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Robust Design of an Automatic Emergency Braking System Considering Sensor Measurement Errors

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***Abstract*—Vehicular safety functions that intervene in dangerous driving situations increase automotive safety. As such functions use the measurements of sensors in order to interpret the driving situation, unavoidable sensor measurement errors have a negative impact on both the safety and the satisfaction of the customer. In this paper, a new methodology for the robust design of an automatic emergency braking (AEB) system considering sensor measurement errors is proposed. Based on a stochastic model, the robust design is formulated as an optimization problem, whose solution results in the optimal values for the parameters of the AEB system, i.e., of both the sensor and the function it is based on, with respect to a probabilistic quality measure. Besides this joint function and sensor design, the robust sensor design for a given function and the robust function design for a given sensor considering sensor measurement errors in case of the AEB system are formulated as optimization problems using the same probabilistic quality measure. An expression for this probabilistic quality measure is derived, and the elaborated theoretical results are illustrated and verified by numerical examples. Moreover, the new design methodology provides the designer with a design space for choosing the parameter values.**

I. Introduction

Besides electrification, connectivity and diverse mobility, autonomous driving is one of the disruptive technology-driven trends in the automotive sector [1]. Although a lot of research is still required in order to make the vision of a fully autonomous car come true, several advanced driver assistance systems (ADAS) that perform at least temporarily single aspects of the driving task are already available. Vehicular safety functions[1] that intervene in dangerous driving situations form an important class of those ADAS. They reduce the number as well as the severity of collisions and thus increase automotive safety.

Fig. 1 depicts the general setting in which a vehicular safety function is embedded. Sensors measure quantities like distances, velocities, accelerations, etc. and thus enable the perception of the environment of a vehicle while the function uses the measurements of the sensors in order to interpret the driving situation and trigger appropriate actions in dangerous driving situations, e.g., an emergency brake intervention. Due to unavoidable sensor measurement errors, a false interpretation of the driving situation is possible, which has a negative impact on both the safety and the satisfaction of the customer. Therefore, these sensor measurement errors have to be taken into account when designing such functions and sensors such that the requirements of the customers are met in a robust manner despite the unavoidable sensor measurement errors.

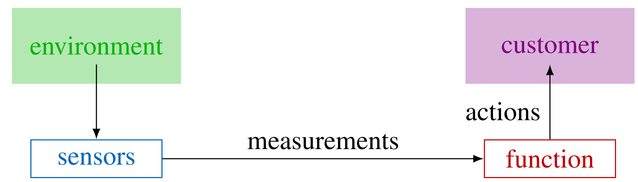

Fig. 1. General setting in which a vehicular safety function is embedded.

A measure for the robustness of decision functions used in active safety systems for deciding on interventions to sensor measurement errors is introduced in [2]. Based on Monte Carlo simulations, [3] investigates the effect of sensor measurement errors on the uncertainty of collision warning criteria used in collision warning systems and [4] analyzes their influence on the accuracy of predicted collision parameters like the time-to-collision (TTC) used in predictive passive safety systems. Closed-form expressions for the probability distributions of criticality measures used in ADAS like the TTC, which are subject to the uncertainty in the prediction of the future evolution of a situation and sensor measurement errors, are derived in [5]. The framework presented there is applied in [6] in order to analyze the impact of prediction uncertainty and sensor measurement errors on the performance of automatic emergency braking (AEB) systems. For vehicle localization, [7] proposes an approach to sensor design, which derives sensor parameters from requirements for the localization accuracy based on a probabilistic model taking sensor measurement errors into account.

In this paper, a new methodology for the robust design of an AEB system considering sensor measurement errors is proposed, which is capable of determining the parameters of both the sensor and the function it is made up of. Based on a stochastic model, the robust design is formulated as an optimization problem, whose solution results in the optimal parameter values with respect to a probabilistic quality mea-

[1]In automotive engineering, the word “function” refers to an application implemented in a vehicle in order to fulfill a particular purpose and is not to be confused with a mathematical function.

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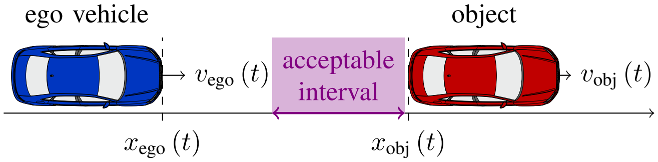


Fig. 2. Considered scenario at time t.

