2018 21st International Conference on Intelligent Transportation Systems (ITSC)
Maui, Hawaii, USA, November 4-7, 2018

Multi Stage Model Predictive Trajectory Set Approach for Collision Avoidance

Andreas Homann
Institute of Control Theory
and Systems Engineering, TU Dortmund
44227 Dortmund, Germany
andreas2.homann@tu-dortmund.de

Markus Buss
Active & Passive Safety Technology
ZF Group
40547 Düsseldorf, Germany

Martin Keller
Active & Passive Safety Technology
ZF Group
40547 Düsseldorf, Germany

Karl-Heinz Glander
Active & Passive Safety Technology
ZF Group
40547 Düsseldorf, Germany

Torsten Bertram
Institute of Control Theory
and Systems Engineering, TU Dortmund
44227 Dortmund, Germany

***Abstract*—The presented approach combines the planning of trajectories and the vehicle control during emergency maneuvers. For this purpose an approach is utilized, which predicts the future behavior of the actuators and the vehicle with a nonlinear model. The input space is roughly discretized and a trajectory set is calculated explicitly. The choice of optimal inputs is performed by a direct comparison of the possible trajectories, in contrast to model predictive control. The discretization is carried out adaptively depending on the current reference input. Issues arising from the limited degree of freedom are solved by an additional transition time within the prediction horizon. Model inaccuracies are taken into account during the objective function evaluation, by utilizing a soft constraint function, which increase the distance to objects and street boundaries.**

***Index Terms*—Intelligent vehicles, Emergency Collision Avoidance, Model Predictive Control**

I. Introduction

In the last decade, intensive efforts have been made to develop and improve driver assistance systems. Due to the technological progress towards sensor technologies and computational capacity more complex and powerful systems can be integrated into vehicles. The development begins with systems assisting the driver in vehicle stabilization (e.g. anti-lock braking systems or electronic stability control), through systems which take control of either longitudinal or lateral vehicle guidance (e.g. adaptive cruise control or lane keeping), all the way to systems which controls the longitudinal and lateral dynamics of the vehicle (e.g. traffic jam assistant). If the assistant system takes control of either the lateral or longitudinal direction, the system relies not only on the vehicle dynamic state but also on environmental information sensors like camera and radar. Due to uncertainties and the limited range of the current sensor technologies one cannot detect obstacles unless they are already close thus autonomous braking systems are not sufficient in all circumstances. In such situations, an evasive maneuver can lead to a collision free trajectory. Though the average driver is incapable of controlling the vehicle in critical situations, an assistant system can support the driver by steering maneuvers [1]. To achieve the highest collision avoidance potential, additionally combined steering and braking interventions can be applied. The first systems are based on path following approaches [1], [2], [3] or [4]. As a first step, a static collision free path is usually planned with the help of a basic function like a polynomial or sigmoid curve. In a second step, underlying controllers are utilized to guide the vehicle along that path. Due to the static path, an adaption on complex and time variable scenarios is not easily feasible. Another methods for collision avoidance systems are trajectory following approaches [5], [6], [7] and [8]. These systems have the benefit that the trajectory can adapt to different situations because of more flexible structure and the periodical recalculation. However underlying controllers are still required to follow the trajectory. The design of such controller system is a time consuming issue. In [9] the planning is based on a vehicle model. Moreover, planning and control can be combined for collision avoidance systems. Frasch et al. [10] present a model predictive control (MPC) algorithm which involves the collision avoidance in the objective function. Whereas Götte et al. [11]. utilize a general model predictive planning and control (MPPC) approach for lateral vehicle guidance. Due to the simultaneous MPPC, the complex task of designing a nonlinear vehicle control system to follow a trajectory is not necessary. The high computational effort is the main disadvantage of MPPC. Therefore Keller et al. [13], [14] presented an approach which reduces the number of calculations. The method bases on the prediction of a trajectory set similar to von Hundelshausen et al. [12] and can be extended to general control-oriented applications [15]. The paper at hand is an enhancement to the model predictive trajectory set approach and is divided into six sections: Section II presents the prediction model utilized for the collision avoidance approach. Afterward, the improvements of the approach and the objective function are derived. Subsequently, in section IV and V the presented approach is analyzed with simulation results. Finally, the paper is summarized and concluded.

