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Mobility Oriented Motion Planning for Cooperative Lane Changing under Partially Connected and Automated Environment

Yu Bai
Key Laboratory of Road and Traffic Engineering of the Ministry of Education
Tongji University
Shanghai, P.R. of China
baiyu@tongji.edu.cn

Yu Zhang
Key Laboratory of Road and Traffic Engineering of the Ministry of Education
Tongji University
Shanghai, P.R. of China
1610734@tongji.edu.cn

Jia Hu
Key Laboratory of Road and Traffic Engineering of the Ministry of Education
Tongji University
Shanghai, P.R. of China
hujia@tongji.edu.cn

***Abstract*—This research proposes a cooperative lane-changing (CLC) controller for connected and automated vehicles under partially connected and automated environment. The goal is to further advance the technology of automated lane change in terms of realizing coordination and cooperation between a lane change vehicle and its following vehicle in order to reduce oscillation and shockwave caused by lane change maneuvers. The proposed controller is formulated as a model predictive control. The optimal control problem is solved by a dynamic programming based numerical algorithm proposed by this research team in a previous study to ensure computation speed. The proposed CLC controller is evaluated against human drivers to quantify its performance. Sensitivity analysis is conducted in terms of the initial headway of receiving gap. The results proved the effectiveness of the CLC controller. For a lane-change vehicle, CLC reduced oscillation by 0.5%-13.9%. For its following vehicle, CLC benefited up to 9.6%. The variation is due to the changes in initial headway of receiving gap. The computation time is around 17-21 ms with 0.1 seconds time step and 20 seconds optimization time horizon.**

***Keywords*—advanced driver assistant systems, connected and automated vehicle, cooperative lane changing, model predictive control, rolling horizon control.**

I. INTRODUCTION

Lane changes impact negatively on transportation safety and mobility. According to the U.S. Department of Transportation, overtaking maneuvers (which could be regarded as two consecutive lane-changing maneuvers) lead to a total of 13,939 fatal crashes in the United States from 1994 to 2005 [1]. In addition, most traffic bottlenecks are where frequent lane changes are observed.

The emerging connected automation technology has shown great potentials in improving safety [2], enhancing capacity [3-5], improving traffic efficiency [6] and reducing fuel consumption [7]. By having vehicles talk to each other and to infrastructure, Connected and Automated Vehicles (CAVs) are able to take advantage of real-time traffic information [8, 9] and drive cooperatively [9-11]. Hence, connected automation could enable the aforementioned lane change cooperation and be a solution to lane changes' adverse effect.

Past studies have rarely investigated how to realize automated cooperative lane change. Most controllers proposed only focus on enabling automated lane change with no surrounding traffic[12-15]. Some recent studies did look at automated lane change with background traffic, including decision making[16], motion planning[17-20] and lower-level control[1, 21]. However, their scopes exclude cooperative automation between lane-changing vehicle and following vehicle. In addition, they only value the safety [1, 16-19, 21] or mobility [20, 22, 23] of lane changing vehicle, instead of mobility of the entire transportation system.

This research proposes an automated lane-changing controller with the following features:

- Reducing lane changes' adverse effect on mobility
- Enabling cooperation between a lane change vehicle and its following vehicle
- Applicable to vehicles with longitudinal automation (level-2 and above)
- Efficient computation for real-time application.
- Taking advantage of the connected and automated vehicles technology.

The remainder of the paper is organized as follows. Section II 'Control logic' presents research scope and the proposed cooperative lane-changing logic. Section III 'Problem formulation and solution' mainly discusses formulation and solution methodology. Section IV 'Case study' demonstrates an evaluation of the proposed controller. Section V provides conclusions and future research opportunities.

II. CONTROL LOGIC

This section presents the control logic of the proposed cooperative lane change design.

A. Scope

Fig. 1 demonstrates the scope of interest. Vehicle D in lane 2 intends to carry out lane-changing maneuver to switch to lane 1 and cut in between vehicle C and vehicle E. In this scenario, it is assumed that vehicle D and E are CAVs, while vehicle A, B, and C could be either CAVs or conventional vehicles.

