



**Florida Atlantic University**

College of Engineering

## **Project 1**

Application of the Direct Integration Method or of Discontinuity (Singularity)

Functions to Statically Determinate Uniform or Stepped Beam Design

Serial Number: 02

Name: Pedro Almeida

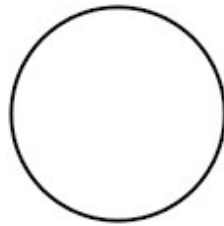
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**September 15, 2021**



# **INTERMEDIATE** **STRENGTH OF** **MATERIALS**

**FALL 2021**

Communication Project 1:

**“Application of the Direct Integration  
Method or of Discontinuity (Singularity)  
Functions to Statically Determinate  
Uniform or Stepped Beam Design”**

Requirements and Deliverables

(assigned: Friday, September 10, 2021; submission date: Friday, October 1,  
2021; not later than 4:00 pm)

**“You will be judged in industry not on  
[only] your work but how well you  
communicate your work,”** Charles R. Cluer “A Survey of  
Industrial Mathematics”, page 277, Dover, 2012. The requirements that should  
be satisfied by the project are given in the document called FORMAT (see  
CANVAS).

### Problem 1

For the beam that is depicted below is subjected to a distributed load  $w_0$ , concentrated load  $P$  and concentrated moment  $M_0$ . The length is specified  $L = 2$  meters. The cross section is represented by an I-beam. Material is aluminum; yield stress = 276 MPa;  $E = 69.9$  GPa; required safety factor is  $k = 1.5$ . The allowable displacement  $v_{allow} = L/500$ . Design the beam so that it satisfied strength condition, as well as stiffness condition. The moments of inertia of I-beams are given on pages 804-805 of the textbook, the number  $s$  is your serial number. Note: don't forget to depict FBD. Write equations to determine reactions. Use singularity functions, not the superposition principle. Please do not use SKYCIV software.

### Problem 2

For the beam that is depicted is subjected to a distributed load  $w_0$ , concentrated load  $P$  and concentrated moment  $M_0$ . The length is specified  $L = 2$  m. The cross section is circular with radius  $c$ . Material is aluminum; yield stress = 276 MPa;  $E = 69.9$  GPa; required safety factor is  $k = 1.5$ . The allowable displacement  $v_{allow} = L/400$ . The number  $s$  is your serial number. Use singularity functions, not the superposition principle. Please do not use SKYCIV software.

- (a) Design the beam—find minimum allowable radius of cross section-- so that it satisfies strength condition, as well as stiffness condition.
- (b) Write resulting expression of  $V_y(x)$  and  $M_z(x)$ .
- (c) Construct diagrams of the shear force  $V_y(x)$  and bending moment  $M_z(x)$  as well as for deflection  $v(x)$ . Note: don't forget to depict FBD. Write equations to determine reactions

### Problem 3

The beam is subjected to concentrated loads as shown in the figure. The number of loads equals  $100-s$ , where  $s$  is your serial number. Find the maximum displacement of the beam. Note: don't forget to depict FBD. Write equations to determine reactions. Use singularity functions, not the superposition principle. Please do not use SKYCIV software.

### Problem 4

For the beam that is depicted is subjected to a distributed load  $w_0$ , concentrated load  $P$  and concentrated moment  $M_0$ . The length is specified  $L = 2$  m. The cross section is represented by a rectangle who base is three times less than its height (or depth). Material is aluminum; yield stress = 276 MPa;  $E = 69.9$  GPa; required safety factor is  $k = 2$ . The allowable displacement  $v_{allow} = L/400$ . The number  $s$  is your serial number. Use singularity functions, not the superposition principle. Please do not use SKYCIV software.

- (d) Design the beam so that it satisfied strength condition, as well as stiffness condition, i. e. find minimum allowable value of the beam's hight (depth).

