figure **??**. Looking at the speed profiles shown in figure **??** one can see that all vehicles can stick to their desired velocity. In figure (3) the collective costs for each approach are compared. The cooperative MIQP approach clearly outperforms individual as well as priority-based motion planning and can significantly lower a collective cost function and therefore increase comfort and travel time in cooperative traffic scenarios.

V. Conclusions and Future Work

In this paper, we present a novel approach to formulate the task of cooperate trajectory planning for an ensemble of communicating automated vehicles as a Mixed-Integer Quadratic Program. The method is well suited for general non-hazardous on-road traffic scenarios. It outperforms currant priority based, as well as *non-cooperative* individual motion planning approaches and guarantees globally optimal solutions. Furthermore, numerical experiments are provided which demonstrate the feasibility of our method as well as the optimization of comfort and travel time.

Future research includes extending the approach to arbitrary road geometry as well as integrating a plan B trajectory for each vehicle, that guarantees reaching a safe-state in case of failure e.g. loss of communication. Although the computation of the cooperative maneuvers is near real-time capable, it doesn't scale well with an increasing number of participants. Therefore methods to reduce the computational complexity e.g. by building cooperation groups, or distributed computing are also subject of further research.

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References

[1] K. Bengler, K. Dietmayer, B. Farber, M. Maurer, C. Stiller, and H. Winner, "Three Decades of Driver Assistance Systems: Review and Future Perspectives", *IEEE Intell. Transp. Syst. Mag.*, vol. 6, no. 4, 2014.

[2] D. González, J. Pérez, V. Milanés, and F. Nashashibi, "A Review of Motion Planning Techniques for Automated Vehicles", *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 4, pp. 1135–1145, Apr. 2016, ISSN: 1524-9050.

[3] B. Paden, M. Čáp, S. Z. Yong, D. Yershov, and E. Frazzoli, "A Survey of Motion Planning and Control Techniques for Self-Driving Urban Vehicles", *IEEE Transactions on Intelligent Vehicles*, vol. 1, no. 1, pp. 33–55, Mar. 2016, ISSN: 2379-8858.

[4] S. M. LaValle, *Planning Algorithms*. Cambridge University Press, 2006.

[5] T. Schouwenaars, B. D. Moor, E. Feron, and J. How, "Mixed integer programming for multi-vehicle path planning", in *2001 European Control Conference (ECC)*, 00417, Sep. 2001, pp. 2603–2608.

[6] X. Qian, F. Altché, P. Bender, C. Stiller, and A. d. L. Fortelle, "Optimal trajectory planning for autonomous driving integrating logical constraints: An MIQP perspective", in *2016 IEEE 19th International Conference on Intelligent Transportation Systems (ITSC)*, 00003, Nov. 2016, pp. 205–210.

[7] J. Nilsson and J. Sjöberg, "Strategic decision making for automated driving on two-lane, one way roads using model predictive control", in *2013 IEEE Intelligent Vehicles Symposium (IV)*, Jun. 2013, pp. 1253–1258.

[8] P. Bender, Ö. Ş. Taş, J. Ziegler, and C. Stiller, "The combinatorial aspect of motion planning: Maneuver variants in structured environments", in *2015 IEEE Intelligent Vehicles Symposium (IV)*, 00012, Jun. 2015, pp. 1386–1392.

[9] M. L. Sichitiu and M. Kihl, "Inter-vehicle communication systems: A survey", *IEEE Communications Surveys Tutorials*, vol. 10, no. 2, pp. 88–105, Second 2008, ISSN: 1553-877X.

[10] L. Chen and C. Englund, "Cooperative intersection management: A survey", *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 2, pp. 570–586, Feb. 2016, ISSN: 1524-9050.

[11] F. Altché and A. de La Fortelle, "Analysis of optimal solutions to robot coordination problems to improve autonomous intersection management policies", in *2016 IEEE Intelligent Vehicles Symposium (IV)*, Jun. 2016, pp. 86–91.