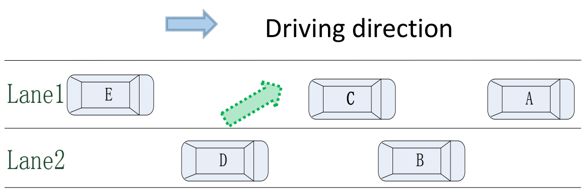


Fig.1. Scenario setting.

B. Cooperation Logic

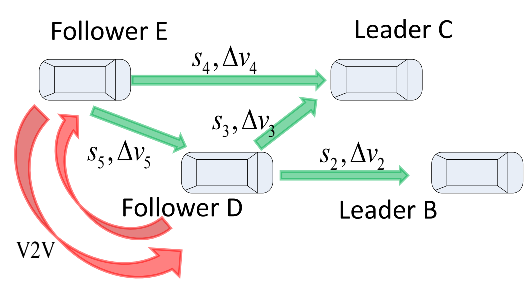
The goal of the proposed cooperative lane change control is to ease lane change process and reduce shockwave caused by lane changes. The means of achieving this goal is by enlarging the target gap of the ego vehicle before lane change process takes place. Fig. 3 is a flow chart of the proposed logic. Vehicles in the green block are controlled by CLC controller while those in the red block are background traffic. Depending on whether vehicle C participates in cooperative lane change, two logic branches are designed:

- CLC-1 controller: In this strategy, vehicle D and E maneuver cooperatively to help vehicle D change lane, as shown in the left branch of Fig.3.
- CLC-2 controller: In this strategy, vehicle C, D, and E maneuver cooperatively to facilitate lane change, as shown in the right branch of Fig. 3.

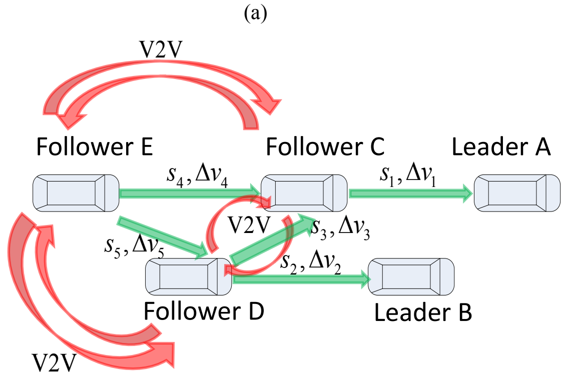
C. Data Flow

Data flow architecture for CLC-1 controller is shown in Fig.2(a). Sensors are also equipped to detect relative distance and speed between CAVs and adjacent vehicles.

Data flow architecture for CLC-2 controller is shown in Fig.2(b). Sensors are also equipped to detect relative distance and speed between CAVs and adjacent vehicles.



(a)



(b)

Fig.2. Data flow architecture for the proposed CLC controller (green arrows refer to sensor detection).

III. Problem Formulation and Solution

The problem is formulated as a model predictive control and is solved by a dynamic programming based numeric algorithm developed by this research team in a previous study.

A. CLC-1 Controller

The state vector and control vector of the CLC-1 controller are:

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{s}_k & \Delta \mathbf{v}_k \end{bmatrix}^T \tag{1}$$

$$\mathbf{u}_k = \begin{bmatrix} a_{\mathrm{D}} & a_{\mathrm{E}} \end{bmatrix}^T \tag{2}$$

With

$$\mathbf{s}_k = \begin{bmatrix} s_2 \\ s_3 \\ s_4 \\ s_5 \end{bmatrix} \quad \Delta \mathbf{v}_k = \begin{bmatrix} \Delta v_2 \\ \Delta v_3 \\ \Delta v_4 \\ \Delta v_5 \end{bmatrix} \tag{3}$$

Where k is control step index, for state $k \in \{0,1,\ldots,N+1\}$ and for control $k \in \{0,1,\ldots,N\}$; N is the total number of control steps; s is distance headway; $\Delta v = v_f - v$ is relative speed, with v_f being the preceding vehicle's speed and v being the ego-vehicle's speed; a is acceleration. The subscripts 1 to 5 and A to E are illustrated in Fig.1 and Fig.2.

Typically, the acceleration of both vehicle D and E is bounded in the maximum deceleration and maximum acceleration, $a_{\min} \le a_{\mathrm{D}} \le a_{\max}$ and $a_{\min} \le a_{\mathrm{E}} \le a_{\max}$; the speed of both vehicle D and E is bounded in zero and the maximum speed, $0 \le v_{\mathrm{D}} \le v_{\max}$ and $0 \le v_{\mathrm{E}} \le v_{\max}$.

