2018 21st International Conference on Intelligent Transportation Systems (ITSC)
Maui, Hawaii, USA, November 4-7, 2018

Criticality Metric for the Safety Validation of Automated Driving using Model Predictive Trajectory Optimization

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***Abstract*—The safety validation of automated driving of SAE level 3 and higher (AD) is still an unsolved issue. In the validation process, criticality metrics can be used for two different purposes. First, for the identification of test scenarios from recorded data that are later tested in simulation. Secondly, for an estimation of the safety of a specific AD system based on the likelihood of critical situations in test drives or in other words as a safety surrogate. In the past, different metrics for those purposes have been defined that work well in specific scenarios such as longitudinal traffic. However, a metric that describes criticality in all situations and is applicable to human and AD traffic is currently not available. In this paper, an approach to define a criticality metric is introduced. The metric is based on the definition of criticality as the level of driving requirements in the specific situation. The computation of the proposed metric uses elements of model predictive control using an objective function that contains four elements that describe the difficulty of the driving task. Based on those demands and a simplified driving dynamics model, the solution with the minimal criticality is computed. Finally, the metric is tested in four test scenarios that are typical for highway traffic. A short parameter variation study is conducted in order to study certain effects of the algorithm and to identify room for improvement.**

Keywords—Safety Validation, Metric, Automated Driving, MPC

I. Motivation and Purpose

While the development of automated driving systems of SAE level 3 or higher (AD) has been further progressing over the last years, the safety validation of those systems is still an open issue. As critical events and accidents are fortunately rare events in a vehicle lifetime, the safety validation of AD is challenging [1]. The metric presented in this paper contributes towards two open challenges in a possible safety validation process explained in subsections A and B: The identification of scenarios and the direct estimation of AD safety performance (SP) in test-drives (Fig. 1). SP is defined in [2] as the average mileage between two accidents of a certain severity.

A. Criticality Metric as Safety Surrogate

In [2] and [3], the required real-world testing in order to prove superior safety of AD compared to today's traffic is analyzed. If the occurring fatal accidents are used as a metric for safety, the expected amount of testing kilometers is in the billions. If general accidents are used, the required mileage is still in the range of several million kilometers. If a surrogate were to be found that is able to describe the level of safety in non-crash situations, the required mileage could be further reduced. A similar concept is to measure road safety by surrogates (e.g. on a specific crossing) [4, 5]. In [6] and [7], extreme value theory (EVT) is used together with the metrics time-to-collision and brake-threat-number (BTN), which is basically the used friction divided by the available friction in front-to-rear near-crash situations. When applying EVT on data from a naturalistic driving study, only BTN could estimate the real number of accidents within a 95% confidence interval. This is considered a validation of the metric for human test drives or naturalistic driving studies (NDS). However, obviously the metric can only be applied to situations that are close to front-to-rear accidents. There is also no information about the potential severity of an avoided accident so the SP cannot be estimated for different severity categories, only for accidents in general. Hence, it is obvious that BTN cannot be used as the only metric in a safety assessment.

B. Identification of Test Scenarios

Another approach for the safety validation of AD is the scenario-based testing. The system is not tested in real traffic but in simulation or on proving grounds. The test scenarios can be derived based on different sources, e.g. accident studies, NDS or by systematic deduction [8]. When deducing scenarios from NDS, a filter to find relevant scenarios is required. Most driving especially on motorways is monotone

This paper is part of the work in PEGASUS, a project funded by the German Federal Ministry for Economic Affairs and Energy.

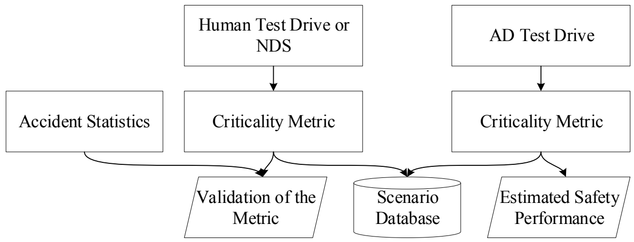

Fig. 1. Double Purpose of the Metric

and does not require a special test case [9]. The metric should detect all scenarios with increased driving requirements in order to derive test cases. It can be applied on data from NDS and during AD test drives to save relevant scenarios into a database. When the metric is applied online during driving, real-time computation is necessary. If this requirement is not met, a metric to filter all definitely irrelevant scenarios could be used in a first assessment step in order to reduce the number of scenarios that need further computational effort. The Worst-Time-to-Collision [9] is a metric designed to work as a first filter to efficiently find those scenarios and could be used here.

