Problems

1) Consider the boundary-value problem (BVP)

$$u'' - 4u = -x \,, \qquad 0 \le x \le 1$$

$$u(0) = 0 = u(1)$$

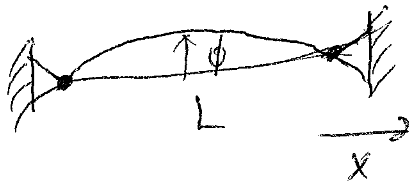
a) Write down a 2-term trial function for this BVP that satisfies the boundary conditions.

b) Using this trial function find approximations of the exact solution according to (i) the Galerkin weighted residual method, (ii) the least squares method and (iii) the collocation method with collocation points $x = \frac{1}{4}$ and $x = \frac{3}{4}$.

c) Derive the variational principle for the BVP and use the variational method to find an approximate solution.

d) Solve the BVP exactly and compare the solution with the approximations found in b) and c).

2) The differential equation governing free (small) transverse vibrations of a string is given by



$$\frac{d^2\phi}{dx^2} + \lambda \phi = 0 \,, \qquad 0 \le x \le L$$

with boundary conditions $\phi(0) = 0 = \phi(L)$, where $\lambda = \frac{\rho w^2 L^2}{T}$ is the eigenvalue, ρ is the mass per unit length, T is the tension in the string, L is the length, and w is the natural frequency of vibration.

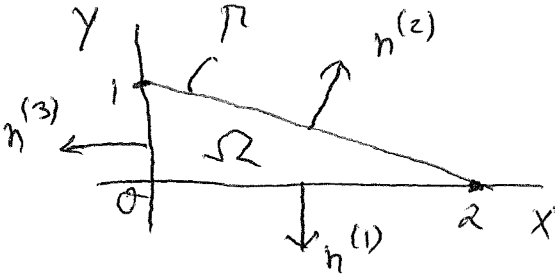
Using the trial function $\tilde{\phi}(x) = a_1 x(L-x) + a_2 x^2(L-x)$ where a_1 and a_2 are constants, determine the eigenvalues of the string using Galerkin's method.

<u>Hint:</u> The eigenvalues are those values of λ for which there is a non-trivial solution.

3) Gauss divergence theorem

Given the vector field

$$v(x,y) = \begin{pmatrix} 3x^2y + y^3 \\ 3x + y^3 \end{pmatrix}$$



on the triangular domain Ω shown, where $n^{(i)}$ are the outward-pointing unit normals to the (counter-clockwise) boundary Γ, verify the divergence theorem

$$\int_\Omega \operatorname{div}(v)\,dx\,dy = \int_\Gamma v\cdot n\,ds$$

Answer: left- and right-hand sides are both 1.5.

4) 1D weak formulation

Find the weak formulation for the boundary-value problem

$$x\frac{d^2u}{dx^2} + \frac{du}{dx} - x = 0 \qquad (0 \le x \le 1)$$

$$u(0) = 0 = u(1)$$