- (e) Write resulting expression of  $V_y(x)$  and  $M_z(x)$ .
- (f) Construct diagrams of the shear force  $V_y(x)$  and bending moment  $M_z(x)$  as well as for deflection  $v(x)$ . Note: don't forget to depict FBD. Write equations to determine reactions

### Problems 5-6

Determine the maximum deflection in the beam subjected to following loading. The cross-section is given by American Standard Channel (p. 810). Make a choice of the material, as well as (nonzero) values of  $w_0$  and  $M_0$ . Determine the maximum deflection. Does it occur in the middle cross-section? Note: don't forget to depict FBD. Write equations to determine reactions. Use singularity functions, not the superposition principle. Please do not use SKYCIV software.

## Problems 7

For the beam that is depicted below is subjected to a distributed load  $w_0$ , concentrated load  $P$  and concentrated moment  $M_0$ . The length is specified  $L= 6$  meters. The cross section of the first part and the third part are represented by identical circular beams with radius  $c$ . The cross-section in the second part is also circular but radius equals  $2c$ . Material is aluminum; yield stress= 276 MPa;  $E= 69.9$  GPa; required safety factor is  $k= 1.5$ . The allowable displacement  $v_{allow}= L/400$ . Design the beam so that it satisfied strength condition, as well as stiffness condition. The moments of inertia of I-beams are given on pages 804-805 of the textbook, the number  $s$  is your serial number. Note: don't forget to depict FBD. Write equations to determine reactions. Use equivalent beam method. Please do not use SKYCIV software.

The copy of these assignment page must be included beneath the title page of the project.

The project ought to be presented in the reader-friendly format: For each problem, if the diagrams are not strictly below each other in the same scale as the Include sections of **Abstract** and **Conclusion**: Summarize what you learned from this personal communication project; provide also recommendation(s) to the lecturer on the project assignment and management.

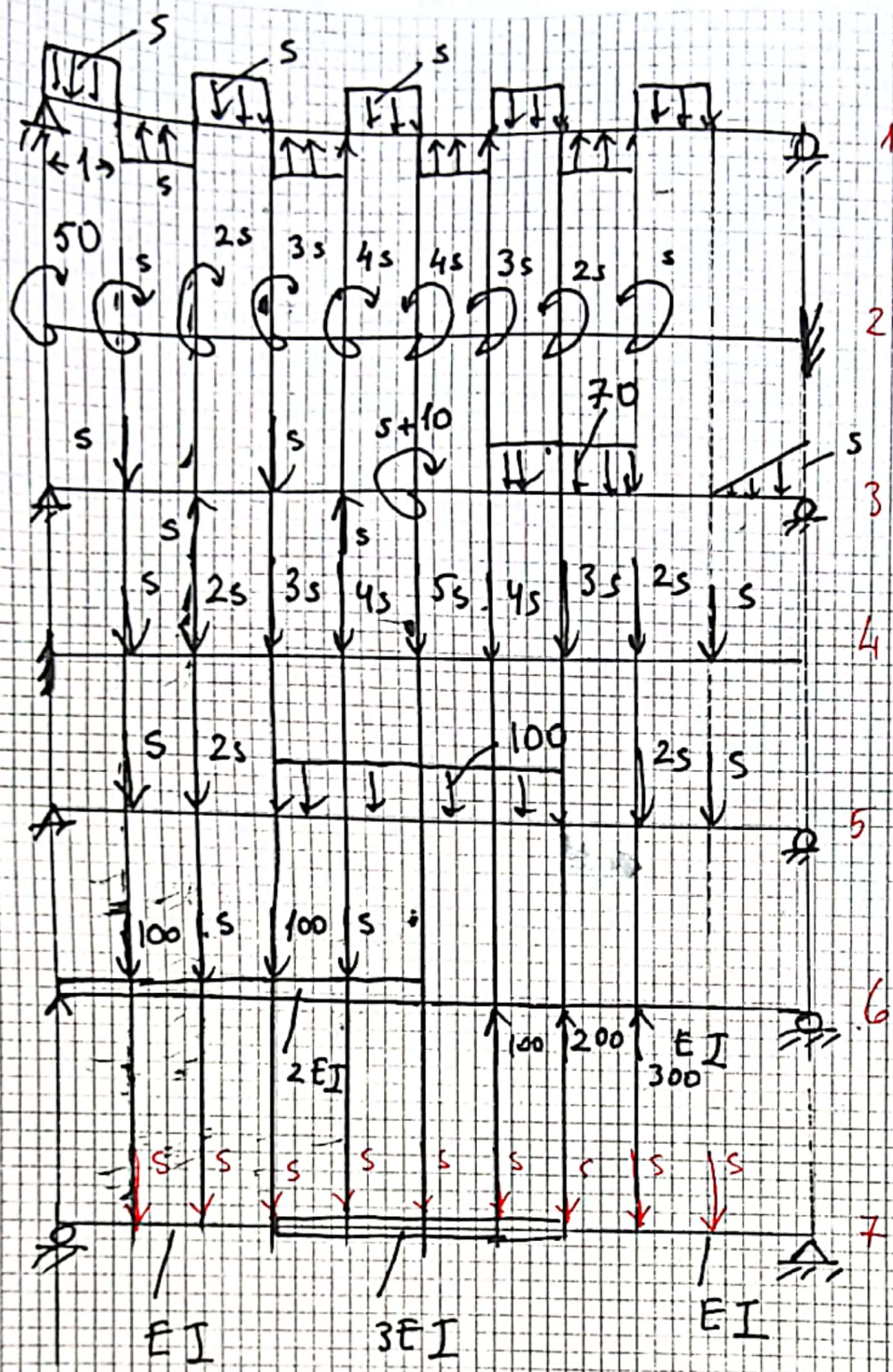
**P.S: Entire work must be neatly hand-written or typed. A grade “zero” will be assigned if not doing the project by yourself. Bonus of 5% for typed project but not computer-generated plots; 5% additional credit points for typed project and computer-generated plots. Students must submit the project via CANVAS ( the submission methodology will be communicated later on). Each project submitted later than October 1, 2021, leads to 10 credits less, for each work day (this does not include weekends, Saturday and Sunday).**

