

Importance of Vertical Dynamics for Accurate Modelling, Friction Estimation and Vehicle Motion Control

Moad Kissai¹, Bruno Monsuez¹, Xavier Mouton², Didier Martinez², and Adriana Tapus¹

Abstract—Autonomous vehicles are round the corner. Most of car manufacturers are racing to become a leader in this promising market. In this paper, we focus on one of the remaining challenges to be overcome, namely, the control of vehicle motion when friction varies. Researchers tend to estimate the friction coefficient using complex observers based on simple mathematical equation. We believe that by taken into account some formerly simplified dynamics, as roll dynamics, we can reduce the estimations' errors, and therefore use simple observers. Co-simulations of the control logic with a high-fidelity vehicle model proved the relevance of this method. Results showed that the vehicle can be controlled in severe conditions while the friction changes using only today's passenger cars sensors. Vertical dynamics should therefore be taken into account even for plane motions.

Index Terms—Vertical Dynamics, Friction Estimation, Vehicle Motion Control, Robust Control, Control Allocation.

I. INTRODUCTION

The automotive sector is facing one of its biggest revolutions. Autonomous vehicles promise a lot of potential in terms of safety, comfort, and time saving. Autonomous driving may even reinvent mobility. Most of the expertise that car manufacturers developed regarding the interaction between human and vehicles is questioned. An autonomous vehicle should be able to perform like a human when the driver's attention is carried away. In this context, active researches are conducted to perceive the vehicle surrounding, take an intelligent decision, and control the vehicle motion. The vehicle should be able for example to detect an obstacle, update its planned trajectory, and control its dynamics to follow the desired trajectory. In [1], trajectories have been calculated using simple clothoids for lane change. What is more interesting about this research, is the fact that both the trajectory planning and the velocity profile generation are closely related to friction constraints. Authors however suppose that friction values are available and no estimation method is provided. The same methods have been detailed in [2] to test an autonomous vehicle at the limits of handling. Here, the controller uses a priori knowledge about the friction. The vehicle relies afterwards on the controller robustness to handle surface variations.

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In addition, few studies have been carried out to use the cameras in order to estimate the road friction rather than object detection. The vehicle however should be able to follow the trajectory planned no matter how the friction varies. However, as the friction coefficient is a representative ratio between the tire force and the vertical load [3], it cannot be directly measured. Even the costly sensors equipped at the tire rubber blocks are usually used for tire force measurement, e.g., a wireless piezoelectric tire sensor for the measurements of tire deformations [4]. Important efforts have been carried out in order to provide an estimate of the friction coefficient. In [5], an attempt to estimate the road friction is presented, which is based on vehicle braking dynamics. Simulation results showed the ability of the observer to provide a good estimation when braking. However, in combined maneuvers, when the vehicle is accelerating in steering at the same time for example, combined slip dynamics should be taken into account [6]. This is mainly due to the ellipse of friction [6] that shows that the lateral tire force penalize the longitudinal one, and vice-versa. Not taking into account the influence of the lateral force may prevent a good diagnosis of longitudinal tire force penalization. One cannot tell if it is due to a combined slip or a friction variation. The same philosophy is exposed in [7] and [8]. However, in [7] a more complicated sliding mode observer have been designed based on Dugoff tire model [9], and in [8] a combination of nonlinear Lipschitz observer and a modified super-twisting algorithm observer is developed. Both methods showed good precision as long as only longitudinal forces are excited. Recently, data-based techniques have received more attention. For example, an Auxiliary Particle Filter (APF) have been combined with the Iterated Extended Kalman Filter (IEKF) in [10] based rather on the lateral dynamics only. Data-based techniques are used to correct the errors generated by the unmodeled dynamics, and fit the signals values to the real ones. Experimental tests showed promising results. We believe however that the initial modeling should be precise enough to limit the number of errors to be corrected by data-based techniques.

In this paper, we focus on the initial modeling putting on the spotlight the importance of roll and pitch dynamics. Our investigations showed that particularly the vertical dynamics have an important influence on variables estimation, but are neglected in most research papers. The purpose is to approximate the friction coefficient rapidly without any complicated observation method, update the control logic, and avoid the vehicle's loss of control even in severe conditions. To test our method, a robust controller with a control allocation strategy

have been designed and implemented in Matlab/Simulink[®]. The overall control has been linked through a co-simulation procedure with a high-fidelity over-actuated vehicle model developed in LMS Imagine.Lab AMESim[®]. Results show good performance using simple estimators when taking into account vertical dynamics. The vehicle can be controlled at severe conditions even if friction varies without any a priori knowledge about it.