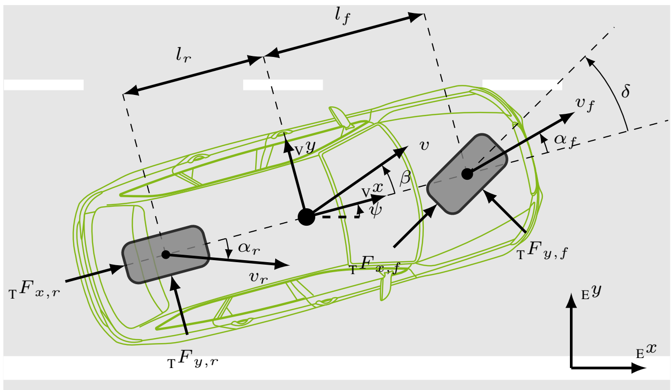

Fig. 1. Top view of single track model utilized for prediction. Lower left indexes E, V and T indicate the earth, vehicle and tire coordinate systems.

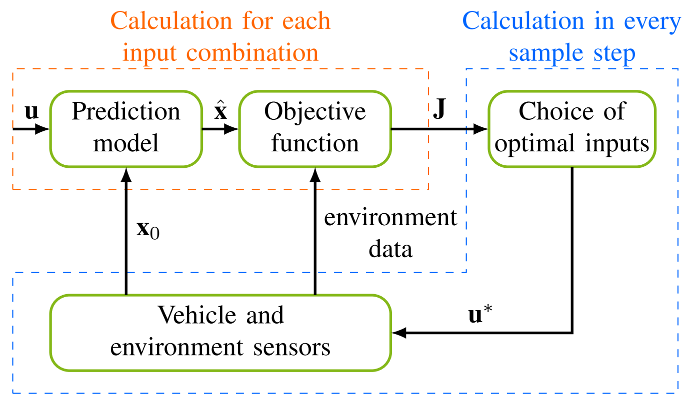

Fig. 2. Block diagramm of the model predictive scheme.

II. Vehicle Dynamics Model

On the one hand a model predictive approach requires a precise model of the plant to achieve realistic predictions of the system behavior. However, on the other hand with an increasing model order the computational effort is rising too. On this account, the nonlinear single track model is chosen to predict the vehicle dynamics as it fit the nonlinearities, and thus represents a balanced compromise between the previously mentioned aspects.
The equations of state are given by rearranging the sum of forces and the torque balance (1) - (3). A visualization of the essential variables is shown in Fig. 1. The position and orientation of the vehicle are determined by the state variables of the single track model (4) - (5).

$$\dot{v} = \frac{\sum {}_{\mathrm{V}}F_x \cos\beta + \sum {}_{\mathrm{V}}F_y \sin\beta}{m}. \tag{1}$$

$$\dot{\beta} = \frac{\sum {}_{\mathrm{V}}F_y \cos(\beta) - \sum {}_{\mathrm{V}}F_x \sin(\beta)}{mv} - \dot{\psi}. \tag{2}$$

$$\ddot{\psi} = \frac{-{}_{\mathrm{V}}F_{y,r}l_r + {}_{\mathrm{V}}F_{y,f}l_f \cos\delta + {}_{\mathrm{V}}F_{x,f}l_f \sin\delta}{J_z}. \tag{3}$$

$$\dot{x} = \cos(\beta + \psi)\, v \tag{4}$$

$$\dot{y} = \sin(\beta + \psi)\, v \tag{5}$$

To respect the nonlinearities at high slips the force in the lateral direction is calculated via a simplified nonlinear Pacejka magic formula tire model [16]. Since the actuator dynamics are limited, these restrictions have to be taken into account during prediction. Therefore, the behavior of the closed control loops of the underlying steering wheel angle (SWA) control system and acceleration control system are approximated. The acceleration controller is modeled by a first order system (6), while a nonlinear second order system approximates the SWA controller (7 - 8). The subscripts ref and act stand for the reference and actual values.

$$\dot{a}_{act} = -\frac{1}{T_a} a_{act} + \frac{K_a}{T_a} a_{ref} \tag{6}$$

$$\ddot{\delta}_{act} = \frac{K_\delta}{T_\delta^2}\delta_{ref} - \frac{1}{T_\delta^2}\delta_{act} - \frac{2d_\delta}{T_\delta} l(\dot{\delta}_{act}) \tag{7}$$

$$l(\dot{\delta}_{act}) = \begin{cases} \dot{\delta}_{min}, & \text{for } \dot{\delta}_{act} \geq \dot{\delta}_{min} \\ \dot{\delta}_{act}, & \text{for } \dot{\delta}_{min} < \dot{\delta}_{act} < \dot{\delta}_{max} \\ \dot{\delta}_{max}, & \text{for } \dot{\delta}_{act} \leq \dot{\delta}_{max} \end{cases} \tag{8}$$