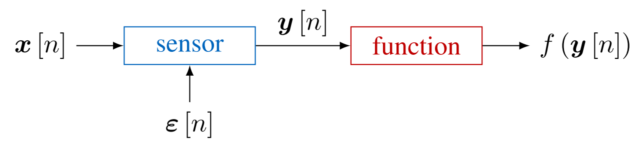


Fig. 3. General mathematical model of a vehicular safety system including sensor measurement errors.

sure inspired by ideas from the approach to the worst-case design of integrated circuits presented in [8]–[10]. Besides this joint function and sensor design, the robust sensor design for a given function and the robust function design for a given sensor considering sensor measurement errors in case of the AEB system are formulated as optimization problems using the same probabilistic quality measure.

The paper is organized as follows. Section II introduces the system model for an AEB system. In Section III, the proposed methodology for the robust design of an AEB system considering sensor measurement errors is described and an expression for the probabilistic quality measure is derived. Section IV presents numerical examples illustrating and verifying the elaborated results. Finally, Section V concludes the paper.

II. System Model

A. Representation of Considered Scenario

In order to describe the basic functionality of an AEB system, we consider the scenario illustrated in Fig. 2, where the ego vehicle is approaching an object, e.g., another vehicle, and make use of a one-dimensional motion model with piecewise constant accelerations as in [6]. At time $t \geq t_0$ within the considered time interval starting at t_0, the front of the ego vehicle moving with the velocity $v_{\text{ego}}(t)$ in longitudinal direction is located at the position $x_{\text{ego}}(t)$ along the longitudinal axis and the back of the object moving with the velocity $v_{\text{obj}}(t)$ in longitudinal direction is located at the position $x_{\text{obj}}(t)$ along the longitudinal axis. The velocity of the object is assumed to be constant over the time t. Assuming $x_{\text{ego}}(t_0) < x_{\text{obj}}(t_0)$ and $v_{\text{ego}}(t_0) > v_{\text{obj}}(t_0)$, a collision would necessarily occur if the ego vehicle moved with constant velocity too. In order to avoid such a collision, an emergency brake intervention is triggered at time $t_{\text{b}} \geq t_0$ by the AEB system of the ego vehicle, which reduces its velocity with a constant deceleration $a > 0$. Since only the relative motion of the ego vehicle and the object is relevant for the decision on whether to trigger an emergency brake intervention, the state of the considered dynamic system at time t is sufficiently described by the state vector $\boldsymbol{x}(t) = [x(t), v(t)]^{\mathrm{T}}$ consisting of only two state variables, namely, the distance

$$\begin{aligned} x(t) &= x_{\text{obj}}(t) - x_{\text{ego}}(t) \\ &= \begin{cases} x_0 + v_0 (t - t_0), & t \leq t_{\text{b}} \\ x_0 + v_0 (t - t_0) + \frac{1}{2} a (t - t_{\text{b}})^2, & t > t_{\text{b}} \end{cases} \end{aligned} \tag{1}$$

between the ego vehicle and the object, and their relative velocity

$$v(t) = v_{\text{obj}}(t) - v_{\text{ego}}(t) = \dot{x}(t) = \begin{cases} v_0, & t \leq t_{\text{b}} \\ v_0 + a (t - t_{\text{b}}), & t > t_{\text{b}} \end{cases} \tag{2}$$

at time t, where $x_0 = x(t_0) = x_{\text{obj}}(t_0) - x_{\text{ego}}(t_0) > 0$ and $v_0 = v(t_0) = v_{\text{obj}}(t_0) - v_{\text{ego}}(t_0) < 0$ are the initial distance and relative velocity, respectively.

B. Stochastic Model of AEB System

Fig. 3 shows the general mathematical model of a vehicular safety system including sensor measurement errors. The sensor takes measurements with a sampling rate f_{s} at time instants $t_n = \frac{n}{f_{\text{s}}}$ with the discrete time index $n = 0, 1, \ldots$ and delivers the measurement vector $\boldsymbol{y}[n] \in \mathbb{R}^M$ consisting of the measurements of M quantities observed by the sensor at the time instant t_n based on the state vector $\boldsymbol{x}[n] = \boldsymbol{x}(t_n) \in \mathbb{R}^N$ consisting of the values of the N relevant quantities representing the state of the considered dynamic system at the time instant t_n under the influence of the measurement errors made by the sensor at the time instant t_n, which are collected in the error vector $\boldsymbol{\varepsilon}[n] \in \mathbb{R}^M$. The vehicular safety function derives safety-relevant information $f(\boldsymbol{y}[n])$ from the measurements $\boldsymbol{y}[n]$ at the time instant t_n using a function

$$f : \mathbb{R}^M \to \mathbb{R}, \boldsymbol{y}[n] \mapsto f(\boldsymbol{y}[n]) \tag{3}$$

in order to interpret the current driving situation and decide on appropriate actions.