The rest of the paper is structured as follows: We start in Section II by presenting the overall control logic adopted for vehicle motion control. In Section III, the vehicle dynamics formulas used for estimation are described. Co-simulation results are shown in Section IV. A discussion about the relevance of this work and the remaining challenges is provided in Section V. Conclusions and future works are outlined in Section VI.

II. VEHICLE MOTION CONTROL STRATEGY

Car manufacturers and equipment suppliers are always trying to make road vehicles safer and more comfortable. They constantly try to propose new active safety systems. Passenger cars have become therefore over-actuated [11]. The number of embedded systems is expected to increase with the arrival of full autonomous vehicles. Therefore, systems' coordination strategies should be ensured in order to avoid internal conflicts. In [12], a review of integrated vehicle dynamics control architectures is provided. Authors concluded that a multi-layered architecture may be a better choice to handle each complication apart. In this way, the motion of the vehicle's center of gravity can be ensured by a high-level robust controller. The generalized efforts required to move the vehicle can be distributed in an optimal manner via optimization-based control allocation strategies to the four tires. These tire forces can be then transformed in actuators commands and activate the system concerned avoiding any internal conflicts. In this paper, a vehicle equipped with an Active Rear Steering (ARS), a braking-based Electronic Stability Program (ESP), and two rear in-wheel electric motors for Torque Vectoring (TV). This choice is motivated by the fact that it represents one of the case studies conducted by our collaborators at the Group Renault.

A. High-Level Controller

The objective here is to calculate the required forces at the vehicle's center of gravity in order to track the desired velocities. The dynamics at this point are characterized by inertial parameters as the mass and moment of inertia. These parameters are subject to several uncertainties [13], which require a certain degree of robustness. Moreover, vehicle motion states are coupled [14]. A Multi-Inputs Multi-Outputs (MIMO) controller is needed to take into account the different couplings. As the vehicle is equipped with an ARS, ESP, and TV, and no access is permitted to active suspensions, only a plane vehicle model is considered at the high-level layer. The importance of vertical dynamics will be emphasized is the vehicle states estimation. The vehicle

high-level model is therefore [14]:

$$\left\{ \begin{array}{l} \begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & V_y \\ 0 & 0 & -V_x \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \psi \end{bmatrix} \\ \quad + \begin{bmatrix} \frac{1}{M} & 0 & 0 \\ 0 & \frac{1}{M} & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} \begin{bmatrix} F_{x_{tot}} \\ F_{y_{tot}} \\ M_{z_{tot}} \end{bmatrix} \\ \begin{bmatrix} V_x \\ V_y \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \psi \end{bmatrix} \end{array} \right. \quad (1)$$

With:

- V_x : longitudinal velocity of the vehicle,
- V_y : lateral velocity of the vehicle,
- ψ : yaw rate of the vehicle,
- $F_{x_{tot}}$: total longitudinal force at the vehicle's Center of Gravity (CoG),
- $F_{y_{tot}}$: total lateral force at the vehicle's CoG,
- $M_{z_{tot}}$: total yaw moment at the vehicle's CoG,
- M : vehicle's overall mass,
- I_{zz} : vehicle's yaw moment of inertia.

Due to parameter uncertainties and the MIMO framework, a MIMO \mathcal{H}_∞ controller has been selected for the high-level control. However, depending on the influence of the coupling terms, Gain Scheduling could be expected to improve the controller performance. Be that as it may, the lateral dynamics are fast, and controller parameters variance can destabilize the overall system [15]. Another alternative would be to design a stationary MIMO \mathcal{H}_∞ controller at a sufficiently high crossover frequency where the velocities variables are decoupled. In the same time, the crossover frequency should be kept sufficiently low so that the \mathcal{H}_∞ synthesis achieves an optimal solution. Details of the design will not be exposed in this paper as they do not represent the main focus. The design algorithm achieved successfully the smallest \mathcal{H}_∞ norm of $\gamma = 1.14$.

B. Control Allocation Strategy

The three chassis systems, namely the ARS, ESP, and TV, can influence the yaw rate. The ESP can decelerate the vehicle, while the TV can accelerate the vehicle and so on. This middle layer aims to coordinate chassis systems in order to avoid conflicts and ensure generation of the total forces calculated at the high-level layer. Total forces should be optimally distributed into the four tires to be able to activate the right system with the right amount of effort. As tires are solicited both longitudinally and laterally, the friction ellipse should be taken into account [6]. This ellipse representing tires' potential is closely related to the friction coefficient and the vertical load, which makes these variables important to estimate. The problem is to find tire forces where:

$$\begin{bmatrix} F_{x_{tot}} \\ F_{y_{tot}} \\ M_{z_{tot}} \end{bmatrix} = \begin{bmatrix} \cos(\delta_f) & \cos(\delta_f) & \cos(\delta_r) & \cos(\delta_r) & -\sin(\delta_r) \\ \sin(\delta_f) & \sin(\delta_f) & \sin(\delta_r) & \sin(\delta_r) & \cos(\delta_r) \\ b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} & b_{3,5} \end{bmatrix} \begin{bmatrix} F_{x_{fl}} \\ F_{x_{fr}} \\ F_{x_{rl}} \\ F_{x_{rr}} \\ F_{y_r} \end{bmatrix} \quad (3)$$