C. Requirements for a Criticality Metric

So what are the requirements for a safety surrogate?

1. The metric or a combination of metrics should be able to identify closeness to all types of accidents.
2. The metric should be validated with existing data, similar to the mentioned EVT study [7]. Hence, it must be applicable to human-driven traffic as well as AD.
3. Information about the potential severity should be given.

In this paper, we focus on the first two requirements. Fulfilling the third requirement is extremely difficult, as the severity not only depends on the speed of the involved accident participants, but also on the type of vehicle, the mass, and the angle and overlap of the collision. It is assumed here that it is sufficient to base the estimation of the severity on the velocities (and hence the potential energy) of the involved participants and a classification of those (e.g. vehicle, truck, two-wheeler, pedestrian). So how the metric should be defined to fulfill the first two requirements? Ideally, the metric should describe the probability of an accident in a specific situation for AD or a human driver. As different drivers and different AD also have different driving performance and therefore different collision probability, the metric shall describe the driving requirements in a situation that is independent of the available driving performance [10]. The driving requirements consist of the following entities:

1. Necessary acceleration in lateral and longitudinal direction
2. Margin for corrections of course angle (side distance)
3. Margin for correction of speed (front distance)

These entities depend on each other. A higher reserve for corrections can be bought with a higher deceleration early on. Hence, the three entities should be normalized and combined into a joint value.

D. Comparison with State of the Art

Various metrics exist for the identification of scenarios, trajectory planning, and the assessment of criticality in general. They can be classified into a posteriori metrics, which are often used to analyze NDS [11, 12], and metrics with deterministic [13–16] and probabilistic trajectory prediction [16–20]. An overview and a classification of metrics with trajectory prediction can be found in [17] and [21]. However, those metrics do not combine all of the mentioned entities. In [22, 23] monte-carlo sampling is used to address combined maneuvers. In [24], the reachable and available free space is assessed to derive criticality. However, these approaches do not address the difficulty of driving on those trajectories. In A combination of different criteria is often done in trajectory planning and optimization [25, 26]. Defining an optimization function to be minimized offers the opportunity to combine different entities and weighting factors. Typically, the resulting trajectory is of interest together with the required control values in case model predictive control (MPC) is used. In this paper, a similar approach is used with an optimization function designed to describe criticality as the driving requirements mentioned above. The minimized value of the optimization function is the criticality in the situation. It is important to note the difference between this approach and most use-cases for trajectory optimization. The metric has no direct feedthrough to the trajectory control. Its only purpose is to optimize possible future trajectories in order to calculate the criticality. This could be either done online during test drive, or a posteriori from recorded data.

II. Model Description

Considering the given motivation, the bases of the optimization problem are shown in this section. Investigations of [27–29] showed the success of the MPC approach for trajectory calculations in mitigation systems. Further enhancements (e.g. in [26]) revealed a performance increase by a factor of 400 so MPC is used in real-time trajectory planning and control. The presented metric does not necessarily require real-time computation as it can be applied online (in a vehicle) or offline (on recorded data). However, in order to optimize data recording, it would be helpful to reduce the data online and only record critical scenarios.

A. System Dymanic Model

To obtain the desired criticality metric, we define the state-space model with state vector $\underline{x} = [x_w\ y_w\ v_v\ \psi_c]$ and input vector $\underline{u} = [a_x\ a_y]$. The problem depends on the current ego velocity in natural coordinates v_v, the course angle ψ_c, and the acceleration a_x, a_y in natural coordinates. Additionally, the position in world coordinates is required because the position of the objects is not updated into vehicle coordinates once the prediction horizon is initialized. For most MPC applications, a linear single-track-model is used. This has the advantage that the actuator inputs can be optimized directly. However, in the application of the criticality metric, the accelerations are of interest. As we do not use the model for actual vehicle control, additional deviations from a simplified model (e.g. because the neglecting of tire cornering stiffness) can be neglected.