The whole nonlinear prediction model can be expressed by:

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + f(\hat{\mathbf{x}}_k, \mathbf{u}_k)\,\Delta t, \quad \hat{\mathbf{x}}_0 = \mathbf{x}_0 \tag{9}$$

with the state vector:

$$\mathbf{x} = \left[\dot{\delta}_{act}, \delta_{act}, a_{act}, v, \beta, \dot{\psi}, \psi, x, y\right]^T \tag{10}$$

and the input vector:

$$\mathbf{u} = \begin{bmatrix} \delta_{ref} \\ a_{ref} \end{bmatrix}. \tag{11}$$

The model is solved using the forward Euler method starting from the initial state $\mathbf{x}_0$.

III. Model Predictive Trajectory Set Approach

The principle of "classic" model predictive control can be described merely as optimizing the input trajectory by predicting the future behavior using a system model and the current state. Therefore an optimal control problem can be formulated:

$$\mathbf{u}^* = \arg\min_{\mathbf{u}} \mathbf{J}, \tag{12}$$

which is usually solved with an iterative gradient based optimization algorithm. Such optimization is time-consuming so that in the suggested approach all possible state trajectories are explicitly calculated and the optimal input is chosen by comparing the objective values of each trajectory. This assumes that the set of possible inputs is sufficiently small. Because of the coarse discretization the approach is only suboptimal. A concept to solve this issue is discussed in section III-A. The method is illustrated in Fig. 2. The objective function to select the optimal inputs evaluates the predicted trajectory candidates and will be defined in section III-C.

A. Adaptive discretization

To achieve stationary accuracy the value range of the input variables have to be discretized with an infinite resolution. Especially in curves the target value has to come close to arbitrary values. However, due to the computational limitation, the amount of discretization steps is finite. Therefore a method is developed, which allows convergence to an almost arbitrary value in spite of a limited vector of possible input values. Assuming, that the optimal value is near to the actual value and only small changes are required, the possible values are discretized depending on the actual value x_0. To achieve a higher resolution of discretized values around the actual value, two second degree polynomials $g_{max}(k)$ and $g_{min}(k)$ are utilized to calculate an adaptive discretized vector of possible values:

$$g(k) = c_0 + c_1 \cdot k + c_2 \cdot k^2. \tag{13}$$

To determine the polynomial coefficients following conditions are expressed:

$$g(\bar{k}) = k_0 \tag{14}$$

$$g'(\bar{k}) = 0 \tag{15}$$

$$g(k_{lim}) = k_{lim}. \tag{16}$$

Herein $\bar{k}$ is the mean of the value range and k_{lim} defines for $g_{max}(k)$ the upper limit k_{max} and for $g_{min}(k)$ the lower limit k_{min}, respectively.

$$f(k) = \begin{cases} g_{min}(k), & \text{for } k_{min} \leq k < \bar{k} \\ g_{max}(k), & \text{for } \bar{k} \leq k \leq k_{max} \end{cases} \tag{17}$$

The piecewise defined function $f(k)$, which consists of the two polynomials are evaluated for a linear discretized vector. The output reveals an adaptive discretized vector with a high resolution around the current value and nevertheless covering the whole value range. Thereby it is possible to converge to an almost arbitrary value with only minor deviations within less time steps. For example the adaptive discretization of the steering wheel angle $f()\delta)$ is illustrated in Fig. 3. In this case the mean is $\bar{\delta} = 0°$ and the value range $[\delta_{min}\ \delta_{max}]$ is symmetrical with an absolute maximum of $240°$.

B. Transition Time

Instead of using the control horizon $n_c = 1$ like Keller et al. [13], we propose to chose the control horizon $n_c = 2$. This is motivated by the fact, that during collision avoidance maneuvers the steering wheel angle is typically variable for the whole prediction horizon for instance of $t_p = 1.25\,\text{s}$. Rather an evasive maneuver is usually composed of at least two characteristic steering interventions. For this reason, it seems to be useful to add a transition time during the prediction of trajectories. Especially, if a subsequent event is more critical in contrast to the first one, it is essential to consider this prematurely.