Where:

- $b_{3,1} = l_f \sin(\delta_f) - \frac{t}{2} \cos(\delta_f)$,
- $b_{3,2} = l_f \sin(\delta_f) + \frac{t}{2} \cos(\delta_f)$,
- $b_{3,3} = -l_r \sin(\delta_r) - \frac{t}{2} \cos(\delta_r)$,
- $b_{3,4} = -l_r \sin(\delta_r) + \frac{t}{2} \cos(\delta_r)$,
- $b_{3,5} = -l_r \cos(\delta_r)$.

And:

- $F_{x_{i,j}}$: $i - j$ longitudinal force, where “ i ” is front or rear, and “ j ” is right or left,
- F_{y_r} : the equivalent lateral tire force of both rear tires,
- δ_i : front or rear steering angle,
- l_i : distance between the front or rear axle and the vehicle’s center of gravity (CoG),
- t : vehicle’s track.

Regarding the online solver, the Weighted Least Squares (WLS) based on one stage Active Set Algorithm is used [16]. The reader can refer to [16],[17] for further details on solver algorithms and their comparison.

C. Low-Level Commands

Once the forces are optimally distributed, tire forces should be transformed into actuators commands before being fed to any embedded system. Here, engine torques, brake torques, and the rear steering angle should be calculated. This layer represent the most inner loop. Hence, it should be the fastest one. In this work, rather than using additional dynamic controllers that can complicate the overall design, a static tire model has been preferred, which is used as an interface between tire forces and actuators commands. The tire model should take into account the combined slip phenomenon, be precise enough in the controllable zone, and invertible. For these reasons, the Linear tire model with Parameter Varying (LPV) developed in [18] has been chosen:

$$\begin{cases} F_x = C_s^*(\alpha, \mu, F_z) \kappa \\ F_y = C_\alpha^*(\kappa, \mu, F_z) \alpha \end{cases} \quad (4)$$

$$(5)$$

where:

- κ : the longitudinal slip,
- α : the side-slip,
- μ : the friction coefficient,
- F_z : the vertical load,
- $C_s^*(\alpha, \mu, F_z)$: the tire varying longitudinal stiffness with respect to the side-slip α , μ , and F_z ,

- $C_\alpha^*(\kappa, \mu, F_z)$: the tire varying cornering stiffness with respect to the longitudinal slip κ , μ , and F_z .

Detailed expressions of $C_s^*(\alpha, \mu, F_z)$ and $C_\alpha^*(\kappa, \mu, F_z)$ can be found in [18]. In order to respect the friction ellipse concept, dynamic constraints are added [18]:

$$\begin{cases} F_x \leq \sqrt{(\mu F_z)^2 - F_y^2} \\ F_y \leq \sqrt{(\mu F_z)^2 - F_x^2} \end{cases} \quad (6)$$

$$(7)$$

III. VEHICLE STATES ESTIMATION

In our case study, the vehicle is equipped with four wheel speed sensors, a steering wheel angle sensor, an accelerometer, and a yaw rate sensor. The yaw rate can be then directly controlled. However, both longitudinal and lateral speeds responses should be estimated. We use then the accelerometer and the yaw rate sensor to get these estimations by means of the following equations:

$$\begin{cases} V_x = \int a_x - V_y \dot{\psi} \\ V_y = \int a_y + V_x \dot{\psi} \end{cases} \quad (8)$$

$$(9)$$

Where a_x and a_y represent the longitudinal and lateral accelerations respectively

The goal is to be able to control the vehicle even when friction changes, without any a priori knowledge about it. The main idea is to estimate the overall acceleration starting from a friction coefficient value of 1 and to compare it with the measured overall acceleration. If any difference is detected, this can be interpreted as a loss of adhesion, and therefore update the value of the friction coefficient as fast as possible to match the measured accelerations. The commands can then be reallocated to respect the friction ellipse and keep the vehicle controllable. In this case, the error between the measured accelerations and the estimated ones should come only from a loss of adhesion and not from a modeling error. As we go through a series of estimations, each estimation should be precise enough so modeling errors do not propagate, or worse, get amplified.

A. Tire Slip

The first stop is the tire slip. Tire forces originate from this phenomenon [6],[9]. We differentiate between the longitudinal slip and the side-slip.