With wishes of success,

***Isaac Elishakoff***

Prof. Isaac Elishakoff and TA Abhishek Ratanpara







*The report will apply the Direct Integration Method of Singularity Functions to statically determine uniform or stepped beam designs to determine the load's effect on the beam's deflection. The resulting functions following the integration will be plotted using MATLAB software to produce simplified renderings of the beam's shape post loading.*

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## 1 Introduction

One of the most important processes in designing beams is the structural analysis of the part under realistic loading conditions. A proper, realistic, theoretical model allows the engineer or designer to understand the expected stresses and deflections of a beam under expected loading conditions, thus removing the need to conduct experiments for each attempted revision. To properly construct the theoretical models, engineers often rely on extensive sets of equations.

The standard method to solve for beam deflection for beams with continuous cross sectional areas relies on partitions; the designer splits the beam into multiple subsections where loads are applied. The resulting cross sections are then each modeled separately. The singularity equations for beam deflection, by contrast, allow for the reduction of the multiple deflection equations into a single continuous equation. The method will be discussed thoroughly in the Background Theory section.

The report will comprise of the Background Theory section discussing the required equations and methods to solve the problems, the Problem Definition section discussing each of the problems presented in the project, the Problem Solution section discussing the steps used to arrive to each solution, the Results and Discussion containing the important plots for each of the problems given, and the Conclusions section to summarize findings and discuss improvements.

## 2 Background Theory

To find the deflection equation for a statically determinate beam under loading, there are two principal ways to approach the problem, both of which lead to the same result. The first method, shown in Eq. 1, uses the loading function  $w(x)$  to reach the deflection. The form is especially useful for beams under non-linear loading functions. However, due to the required

quadruple integration to reach  $v(x)$ , engineers commonly use a second form.

$$EI \frac{\delta^4 v}{\delta x^4} = w(x) \quad (1)$$

The second form of the deflection equation can be given by Eq. 2. The function utilizes the moment function  $M(x)$  and a double integration to model the deflection, generating 2 constants of integration rather than 4. As such, if possible, the second form is preferred. With the form of integration determined, the engineering must then determine the moment function applied onto the beam as a function of the loads, which begins by statically solving the problem.