The transition time τ is defined in relation to the prediction horizon:

$$\tau \in [0.3\ 0.7] \cdot t_p. \tag{18}$$

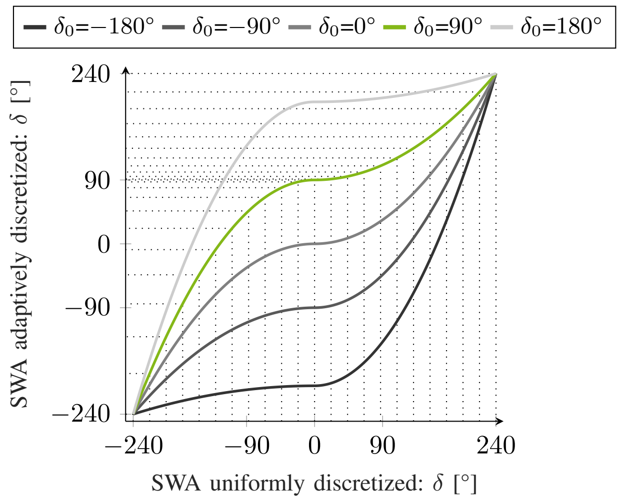

Fig. 3. Illustration of adaptive steering wheel angle discretization for different actual values.

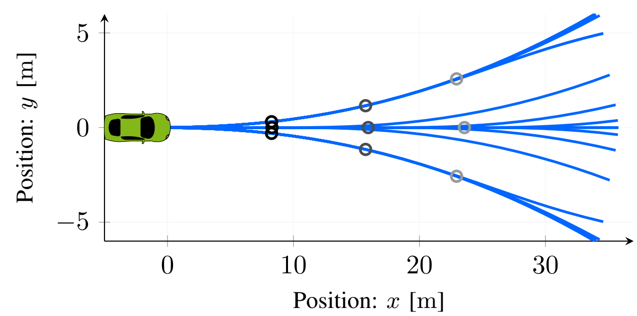

Fig. 4. Illustration of the trajectory utilizing a transition time. The circles highlight three possible transition times.

The interval does not include the boundary values 0 and 1 since at that times a transition to another input provides no valuable gain of information. Because the optimal time of transition depends on the current situation, no fixed relation can be determined from the interval. For this reason, several transition times have to be predicted and compared one against each other. To allow convergence through consecutive time steps, the transition time is adaptively discretized as described in the previous section III-A. Due to the advance of the time, the old transition time is not optimal for the next planning cycle. Therefore the previous transition time is reduced by the time increment for the adaptive discretization:

$$\tau_0 = \tau_{k-1} - \Delta t. \tag{19}$$

Fig. 4 shows the trajectory set for three steering wheel angles and three different transition times. The maximal space spanned by the trajectories remains the same, however, inside this sector, higher accessibility can be ensured. In the present contribution, the transition is only performed for the steering wheel angle, since a additional transition of the acceleration cause a strong increase of the combinatorial diversity.

After the transition, the optimal inputs are not dependent on the current values, so a non-varying vector is utilized.

C. Objective Function

The objective function must consider the information of the environmental sensors and the vehicle dynamics. In critical situations, the primary aim is to avoid the collision with obstacles and the street boundary. All efforts, like high accelerations, are subject to this goal. The objective function is given by:

$$J = \sum_{k=1}^{n_p} \left(\gamma_1 \Gamma_C + \gamma_2 \Gamma_\psi + \gamma_3 \Gamma_a + \gamma_4 \Gamma_{sc} \right). \qquad (20)$$

with the objective terms

$$\Gamma_C = \begin{cases} 2^{(n_p - k)} & \text{if a collision is predicted,} \\ 0 & \text{if no collision is predicted,} \end{cases} \qquad (21)$$

$$\Gamma_\psi = \left(\psi_k - \xi_k \right)^2 \qquad (22)$$

and

$$\Gamma_a = a_{ref}^2. \qquad (23)$$

The objective function consists of four parts. The first one integrates the collision time into the evaluation and ensures that the chosen trajectory is the longest possible without a collision.

The second part minimizes the orientation of the vehicle ψ with respect to the road orientation ξ_k. As a consequence also the dynamic reaction of the car is reduced.

The third one rates the deceleration effort and ensures, that the vehicle only reduces the velocity in critical situations. Due to a situation-dependent choice of the weighting γ_3, unnecessary braking actions can be suppressed. Because the objective function is defined similarly to Keller et al. [13] the dimensioning of weights can be looked up in detail in that contribution.