1) *Longitudinal slip*: The longitudinal slip is defined most of the time as follows [6]:

$$s_{ij} = \frac{R_{ij}\omega_{ij} - V_x}{\max(R_{ij}\omega_{ij}, V_x)} \quad (10)$$

Where:

- R_{ij} : $i - j$ effective rolling radius of the tire,
- ω_{ij} : wheels' angular speed.

The “max” function is used to avoid singularity.

To simplify, V_x is usually considered as the vehicle's velocity at its CoG. But as several authors have reported [9],[19], it is the component of the tire velocity along its longitudinal axis that should be considered. While several works did take into account this aspect by adding the influence of the yaw rate [14], the tire velocity should be actually calculated at the tire/road interface as long as the slip is concerned. The vertical distance between the vehicle's CoG and the road level introduces therefore roll and pitch dynamics. This is mainly caused by the sprung mass motions by means of suspensions, inducing rubber crushing into the road surface. This modifies slip values and therefore tire forces. The longitudinal slip is then calculated here as follows:

$$s_{ij} = \frac{R_{ij}\omega_{ij} - v_{wx_{ij}}}{\max(R_{ij}\omega_{ij}, v_{wx_{ij}})} \quad (11)$$

Where $v_{wx_{ij}}$ is the component of the tire velocity along its longitudinal axis at the tire/road interface. It should be noted that this variable is calculated at the steered non-cambered frame. Camber has little influence in our case and the vehicle is not equipped by any camber sensor. Calculation of $v_{wx_{ij}}$ is done in two steps. First, we express the tire velocity in the non-steered frame using Varignon's theorem, then we determine the velocity value at the steered frame. Let us call “G” the vehicle's CoG, and “ P_{fl} ” the center of the contact area of the front-left tire and the road surface. We have then:

$$\overrightarrow{V(P_{fl})} = \overrightarrow{V(G)} + \overrightarrow{P_{fl}G} \wedge \overrightarrow{\Omega} \quad (12)$$

With $\overrightarrow{\Omega}$ is the rotational vector of the sprung mass. This gives:

$$\begin{cases} V_x(P_{fl}) \\ V_y(P_{fl}) \\ V_z(P_{fl}) \end{cases} = \begin{cases} V_x - \frac{t}{2}\dot{\psi} - h_G\dot{\theta} \\ V_y + l_f\dot{\psi} + h_G\dot{\phi} \\ V_z - l_f\dot{\theta} + \frac{t}{2}\dot{\phi} \end{cases} \quad (13)$$

Where:

- $\dot{\phi}$: the roll velocity,
- $\dot{\theta}$: the pitch velocity,
- h_G : height of the CoG with respect to the ground.

Consequently, in the steered frame we have:

$$\begin{cases} v_{wx_{fl}} = V_x(P_{fl}) \cos(\delta_f) + V_y(P_{fl}) \sin(\delta_f) \\ v_{wy_{fl}} = -V_x(P_{fl}) \sin(\delta_f) + V_y(P_{fl}) \cos(\delta_f) \end{cases} \quad (14)$$

The same method is adopted for the remaining wheels. Care should be given to the signs.

2) *Side-slip*: The side-slip, noted here α_{ij} , can be defined as follows [20]:

$$\tan(\alpha_{ij}) = \frac{v_{wy_{ij}}}{v_{wx_{ij}}} \quad (15)$$

Once again, several papers consider only the longitudinal velocity of the vehicle's CoG, and calculate the lateral velocity at the non-steered tire frame [20],[21]. Even the

most rigorous calculations that take into account the steered tire frame do not take into account vertical dynamics in the side-slip calculation [14]. Here, the side-slip is calculated using expressions as it was shown in equations: (12)-(14).

3) *Roll dynamics*: Slip calculations are based on additional variables. These vertical dynamics related variables are not measured in our case-study vehicle. Effect-based method are used to estimate these variables. Roll dynamics are mainly caused by the vehicle's lateral acceleration [22]. By taking into account the spring and damper ratings of the four suspensions¹, roll dynamics can be expressed as follows:

$$I_{xx}\ddot{\phi} = Ma_y h_G + M_s g h_G - 4K_{susp} \left(\frac{t}{2}\right)^2 \phi - 4C_{susp} \left(\frac{t}{2}\right)^2 \dot{\phi} \quad (16)$$

Where:

- K_{susp} : suspension's spring rate (N/m),
- C_{susp} : suspension's damper rate (N/(m/s)),
- M_s : sprung mass,
- g : gravitational acceleration,
- I_{xx} : vehicle's roll moment of inertia.

The advantage is that only lateral acceleration measurement is needed. A second-order transfer function is then used to determine the roll angle. The same method can be adopted for the pitch dynamics using this time the longitudinal acceleration measurement. It should be noted that the accelerometer do not differentiate between accelerations induced by the vehicle and the gravitational acceleration. Care should be given to signal processing in case of roads with slopes. Moreover, the considered vehicle is not equipped by anti-roll bars. This aspect should be considered for passenger cars.