The system's dynamics is

$$\begin{gathered} \mathbf{x}(k+1) = \mathbf{A}_k \mathbf{x}(k) + \mathbf{B}_k \mathbf{u}(k) + \mathbf{C}_k \\ k \in \{0,1,\ldots,N\} \end{gathered} \tag{4}$$

Where

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{I}_{4\times4} & \mathbf{0}_{4\times4} \\ \mathbf{0}_{4\times4} & \mathbf{I}_{4\times4} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{4\times4} & \mathbf{I}_{4\times4} \\ \mathbf{0}_{4\times4} & \mathbf{0}_{4\times4} \end{bmatrix} \times x_r \tag{5}$$

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{0}_{4\times1} & \mathbf{0}_{4\times1} \\ -1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix} \times x_r \tag{6}$$

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{0}_{4\times1} & a_{\mathrm{B}} & a_{\mathrm{C}} & a_{\mathrm{C}} & 0 \end{bmatrix}^T \times x_r \tag{7}$$

Where $\mathbf{I}_{r\times r}$ is an identity matrix with a size of $r \times r$ and $\mathbf{0}_{r\times s}$ is a zero matrix with a size of $r \times s$, x_r is the control time step length.

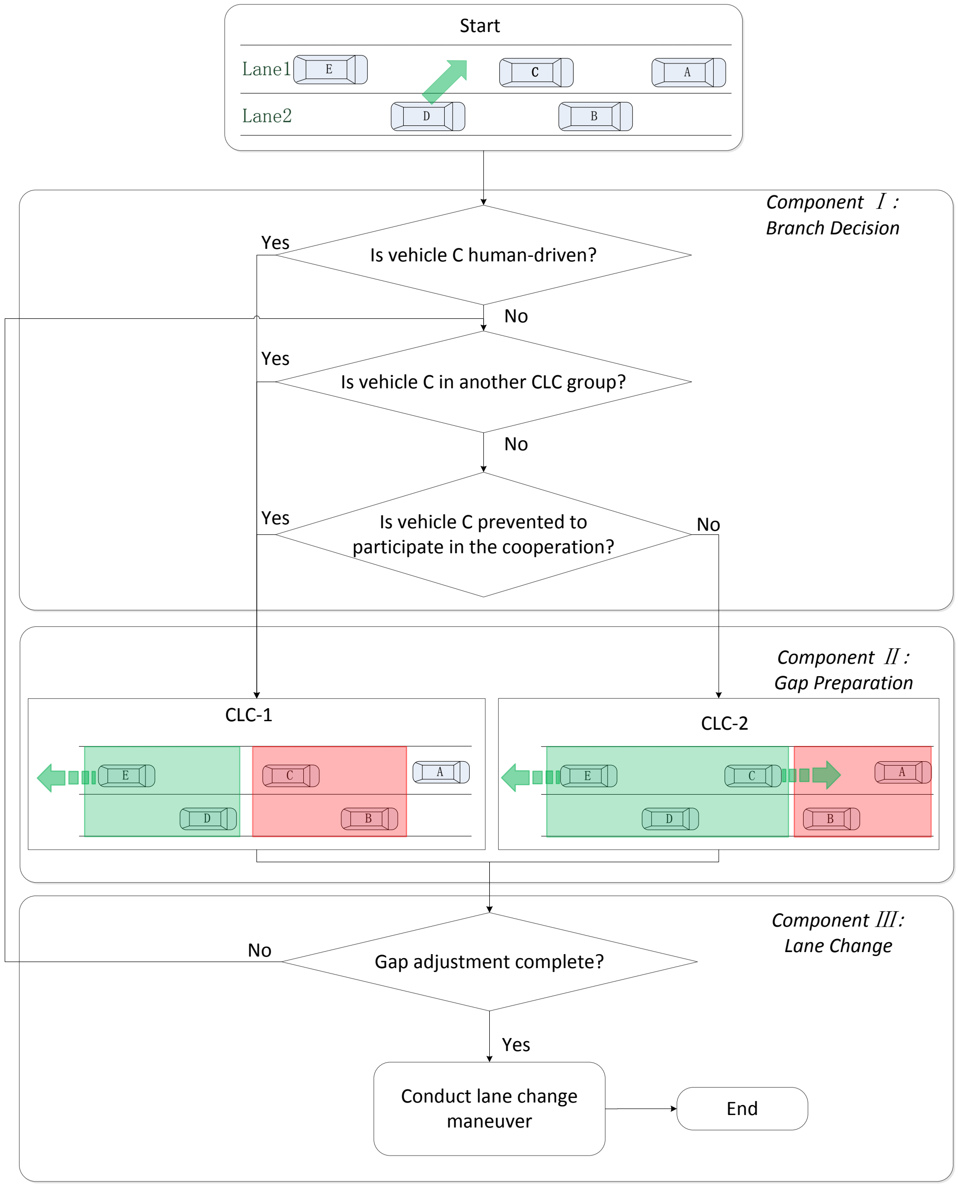


Fig.3. Control structure of cooperative lane changing.

The cost function is formulated in quadratic form with terminal cost set to zero:

$$L = \frac{1}{2}(\mathbf{s}_k - \mathbf{s}_k^*)^T \boldsymbol{\beta}_1 (\mathbf{s}_k - \mathbf{s}_k^*) + \frac{1}{2}\Delta\mathbf{v}_k^T \boldsymbol{\beta}_2 \Delta\mathbf{v}_k + \frac{1}{2}\mathbf{u}_k^T \boldsymbol{\beta}_3 \mathbf{u}_k \tag{8}$$

Where

$$\mathbf{s}_k^* = \begin{bmatrix} s_2^* & s_3^* & s_4^* & s_5^* \end{bmatrix}^T \tag{9}$$