Since the prediction model is only an approximation of the real plant model, inaccuracies occur. These errors have to be compensated, because the previously defined terms of the objective function force the vehicle to drive as close as possible along obstacles. Therefore a soft constraint is defined:

$$\Gamma_{sc} = \Gamma_{obs} + \Gamma_{street}. \qquad (24)$$

To keep a proper distance to the street boundary, a soft constraint is characterized by the square of the shifted distance:

$$\Gamma_{street} = \left(\sqrt{\left(x - x_{street}\right)^2 + \left(y - y_{street}\right)^2} - \epsilon \right)^2. \qquad (25)$$

Moreover, to ensure a single-sided restriction of the available area, the function is limited in dependence on which side of the ego vehicle the street boundary is located. Additional an exponential barrier for obstacles is defined in dependence on the velocity v_{ego} and the free space between the edge of an obstacle and the street boundary d_f is defined:

$$\begin{aligned} \Gamma_{obs} = \exp(&-c_{lon}(\left(x_k - x_{obs,k}\right) \cos \xi_k + \\ &+ \left(y_k - y_{obs,k}\right) \sin \xi_k)^2 + \\ &-c_{lat}(- \left(x_k - x_{obs,k}\right) \sin \xi_k + \\ &+ \left(y_k - y_{obs,k}\right) \cos \xi_k)^2) \end{aligned} \qquad (26)$$

with the scaling factors:

$$c_{lat} = \sqrt{-\ln{(0.05)}} / \left(w_{obs}/2 + d \right), \qquad (27)$$

$$c_{lon} = \sqrt{-\ln{(0.05)}} / \left(\left(v_{ego}/c_1 \right) / \left(c_2/d_f \right) \right). \qquad (28)$$

The soft constraint function considers the orientation of the road in relation to the current vehicle coordinate system. The obstacle width and a safety distance determine the expansion to the lateral direction. In addition to that, the barrier expands with rising velocity and shrinks with increasing space beside the obstacle. Fig. 5 depicts the soft constraint function for the left street boundary and an obstacle on the right lane.

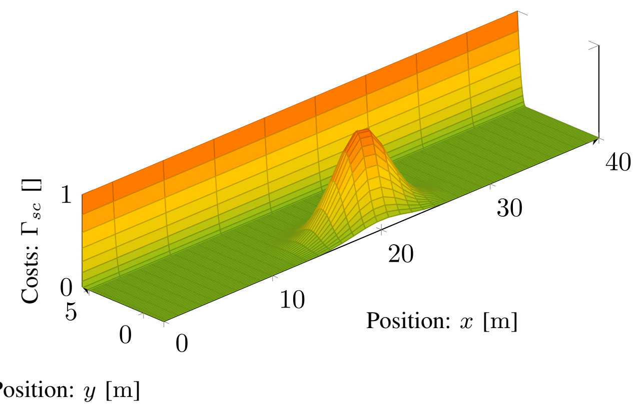

Fig. 5. Visualization of the soft constraint function for the left street boundary and an obstacle on the right.

IV. Driving along curves

To demonstrate the effectiveness of the adaptive discretization for SWA an exemplary situation of a curved street is simulated on the one hand utilizing a fixed steering wheel angle and on the other hand the adaptive discretization. Initially, the road is straight and transitions to a left clothoid curve with a minimal curve radius of $250\,\mathrm{m}$. The vehicle drives with a velocity of $100\,\mathrm{km/h}$. Both variants employ 19 possible steering wheel angles, whereby the fixed discretization choose from the following set:

$$\begin{aligned} \delta_{ref} \in A := \{ &-240° \; -190° \; -140° \; -100° \; -70° \\ &-45° \; -25° \; -15° \; -5° \; 0° \; 5° \; 15° \\ &25° \; 45° \; 70° \; 100° \; 140° \; 190° \; 240° \}. \end{aligned} \qquad (29)$$

In addition the maneuver is simulated utilizing a fixedly discretized set of 81 SWA with an equidistant spacing of 5°. Fig. 6 shows the courses of the reference steering wheel angle chosen from the fixedly and adaptively discretized sets. From the figure, it is apparent that the rough discretization of the fixed set has a more significant impact on the optimality. Since the steering wheel angle to follow the curve is not available in the set the chosen steering wheel angle alternates among two values. The results reveal that the adaptive discretization allows the steering wheel angle to converge to an almost arbitrary value enabling a continual course. Similar smooth results can be obtained using the fine equidistant discretization of

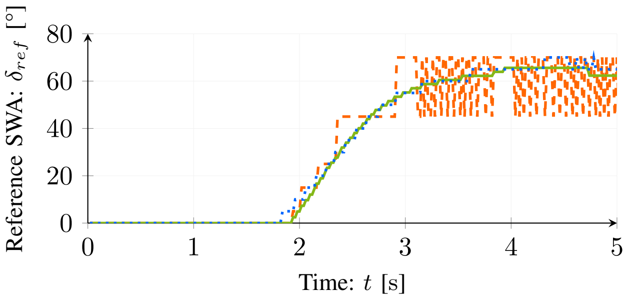

Fig. 6. Reference steering wheel angle fixedly and adaptively discretized.

