Sheet 9 - FEM for a 1-D heat problem

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Exercise 1.
Consider the one-dimensional heat equation

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} + 2 = 0 \qquad 0 < x < 6 \tag{1}$$

along with the boundary conditions

$$T(0) = 0, \;\; T'(6) = -3. \tag{2}$$

Find the exact solution to this equation.

Now write a code to implement a finite-element method for this problem. Use nodes at $x = \{0, 2, 4, 6\}$ and piecewise-linear hat functions for your basis. The basic structure of your code should be as follows:

- Define the grid
- Define the basis functions and compute the integrals (see hint).
- Build a mass-matrix A and a right-hand side vector $\boldsymbol{b}$ – at this point you should switch away from symbolics if you are using them.
- Build your solution T_{num} from the basis functions and coefficients found by solving the matrix inversion.

Compare your exact and numerical solutions.

Hint: There are two contrasting ways to approach this – using symbolic functions and integration (takes more computational time but requires less work from you) or by writing some clever code to implement all of the integrations automatically and quickly.

Exercise 2.
Can you adapt your code to cope with an arbitrary number or spacing of nodes? For a finer mesh, you might want to use some of your matrix inversion codes from previous questions to compute the solution at the final stage.