$\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$, and $\boldsymbol{\beta}_3$ are weighting factor matrices for headway error, speed error, and control input respectively. s^* is desired distance headway.

$$\boldsymbol{\beta}_1 = \begin{bmatrix} \beta_{12} & 0 & 0 & 0 \\ 0 & \beta_{13} & 0 & 0 \\ 0 & 0 & \beta_{14} & 0 \\ 0 & 0 & 0 & \beta_{15} \end{bmatrix} \tag{10}$$

$$\boldsymbol{\beta}_2 = \begin{bmatrix} \beta_{22} & 0 & 0 & 0 \\ 0 & \beta_{23} & 0 & 0 \\ 0 & 0 & \beta_{24} & 0 \\ 0 & 0 & 0 & \beta_{25} \end{bmatrix} \tag{11}$$

$$\boldsymbol{\beta}_3 = \begin{bmatrix} \beta_{3D} & 0 \\ 0 & \beta_{3E} \end{bmatrix} \tag{12}$$

B. CLC-2 Controller

The state vector and control input of the CLC-2 controller are as follows:

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{s}_k & \Delta\mathbf{v}_k \end{bmatrix}^T \tag{13}$$

$$\mathbf{u} = \begin{bmatrix} a_{\mathrm{C}} & a_{\mathrm{D}} & a_{\mathrm{E}} \end{bmatrix}^T \tag{14}$$

Where

$$\mathbf{s}_k = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{bmatrix} \Delta\mathbf{v}_k = \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \\ \Delta v_3 \\ \Delta v_4 \\ \Delta v_5 \end{bmatrix} \tag{15}$$

Like CLC-1, the control of CLC-2 is also restrained as $a_{\min} \le a_{\mathrm{C}} \le a_{\max}$ $a_{\min} \le a_{\mathrm{D}} \le a_{\max}$, and $a_{\min} \le a_{\mathrm{E}} \le a_{\max}$; and the speed of all the controlled vehicle is bounded in zero and the maximum speed, $0 \le v_{\mathrm{C}} \le v_{\max}$, $0 \le v_{\mathrm{D}} \le v_{\max}$ and $0 \le v_{\mathrm{E}} \le v_{\max}$. Similarly, the system's dynamics is

$$\mathbf{x}(k+1) = \mathbf{A}_k\mathbf{x}(k) + \mathbf{B}_k\mathbf{u}(k) + \mathbf{C}_k \quad k \in \{0,1,\ldots,N\} \tag{16}$$