In case of the considered AEB system, the state vector is given by

$$\boldsymbol{x}[n] = \begin{bmatrix} x[n] \\ v[n] \end{bmatrix} = \begin{bmatrix} x(t_n) \\ v(t_n) \end{bmatrix} = \boldsymbol{x}(t_n). \tag{4}$$

Following [3], the measurement errors made by the distance sensor at the time instants t_n are modeled by i.i.d. zero-mean Gaussian random variables $\varepsilon[n] \sim \mathcal{N}(0, \sigma^2)$ of standard deviation σ and it is assumed that the measured relative velocity is error-free such that the measured distance and relative velocity at the time instant t_n read

$$\hat{x}[n] = x[n] + \varepsilon[n] \sim \mathcal{N}(x[n], \sigma^2) \text{ and } \hat{v}[n] = v[n], \tag{5}$$

respectively. Therefore, the measurement vector at time instant t_n can be written as

$$\boldsymbol{y}[n] = \begin{bmatrix} \hat{x}[n] \\ \hat{v}[n] \end{bmatrix} = \boldsymbol{x}[n] + \boldsymbol{\varepsilon}[n] \tag{6}$$

with the error vector $\boldsymbol{\varepsilon}[n] = \left[\varepsilon[n], 0\right]^{\mathrm{T}}$. At the time instant t_n, the function computes the TTC

$$t_{\mathrm{TTC}}[n] = -\frac{\hat{x}[n]}{\hat{v}[n]} = f(\boldsymbol{y}[n]), \tag{7}$$

i.e., the time that remains until the ego vehicle and the object collide under the assumption that they move with constant relative velocity $\hat{v}[n]$ from the time instant t_n on, from the measurements $\boldsymbol{y}[n]$. Based on this TTC, which serves as safety-relevant information $f(\boldsymbol{y}[n])$, an emergency brake intervention is triggered at the time instant t_n if it has become too small or, more specifically, if it does not lie above a threshold τ, i.e.,

$$t_{\mathrm{TTC}}[n] = f(\boldsymbol{y}[n]) \leq \tau. \tag{8}$$

The smallest n for which this condition is fulfilled is the discrete time index n_{b} that corresponds to the time instant

$$t_{\mathrm{b}} = t_{n_{\mathrm{b}}} = \frac{n_{\mathrm{b}}}{f_{\mathrm{s}}} \tag{9}$$

at which the emergency brake intervention is triggered.

III. Robust Design of AEB System

A. Joint Function and Sensor Design

The considered AEB system has two parameters, namely, the standard deviation σ of the measurement errors in (5) and the threshold τ in (8), which parameterize the sensor and the function, respectively. The robust design of the AEB system should determine the values of these parameters such that it meets the requirements of the customers in a robust manner despite the unavoidable sensor measurement errors. Therefore, this joint function and sensor design is formulated as determining the maximal tolerable standard deviation σ_{max} of the sensor measurement errors and the optimal threshold τ_{opt} of the function with respect to a quality measure $Q(\sigma, \tau)$, which depends on σ and τ, and is not allowed to lie below a required minimum quality level Q_{min}, by solving the optimization problem

$$(\sigma_{\mathrm{max}}, \tau_{\mathrm{opt}}) = \underset{\sigma \in \mathbb{R}^+, \tau \in \mathbb{R}}{\operatorname{argmax}}\ \sigma \quad \text{s.t.} \quad Q(\sigma, \tau) \geq Q_{\mathrm{min}}. \tag{10}$$

We adopted the worst-case design approach presented in [8]–[10], and found an appropriate quality measure $Q(\sigma, \tau)$ that reflects the customer satisfaction and takes sensor measurement errors into account. The customer is willing to accept the intervention by the AEB system if the final distance x_{end} between the ego vehicle and the object after the emergency brake intervention is neither too small nor too large. In other words, the final distance x_{end} has to lie in an acceptable interval $[x_{\mathrm{min}}, x_{\mathrm{max}}]$, which is visualized in Fig. 2 by the violet area and can be chosen in a user-specific way. The probability $P(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}})$ that the final distance x_{end}, which is subject to random sensor measurement errors, fulfills the given specification

$$x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}} \tag{11}$$

is used as probabilistic quality measure

$$Q(\sigma, \tau) = P(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}) \tag{12}$$

in the following. Consequently, the optimization problem (10) to be solved becomes

$$(\sigma_{\mathrm{max}}, \tau_{\mathrm{opt}}) = \underset{\sigma \in \mathbb{R}^+, \tau \in \mathbb{R}}{\operatorname{argmax}}\ \sigma \text{ s.t. } P(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}) \geq P_{\mathrm{min}} \tag{13}$$

with the required minimum probability P_{min} of fulfilling the specification as required minimum quality level $Q_{\mathrm{min}} = P_{\mathrm{min}}$.