Instead, a point mass model is used that is extended by a course angle that is required to transform the vehicle motion into world coordinates. The vehicle's dimensions are often described with circles along the center of the vehicle because the computation of circles is faster compared to a rectangle. In [25], three circles are used in order to approximate the vehicle. However, this would require a determination of the yaw center. Instead, only one circle is used with the diameter of the vehicle's width. As a result, we cannot be sure that a trajectory is optimized that does not collide with the front or rear part of the vehicle. As the metric is not designed for actual trajectory control, this is acceptable because near-misses will also be punished with increased criticality. A non-linear vehicle model is described by:

$$\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{v}_v \\ \dot{\psi}_c \end{bmatrix} = \begin{bmatrix} v_v \cdot \cos(\psi_c) \\ v_v \cdot \sin(\psi_c) \\ a_x \\ \frac{a_y}{v_v} \end{bmatrix} \tag{1}$$

Assuming small course angle changes and small changes in velocity allows linearization of the model. Initializing the model at course angle zero and using a constant velocity v_{old}, which is updated only at the beginning of the prediction horizon (comp. [25]), the equations simplify to:

$$\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{v}_v \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v_v \\ v_{\text{old}} \cdot \psi_c \\ a_x \\ \frac{a_y}{v_{\text{old}}} \end{bmatrix} \tag{2}$$

resulting in the system matrices:

$$\underline{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & v_{\text{old}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underline{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1/v_{\text{old}} \end{bmatrix} \tag{3}$$

B. Objective function & Constraints

The goal of the MPC is to optimize the trajectory subject to the criticality within a given prediction horizon N. Hereby, the optimization function should describe criticality according to the definition of criticality in section I.C. The first two influence factors are the accelerations that are minimized. The reserve for corrections of angle is dependent on the lateral distance and the velocity. A higher distance gives more space to correct the course angle. A lower velocity has two influence factors: A disturbance (e.g. side wind) has a smaller influence towards a deviation in the position (because the velocity is integrated) and the course angle can be corrected with less effort in a_y. As we want to minimize a term that includes maximization of distances, we use the position with the maximum lateral distance to all obstacles as reference r_y and a maximum deviation d_y resulting in the following term:

$$R_y^2 = \frac{\left(y(k) - r_y(k)\right)^2}{d_y^2(k)} \cdot \frac{v_{\text{old}}}{v_{\max}} \tag{4}$$

The reserve in longitudinal direction cannot be used in such way, because only front distance is relevant. As reference r_x we use the recommended following distance in Germany (0.5·v/(km/h)). We use the piecewise linear function:

$$R_x = \max\left(0, x(k) - r_x(k)\right) / d_x(k) \tag{5}$$

With the minimization of an objective function alone, the collision with other objects is not excluded. Hence, the outputs are bound by constraints that are updated at the start of the prediction horizon assuming constant velocity and course angle. Again, we neglect objects coming from behind and represent the left, right, and front boundaries to be constrained by c_l, c_r, and c_f respectively. Additionally, the inputs must be bound because the dynamics of a vehicle are limited by the available friction coefficient. According to Kamm's circle:

$$a_x^2(k) + a_y^2(k) \leq \mu_{\max} g \tag{6}$$

However, to allow efficient solving of the optimization problem, Kamm's circle is approximated with 12 linear constraints using L_x, L_y, and M taken from [25]. Modeling the objective function with the sum of the defined elements with constant weighting factors w gives us following optimization problem:

$$\begin{aligned} \min_u J = \sum_{k=1}^{N-1} w_x R_x(k) + \sum_{k=1}^{N-1} \mathrm{w_y} R_y^2(k) \\ + \sum_{k=1}^{N-1} w_{ax} \frac{a_x^2(k)}{(\mu_{\max}\mathrm{g})^2} \\ + \sum_{k=1}^{N-1} w_{ay} \frac{a_y^2(k)}{(\mu_{\max}\mathrm{g})^2} \end{aligned} \tag{7}$$

