GM04: Exercise sheet 3, forward Euler method

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1 The basic program

Consider the differential equation

$$y' - 3y = \mathrm{e}^x, \qquad y(0) = 0.$$

1. Find the exact solution to this equation and plot it in `Matlab`.
2. Write a function that solves this equation using the forward Euler method up to some maximum value $x = X_m$. The function should take the time-step dx as an input, and should return two vectors, x and y (where the i-th entry of y is the value of the solution at the i-th entry of x). The function should also plot the solution.
3. Run your function with $X_m = 3$ and $dx = 0.1$. Plot the result against the exact solution, and add an appropriate title and legend to the graph.
4. Now try various different values of X_m and dx. Try to produce a graph that shows that the global error (difference between numerical and exact solutions at the last time-step) scales with dx. Note that the scaling approximations derived in lectures are valid for large N (*i.e.* when you take a lot of steps). This can be achieved either by taking very small steps, or by running the program for large X_m.

2 A nonlinear equation

Consider now the nonlinear equation

$$y' = \sin y, \qquad y(0) = 1.$$

1. Write a function that solves this equation on the interval $0 < x < 5$ using the forward Euler method.

2. Write down a second-order, centered, finite-difference approximation for y'. Try and set up a function that uses this approximation instead of the usual forward-Euler one. Think carefully about how the integration should be started, and where the right-hand side should be evaluated.

3 A second order problem

Simple harmonic motion is given by the second-order equation

$$y'' + y = 0, \qquad y(0) = y'(0) = 0.5.$$

1. Calculate the exact solution to this equation

2. Re-write the equation as a first order system

3. Adapt Euler's method to work for a system, and write a function that can implement it for a given time-step. The function should produce three output vectors, x, y and y' and plot y against x.

4. Vary the initial conditions and produce a phase-plane diagram.