The final distance x_{end} between the ego vehicle and the object after an emergency brake intervention is defined as the distance $x(t_{\mathrm{end}})$ at the time t_{end} when the emergency brake intervention ends, which is assumed to be the case when their relative velocity finally vanishes. With $v(t_{\mathrm{end}}) = 0$, it follows from (2) that $t_{\mathrm{end}} = t_{\mathrm{b}} - \frac{v_0}{a} > t_{\mathrm{b}}$ and from (1) that

$$x_{\mathrm{end}} = x(t_{\mathrm{end}}) = x\left(t_{\mathrm{b}} - \frac{v_0}{a}\right) = x_0 + v_0(t_{\mathrm{b}} - t_0) - \frac{v_0^2}{2a}. \tag{14}$$

Since $t_0 = 0$ and the time instant t_{b} at which the emergency brake intervention is triggered according to the criterion (8) is given by (9), the final distance after the emergency brake intervention can be rewritten as

$$x_{\mathrm{end}} = x_0 + \frac{n_{\mathrm{b}} v_0}{f_{\mathrm{s}}} - \frac{v_0^2}{2a}. \tag{15}$$

Hence, $x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}$ iff

$$\frac{f_{\mathrm{s}}}{v_0}\left(x_{\mathrm{max}} - x_0 + \frac{v_0^2}{2a}\right) \leq n_{\mathrm{b}} \leq \frac{f_{\mathrm{s}}}{v_0}\left(x_{\mathrm{min}} - x_0 + \frac{v_0^2}{2a}\right). \tag{16}$$

As the discrete time index n_{b} that corresponds to the time instant t_{b} at which the emergency brake intervention is triggered is a non-negative integer, the specification (11) for the final distance x_{end} after the emergency brake intervention is equivalent to the specification $n_{\mathrm{min}} \leq n_{\mathrm{b}} \leq n_{\mathrm{max}}$ for n_{b} with the lower bound

$$n_{\mathrm{min}} = \max\left(0, \left\lceil \frac{f_{\mathrm{s}}}{v_0}\left(x_{\mathrm{max}} - x_0 + \frac{v_0^2}{2a}\right)\right\rceil\right) \tag{17}$$

and the upper bound

$$n_{\mathrm{max}} = \left\lfloor \frac{f_{\mathrm{s}}}{v_0}\left(x_{\mathrm{min}} - x_0 + \frac{v_0^2}{2a}\right)\right\rfloor, \tag{18}$$

and to the specification

$$\frac{n_{\mathrm{min}}}{f_{\mathrm{s}}} = t_{n_{\mathrm{min}}} \leq t_{\mathrm{b}} \leq t_{n_{\mathrm{max}}} = \frac{n_{\mathrm{max}}}{f_{\mathrm{s}}} \tag{19}$$

for t_{b}. Therefore, the probability of fulfilling the specification (11) for the final distance x_{end} after the emergency brake intervention is identical to the probability of fulfilling the specification (19) for the time instant t_{b} at which the emergency brake intervention is triggered:

$$\begin{aligned} P(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}) &= P(t_{n_{\mathrm{min}}} \leq t_{\mathrm{b}} \leq t_{n_{\mathrm{max}}}) \\ &= \sum_{n=n_{\mathrm{min}}}^{n_{\mathrm{max}}} P(t_{\mathrm{b}} = t_n). \end{aligned} \tag{20}$$

In order to find an expression for the probability $P(t_\text{b} = t_n)$ that the time instant t_b at which the emergency brake intervention is triggered is the time instant t_n, the condition (8) for triggering an emergency brake intervention at a time instant t_n is considered. With (5) and (7), this condition reads

$$t_\text{TTC}[n] = -\frac{\hat{x}[n]}{\hat{v}[n]} = -\frac{x[n] + \varepsilon[n]}{v[n]} \leq \tau. \tag{21}$$