B. Tire Forces

A tire model precise enough is needed in the estimation process. As tire forces depends on the vertical loads [6], not only ground forces should be estimated bu also vertical dynamics variations.

1) *Ground forces*: Several tire models exist in the literature. Reviews can be found in [6] and [19]. In contrast with the control strategy where a simple tire model is needed, the objective here is to select a precise model, even if it presents some nonlinearities. The most precise one is Pacejka's model, or what is referred to as the *Magic Formula* [6]. It is a semi-empirical model fitted to several experimental data. However, this model depends on numerous parameters with no physical significance. As our goal is to estimate the friction coefficient, we prefer to choose a physical model that depends on one parameter representing fiction variations. Consequently, Dugoff's model has been chosen here [9]. This model depicts well the combined slip phenomenon, but only in the stable zone of the tire. In this work, this do not represent any drawback, because the control allocation strategy is based on the friction ellipse. This prevents the

¹Considered here identical.

vehicle from entering the non stable zone of the tire. Dugoff's model can be expressed as follows:

$$\begin{cases} F_{x_{ij}} = C_s \frac{s_{ij}}{1 - s_{ij}} \tau_{ij} \\ F_{y_{ij}} = C_\alpha \frac{\tan(\alpha_{ij})}{1 - s_{ij}} \tau_{ij} \end{cases} \quad (17)$$

$$\begin{cases} F_{x_{ij}} = C_s \frac{s_{ij}}{1 - s_{ij}} \tau_{ij} \\ F_{y_{ij}} = C_\alpha \frac{\tan(\alpha_{ij})}{1 - s_{ij}} \tau_{ij} \end{cases} \quad (18)$$

τ is introduced to take account of the combined slip:

$$\tau_{ij} = \begin{cases} (2 - \sigma_{ij}) \sigma_{ij} & \text{if } \sigma < 1 \\ 1 & \text{otherwise} \end{cases} \quad (19)$$

With:

$$\sigma_{ij} = \frac{(1 - s_{ij}) \mu F_{z_{ij}}}{2\sqrt{C_s^2 s_{ij}^2 + C_\alpha^2 \tan^2(\alpha_{ij})}} \quad (20)$$

Where:

- C_s : longitudinal stiffness of the tire,
- C_α : lateral stiffness of the tire,
- $F_{z_{ij}}$: vertical load.

2) *Vertical loads*: Vertical loads vary with accelerations. Most of the works use directly accelerations measurement to represent vertical loads variations [14],[23],[24]. However,

the vertical load impact first the suspensions, then the tires. Using accelerations signals, which vary sometimes very quickly, give noisy and imprecise vertical loads signals. Here, we use rather the roll and pitch dynamics, which give filtered and more representative signals according to equation: (16). Vertical loads are then expressed as:

$$\begin{cases} F_{z_{fl}} = \frac{1}{2} M_s g \frac{l_r}{L} + K_{susp} \frac{t}{2} \phi + C_{susp} \frac{t}{2} \dot{\phi} \\ \quad - K_{susp} l_f \theta - C_{susp} l_f \dot{\theta} \end{cases} \quad (21)$$

$$\begin{cases} F_{z_{fr}} = \frac{1}{2} M_s g \frac{l_r}{L} - K_{susp} \frac{t}{2} \phi - C_{susp} \frac{t}{2} \dot{\phi} \\ \quad - K_{susp} l_f \theta - C_{susp} l_f \dot{\theta} \end{cases} \quad (22)$$

$$\begin{cases} F_{z_{rl}} = \frac{1}{2} M_s g \frac{l_f}{L} + K_{susp} \frac{t}{2} \phi + C_{susp} \frac{t}{2} \dot{\phi} \\ \quad + K_{susp} l_r \theta + C_{susp} l_r \dot{\theta} \end{cases} \quad (23)$$

$$\begin{cases} F_{z_{rr}} = \frac{1}{2} M_s g \frac{l_f}{L} - K_{susp} \frac{t}{2} \phi - C_{susp} \frac{t}{2} \dot{\phi} \\ \quad + K_{susp} l_r \theta + C_{susp} l_r \dot{\theta} \end{cases} \quad (24)$$

With $L = l_f + l_r$ being the wheelbase.

