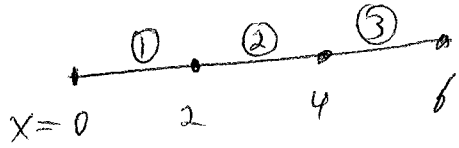
Exercises:

1) Solve the convective heat flow problem

$$\begin{cases} \frac{d}{dx}\left(Ak\frac{dT}{dx}\right) - \alpha T + \beta = 0 \\ T(0) = T_0 \\ q(6) = \bar{q} \end{cases} \qquad (A, k, \alpha, \beta, T_0, \bar{q} \text{ constants})$$

on a mesh of three 2-node elements

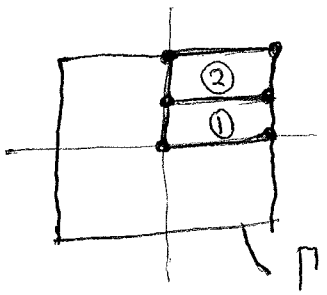


2) Solve the BVP for the torsion of a square cross-section shaft

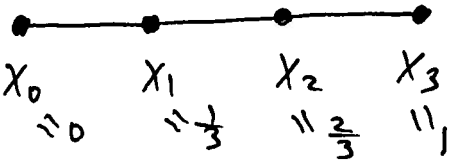
$$\begin{cases} \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2G\theta = 0 \\ \phi = 0 \quad \text{on } \Gamma \end{cases}$$

on a mesh of two 4-node rectangular elements for a quarter of the shaft's cross-section.

Compute an estimate for the shaft's torsional stiffness.



3) Comparison FE and FD:



Solve the Dirichlet boundary-value problem

$$\frac{d^2\phi}{dx^2} - \phi = 0, \quad 0 \le x \le 1$$

$$\phi(0) = 0, \quad \phi(1) = 1$$

on a uniform grid of 4 nodes using both finite elements and finite differences and compare the results to the exact solution.

[Answer: FE: $\phi_1 = \frac{53^2}{112^2 - 53^2} = 0.2885$, $\phi_2 = \frac{112 \cdot 53}{112^2 - 53^2} = 0.6098$

FD: $\phi_1 = \frac{q^2}{1q^2 - q^2} = 0.2893$, $\phi_2 = \frac{1q \cdot q}{1q^2 - q^2} = 0.6107$

exact: $\phi_1 = \frac{e^{1/3} - e^{-1/3}}{2\sinh(1)} = 0.2889$, $\phi_2 = \frac{e^{2/3} - e^{-2/3}}{2\sinh(1)} = 0.6102$]