Project 3 (15 points)

EOC 6189 - Computational Fluid Dynamics
Florida Atlantic University

February 24, 2019

Collaboration policy: *You are strongly encouraged to discuss your work with fellow students, to brainstorm ideas, and to learn from each other. However, you are not allowed to copy solutions/code from each other, from the internet, or from any other source. All work shown must be your own. You are expected to follow the University Honor Code at all times.*

Instructions: Please create a single zip file containing all your PDFs, movies, etc., and upload it on Canvas by the deadline. Your code source files must be submitted via git repository. Try to keep your code as modular as possible, which involves writing separate functions, where one function does only one job (e.g., discretization, derivative computations, time integration, etc.). This will help you reuse your functions with minimal modification in future projects. Make sure to write short, but useful comments in the code and make regular git commits. Undocumented code will lose points. Please give the instructor access to your git repository when you are ready for submission.

2D Advection - second order upwind

1. For this problem, you will solve the 2-dimensional pure advection problem using the 2nd order upwind method. The variable being advected is a passive scalar, i.e., it does not impact the flow velocity.

   (a) Write out the appropriate discretization for the 2nd order upwind 2D advection equation on paper.

   (b) Create a quiver plot with the following 2D velocity profile that is constant in time, but varies in both x and y. Use $L_x = L_y = 2\pi$, and $N_x = N_y = 31$ points.

   $$u = \cos(x)\sin(y) \tag{1}$$
   $$v = -\sin(x)\cos(y) \tag{2}$$

   (c) Create a 2D scalar field, using the following cosine distribution in space (hint: use *contourf* in Matlab):

   $$\phi_0 = cos(x) \tag{3}$$

Create an image of the scalar contour plot, with the velocity quiver plot overlaid on top of it.

(d) Write code to solve the 2D advection equation, with the initial scalar ϕ_0 in Eq. 3 being advected by the velocity profile given in Eqs. 1 and 2. Use periodic boundary conditions, and $dt = 1e - 2$. Be sure to make regular git commits of your code. Create a video showing the scalar being advected from $t = 0$ to $t_{max} = 10$ seconds.

(e) Rerun your simulation with 101 grid points. What do you observe?

(f) Rerun your simulation with 501 grid points. What do you observe? Why?

2. Create a second video of the advection with $N_x = N_y = 31$ and $t_{max} = 10$, but with a velocity profile that is varying in both space and time as follows:

$$u = sin(2\pi t/t_{max}) \cos(x) \sin(y) \tag{4}$$

$$v = -sin(2\pi t/t_{max}) \sin(x) \cos(y) \tag{5}$$

What do you expect the final scalar profile to look like? Is this true for your solution? Why, or why not?