SWA at the expense of a higher computation time, especially if a transition time is utilized.

V. Automated Collision Maneuver

In the following section, the suggested modifications to the model predictive trajectory set approach are evaluated by simulating a collision avoidance maneuver. In the selected scenario the weaknesses of the original approach are revealed. In the investigated situation the ego vehicle approaches a static obstacle at a speed of approximately $100\,\mathrm{km/h}$. To prevent a collision, the ego vehicle has to change to a narrower lane, since pure braking is not sufficient. The time to collision is approximately $1\,\mathrm{s}$. To make the benefits resulting from the modified method more visible and clearly presentable only steering interventions are permitted, because a deceleration eases the evasive maneuver. Fig. 9 shows the chronological sequence of the evasive maneuver. For the sake of clarity, only the selected trajectory from the set is depicted. At the first time step, the optimal trajectory for the trajectory set without transition time is shown also. According to the depiction of this time step a larger steering wheel angle is chosen if a transition time is utilized. Consequently, the vehicle accelerates faster in the lateral direction, as the street boundary is already taken into account for the selection of the optimal trajectory. Due to the smaller steering wheel angle at the beginning, the collision cannot be avoided without the transition time. Fig. 7 shows the acceleration in the lateral direction. Since

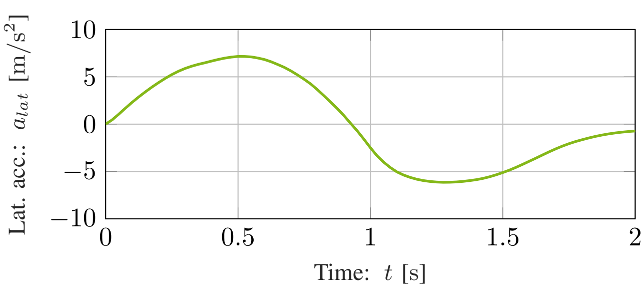

Fig. 7. Lateral acceleration during emergency evasive maneuver.

only steering interventions are allowed a lateral acceleration solely, with an absolute maximum value of $7.2\,\mathrm{m/s^2}$, arises. The data reveal that even larger accelerations are possible in collision avoidance maneuvers since the physical limit is not reached. An emergency steering system has to solve this situation without a collision so that the disadvantages of the original approach compared to the extension with the transition time also get aware at this point. Fig. 8 shows the transition

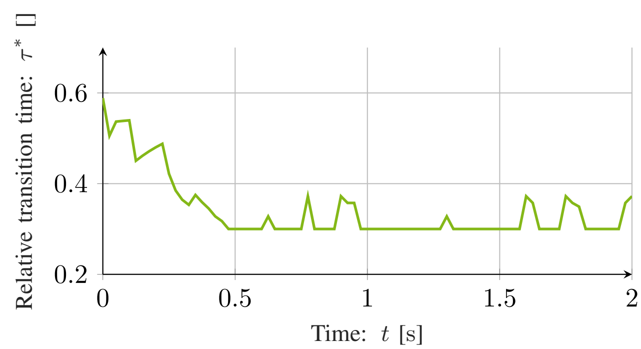

Fig. 8. Relative transition time during emergency evasive maneuver.

time in relation to the simulation time. There is a clearly defined correlation between $0.00\,\mathrm{s}$ and $0.45\,\mathrm{s}$. The two relative transition times 0.59 and 0.3 correspond to $0.74\,\mathrm{s}$ and $0.37\,\mathrm{s}$ at a prediction horizon of $1.25\,\mathrm{s}$. The numbers show that the transition time decreases approximately with the same step size like the simulation time as expected. It is apparent that the approach detects the optimal transition time for that maneuver. In the further course, the transition time is approximately the lower limit, since the proximity to the street boundary requires an early transition. Fig. 10 shows the evasive trajectory for a combined braking and steering maneuver. The deceleration is chosen from 5 values. Because the street is blocked by two obstacles the velocity is reduced until the vehicle comes to a complete halt, which is shown in Fig. 11.