C. Accelerations Estimation

Once tire forces are estimated for each tire, accelerations can be estimated as follows:

$$\begin{bmatrix} a_{x_{est}} \\ a_{y_{est}} \end{bmatrix} = \begin{bmatrix} \cos(\delta_f) & \cos(\delta_f) & \cos(\delta_r) & \cos(\delta_r) & -\sin(\delta_f) & -\sin(\delta_f) & -\sin(\delta_r) & -\sin(\delta_r) \\ \sin(\delta_f) & \sin(\delta_f) & \sin(\delta_r) & \sin(\delta_r) & \cos(\delta_f) & \cos(\delta_f) & \cos(\delta_r) & \cos(\delta_r) \end{bmatrix} \begin{bmatrix} F_{x_{fl}} \\ F_{x_{fr}} \\ F_{x_{rl}} \\ F_{x_{rr}} \\ F_{y_{fl}} \\ F_{y_{fr}} \\ F_{y_{rl}} \\ F_{y_{rr}} \end{bmatrix} \quad (25)$$

The global horizontal acceleration can be then calculated as follows:

$$a_{g_{est}} = \sqrt{a_{x_{est}}^2 + a_{y_{est}}^2} \quad (26)$$

D. Friction Estimation Strategy

All these estimation layers serve to generate a sufficiently accurate global acceleration estimation. The idea is then to compare this estimation to the measured one. If the difference between the two signals starts to grow, this would mean that friction is no longer sufficient to generate the desired acceleration, and we can therefore update the control logic to avoid the loss of control of the vehicle. In fact, as it was reported in [1] and [2], the speed profile of a path should be generated while respecting the friction circle², which can be represented as:

$$\sqrt{a_x^2 + a_y^2} \leq \mu g \quad (27)$$

²Considering an isotropic friction.

“ μg ” represents then the maximum achievable global acceleration. Therefore, by using an effect-based estimation method, μ can only be estimated at the limits of handling. In other words, if the estimated acceleration exceeds the measured one $a_{g_{mes}}$, this means that the real acceleration has reached its maximum “ μg ”. We can simply divide $a_{g_{mes}}$ by g to get μ . Moreover, by updating the control in low friction surfaces, the performance would be limited. The algorithm should be able to improve the vehicle's performances when regaining a high friction surface. In this paper, we simply increase gradually the value of the friction coefficient without destabilizing the overall system. The idea is to continually test the surface potential, and benefit from its maximum when needed, without jeopardizing vehicle's stability. It should be noted that a threshold that takes account of the remaining modeling errors has been added.

IV. CO-SIMULATION RESULTS

To test the control logic and the estimation benefits, a co-simulation procedure is followed. Our motivation lays on the fact that we want to test advanced control algorithms on complex high-fidelity vehicle models. Matlab/Simulink® is chosen to incorporate the control logic with the different estimators due to its efficient numerical computations. The high-fidelity vehicle model is developed in LMS Imagine.Lab AMESim® thanks to its large vehicle dynamics library. The interesting use-cases to evaluate manifest when vertical dynamics vary and when the friction changes. The double lane change maneuver referring to the ISO 3888-1:1999(E) standard is then selected to excite vertical dynamics in a severe maneuver. Moreover, to test the controller adaptability thanks to the estimation process, we force the friction to change in the middle of the maneuver.

The severe double lane-change maneuver is a dynamic process consisting of rapidly driving a vehicle from its initial lane to another lane parallel to the initial one, and returning to the initial lane. According to the ISO 3888-1:1999(E) standard, the vehicle's speed should keep the value of 80 km/h during all the maneuver. We reproduce this in AMESim, and we add a little twist, we change the friction coefficient from 1 to 0.4 at the second lane (at the time $t=3.5s$), then we bring it back to 1 at the final lane (Fig. 1).

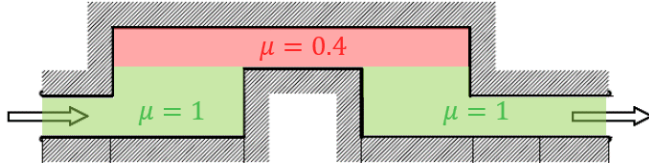


Fig. 1. ISO 3888-1:1999(E) standard with friction variation.

We compare performance of the control logic without friction change, with friction change and without vertical dynamics estimation, then by taking them into account. We get the results for the controlled variables when vertical dynamics are ignored, namely, the lateral speed (Fig. 2), and the yaw rate (Fig. 3).

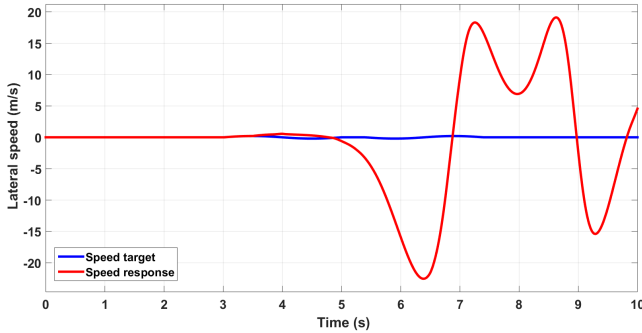


Fig. 2. Lateral speed control when vertical dynamics are ignored.