For $t_n = \frac{n}{f_\text{s}} \leq t_\text{b} = \frac{n_\text{b}}{f_\text{s}}$, i.e., $n \leq n_\text{b}$, it follows from (1) and (2) that the state vector at the time instant t_n from (4) is given by

$$\boldsymbol{x}[n] = \begin{bmatrix} x[n] \\ v[n] \end{bmatrix} = \begin{bmatrix} x_0 + \frac{nv_0}{f_\text{s}} \\ v_0 \end{bmatrix} \tag{22}$$

such that the condition for triggering an emergency brake intervention at a time instant t_n becomes

$$t_\text{TTC}[n] = -\frac{x_0 + \frac{nv_0}{f_\text{s}} + \varepsilon[n]}{v_0} \leq \tau. \tag{23}$$

This condition for the TTC $t_\text{TTC}[n]$ at the time instant t_n is equivalent to the condition

$$\varepsilon[n] \leq -x_0 - \left(\frac{n}{f_\text{s}} + \tau\right) v_0 \tag{24}$$

for the sensor measurement error $\varepsilon[n]$ at the time instant t_n. As a consequence, the probability that an emergency brake intervention is triggered at the time instant t_n, i.e., $t_\text{TTC}[n] \leq \tau$, is identical to the probability that this condition for $\varepsilon[n]$ is fulfilled:

$$P(t_\text{TTC}[n] \leq \tau) = P\left(\varepsilon[n] \leq -x_0 - \left(\frac{n}{f_\text{s}} + \tau\right) v_0\right). \tag{25}$$

Since $\varepsilon[n] \sim \mathcal{N}(0, \sigma^2)$, this probability can be expressed as

$$P(t_\text{TTC}[n] \leq \tau) = \Phi\left(-\frac{x_0 + (n/f_\text{s} + \tau) v_0}{\sigma}\right) \tag{26}$$

with the cumulative distribution function (cdf) $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-\xi^2/2} \mathrm{d}\xi$ of the standard normal distribution. The time instant t_b at which the emergency brake intervention is triggered is the time instant t_n if an emergency brake intervention is not triggered, i.e., $t_\text{TTC}[i] > \tau$, at all time instants t_i, $i = 0, 1, \ldots, n-1$, before the time instant t_n and, finally, an emergency brake intervention is triggered at the time instant t_n, i.e., $t_\text{TTC}[n] \leq \tau$. Therefore, the probability that the time instant t_b at which the emergency brake intervention is triggered is the time instant t_n reads

$$\begin{aligned} &P(t_\text{b} = t_n) = \\ &P(t_\text{TTC}[n] \leq \tau \wedge t_\text{TTC}[i] > \tau, i = 0, 1, \ldots, n-1). \end{aligned} \tag{27}$$

This probability can be split into the product of the probabilities of the individual events $\{t_\text{TTC}[n] \leq \tau\}$ and $\{t_\text{TTC}[i] > \tau\}$, $i = 0, 1, \ldots, n-1$, as these events are statistically independent due to the statistical independence of the predicted TTCs in the condition (21) for triggering an emergency brake intervention at different time instants t_n, which the assumed statistical independence of the sensor measurement errors $\varepsilon[n] \sim \mathcal{N}(0, \sigma^2)$ at different time instants t_n implies:

$$\begin{aligned} P(t_\text{b} = t_n) &= P(t_\text{TTC}[n] \leq \tau) \prod_{i=0}^{n-1} P(t_\text{TTC}[i] > \tau) \\ &= P(t_\text{TTC}[n] \leq \tau) \prod_{i=0}^{n-1} 1 - P(t_\text{TTC}[i] \leq \tau). \end{aligned} \tag{28}$$

Substituting this equation and (26) into (20) finally yields the expression

$$\begin{aligned} P(x_\text{min} \leq x_\text{end} \leq x_\text{max}) = &\sum_{n=n_\text{min}}^{n_\text{max}} \Phi\left(-\frac{x_0 + (n/f_\text{s} + \tau) v_0}{\sigma}\right) \\ &\cdot \prod_{i=0}^{n-1} 1 - \Phi\left(-\frac{x_0 + (i/f_\text{s} + \tau) v_0}{\sigma}\right) \end{aligned} \tag{29}$$

for the probability of fulfilling the specification (11) used as probabilistic quality measure $Q(\sigma, \tau)$.

