# GM04: Exercise sheet 4, RK4 method

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## 1 RK4 for a simple equation

Consider the first order differential equation

$$y' = \frac{\sin t - 2ty}{t^2}, \qquad 1 \leq t \leq 2, \qquad y(1) = 2.$$

1. Write a `MATLAB` code to solve this problem for stepsize $h = 0.02$ using the forward Euler method.
2. Write another code that uses the same step-size and the RK4 scheme.
3. Find the exact solution (using `dsolve`, Wolfram or by hand...!) Calculate the average global truncation errors from both methods and confirm their orders of accuracy.

## 2 RK4 for a system

Consider the system of equations given by

$$x'(t) = x - y, \qquad x(0) = 0.y'(t) = 1 - xy, \qquad y(0) = 0.$$

1. Write a program that solves the system using RK4. Run your solution and plot the outcome as $x$ against $y$.
2. What is the long-term behaviour of the solution?
3. Change the initial conditions to $x(0) = 0.8$, $y(0) = 1.1$. What is the long-term behaviour of the system now?
4. Run the code for several different initial conditions and plot a phase-plane diagram (that is, plot $x$ against $y$ on the same set of axes for different initial conditions. Try to work out the nature of the critical points, and confirm this analytically.