GM04: MATLAB Exercise 1
Introduction—arrays and plotting

Autumn 2015

1. Create the following matrices in Matlab

$$\mathcal{A} = \begin{pmatrix} 5 & 3 & 1 & -1 \\ 2 & -3 & 6 & 1 \\ 6 & 4 & 1 & 3 \end{pmatrix} \quad \mathcal{B} = \begin{pmatrix} 1 & 3 \\ 3 & -4 \\ 2 & 0 \end{pmatrix} \quad \mathcal{C} = \begin{pmatrix} 6 & 0 & 6 & -1 \\ 9 & 2 & -1 & 5 \end{pmatrix}$$

   and combine the three matrices in all possible ways using the constructor operator [ ] and the operators **;** and **,**.

2. Take the following matrix

$$\mathcal{D} = \begin{pmatrix} 2 & 3 & 4 & 5 & 6 & 7 \\ 29 & 26 & 23 & 20 & 17 & 14 \end{pmatrix}.$$

   Write a code to

   (a) create this matrix using the colon (:) operator

   (b) add the elements of the first row to the elements of the second row and place the result in a third row

   (c) take the average value of the elements of the first and second rows and place the result in a fourth row.

   (d) add a positive random 10% error to all the elements of the matrix.

3. Create the following three matrices

$$\mathcal{A} = \begin{pmatrix} 5 & 2 & 4 \\ 1 & 7 & -3 \\ 6 & -10 & 0 \end{pmatrix} \quad \mathcal{B} = \begin{pmatrix} 11 & 5 & -3 \\ 0 & -12 & 4 \\ 2 & 6 & 1 \end{pmatrix} \quad \mathcal{C} = \begin{pmatrix} 7 & 14 & 1 \\ 10 & 3 & -2 \\ 8 & -5 & 9 \end{pmatrix}$$

(a) Find their inverses and check that multiplying each by their inverse leads to the identity matrix

(b) Calculate $\mathcal{A}(\mathcal{B}+\mathcal{C})$ and $\mathcal{AB}+\mathcal{AC}$ to show that matrix multiplication is distributive

(c) Use the function **eig(X)** in Matlab (look up its use in help) to find the eigenvectors and eigenvalues of the matrix $\mathcal{A}$ and hence find the matrices $\mathcal{P}$ and $\mathcal{D}$ (where $\mathcal{D}$ is a diagonal matrix) that satisfy

$$\mathcal{PDP}^{-1} = \mathcal{A}$$

4. Use matrix operations to solve the following systems of linear equations

   (a)

$$\begin{aligned} 4x - 2y + 6z &= 8 \\ 2x + 8y + 2z &= 4 \\ 6x + 10y + 3z &= 0 \end{aligned}$$

   (b)

$$\begin{aligned} 2p + 3q - r + 4s &= 23 \\ p + q - 3r + 5s &= 11 \\ 7p + q + 3r + 4s &= 12 \\ 5p + 4q + 3r - 11s &= 14 \end{aligned}$$

   In each case, form the matrix equation $\mathcal{A}\mathbf{x} = \mathbf{b}$ and find the inverse of the matrix $\mathcal{B} = \mathcal{A}^{-1}$. Check that $\mathcal{AB}$ is equal to the identity matrix. Once you have the solution, verify that the values obtained satisfy the equations given.

5. For the function $y = (x^2 + 1)^3 x^3$, calculate the value of y for twenty-one values of x equally spaced between $x = -3$ and $x = +3$. Solve the problem by creating two vectors of x and y values and using vectorised (element-by-element) operations. Plot this function.

6. Plot the functions $\cosh(x)$, $\sinh(x)$ and $\tanh(x)$ between $[-2, 2]$ on the same axes. Try to rescale the plot so that only the range $x \in [-1, 1]$ is seen, label the plots accordingly, and export the figure as a JPEG file.

7. Use vectorised operations to approximate the sum of the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Do this by computing the sum for $n = 100, 1000$ and 10000. *Hint: start by creating a vector whose elements are the integers from* 1 *up to* n*, then use vectorised operations and the function sum().* Try to do a similar experiment to show that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2.$$

8. Write a function called *spherecircle* that given a radius r will calculate the area and perimeter and a circle of that radius, as well as the volume and surface area of a sphere of the same radius. Write another function called *cylinder* that given the cylinder's radius and length will calculate its volume and surface area.

9. Write a function that will take the cross product of two three-dimensional vectors. Check that it agrees with the built in function $\mathbf{cross}(A, B)$.

10. Write a function that will take a function handle e.g.

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f = @(x)(x.^2)
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and integrate it numerically over an interval $[a, b]$ using *Simpson's Rule.* Use vectorised operations.

11. Plot the function

$$z = 1.8^{-\sqrt{x^2+y^2}} \sin(x) \cos(y/2)$$

over the domain $-4 \le x \le 4$ and $-4 \le y \le 4$ using $\mathbf{mesh}(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$. Label the axes correctly and add a suitable title. Plot the function over the same range as a contour plot (with twenty evenly spaced contour lines between the maximum and minimum values attained) and as a surface plot.