B. Sensor Design

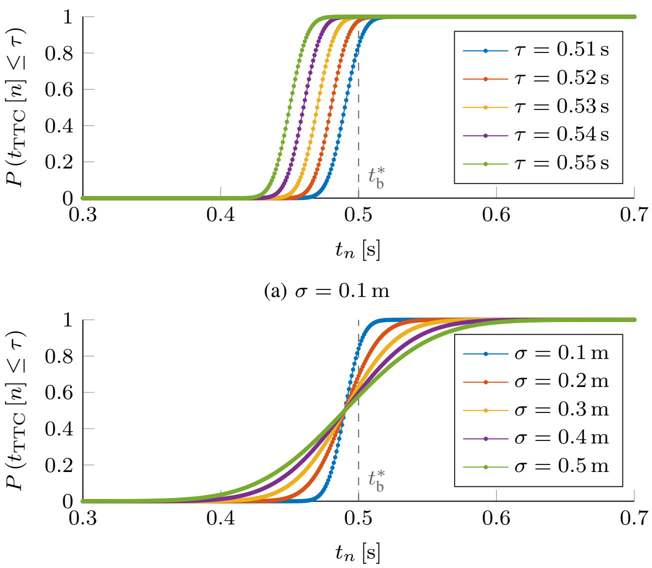
The joint function and sensor design for the AEB system determines the optimal values for the parameters of both the function and the sensor, namely, the optimal threshold τ_opt of the function and the maximal tolerable standard deviation σ_max of the sensor measurement errors, by solving the optimization problem (13). However, a value for the parameter of the function, i.e., a threshold τ, might already be given. In such a situation where the function is already given, a relevant question is which requirements the sensor has to fulfill such that the AEB system with the given function meets the requirements of the customers in a robust manner despite the unavoidable sensor measurement errors. This question can be answered by deriving sensor accuracy requirements in terms of the maximal tolerable standard deviation σ_max of the sensor measurement errors for the given threshold τ of the function by solving the optimization problem

$$\sigma_\text{max} = \underset{\sigma \in \mathbb{R}^+}{\operatorname{argmax}}\, \sigma \text{ s.t. } P(x_\text{min} \leq x_\text{end} \leq x_\text{max}) \geq P_\text{min}, \tag{30}$$

which is a slight modification of the optimization problem (13) to be solved in the joint function and sensor design for a given threshold τ.

C. Function Design

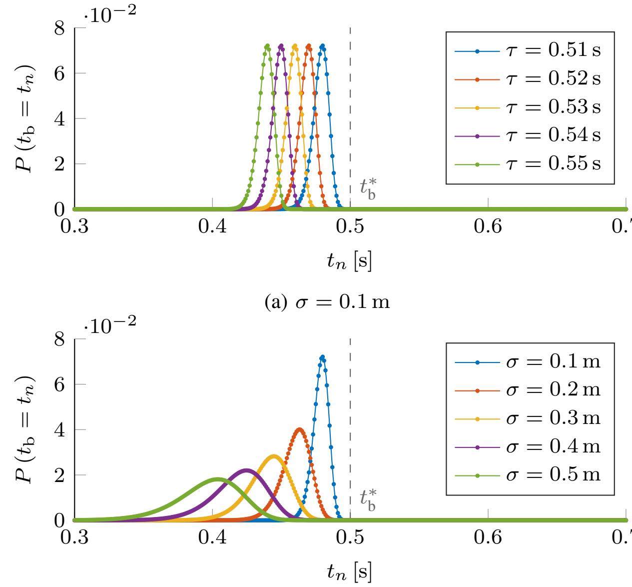
Instead of the function, the sensor to be used in the AEB system and, in particular, the standard deviation σ of its measurement errors might be given already. A relevant question that arises in such a situation is how the function has to be adapted to the given sensor such that the AEB system meets the requirements of the customers in a robust manner despite the unavoidable sensor measurement errors. The problem of optimally adapting the function to the given sensor can be tackled by choosing the threshold of the function



(a) $\sigma = 0.1\,\mathrm{m}$

(b) $\tau = 0.51\,\mathrm{s}$

Fig. 4. Probability that an emergency brake intervention is triggered at the time instant t_n for $x_0 = 10\,\mathrm{m}$, $v_0 = -10\,\frac{\mathrm{m}}{\mathrm{s}}$ and $f_{\mathrm{s}} = 1\,\mathrm{kHz}$.