We notice the complete loss of control of the vehicle as soon as we enter the low friction surface. We choose to represent results when taking into account vertical dynamics

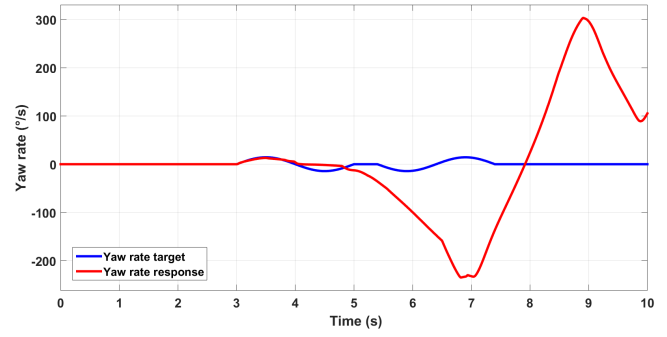


Fig. 3. Yaw rate control when vertical dynamics are ignored.

in a separate figure to show the effectiveness of the control logic by zooming into the signals (Fig. 4-5).

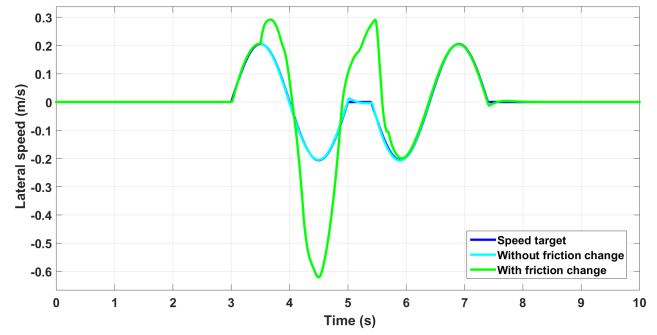


Fig. 4. Lateral speed control when vertical dynamics are considered.

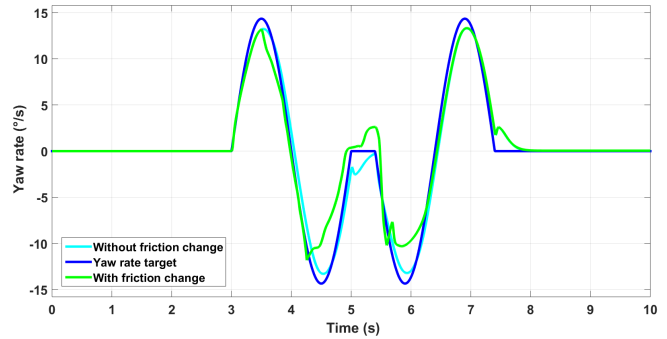


Fig. 5. Yaw rate control when vertical dynamics are considered.

We can see then the ability of the control strategy to keep the vehicle controllable. The reason is simply that with a better estimation of the vehicle dynamics by taking into account the vertical ones, we can get a better approximation of the road friction (Fig. 6). This enables updating the friction ellipse that represents the limits for the control allocation strategy. The different chassis systems can then produce commands within the limits of adhesion, and keep the vehicle stable. Note that ignoring vertical dynamics leads to a drop in friction estimation at the beginning of the double lane change maneuver and before the real drop of friction. This shows that a bad estimation of the lateral acceleration can simply lead to a bad interpretation.

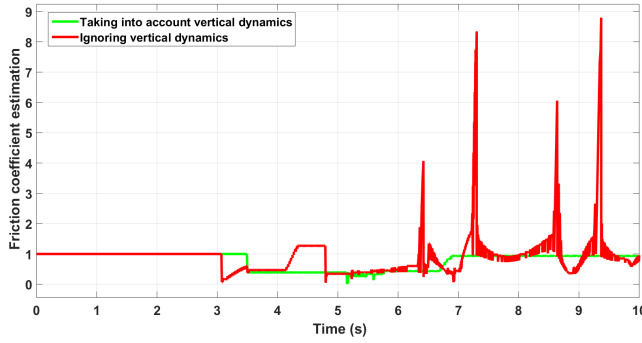


Fig. 6. Importance of vertical dynamics for friction estimation.

It should be noted that vertical dynamics do not interfere in the calculation of the vertical loads only, but also in the slip calculations. These latter change even the shape of the tire forces while the vertical loads influence their amplitude. Therefore, taking into account vertical dynamics in the vertical loads only is not sufficient.

