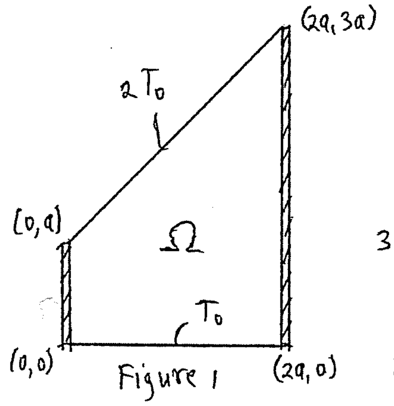
Problem 1:

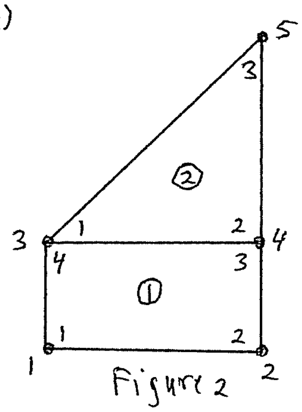
Steady-state heat transfer in a nonuniform plate in the (x, y) plane is governed by

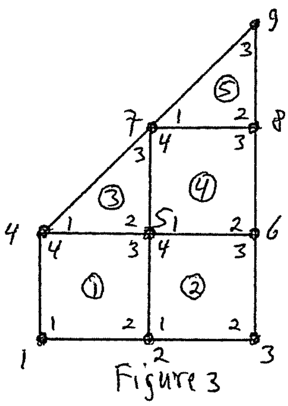
$$-\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(k(1+y)\frac{\partial T}{\partial y}\right) = f(x, y), \qquad (x, y) \in \Omega,$$

where T is the temperature, $f(x, y)$ is the internal heat generation per unit volume, and k is the thermal conductivity. We consider the heat flow problem on the quadrilateral domain Ω of Figure 1. The bottom and top boundaries are kept at temperatures T_0 and $2T_0$, respectively, while the other boundaries are insulated, meaning that the heat flux through these boundaries is zero, i.e., $\partial T/\partial x = 0$.

We shall assume a uniform heat source and take units such that $a = 1$, $k = 1$ and $f = 1$.







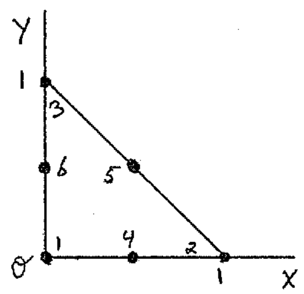
a) Solve the FE equation for the nodal temperatures on the mesh depicted in Figure 2. Use the local and global node numbering indicated.

b) Do the same for the refined mesh of Figure 3. Compare the two solutions.

c) For the solution in b) compute the heat flux through the boundary at global nodes 2 and 7.

Problem 2:

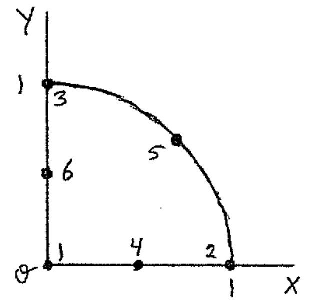
a) Show that the shape functions for the normal 6-node triangular element



are given by

$$
\begin{aligned}
N_1(x,y) &= (1-x-y)(1-2x-2y),\\
N_2(x,y) &= x(2x-1),\\
N_3(x,y) &= y(2y-1),\\
N_4(x,y) &= 4x(1-x-y),\\
N_5(x,y) &= 4xy,\\
N_6(x,y) &= 4y(1-x-y).
\end{aligned}
$$

b) Use the element in a) as the parent element in an isoparametric transformation to compute the stiffness matrix for the Laplacian $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ on the following 6-node curved element in the shape of a quarter of a circular disc:



c) The equation governing small lateral deflections z of a uniform membrane subjected to a lateral (dimensionless) pressure p is given by

$$
\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + p = 0.
$$

Solve an appropriate FE equation to find the deflection at the centre of a circular membrane of radius 1 that is fixed (i.e., $z = 0$) at the boundary. Assume uniform pressure $p = 1$.

Hint: use the result in b).

d) Discuss the solution. Compute and physically interpret the boundary vector.

(Due in date: 27 January 2017)