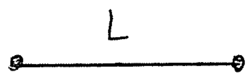
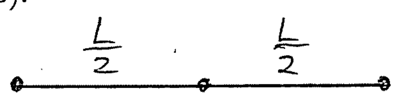
Finite element shape functions

Linear 1D element ($0 \leq x \leq L$):



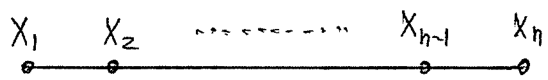
$$N_1^e(x) = 1 - \frac{x}{L}, \qquad N_2^e(x) = \frac{x}{L}$$

Quadratic 1D element ($0 \leq x \leq L$):



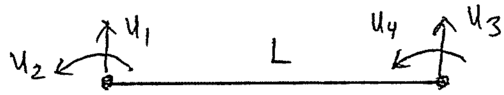
$$N_1^e(x) = \left(1 - \frac{x}{L}\right)\left(1 - \frac{2x}{L}\right), \qquad N_2^e(x) = \frac{4x}{L}\left(1 - \frac{x}{L}\right), \qquad N_3^e(x) = -\frac{x}{L}\left(1 - \frac{2x}{L}\right)$$

General Lagrange element of order n ($x_1 \leq x \leq x_n$):



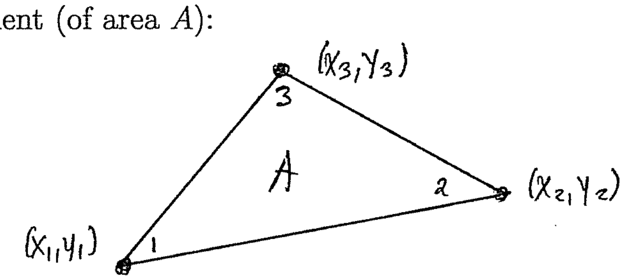
$$N_i^e(x) = \frac{(x - x_1)(x - x_2)\cdots(x - x_{i-1})(x - x_{i+1})\cdots(x - x_n)}{(x_i - x_1)(x_i - x_2)\cdots(x_i - x_{i-1})(x_i - x_{i+1})\cdots(x_i - x_n)} \qquad (i = 1, 2, ..., n)$$

2-node beam element ($0 \leq x \leq L$):



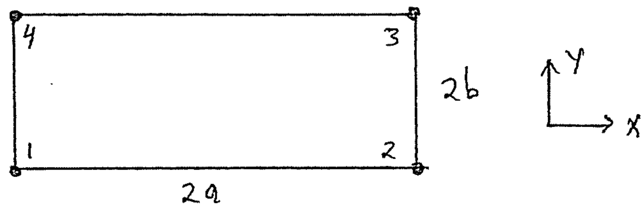
$$N_1^e(x) = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}, \qquad N_2^e(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2},$$
$$N_3^e(x) = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}, \qquad N_4^e(x) = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

Linear triangular element (of area A):



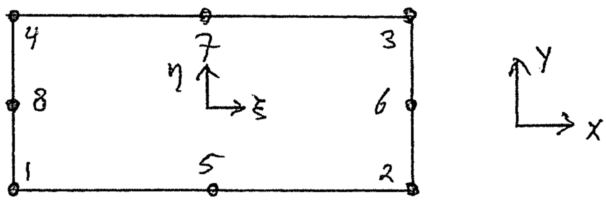
$$N_i^e(x, y) = \frac{1}{2A}\left[x_j y_k - x_k y_j + (y_j - y_k)x + (x_k - x_j)y\right] \qquad (i, j, k \text{ cyclic})$$

4-node rectangular (Lagrange) element:



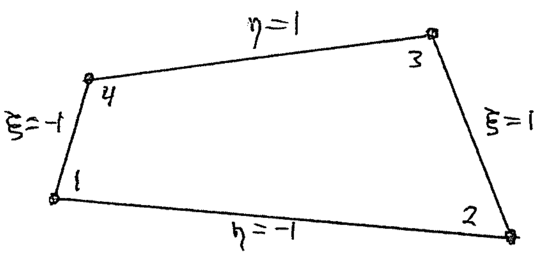
$$N_1^e(x,y) = \frac{1}{4ab}(x-x_2)(y-y_4), \qquad N_2^e(x,y) = -\frac{1}{4ab}(x-x_1)(y-y_3),$$
$$N_3^e(x,y) = \frac{1}{4ab}(x-x_4)(y-y_2), \qquad N_4^e(x,y) = -\frac{1}{4ab}(x-x_3)(y-y_1)$$

(Isoparametric) 8-node serendipity element:



$$N_1^e(\xi,\eta) = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta), \qquad N_2^e(\xi,\eta) = -\frac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta),$$
$$N_3^e(\xi,\eta) = -\frac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta), \qquad N_4^e(\xi,\eta) = -\frac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta),$$
$$N_5^e(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1-\eta), \qquad N_6^e(\xi,\eta) = \frac{1}{2}(1+\xi)(1-\eta^2),$$
$$N_7^e(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1+\eta), \qquad N_8^e(\xi,\eta) = \frac{1}{2}(1-\xi)(1-\eta^2)$$

4-node isoparametric quadrilateral element:



$$N_1^e(\xi,\eta) = \frac{1}{4}(\xi-1)(\eta-1), \qquad N_2^e(\xi,\eta) = -\frac{1}{4}(\xi+1)(\eta-1),$$
$$N_3^e(\xi,\eta) = \frac{1}{4}(\xi+1)(\eta+1), \qquad N_4^e(\xi,\eta) = -\frac{1}{4}(\xi-1)(\eta+1)$$

GH/05