[12] A. Colombo and D. D. Vecchio, "Least restrictive supervisors for intersection collision avoidance: A scheduling approach", *IEEE Transactions on Automatic Control*, vol. 60, no. 6, pp. 1515–1527, Jun. 2015, ISSN: 0018-9286.

[13] M. R. Hafner, D. Cunningham, L. Caminiti, and D. D. Vecchio, "Cooperative collision avoidance at intersections: Algorithms and experiments", *IEEE Transactions on Intelligent Transportation Systems*, vol. 14, no. 3, pp. 1162–1175, Sep. 2013, ISSN: 1524-9050.

[14] J. Grégoire, S. Bonnabel, and A. de La Fortelle, "Priority-based intersection management with kinodynamic constraints", in *2014 European Control Conference (ECC)*, Jun. 2014, pp. 2902–2907.

[15] D. Bevly, X. Cao, M. Gordon, G. Ozbilgin, D. Kari, B. Nelson, *et al.*, "Lane Change and Merge Maneuvers for Connected and Automated Vehicles: A Survey", *IEEE Transactions on Intelligent Vehicles*, vol. 1, no. 1, pp. 105–120, Mar. 2016, 00000, ISSN: 2379-8858.

[16] M. G. Plessen, D. Bernardini, H. Esen, and A. Bemporad, "Multi-automated vehicle coordination using decoupled prioritized path planning for multi-lane one- and bi-directional traffic flow control", in *2016 IEEE 55th Conference on Decision and Control (CDC)*, Dec. 2016, pp. 1582–1588.

[17] S. Manzinger, M. Leibold, and M. Althoff, "Driving strategy selection for cooperative vehicles using maneuver templates", in *2017 IEEE Intelligent Vehicles Symposium (IV)*, Jun. 2017, pp. 647–654.

[18] C. Frese and J. Beyerer, "A comparison of motion planning algorithms for cooperative collision avoidance of multiple cognitive automobiles", in *2011 IEEE Intelligent Vehicles Symposium (IV)*, Jun. 2011, pp. 1156–1162.

[19] J. B. Tomas-Gabarron, E. Egea-Lopez, and J. Garcia-Haro, "Vehicular trajectory optimization for cooperative collision avoidance at high speeds", *IEEE Transactions on Intelligent Transportation Systems*, vol. 14, no. 4, pp. 1930–1941, Dec. 2013, ISSN: 1524-9050.

[20] J. Eilbrecht and O. Stursberg, "Cooperative driving using a hierarchy of mixed-integer programming and tracking control", in *2017 IEEE Intelligent Vehicles Symposium (IV)*, 00001, Jun. 2017, pp. 673–678.

[21] M. During and K. Lemmer, "Cooperative maneuver planning for cooperative driving", *IEEE Intelligent Transportation Systems Magazine*, vol. 8, no. 3, pp. 8–22, Fall 2016, ISSN: 1939-1390.

[22] C. Burger, P. F. Orzechowski, Ö. Ş. Taş, and C. Stiller, "Rating cooperative driving: A scheme for behavior assessment", in *2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC)*, Oct. 2017, pp. 1–6.

[23] J. Nilsson, M. Brännström, J. Fredriksson, and E. Coelingh, "Longitudinal and lateral control for automated yielding maneuvers", *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 5, pp. 1404–1414, May 2016, ISSN: 1524-9050.

[24] J. Nocedal and S. J. Wright, *Numerical optimization*, 2. ed., ser. Springer series in operation research and financial engineering. New York, NY: Springer, 2006, ISBN: 0-387-30303-0; 978-0-387-30303-1.

[25] W. L. Winston, Ed., *Operations research*, 4. ed. Belmont, Calif.: Brooks/Cole, Cengage Learning, 2003, vol. 1: Introduction to mathematical programming, ISBN: 0-534-35964-7; 978-0-534-35964-5; 0-534-42357-4; 0-534-42354-X; 978-0-534-42354-4.

[26] I. Gurobi Optimization, *Gurobi optimizer reference manual*, 2016.