Where

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{I}_{5\times5} & \mathbf{0}_{5\times5} \\ \mathbf{0}_{5\times5} & \mathbf{I}_{5\times5} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{5\times5} & \mathbf{I}_{5\times5} \\ \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} \end{bmatrix} \times x_r \tag{17}$$

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{0}_{5\times1} & \mathbf{0}_{5\times1} & \mathbf{0}_{5\times1} \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \times x_r \tag{18}$$

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{0}_{5\times1} & a_{\mathrm{A}} & a_{\mathrm{B}} & 0 & 0 & 0 \end{bmatrix}^T \times x_r \tag{19}$$

The cost function for CLC-2 is in similar form to that of CLC-1 controller. The only difference is the desired distance vector:

$$\mathbf{s}_k^* = \begin{bmatrix} s_1^* & s_2^* & s_3^* & s_4^* & s_5^* \end{bmatrix}^T \tag{20}$$

C. Solution Approach

A solution approach inspired by dynamic programming is adopted to solve the CLC problem. This approach was proposed by this research group in a previous study [24]. It can accelerate computation speed greatly by predetermine the terminal state associated with the optimal solution. This approach involves a backward calculation of concomitant matrices and a forward calculation of control vector and state vector.

1. Calculate $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$ for $k \in \{0,1,\ldots,N\}$. Where N is the total number of control steps.

2. Calculate $\mathbf{Q}_k$ and $\mathbf{R}_k$ for $k \in \{0,1,\ldots,N+1\}$. For the CLC controllers proposed, they are defined as follows:

$$\mathbf{Q}_k = \begin{bmatrix} \boldsymbol{\beta}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\beta}_2 \end{bmatrix} \tag{21}$$

$$\mathbf{D}_k = -\mathbf{s}_k^* \tag{22}$$

$$\mathbf{R}_k = \boldsymbol{\beta}_3 \tag{23}$$

3. For $k = N+1$, it has been proven that, when the cost function is in quadratic form, the optimal solution is associated with state [24]:

$$\mathbf{Q}_{N+1} = \mathbf{Q}_{N+1} \tag{24}$$

$$\mathbf{D}_{N+1} = \mathbf{0} \tag{25}$$

$$\mathbf{E}_{N+1} = \mathbf{0} \tag{26}$$

4. For $k \in \{N, N-1, \ldots, 0\}$, calculate the concomitant matrices backward:

$$\tilde{\mathbf{Q}}_k = \mathbf{G}_k^T\mathbf{R}_k\mathbf{G}_k + \mathbf{S}_k^T\mathbf{Q}_{k+1}\mathbf{S}_k + \mathbf{Q}_k \tag{27}$$

$$\tilde{\mathbf{E}}_k = \frac{1}{2}\mathbf{H}_k^T\mathbf{R}_k\mathbf{H}_k + \frac{1}{2}\mathbf{T}_k^T\mathbf{Q}_{k+1}\mathbf{T}_k \tag{28}$$

$$\tilde{\mathbf{D}}_k = \mathbf{G}_k^T\mathbf{R}_k\mathbf{H}_k + \mathbf{S}_k^T\mathbf{Q}_{k+1}\mathbf{T}_k + \mathbf{S}_k^T\mathbf{D}_{k+1} \tag{29}$$

With

$$\mathbf{P}_k = (\mathbf{R}_k + \mathbf{B}_k^T\mathbf{Q}_{k+1}\mathbf{B}_k)^{-1} \tag{30}$$

$$\mathbf{G}_k = -\mathbf{P}_k\mathbf{B}_k^T\mathbf{Q}_{k+1}\mathbf{A}_k \tag{31}$$

$$\mathbf{H}_k = -\mathbf{P}_k\mathbf{B}_k^T(\mathbf{Q}_{k+1}\mathbf{C}_k + \mathbf{D}_{k+1}) \tag{32}$$

$$\mathbf{S}_k = \mathbf{A}_k + \mathbf{B}_k\mathbf{G}_k \tag{33}$$

$$\mathbf{T}_k = \mathbf{B}_k\mathbf{H}_k + \mathbf{C}_k \tag{34}$$

5. For $k \in \{0,1,\ldots,N\}$, calculate control vector and state vector forward:

$$\mathbf{u}_k = \mathbf{G}_k\mathbf{x}_k + \mathbf{H}_k \tag{35}$$

$$\mathbf{x}(k+1) = \mathbf{S}_k\mathbf{x}(k) + \mathbf{T}_k \tag{36}$$

IV. CASE STUDY

The proposed CLC controller is evaluated against human lane-changing behavior. The Measurements of Effectiveness (MOE) adopted are individual vehicle speed and time headway.