$$\text{s.t.} \quad \underline{\dot{x}} = \underline{A}(k)\underline{x}(k) + \underline{B}(k)\underline{u}(k) \tag{8}$$

$$c_r(k) \leq y(k) \leq c_l(k) \tag{9}$$

$$x(k) \leq c_f(k) \tag{10}$$

$$L_x a_x(k) + L_y a_y(k) \leq M \tag{11}$$

$$L_x a_x(k) - L_y a_y(k) \leq M \tag{12}$$

The weighting factors are chosen in a way that allows equal weight of the four elements of the optimization function. However, R_x typically results in values 10 to 20 times higher than the other elements. Because the ego vehicle is approaching, the elements of the objective function are summed up for subsequent steps. Hence, we choose a weighting factor of 1/10 here, while all other factors remain 1. The different factors and the fact that R_x is defined using the recommended distance in Germany show a certain degree of arbitrariness. An alternative approach is discussed in the outlook. The model was implemented in Matlab using the optimization toolbox Yalmip [30] on an office computer. The duration of the computation of 5 s scenarios lasted between 5.5 s to 7 s. When the metric is implemented in a test vehicle and not on offline data, the code should either be optimized further, or only activated when other, simple metrics detect a scenario that could be of interest (comp. section I.B).

III. Application to Test Scenarios

In order to verify the model, it is applied to four standard scenarios that could happen in daily traffic. We close the loop to the trajectory control to allow the verification of the found optimal trajectory as well. Hence, it is expected that the criticality will always be reduced after the first seconds of the scenario. However, the prediction horizon of 2 s is too low for a potent trajectory control at higher velocities so the reaction of the vehicle will happen relatively late.

A. Introduction of the Scenarios

In the following, the four test scenarios are described together with the expected outcome regarding trajectory and criticality.

1) Static object in lane

A static object is located in the right lane in front of the ego vehicle, which is driving at a speed of 40 km/h. The road has two lanes with crash barriers alongside and the left lane is clear. A usual behavior would be to evade the object and additionally lower the velocity. The total criticality should reach its maximum just before the lateral distance to the obstacle is reached.

2) Reaching the end of a tailback

The road in this scenario has three lanes. While the ego vehicle is driving in the right lane at a speed of 120 km/h, the right and the middle lane are blocked by vehicles. To differ this scenario even more from the first one, these vehicles are driving 20 km/h and the left lane is clear to leave some space for an evasive maneuver. Therefore, again, an evasion while also braking is assumed an option. The difference to the first scenario is that the ego vehicle would need to cross two lane markings, and the vehicles are moving. Hence, braking without an evasive maneuver might result in a lower criticality depending on the distance left to the upcoming vehicles. The maximum criticality is expected just before maximum deceleration or evasion angle is reached, when the objects dive into the prediction horizon.

3) Overtaking

The road has two lanes wherein an object vehicle is driving in the right lane at a constant speed of 75 km/h. The ego vehicle is overtaking this object starting at a speed of 85 km/h and driving in the left lane. This scenario is typical for driving through a construction site on a highway. We assume from own experiences that most human drivers would accelerate while overtaking the object due to narrow lanes in construction sites. This behavior might come from a lower total criticality because the exposure time in which the vehicles are next to each other is shorter. However, an increased velocity might also result in an increased criticality. In total, low to medium criticality is expected.

4) Double-merge into middle lane

The road has three narrow lanes wherein an object vehicle is driving in the right lane at a speed of 55 km/h. The object is merging into the left lane wherein the ego vehicle is coming from behind at a speed of 70 km/h. Braking and keeping the lane until the other vehicle has passed is assumed to be the option with the lowest criticality. Maximum total criticality is expected either just before the velocities are equal after braking, or when the vehicles pass each other in lateral direction.