V. RESEARCH TRACKS FOR A BETTER CONTROL

The aim of this paper is to show that thanks to a better comprehension of vehicle dynamics, a better estimation of vehicle's states can be provided. This can help to keep the vehicle controllable even in extreme and unpredictable situations. However, there is still some work to be done in this context. The use of a threshold in the μ estimation algorithm shows that there are still some modeling errors that should be separated from environmental changes. Two different causes can be separated. The first one is the simplification of static nonlinear phenomena, e.g., camber angles, aerodynamics effects and so on. And the second one is more related to dynamic phenomena, e.g., tire wear, actuators degradation and so on. Even if a robust controller is used in the high-level layer to overcome parameter uncertainties, the estimation process makes use of the vehicle's initial parameter. Consequently, the additional phenomena would affect directly the precision of the estimation.

A. Machine Learning

In this paper, we suppose that the lack of acceleration ability is due only to friction variation. In real life, accelerations can be affected by the wind velocity and slopes [25]. In addition, slopes add the influence of the gravitational acceleration in the acceleration measurement. This makes the velocities estimation and the problem diagnosis more challenging. To top it all off, even the suspensions' spring rate and damper rating are variable, axles' elastokinematics add additional nonlinearities, the mass varies with respect to the number of passengers and their objects which can change the position of the center of gravity, the wheels suffer from the camber effects that influence slip value [6] and so on.

One possible solution in this case is to consider the effect of the remaining not modeled phenomena as nonlinear unknown functions. The estimators presented in this paper can be separated into simple functions, where each function

can represent a neuron. A Neural Network can be developed where its inputs could be the online measured variables, its output would be the global acceleration, and the different estimation layers would represent hidden layers [26]. As the proposed inputs and outputs are measurable data, the neural network can be trained using real experiments data to estimate the remaining not modeled variations [27].

As the previous method is more adapted to the static not modeled phenomena, a different method could be needed for the neglected dynamic variations. A model-based Monte Carlo method could fill the gap in this case. In [28], this method has been used for estimating failure probability of a component subject to degradation. Particularly, the fatigue degradation is modeled by the Paris-Erdogan law. The robustness of the results suggest applying these methods to estimate the remaining life of actuators and updating the prediction to avoid hazardous situations. Both methods can be combined for eventual better performances using Supervised Machine Learning as authors of [29] propose.

B. Robotic Vision

Most of autonomous vehicles researches are actually based on Model Predictive Control (MPC) [30]. This approach does not need only an accurate vehicle model, but also some *prediction* of future events. In [30], an a priori knowledge about μ is assumed. The problem that raises in this configuration is the fact that we do not need to know the friction coefficient at the current situation only, but future variations of the friction should also be known. This cannot be ensured by means of effect-based estimation techniques. Robotic Vision could give insights about friction changes in front of the vehicle. Recently, authors of [31] have proposed a method to estimate the coefficient of friction by analyzing photo images using at first a vertical camera, and then a front camera that could that could be used by a robot by applying a discriminant analysis. The method consists on taking preliminary pictures and measure the coefficient friction, and then train the image processor in order to match new photos to a coefficient friction value. This was tested in twelve samples of the floor tiles that the authors have found on their campus at low velocities. Additional difficulties could be encountered for high velocities and for real roads going from icy to dry ones. We believe that a combination of effect-based methods and robotic vision could be needed. Robotic vision would warn of a potential friction change, and the effect-based method would give the right estimation of friction.

C. New Sensor Technology

For more accurate vertical dynamics signals, measured signals related to the wheel's load could provide better performances. In fact, load-sensing bearings and intelligent tires are capable of providing additional information regarding tire/road contact per wheel. In [32] for example, a novel model based bearing load measurement approach is presented. Instead of strain filtering and in-situ mapping, the paper expose a model based reconstruction approach. An unscented Kalman Filter is used to reconstruct the unknown

wheel loads by analysis of the bearing's outer-ring. Results showed good reconstruction of tire forces and moments signals. Therefore, the remaining states could be more accurate to ensure a better friction coefficient estimation.

Intelligent tires as in [33] are developed by placing accelerometers on the inner liner of the tire. A noticeable difference has been shown regarding the acceleration response when the tire was tested on different surface conditions. This variation may present an opportunity to characterize directly the friction coefficient.

VI. CONCLUSIONS

In this paper we showed that more accurate vehicle dynamics modeling is needed for control problems. Spotlight has been especially put on vertical dynamics that interfere in longitudinal slip and side-slip calculations. This rigor in calculations enabled us to get better estimations of vehicle states. Therefore, estimations can be compared to available measures provided by in-vehicle sensors to detect nonlinear irregularities, e.g. friction variations. The control logic can be then adapted to control the vehicle in severe conditions.

The simulations presented in this paper consider availability of accurate measurements. Additional difficulties are expected in case of real experiments that may need additional efforts to adapt this strategy. Due to the importance of friction estimation, we expect more collaboration from car manufacturers to carry experiments campaigns.

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