A. Experiment Design

The scenario of one vehicle making lane change on arterials is evaluated, as shown in Fig.1. Vehicle D is the lane-changing vehicle. The receiving gap is the gap between vehicle C and E. Vehicle D is assumed to be in the middle of vehicle C and E at the start of the experiment. Speed limit is 20 m/s (45 mph). The following two controller are tested:

<u>Proposed Cooperative Lane Change</u> (CLC): In this case, the proposed CLC controls all CAVs participating in a lane change.

Baseline: In the baseline scenario, the driver model adopted is Intelligent Driver Model (IDM) [25].

B. Sensitivity Analysis

Sensitivity analysis is conducted in terms of the initial headway of receiving gap (gap between vehicle C and E). Levels tested includes 3.00 seconds (distance headway 60 m), 3.75 seconds (75 m), 4.50 seconds (90 m), 5.25 seconds (105 m), 6.00 seconds (120 m), and 6.75 seconds (135 m).

C. Control Settings

Parameters for CLC and IDM are carefully tuned for a fair comparison. Factors are calibrated to match CLC and IDM, including desired headway, acceleration/deceleration bounds, and desired speed. The following settings are made for the proposed CLC controllers:

- Optimization time horizon T is 20 s.
- Time step x_r is 0.1 s.
- Acceleration range is [-5,3] m/s^2.
- The weighting factors: $\beta_{1i} = 0.01$ for $i \in \{1,2,3,4,5\}$, $\beta_{2j} = 1$ for $j \in \{1,2,3,4,5\}$, $\beta_{3m} = 20$ for $m \in \{\mathrm{C,D,E}\}$.
- Desired distance headway:

  $$\mathbf{s}_k^* = \begin{bmatrix} 75 & 75 & 75 & 150 & 75 \end{bmatrix}^T.$$

- Initial speed of all the five vehicles are 20 m/s, and vehicle A and B drives at constant speed.

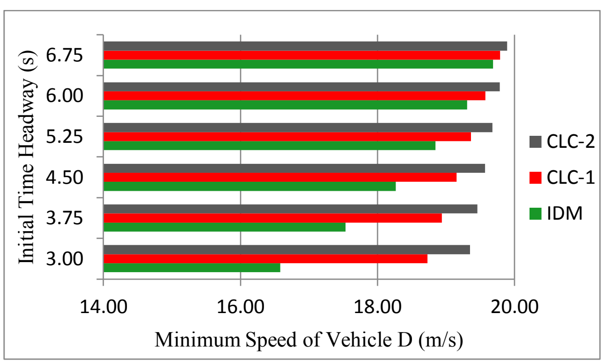
The settings for IDM are as following:

- Maximum acceleration is 3 m/s^2.
- Deceleration bound is -5 m/s^2.
- Acceleration exponential parameter is 4.
- Minimum distance headway is 7 m.
- Time headway is 1.8 s.