B. Results

In this section, the simulation results are analyzed. All results are depicted in Fig. 2, maximum and mean criticality can be found in TABLE I. In addition to the objective function, the maximum of the absolute value (2-norm) of the acceleration that is predicted at each position during the prediction horizon is determined. This could be used as an alternative criticality because it describes the maximum expected driving dynamics demand for driving. The first two scenarios were simulated as predicted. Both result in a combined braking and evading maneuver. What has to be pointed out is that braking is typically performed over several consecutive steps leading to higher criticality. Evasion only requires a shorter period of acceleration. It should be considered to increase w_{ay}. The effects of which are analyzed in the next section. The second scenario has the highest criticality of all, which was expected, as the relative velocity is the highest combined with two blocked lanes. The third scenario has very low criticality that is almost exclusively the result of R_y. The velocity is not increased to speed up the overtaking maneuver. This is because the change in updated velocity is not part of the optimization function for the sake of using a quadratic function. However, it is expected that the criticality is lower with increased velocity anyway. This will be analyzed in the next section. What is surprising in the fourth scenario is that there is almost no change in trajectory from the ego vehicle.

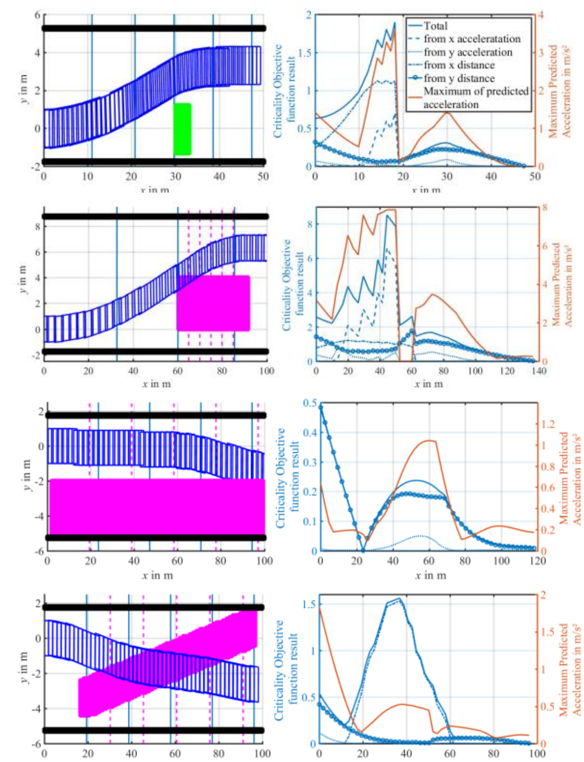

Fig. 2. Simulation results of the four test scenarios; Left: Trajectories topview: Moving Vehicles are color coded in pink, static obstacles in green, position of ego vehicle is indicated in 1 s intervals in solid lines, rear bumper of object in dashed lines; Road boundaries are black; Right: Objective function results on left ordinate axes, Maximum of predicted acceleration during the prediction horizon on right ordinate axes

A deceleration until the ego vehicle becomes slower than the object would increase the criticality more than the relatively short overlap of the trajectories' lateral position does.

C. Parameter Variation

In the previous section, some interesting effects were highlighted. In this section, a parameter variation is conducted to analyze those effects further. The results are summarized in TABLE I. The first entry for each scenario always describes the standard parameters that where already analyzed above. Parameter variation of the first two scenarios showed the influence of a higher w_{ay}. At least for human driving, this parameterization should be considered, because it is easier for a human to do a full brake compared to an emergency evasive maneuver. For the original speed of 40 km/h, the influence is not that strong, because the collision can be avoided by a moderate evasion maneuver compared to a strong braking. However, when reducing the velocity to 20 km/h, the least critical situation is coming to a full stop. The maximum criticality is not much higher despite the increased weighting factor. In the overtaking scenario, the influence of the accelerations is only minor. However, the influence of velocity has to be pointed out here. What is surprising at first is that the criticality is reduced at higher speeds. As it was pointed out above, this matches the intuitive driving style of the human driver. The higher velocity results in a shorter exposure of the potential threat because overtaking takes less time. Especially at lower speeds, we can also see a difference when varying the prediction horizon, because the total exposure is longer than 2 s. Varying the fourth scenario proves that the metric is not yet perfect for more complicated scenarios. There is almost no braking of the vehicle despite this would reduce the criticality enormously. The reason is in the one-dimensional nature of the terms R_x and R_y. Obstacles that pose a combined threat are not handled accordingly. The optimization function should be improved further, to cover that kind of scenarios as well.