VI. Conclusion

The proposed transition time is an efficient and useful modification on the model predictive trajectory set approach. By utilizing the transition time, the algorithm increases the degree of freedom during the planning which allows a more suitable situation handling. The amendment allows considering meaningful information of periods within the prediction horizon further in the future so that a generalizable application of the approach is possible. Additional, the potential of the method is increased by the adaptive discretization. By utilizing these, it is possible to converge to an almost arbitrary value during several time steps. The suboptimality caused by the rough discretization is decreased. As shown this allows the vehicle to follow curved roads with a random radius for instance. The effects of errors in the prediction model, engendered by inaccuracies during the modeling process, are alleviated by a soft constraint. Utilizing these the vehicle increases the distance to obstacles and the street boundary. The subsequent steps will include the implementation in a test vehicle, when the approach has to react to real environment information.

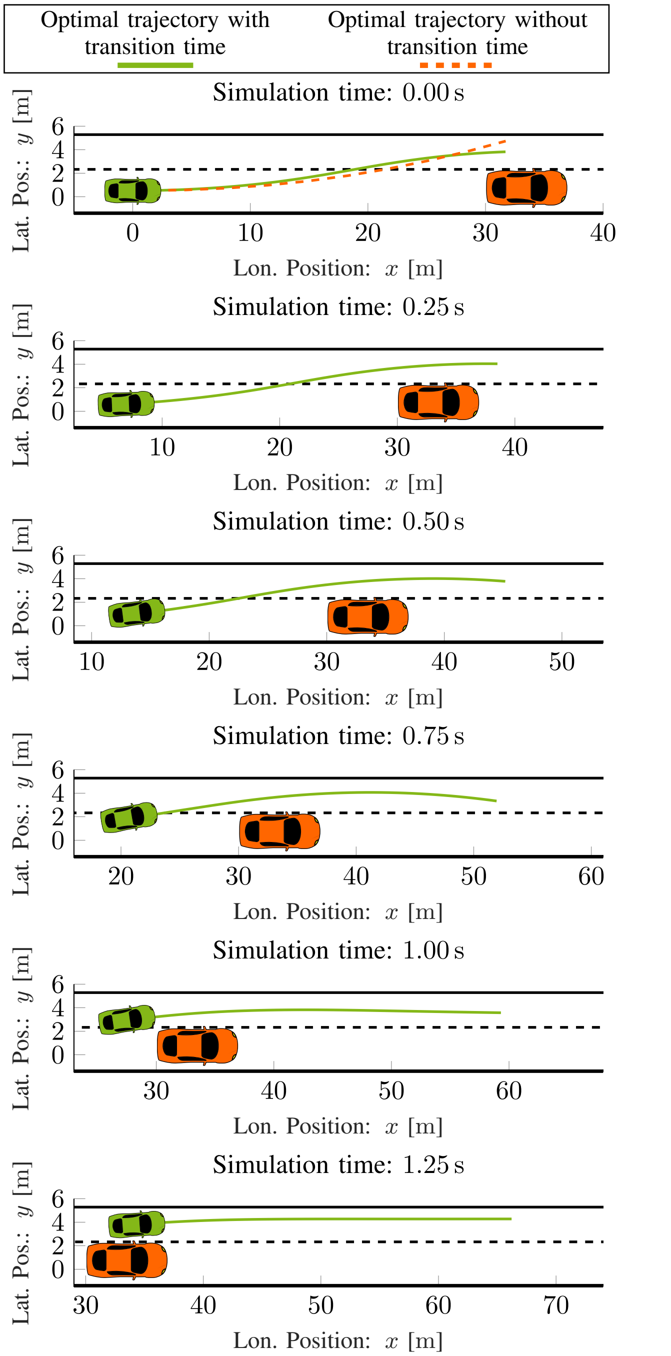

Fig. 9. Sequence of the emergency evasive maneuver with an interval of 0.25 s.

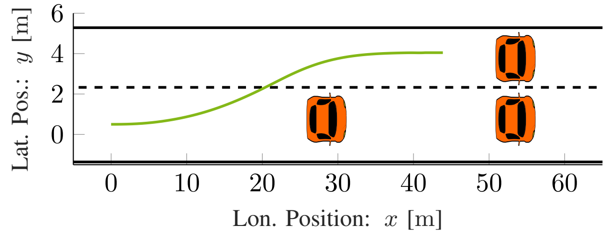

Fig. 10. Evasive trajectory for an combined braking and steering maneuver.

