Pallak Singh (NUID: 001063095) Section 02

INFO 6205
Program Structures & Algorithms
Fall 2020
Assignment No 1

- **Task**

  Our task was to deduce a relationship between the number of steps and the mean Euclidean distance walked by a drunkard in a random walk experiment. The drunkard can walk in either north, south, east or west directions during each step. The task was completed in the following steps:

  1. Implementing the unimplemented functions in RandomWalk.java
  2. Plotting a steps vs mean Euclidean distance graph
  3. Making a general guess about the relationship between the number of steps and the mean Euclidean distance
  4. Deriving a relationship between the number of steps and the mean Euclidean distance
  5. Proving the relationship using a graph
  6. Providing evidence and findings in the report

- **Output**

  In RandomWalk.java, I ran a loop that runs the experiment 500 times for steps from 1 to 100 (a total of 50,000 runs), found the mean Euclidean distance for each step and plotted the result on a steps vs mean Euclidean distance graph. I got the following graph:

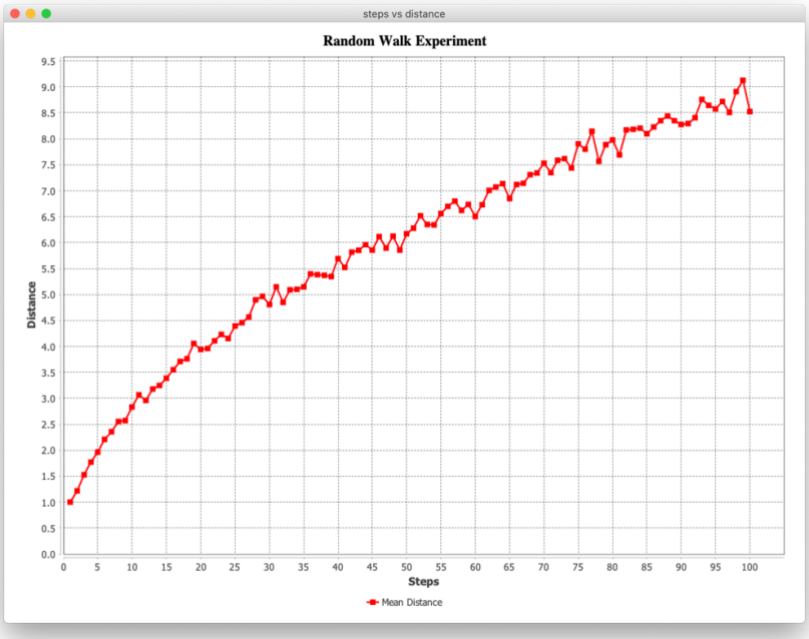


Table 2.1: Line graph of steps vs mean Euclidean for n = 500

I also printed the mean Euclidean distance for each step which is provided below:

1 steps: 1.0 over 500 experiments
2 steps: 1.2233077913133283 over 500 experiments
3 steps: 1.6085386471799226 over 500 experiments
4 steps: 1.7889321455572662 over 500 experiments
5 steps: 2.0316728002502544 over 500 experiments
6 steps: 2.173125983831428 over 500 experiments
7 steps: 2.39605107521097 over 500 experiments:
:
:
94 steps: 8.647473253975608 over 500 experiments
95 steps: 8.574440088050979 over 500 experiments
96 steps: 8.720069168786308 over 500 experiments
97 steps: 8.51024165014071 over 500 experiments
98 steps: 8.907587072035422 over 500 experiments
99 steps: 9.12835897151636 over 500 experiments
100 steps: 8.5259415204996 over 500 experiments

The RandomWalk.java class and the RandomWalkPlotter.java class (for plotting the graphs) are provided in the folder 'Code' attached with the submission.

- **Relationship conclusion**

  Upon looking at the steps vs mean distance graph (Figure 2.1), it was easy to deduce that there must be a relationship between the $\sqrt{step}$ and the mean Euclidean distance. So, in order to confirm my suspicions, I plotted the steps vs $\sqrt{step}$ and the steps vs mean distance graph together to find the following:

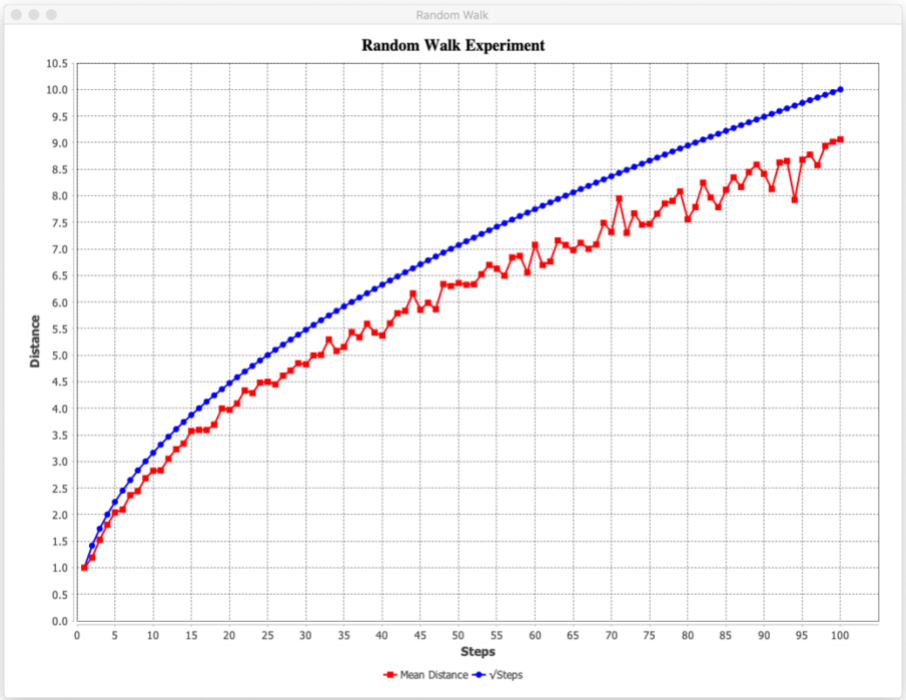


Table 3.1: Line graph of steps vs mean Euclidean Distance and steps vs $\sqrt{steps}$ for n = 500

We can observe that there is a relationship indeed!
We find that,

$$\bar{d} \propto \sqrt{m}$$

$d =$ mean Euclidean distance travelled by drunkard
$m =$ step

$$\bar{d} = c\sqrt{m}$$

In order to find the average c over each step, I used the following formula:

$$\bar{c} = \frac{\sum_{m=1}^{k} \frac{\sqrt{i}}{\bar{d}}}{k}$$

The average c came out to be 0.88716.

That is,

$$\bar{d} = 0.88716 \times \sqrt{m}$$

Upon research [1], I found that the relationship between the mean Euclidean distance and the number of steps is:

$$\bar{d} = \frac{\sqrt{m\pi}}{2}$$

And,

$$\frac{\sqrt{\pi}}{2} \approx 0.88622692545$$

There is a mathematical basis to our hypothesis!

- **Evidence to support relationship**

To prove the relationship mentioned in the section above, I plotted more graphs!

The first graph plots the steps vs mean distance, steps vs $\sqrt{\text{steps}}$ and steps vs $0.88716 \times \sqrt{\text{steps}}$ (from our equation above). I got the following result:

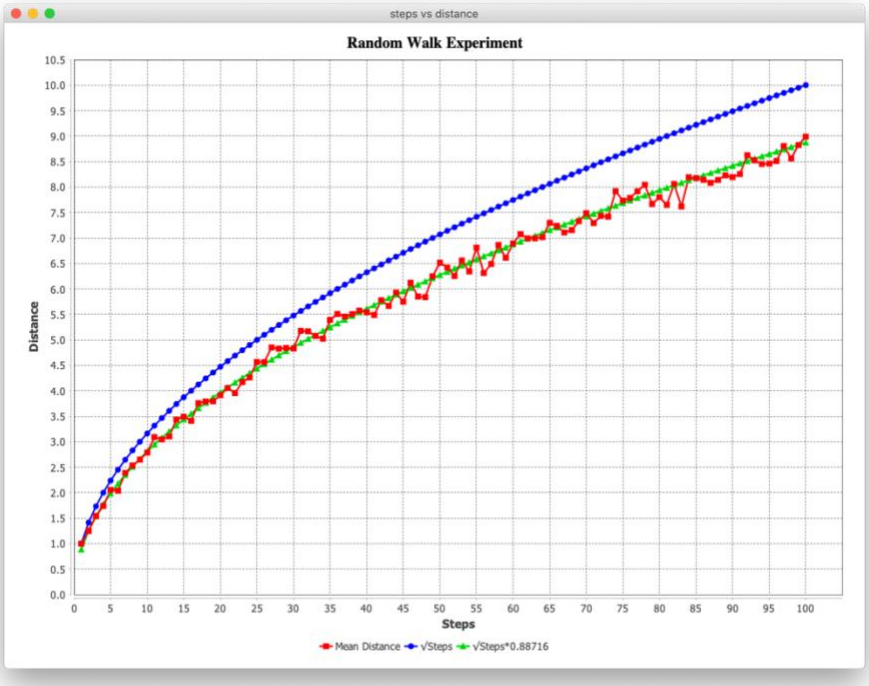


Table 4.1: Line graph of steps vs mean Euclidean Distance, steps vs $0.88716 \times \sqrt{steps}$ and steps vs $\sqrt{steps}$ for n = 500

Since there was variance in the distances calculated from 500 experiments for each step, I increased n to 10,000 and found the following result:

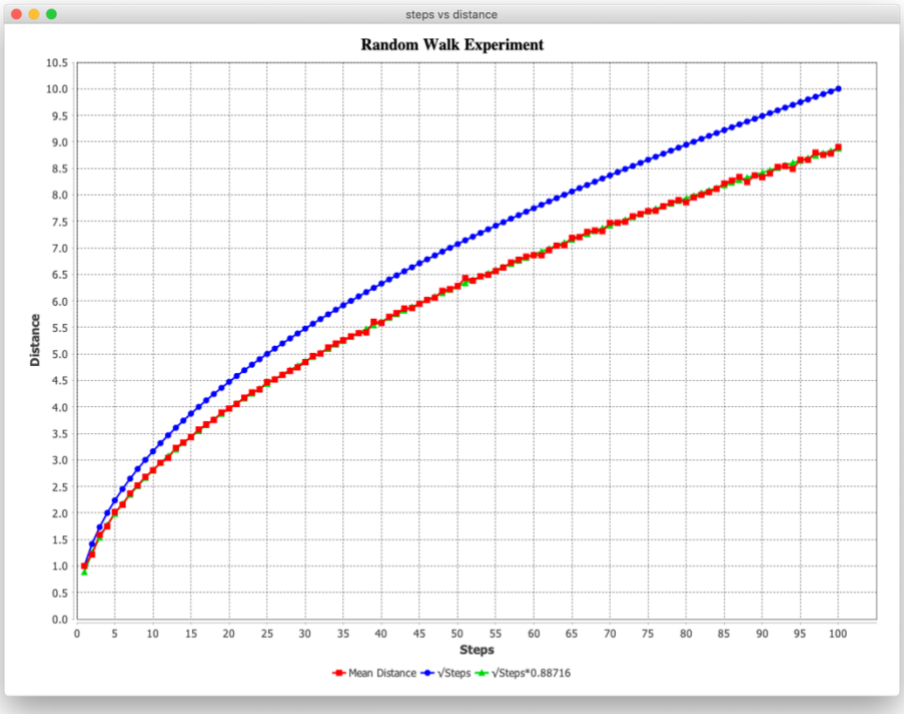


Table 4.2: Line graph of steps vs mean Euclidean Distance, steps vs $0.88716 \times \sqrt{steps}$ and steps vs $\sqrt{steps}$ for n = 10000

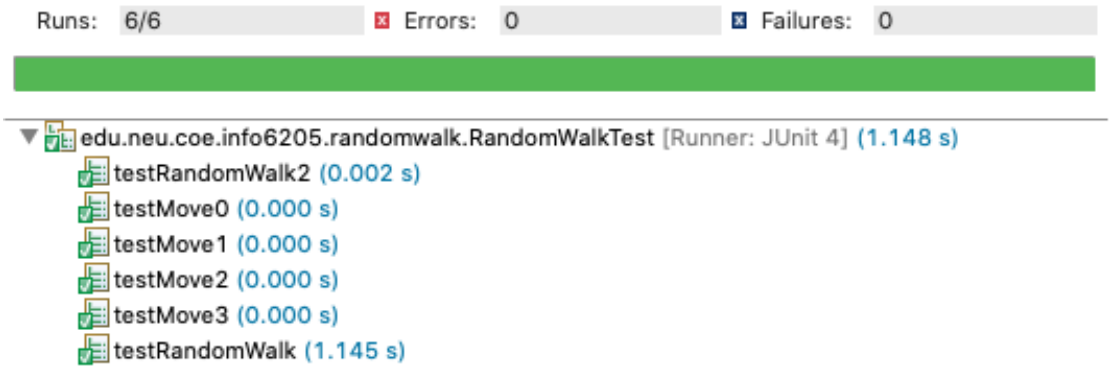
Our expected value of mean distance overlaps the mean distance that was found!
Here is a table of the mean distance, $0.88716 \times \sqrt{\text{step}}$ for steps from 1 to 10 for n = 500:

Steps	Mean Distance	$0.88716 \times \sqrt{\text{step}}$
1	1.0	0.88716
2	1.22330	1.25463
3	1.60853	1.53660
4	1.78893	1.77432
5	2.03167	1.98375
6	2.17312	2.17308
7	2.39605	2.34720
8	2.52652	2.50926
9	2.68637	2.66148
10	2.76853	2.80544

Table 1: Comparison of Mean Euclidean Distance and $0.88716 \times \sqrt{\text{steps}}$

The evidence points to the derived relationship being an accurate representation of the mean distance and steps.

- **Screenshot of Unit test passing**



Works Cited

[1] WIRED Staff. (2017, March 14). How To Calculate Pi on a Random Walk. Retrieved September 19, 2020, from WIRED website: https://www.wired.com/2017/03/hey-can-find-pi-random-walk-heres/