(a) $\sigma = 0.1\,\mathrm{m}$

(b) $\tau = 0.51\,\mathrm{s}$

Fig. 5. Probability that the time instant t_{b} at which the emergency brake intervention is triggered is the time instant t_n for $x_0 = 10\,\mathrm{m}$, $v_0 = -10\,\frac{\mathrm{m}}{\mathrm{s}}$ and $f_{\mathrm{s}} = 1\,\mathrm{kHz}$.

for the given standard deviation σ of the sensor measurement errors to be

$$\tau_{\mathrm{opt}} = \underset{\tau \in \mathbb{R}}{\operatorname{argmax}}\, P\left(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}\right), \tag{31}$$

which maximizes the probability of fulfilling the specification (11) that the final distance after the emergency brake intervention lies in the acceptable interval, which has already been used as probabilistic quality measure in the optimization problem (13) of the joint function and sensor design, and the optimization problem (30) of the sensor design.

IV. Numerical Examples

In all presented numerical examples, the same scenario with initial distance $x_0 = 10\,\mathrm{m}$, initial relative velocity $v_0 = -10\,\frac{\mathrm{m}}{\mathrm{s}}$ and sampling rate $f_{\mathrm{s}} = 1\,\mathrm{kHz}$ is considered.

In Fig. 4, the probability $P\left(t_{\mathrm{TTC}}[n] \leq \tau\right)$ that an emergency brake intervention is triggered at the time instant t_n given by (26) is plotted over the time instants t_n for a fixed standard deviation $\sigma = 0.1\,\mathrm{m}$ of the sensor measurement errors and various values of τ as well as a fixed threshold $\tau = 0.51\,\mathrm{s}$ of the function and various values of σ. It can be observed that it is close to 0 at earlier time instants, increases over time and approaches 1 at later time instants for all values of τ and σ. However, it increases later if τ is smaller and in a shorter time if σ is smaller.

As can be seen in Fig. 5, where the probability $P\left(t_{\mathrm{b}} = t_n\right)$ that the time instant t_{b} at which the emergency brake intervention is triggered is the time instant t_n given by (28) is plotted over the time instants t_n for a fixed $\sigma = 0.1\,\mathrm{m}$ and various values of τ as well as a fixed $\tau = 0.51\,\mathrm{s}$ and various values of σ, there is a time interval in which this probability is significantly larger than zero for all values of τ and σ. Decreasing τ shifts this interval forth in time whereas decreasing σ not only shifts it forth in time but also reduces its width.

For the rest of this section, the deceleration is $a = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$, and the minimal and maximal tolerable final distances after the emergency brake intervention are $x_{\mathrm{min}} = 0$ and $x_{\mathrm{max}} = 0.5\,\mathrm{m}$. From (14), it follows that the time instant t_{b} at which the emergency brake intervention has to be triggered at the latest in order to avoid a collision such that $x_{\mathrm{end}} = 0$ is $t_{\mathrm{b}}^* = 0.5\,\mathrm{s}$, which is highlighted in the plots of Fig. 4 and Fig. 5.

In Fig. 6, the probability $P\left(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}\right)$ of fulfilling the specification (11) given by (29) is plotted over σ for various values of τ and over τ for various values of σ with solid lines. In order to verify these theoretical results elaborated in the previous section, the AEB system has been simulated based on the description of the system model in Section II. In the simulation, the final distance x_{end} has been obtained 10^6 times by drawing realizations of the i.i.d. sensor measurement errors from $\mathcal{N}\left(0, \sigma^2\right)$. The frequency of fulfilling the specification (11) is represented by crosses in Fig. 6. It can be concluded that the simulation results and the theoretical results accurately match.

Fig. 7 shows the contour lines of $P\left(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}\right)$ for constant values p. The gray area represents the set of all pairs (σ, τ) for which $P\left(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}\right) \geq P_{\mathrm{min}} = 0.99$. This is the design space the designer is provided with for choosing the parameter values. The solution $\left(\sigma_{\mathrm{max}}, \tau_{\mathrm{opt}}\right)$ of the optimization problem (13) to be solved in the joint function and sensor design for a required minimum probability $P_{\mathrm{min}} = 0.99$ is marked with a cross. It is the pair (σ, τ) for which $P\left(x_{\mathrm{min}} \leq x_{\mathrm{end}} \leq x_{\mathrm{max}}\right) \geq P_{\mathrm{min}} = 0.99$ and σ is maximum. This σ is the maximal tolerable standard deviation σ_{max} of the sensor measurement errors and the corresponding