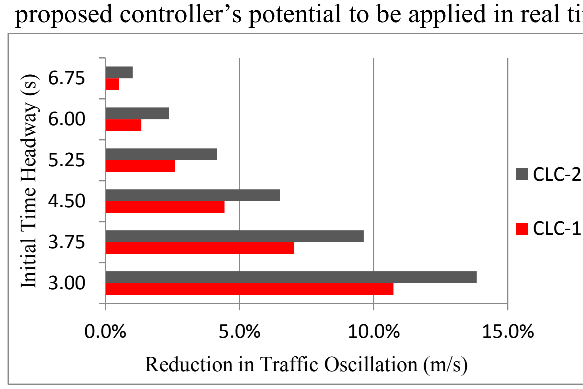
D. Simulation Result

The proposed controller generally shows a positive effect on lane change process. The magnitude of the benefit is greatly affected by the initial headway of receiving gap. Fig.4 presents the benefit of the proposed CLC controller in terms of reduction in traffic oscillation. The reduction is quantified by the speed change of lane-changing vehicle (vehicle D) and following vehicle (vehicle E). As shown in Fig.4, for the lane-changing vehicle, the proposed controller is always beneficial, depending on initial headway, the benefit ranges from 0.5% to 13.9%. For the following vehicle, the benefit is not guaranteed. It ranges between -2.3% to 9.6%. When applying CLC-1, the initial headway of receiving gap is required to be less than 3.75 seconds in order to be beneficial. When applying CLC-2, the initial headway should be less than 4.50 seconds. Intuitively, the smaller the initial headway is, the greater the benefit is. As the initial gap increases towards twice the desired headway, the proposed CLC gradually loses its positive effect.

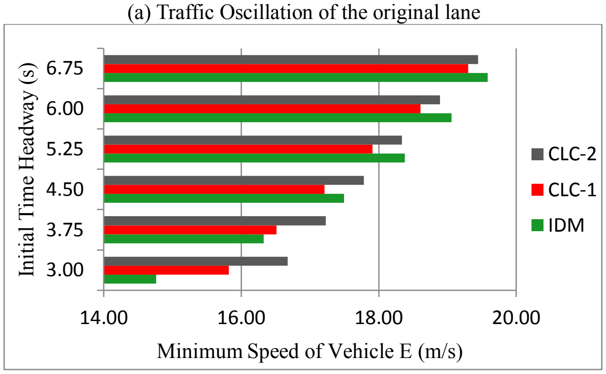
Computation speed of the proposed CLC is quite fast. CLC-1 completes computation in about 17 ms and CLC-2 about 21ms. The computation speed could be further improved by applying a greater time step, which is set to 0.1 sec in this case study. The computation speed indicates the proposed controller's potential to be applied in real time



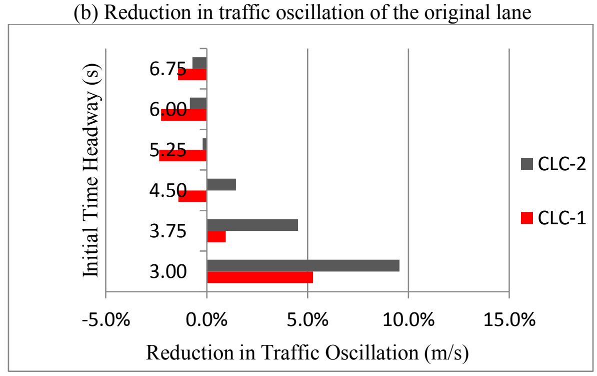
(a) Traffic Oscillation of the original lane



(b) Reduction in traffic oscillation of the original lane



(c) Traffic Oscillation of the target lane



(d) Reduction in traffic oscillation of the target lane

Fig.4. Traffic oscillation comparison

V. Conclusion

This research proposes a cooperative lane-changing (CLC) controller for connected and automated vehicles under partially connected and automated environment. The goal is to further advance the technology of automated lane change in terms of realizing coordination and cooperation between a lane change vehicle and its following vehicle in order to reduce oscillation and shockwave caused by lane change maneuver. The proposed controller is formulated as a model predictive control. The optimal control problem is solved by a dynamic programming based numerical algorithm proposed by this research team in a previous study to ensure computation speed. The proposed CLC controller is evaluated against human drivers to quantify its performance. Sensitivity analysis is conducted in terms of the initial headway of receiving gap. Detailed analysis on the CLC controller reveals:

- The proposed controllers is very efficient. Its computation time is around 17-21 ms with 0.1 seconds time step and 20 seconds optimization time horizon.
- For the lane-changing vehicle, the proposed controller is always beneficial, depending on initial headway, the benefit ranges from 0.5% to 13.9%.
- For the following vehicle, the benefit is not always guaranteed. It ranges between -2.3% to 9.6%. When applying CLC-1, the initial headway of receiving gap is required to be less than 3.75 seconds in order to be beneficial. When applying CLC-2, the initial headway should be less than 4.50 seconds.

In this study, only one set of weighting factor value is tested. Future studies could conduct sensitivity analysis on weighting factors to reveal the rationale of the trade-off between lane-change efficiency and mobility gain. It would provide a guidance for future users to pick weighting factor value according to their needs.

Acknowledgment

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