$$EI \frac{\delta^2 v}{\delta x^2} = M(x) \quad (2)$$

The first step to find the static forces acting on a beam includes the decomposition of the applied and reaction forces onto a Free Body Diagram (FBD). To define the FBD, the report must first determine the type of supports acting on the beam. For a simply supported beam, the pin support creates a reaction  $x$  and  $y$  directions, while the roller support only creates a reaction in the  $y$  direction. For a wall support, the beam must have a reaction in the  $x$  and  $y$  directions in addition to a moment reaction. Fig. 1 shows an example of a FBD decomposition.

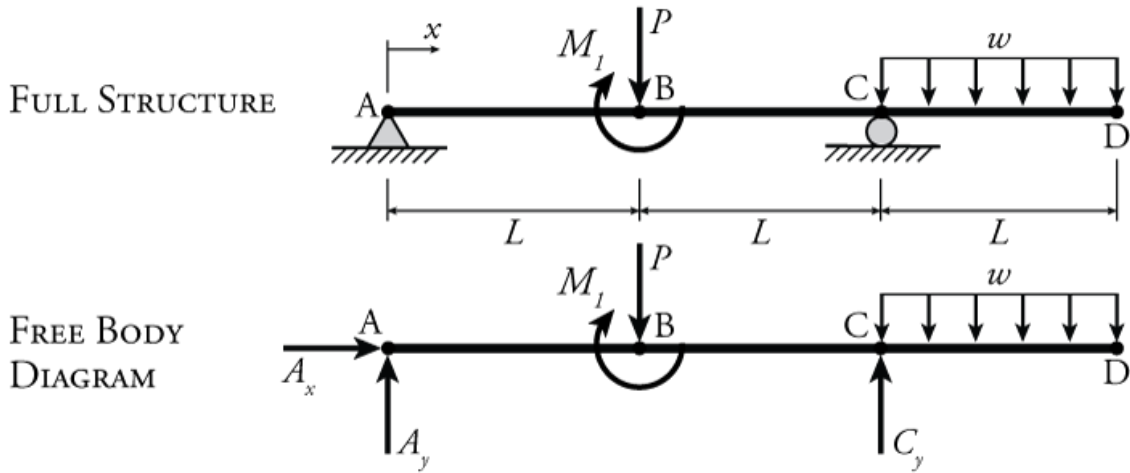


Figure 1: Example Free Body Diagram

With the FBD drawn, the report uses the set of equations shown in Eq. 3 to solve for the reaction forces. In most scenarios, the horizontal force  $F_x$ , and therefore the horizontal reaction, will be equal to zero, leaving 2 main equations to be used to solve for the vertical reactions at each of the supports.

$$\begin{aligned}
&\uparrow \sum F_y = 0 \\
&\rightarrow \sum F_x = 0 \\
&\circlearrowleft \sum M_A = 0
\end{aligned} \tag{3}$$

Once the support reactions have been found, the engineer proceeds by selecting Eq. 1 or Eq. 2 to apply. For the purposes of the example, Eq. 2 will be used. For that form, the report sets an  $x$ - $y$  coordinate system on the left side of the beam such that the  $x$  axis extends along the beam and the  $y$  axis extends perpendicular to the length of the beam. With the coordinate system established, the report constructs the Moment equation based on Eq. 4 where the moment  $M$  can be found as the product of the force  $F$  and the radius of the arm  $r$  given as the distance between the force to the axis of rotation.

$$M = F * r \tag{4}$$

For the given example, the moment equation is as shown in Eq. 5. In the equation, any term found containing  $\langle x - x_0 \rangle^n$  conveys the force represented only acts starting from point  $x_0$ ; that is, if the beam were to be segmented prior to  $x_0$  such that  $x < x_0$ , the moment generated would be equal to zero. As such,  $\langle x - x_0 \rangle^n$  evaluates to  $(x - x_0)^n$  if  $x \geq x_0$  or 0 if  $x < x_0$ .