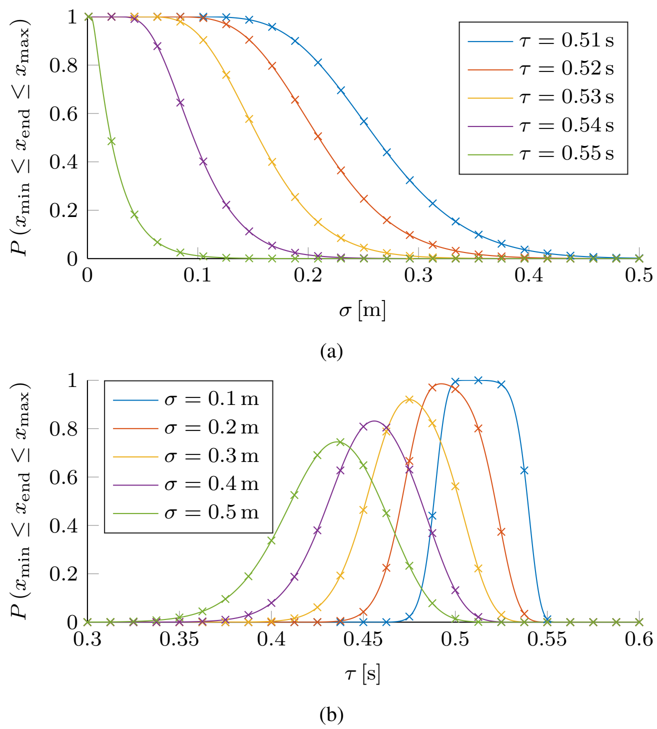


Fig. 6. Probability that the specification $x_{\min} \leq x_{\text{end}} \leq x_{\max}$ is fulfilled vs. σ (a) and τ (b) for $x_0 = 10\,\mathrm{m}$, $v_0 = -10\,\frac{\mathrm{m}}{\mathrm{s}}$, $f_\mathrm{s} = 1\,\mathrm{kHz}$, $a = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$, $x_{\min} = 0$ and $x_{\max} = 0.5\,\mathrm{m}$ with solid lines and crosses representing theoretical and simulation results, respectively.

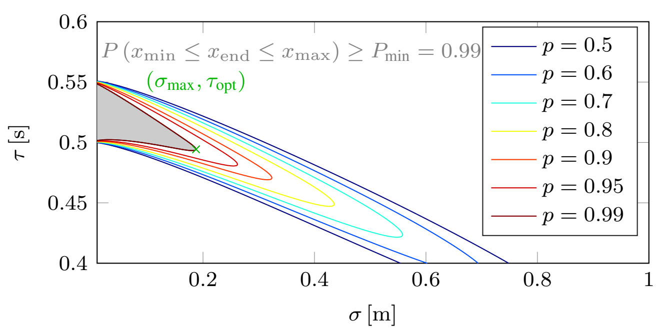


Fig. 7. Contour lines of $P(x_{\min} \leq x_{\text{end}} \leq x_{\max})$ at different heights p for $x_0 = 10\,\mathrm{m}$, $v_0 = -10\,\frac{\mathrm{m}}{\mathrm{s}}$, $f_\mathrm{s} = 1\,\mathrm{kHz}$, $a = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$, $x_{\min} = 0$ and $x_{\max} = 0.5\,\mathrm{m}$ with the optimal parameter value pair $(\sigma_{\max}, \tau_{\text{opt}})$ determined by the joint function and sensor design for $P_{\min} = 0.99$.

τ is the optimal threshold τ_{opt} of the function.

V. Conclusion

In this paper, a new methodology for the robust design of an AEB system considering sensor measurement errors has been proposed. Based on a stochastic model, the robust design has been formulated as an optimization problem, whose solution results in the optimal values for the parameters of the AEB system, i.e., of both the sensor, which measures the distance between the ego vehicle and the object, and the function, which interprets the driving situation and decides on whether to trigger an emergency brake intervention based on the measurements, with respect to a probabilistic quality measure inspired by ideas from an approach to the worst-case design of integrated circuits. Besides this robust design of the whole AEB system, which can be considered as a joint function and sensor design, the robust sensor design for a given function and the robust function design for a given sensor considering sensor measurement errors in case of the AEB system have been formulated as optimization problems using the same probabilistic quality measure as well. An expression for this probabilistic quality measure has been derived, and the elaborated theoretical results have been illustrated and verified by numerical examples. Moreover, the new design methodology provides the designer with a design space for choosing the values of the parameters of both the sensor and the function.

In order to basically describe the proposed design methodology for an AEB system, a simplified and idealized model has been used, where only two designable parameters and one source of measurement errors are considered. Future work includes the extension to more designable parameters and sources of measurement errors, where, in general, computations in closed form won't be possible. The proposed design methodology for an AEB system can be understood as special case of a more general methodology for the robust function and sensor design considering sensor measurement errors, which can be applied to design other vehicular safety systems as well.

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