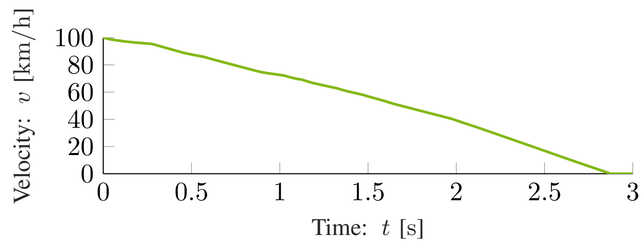

Fig. 11. Velocity course during combined braking and steering maneuver.

References

[1] M. Keller, C. Hass, A. Seewald and T. Bertram, “Driving Simulator Study on an Emergency Steering Assist”, IEEE International Conference on Systems, Man and Cybernetics, pp. 3039–3044, 2014.

[2] J. Choi, K. Kim and K. Yi, “Emergency Driving Support algorithm with steering torque overlay and differential braking”, IEEE 14th International Conference on Intelligent Transportation Systems (ITSC), pp. 1433–1439, 2011.

[3] D. Soudbakhsh, A. Eskandarian and J. Moreau, “An emergency evasive maneuver algorithm for vehicles”, IEEE 14th International Conference on Intelligent Transportation Systems (ITSC), pp. 973–978, 2011.

[4] K. Berntorp, “Path Planning and Integrated Collision Avoidance for Autonomous Vehicles”, American Control Conference, pp. 4023–4028, 2017.

[5] M. Keller, F. Hoffmann, C. Hass, A. Seewald and T. Bertram, “Planning of Optimal Collision Avoidance Trajectories with Timed Elastic Bands”, IFAC World Congress, pp. 9822–9827, 2014.

[6] M. Werling, S. Kammel, J. Ziegler and L. Gröll,, “Optimal trajectories for time-critical street scenarios using discretized terminal manifolds”, International Journal of Robotics Research, Vol. 31, No. 3, pp. 346–359, March 2012.

[7] J. Ziegler, P. Bender, T. Dang and C. Stiller, “Trajectory Planning for Bertha - a Local Continuous Method”, IEEE Intelligent Vehicles Symposium, pp. 450–457, 2014.

[8] Z. Wang, S. Ramyar, S. M. Salaken, A. Homaifar, S. Nahavandi and A. Karimoddini, “A Collision Avoidance System with Fuzzy Danger Level Detection”, IEEE Intelligent Vehicles Symposium (IV), pp. 283–288, 2017.

[9] M. Werling and D. Liccardo, “Automatic Collision Avoidance Using Model-predictive Online Optimization”, IEEE Conference in Decision and Control, pp. 6309–6314, 2012.

[10] J. V. Frasch, A. Gray, M. Zanon, H. J. Ferreau, S. Sager, F. Borrelli and M. Diehl, “An Auto-generated Nonlinear MPC Algorithm for Real-Time Obstacle Avoidance of Ground Vehicles”, European Control Conference, pp. 4136–4141, 2013.

[11] C. Götte, M. Keller, C. Rösmann, T. Nattermann, C. Hass, K.-H. Glander, A. Seewald and T. Bertram, “A Real-Time Capable Model Predictive Approach to Lateral Vehicle Guidance”, IEEE 19th International Conference on Intelligent Transportation Systems (ITSC), pp. 1908–1913, 2016.

[12] F. von Hundelshausen, M. Himmelsbach, F. Hecker, A. Mueller, and H.-J. Wuensche, “Driving with Tentacles: Integral Structures for Sensing and Motion”, Journal of Field Robotics, vol. 25(9), pp. 640–673, 2008.

[13] M. Keller, C. Hass, A. Seewald and T. Bertram, “A model predictive approach to emergency maneuvers in critical traffic situations”, IEEE 18th International Conference on Intelligent Transportation Systems (ITSC), pp. 369–374, 2015.

[14] M. Keller, “Trajektorienplanung zur Kollisionsvermeidung im Straßenverkehr”,Dissertation, TU Dortmund, Germany, 2017.

[15] A. Makarow, M. Keller, C. Rösmann, T. Bertram, G. Schoppel and I. Glowatzky,“Model predictive trajectory set control for a proportional directional control valve”, 1st IEEE Conference on Control Technology and Applications (CCTA), pp. 1229–1234, 2017.

[16] H. B. Pacejka, “Tyre and vehicle dynamics”, Elsevier/Butterworth-Heinemann, Amsterdam, 2.ed., 2007.