An Example of the Assembly Process

In Figure 2.12 is shown a small mesh containing three elements all of which have properties defined by (2.51). The problem is to assemble the element matrices into the complete system matrix. Assuming that there is only one unknown at each node the global node numbers will correspond to the global freedom numbers and the element node numbers will correspond to the local freedom numbers.

Each element possesses local freedom numbers (shown in parentheses) which follow the standard scheme, namely a consistent clockwise numbering.

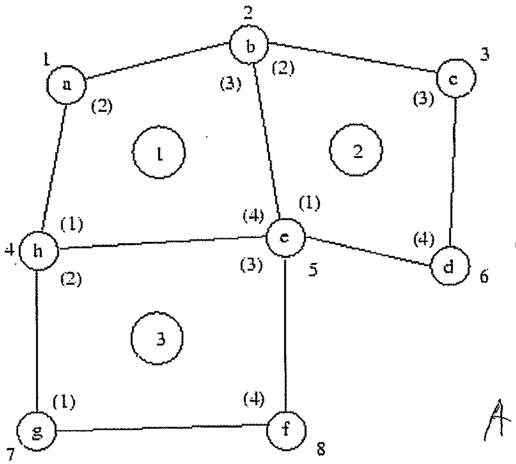

Figure 2.12: Mesh of quadrilateral elements

Boolean matrices:

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Each individual element equation can then be written as

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} \qquad (2.52)$$

Connectivity matrix:

$$C = \begin{pmatrix} 4 & 1 & 2 & 5 \\ 5 & 2 & 3 & 6 \\ 7 & 4 & 5 & 8 \end{pmatrix}$$

However, in the global freedom numbering system (not in parentheses) freedom 4 at node (h) corresponds to local freedom (1) in element 1 and local freedom (2) in element 3. In the assembled system matrix, term K_{11} from element 1 and K_{22} from element 3 would be added together and would appear in location (4,4) of the system matrix and so on. Similarly the contributions to global freedom 5 at node (e) will come from three elements. The process is essentially additive. The only complication is in ensuring that the correct element contributions are added together. The total system matrix for Figure 2.12 is given in Table 2.3 where the superscripts refer to element numbers.

Table 2.3: System stiffness matrix for mesh Figure 2.12

K^1_{22}	K^1_{23}	0	K^1_{21}	K^1_{24}	0	0	0
K^1_{32}	$K^1_{33}+K^2_{22}$	K^2_{23}	K^1_{31}	$K^1_{34}+K^2_{21}$	K^2_{24}	0	0
0	K^2_{32}	K^2_{33}	0	K^2_{31}	K^2_{34}	0	0
K^1_{12}	K^1_{13}	0	$K^1_{11}+K^3_{22}$	$K^1_{14}+K^3_{23}$	0	K^3_{21}	K^3_{24}
K^1_{42}	$K^1_{43}+K^2_{12}$	K^2_{13}	$K^1_{41}+K^3_{32}$	$K^1_{44}+K^2_{11}+K^3_{33}$	K^2_{14}	K^3_{31}	K^3_{34}
0	K^2_{42}	K^2_{43}	0	K^2_{41}	K^2_{44}	0	0
0	0	0	K^3_{12}	K^3_{13}	0	K^3_{11}	K^3_{14}
0	0	0	K^3_{42}	K^3_{43}	0	K^3_{41}	K^3_{44}

$$K = A_1^T K^1 A_1 + A_2^T K^2 A_2 + A_3^T K^3 A_3$$

This matrix will be symmetric if its constituent matrices are symmetric, and also possesses the useful property of *bandedness*. That is, the terms making up Table 2.3 are concentrated around the *leading diagonal* which stretches from th upper left to the lower right of the table. In fact no term in any row can be more than 4 locations removed from the leading diagonal so the system is said to have a *semi-bandwidth* of 5. The semi-bandwidth is usually denoted by HBAND.

The semi-bandwidth can be obtained by inspection of Figure 2.10 by subtracting lowest freedom number from the highest i each element and adding one. Complex meshes have variable bandwidths and computer programs make use of bandednes when storing the system matrices.

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