$$M(x) = -P \langle x - L \rangle^1 - M_I \langle x - L \rangle^0 + C_y \langle x - 2L \rangle^1 - \frac{w}{2} \langle x - 2L \rangle^2 \tag{5}$$

With the moment equation found, the report will integrate it twice as given per Eq. 2 to find  $v(x)$ , which can be shown in Eq. 6.

$$\begin{aligned}
EI \frac{\delta^2 v}{\delta x^2} &= -P \langle x - L \rangle^1 - M_I \langle x - L \rangle^0 + C_y \langle x - 2L \rangle^1 - \frac{w}{2} \langle x - 2L \rangle^2 \\
EI \frac{\delta v}{\delta x} &= \frac{-P}{2} \langle x - L \rangle^2 - M_I \langle x - L \rangle^1 + \frac{C_y}{2} \langle x - 2L \rangle^2 - \frac{w}{6} \langle x - 2L \rangle^3 + C_1 \\
EI v(x) &= \frac{-P}{6} \langle x - L \rangle^3 - \frac{M_I}{2} \langle x - L \rangle^2 + \frac{C_y}{6} \langle x - 2L \rangle^3 - \frac{w}{24} \langle x - 2L \rangle^4 + C_1 x + C_2
\end{aligned} \tag{6}$$

The last step in finding the displacement equation is to apply the boundary conditions to find the coefficients of integration  $C_1$  and  $C_2$ , which use the properties of the supports. As given

in Fig. 1, there are two supports that provide vertical reactions to the beam's displacement. In other words, at the point of contact with each of the supports, the vertical displacement must be equal to zero. As such, we can use the conditions shown in Eqs. 7 and 8 to solve for  $C_1$  and  $C_2$ .

$$\begin{aligned}
v(x=0) &= 0 \\
\Rightarrow \frac{-P}{6} < (0) - L >^3 - \frac{M_I}{2} < (0) - L >^2 + \frac{C_y}{6} < (0) - 2L >^3 - \frac{w}{24} < (0) - 2L >^4 + C_1(0) + C_2 \\
\Rightarrow C_2 &= 0
\end{aligned} \tag{7}$$

$$\begin{aligned}
v(x=2L) &= 0 \\
\Rightarrow \frac{-P}{6} < (2L) - L >^3 - \frac{M_I}{2} < (2L) - L >^2 + \frac{C_y}{6} < (2L) - 2L >^3 - \frac{w}{24} < (2L) - 2L >^4 + C_1(2L) + C_2 \\
\Rightarrow C_1 &= \frac{\frac{PL^3}{6} + \frac{M_IL^2}{2}}{2L} = \frac{PL^2}{12} + \frac{M_IL}{4}
\end{aligned} \tag{8}$$

Therefore, the final equation for the deflection of the beam can be found as shown in Eq. 9 where the unknown coefficients have been solved for.

$$EIv(x) = \frac{-P}{6} < x-L >^3 - \frac{M_I}{2} < x-L >^2 + \frac{C_y}{6} < x-2L >^3 - \frac{w}{24} < x-2L >^4 + \left( \frac{PL^2}{12} + \frac{M_IL}{4} \right) x \tag{9}$$

### 3 Problem Definition

#### 3.1 Problem 1

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### **3.2 Problem 2**

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### **3.3 Problem 3**

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### 3.4 Problem 4

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### 3.6 Problem 6

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### 3.7 Problem 7

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## 4 Problem Solution

### 4.1 Problem 1

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## 4.2 Problem 2

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## 4.3 Problem 3

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## 4.4 Problem 4

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#### 4.5 Problem 5

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#### 4.6 Problem 6

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## 4.7 Problem 7

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## 5 Results and Discussion

### 5.1 Problem 1

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### 5.2 Problem 2

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### 5.3 Problem 3

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### 5.4 Problem 4

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## 5.5 Problem 5

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## 5.6 Problem 6

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## 5.7 Problem 7

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## 6 Conclusions

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## 7 Appendix

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