IV. OUTLOOK

In this paper, a quadratic cost function with linear constraints was chosen. This has the advantage that efficient solvers exist. However, the objective function cannot be designed freely. The disadvantages of this approach were analyzed in the last two scenarios. The velocity is not a variable part of the objective function. However, the parameter variation showed that higher velocities would result in a lower criticality as it was expected based on human driving behavior. Additionally, minor acceleration should not be punished, especially when precision is not required due to obstacles in longitudinal direction. For the position in x- and y-direction, a "bathtub-curve" would be an improvement compared to the quadratic functions. We would suggest a convex function in the following form:

$$R_y = -\ln\left(\frac{d_y}{2} + r_y - y\right) - \ln\left(y - r_y + \frac{d_y}{2}\right) \quad (13)$$

Another approach to overcome the arbitrariness of the longitudinal and lateral margin is modelling the causes of an increased criticality. As described in section I.C, the terms describe the margin necessary in case of uncertainties. In lateral direction, they could be described by disturbances in lateral acceleration (object and ego vehicle). This would lead to increased criticality for small safety margins. For dynamic objects, a substitution with disturbances is more complicated. The uncertainties are especially in the unknown future object trajectory. High decelerations, despite being unlikely, have a great impact on the necessary reaction. A way out could be to compute a worst-case and a best-guess assumption for the trajectory prediction. In the worst case, the object performs a full brake depending on the maximum available friction. However, both results have to be weighted.

TABLE I. RESULTS OF THE PARAMETER VARIATION (VARIATIONS ARE BOLD)

Test Scenario	v_{ego} in km/h	v_{obj} in km/h	w_x	w_y	w_{ax}	w_{ay}	N	Maximum criticality	Mean criticality
Static obstacle	40	0	1	1	0.1	1	20	1.90	0.50
	40	0	1	1	0.1	**10**	20	4.58	1.14
	20	0	1	1	0.1	**10**	20	0.40	0.23
	20	0	1	1	0.1	1	20	0.22	0.04
Tailback	120	20	1	1	0.1	1	20	8.53	2.14
	120	20	1	1	0.1	**10**	20	18.04	5.37
	80	20	1	1	0.1	1	20	4.27	1.23
	80	20	1	1	0.1	**10**	20	11.70	2.66
	60	20	1	1	0.1	1	20	1.02	0.43
	60	20	1	1	0.1	**10**	20	1.27	0.91
Overtaking	85	75	1	1	0.1	1	20	0.49	0.14
	100	75	1	1	0.1	1	20	0.45	0.11
	77	75	1	1	0.1	1	20	0.62	0.18
	85	75	1	1	0.1	1	**30**	0.49	0.19
	77	75	1	1	0.1	1	**30**	0.77	0.22
Double-merge	70	55	1	1	0.1	1	20	1.56	0.51
	75	55	1	1	0.1	1	20	43.8	4.15

Additional enhancement is necessary for scenarios that are more complex. Objects can be passed on different sides, so there are two (or more) local minima available. In convex optimization, the decision on which side the object is passed must be done a priori (comp. [25][26]). Often, the best choice is not obvious as in scenario four. Therefore, either the possible variants should be determined first, before starting an optimization for each variant, or a non-convex optimization problem should be formulated.

Finally, the metric needs calibration according to the driving capabilities of a typical human driver and AD function. The parameter variation showed a significant impact varying the weighting factors, especially for higher speeds. Once a final parametrization is determined, the metric should be validated as explained in section I.A.

V. Conclusion

In this paper, the role of criticality metrics in the validation of automated driving was explained. The two purposes are the identification of critical scenarios for a scenario-based test and the assessment of test drives with the criticality metric as safety surrogate. As long as we do not have enough data for statistical analysis of automated driving accidents, metrics should be validated in human-driven vehicles. Based on requirements for such metrics, a metric based on an optimization of a defined criticality is introduced using techniques from MPC application. Instead of using MPC for trajectory control, the optimization function was designed in a way to describe criticality based on the driving requirements in each particular situation. As the driving task consists of several individual elements, those are multiplied by weighting factors requiring calibration to the driving skills of human or automated driving.

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