**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol: option : B

Explanation: for normal distribution :-

z = (X-μ)/б

= (60-55)/8

= 0.625

In R software for probability finding we use function called pnorm

As we want to find the probability of service manager cannot meet his commitment, So we should write below command.

1-pnorm(0.625)

=0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol:Mean = 38,SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE**

B) Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Sol:

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| As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .  Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).  2X1~ N(2 u,4 σ^2) and  =X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )  = 2X1-(X1+X2) = N( 4µ,6 σ^2) |

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Sol:

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| SSince we need to find out the values of a and b, which are symmetric about ththe mean, such that the probability of random variable taking a value bebettween them is 0.99, we have to work out in reverse order.   |  | | --- | |  | |  | The Probability of getting value between a and b should be 0.99. | | |  | So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99). | | |  | The Probability towards left from a = -0.005 (ie. 0.01/2). | | |  | The Probability towards right from b = +0.005 (ie. 0.01/2). | | |  | So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities. | | |  | By finding the Standard Normal Variable Z (Z Value), we can calculate the X values. | | |  | Z=(X- μ) / σ | | |  | For Probability 0.005 the Z Value is -2.57 (from Z Table). | | |  | Z \* σ + μ = X | | |  | Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4 | | |  | Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6 | | |

5) Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
2. Specify the 5th percentile of profit (in Rupees) for the company
3. Which of the two divisions has a larger probability of making a loss in a given year?

Sol:

Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD = (3^2)+(4^2)

print('Standard Deviation is Rs', SD\*45, 'Million')

Standard Deviation is Rs 315 Million

A) Range is Rs (-77.38865513011706, 1157.388655130117) in Millions.

B) 5th percentile of profit (in Million Rupees) is 23.4.

C) Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3) = 0.048

Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4) = 0.040

Inference: Probability of Division 1 making a loss in a given year is more than Division 2.