Project 1

EOC 6189 - Computational Fluid Dynamics
Florida Atlantic University

January 30, 2019

Collaboration policy: *You are strongly encouraged to discuss your work with fellow students, to brainstorm ideas, and to learn from each other. However, you are not allowed to copy solutions/code from each other, from the internet, or from any other source. All work shown must be your own. You are expected to follow the University Honor Code at all times.*

Instructions: Please create a single zip file containing all your PDFs, movies, etc., and upload it on Canvas by the deadline. Your code source files must be submitted via git repository. Try to keep your code as modular as possible, which involves writing separate functions, where one function does only one job (e.g., discretization, derivative computations, time integration, etc.). This will help you reuse your functions with minimal modification in future projects. Make sure to write short, but useful comments in the code and your git commits. Undocumented code will lose points.

1. 1-Dimensional Heat Equation

   (a) Write code to solve the inhomogeneous 1D heat equation given below. Use a first order explicit discretization in time, and second order discretization in space. Make sure to commit your changes to git as you build up your code.

   $$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \sin(5x) \tag{1}$$

   Use the following Initial and Boundary Conditions:

   $$IC: \; T(x,0) = x(\pi - x) \tag{2}$$
   $$BC: \; T(0,t) = T(L,t) = 0 \tag{3}$$

   Here, $\alpha = 0.25$ is the thermal diffusivity of the rod material, and $L = \pi$ is its length. Discretize the rod into 100 separate sections (i.e., using 101 points), and use a time step size of 10^{-4}. Run your simulation up until $t = 50$. Plot the time evolution of $T(x = 0.5L, t)$, and the final temperature profile in the rod at $T(x, t = 50)$.

(b) Rerun the simulation with $\Delta t = 10^{-2}$, and make the two plots for $T(x = 0.5L, t)$ and $T(x, t = 50)$ again. What do you observe? Why? Support your answer by referring to the stability condition for the discretization.

(c) You are given that the analytical solution for the steady state temperature profile in the rod is as follows:

$$T_{exact}(x) = \frac{1}{25\alpha} \sin(5x) \tag{4}$$

Using this knowledge, you may compute the error in the numerical solution as follows:

$$\varepsilon = \sum_{i=1}^{n} (T_{exact}(x_i) - T_i)^2 \tag{5}$$

Run your simulation with the time step fixed at $1e - 4$, but with different grid refinements, and compute this error for each run. For instance, you could choose to discretize the rod using 10 segments, 50 segments, 100 segments, 200 segments, and 400 segments. Plot the log of the error (y axis) vs the log of number of points (x axis). The slope of this curve should give you the order of accuracy of the discretization. What is the order of accuracy of the spatial discretization according to your graph? Is this expected?

(d) Run your simulation with 1000 segments for the rod, and $\Delta t = 1e - 4$. What happens to your result? Why? Mention two different ways that you can fix this issue.

(e) ~~Run your simulation with 100 segments for the rod, and compute the error in your solution using various time step sizes. For instance you could try $\Delta t = 1e - 3,\ 5e - 4,\ 1e - 4,\ 5e$ Plot the log of the error vs the log of total number of timesteps taken (e.g., $50/(1e - 3)$ steps, for the case when $\Delta t = 1e - 3$). What is the order of accuracy of the time discretization according to your graph? Is this expected?~~

2. For this problem, you will create a movie showing the time decaying solution of the heat equation. Reuse components from the code you wrote for the previous problem to minimize development effort. This time you will solve the homogeneous heat equation, with the following ICs and BCs:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{6}$$

$$IC: \ T(x, 0) = \sin\left(\frac{\pi x}{L}\right) \tag{7}$$

$$BC: \ T(0, t) = T(L, t) = 0 \tag{8}$$

Here, $\alpha = 0.1$, and the length of the rod is 1. The exact time-decaying solution for this problem is given as:

$$T_{exact}(x, t) = \sin\left(\frac{\pi x}{L}\right) \exp\left(-\frac{\alpha \pi^2 t}{L^2}\right) \tag{9}$$

Select an appropriate grid size and time step size, and run your numerical simulation until $t = 10$. Create a video of the solution, by plotting the evolution of both the exact solution and the numerical solution simultaneously.

3. Create a movie showing the time-varying numerical solution for the following problem:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{10}$$

$$T(x, 0) = 0 \tag{11}$$

$$T(L, t) = 0 \tag{12}$$

$$T(0, t) = \sin(2\pi t) \tag{13}$$

Here, $\alpha = 0.1$, and $L = 1$. Run your simulation until $t = 2$. ~~$t = 10$~~ Provide a physical interpretation of your animation, in terms of heat transfer through the rod.