Florida Atlantic University

College of Engineering

Project 2

Application of the Direct Integration Method or of Discontinuity (Singularity) Functions to Statically Indeterminate Uniform or Stepped Beam Design

Serial Number: 02

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October 13, 2021

INTERMEDIATE STRENGTH OF MATERIALS

FALL 2021

Communication Project 2:

"APPLICATION OF THE DIRECT INTEGRATION METHOD OR OF DISCONTINUITY (SINGULARITY) FUNCTIONS TO STATICALLY INDETERMINATE UNIFORM OR STEPPED BEAM DESIGN"

Requirements and Deliverables

(assigned: Friday, October 1, 2021; submission date: Friday, October 15, 2020; not later than 4:00 pm)

"YOU WILL BE JUDGED IN INDUSTRY NOT ON [ONLY] YOUR WORK BUT HOW WELL YOU COMMUNICATE YOUR WORK," Charles R. Cluer "A Survey of Industrial Mathematics",

page 277, Dover, 2012. The requirements that should be satisfied by the project are given in the document called FORMAT (see CANVAS).

Problem 1

For the beam that is depicted below is subjected to a distributed load w_0, concentrated load P and concentrated moment M_0.. The length is specified L= 2 meters. The cross section is represented by an I-beam. Material is aluminum; yield stress= 276 MPa; E= 69.9 GPa; required safety factor is k= 1.5. The allowable displacement $v_{allow=}$ L/500. Design the beam so that it satisfied strength condition, as well as stiffness condition. The moments of inertia of I-beams are given on pages A25-A26 of the textbook, the number ***s*** is your serial number. You must depict the FBD. Write equations to determine reactions. Depict FBD with loadings in actual numbers. Use singularity functions, not the superposition principle. Using any computer software is not permissible.

Problem 2

For the beam that is depicted is subjected to a distributed load w_0, concentrated load P and concentrated moment M_0.. The length is specified L= 2 m. The cross section is circular with radius c. Material is aluminum; yield stress= 276 MPa; E= 69.9 GPa; required safety factor is k= 1.5. The allowable displacement $v_{allow=}$ L/400. The number s is your serial number. You must depict the FBD. Write equations to determine reactions. Depict FBD with loadings in actual numbers. Use singularity functions, not the superposition principle. Using any computer software is not permissible.

(a) Design the beam—find minimum allowable radius of cross section-- so that it satisfies strength condition, as well as stiffness condition.
(b) Write resulting expression of $V_y(x)$ and $M_z(x)$.

(c) Construct diagrams of the shear force $V_y(x)$ and bending moment $M_z(x)$ as well as for deflection $v(x)$.

Problem 3

The beam is subjected to concentrated loads as shown in the figure. The number of loads equals 100-*s*, where *s* is your serial number; choose the material and the cross section. Find the maximum displacement of the beam. You must depict the FBD. Write equations to determine reactions. Depict FBD with loadings in actual numbers. Use singularity functions, not the superposition principle. Using any computer software is not permissible.

Problem 4

For the beam that is depicted is subjected to a distributed load w_0, concentrated load *P* and concentrated moment $M_{0.}$. The length is specified *L*= 2 m. The cross section is represented by a rectangle who base is three times less than its height (or depth). Material is aluminum; yield stress= 276 MPa; *E*= 69.9 GPa; required safety factor is *k*= 2. The allowable displacement $v_{allow}=$ *L*/400. The number *s* is your serial number. You must depict the FBD. Write equations to determine reactions. Depict FBD with loadings in actual numbers. Use singularity functions, not the superposition principle. Using any computer software is not permissible.

(d) Design the beam so that it satisfied strength condition, as well as stiffness condition, i. e. find minimum allowable value of the beam's hight (depth).
(e) Write resulting expression of $V_y(x)$ and $M_z(x)$.
(f) Construct diagrams of the shear force $V_y(x)$ and bending moment $M_z(x)$ as well as for deflection $v(x)$.

Problems 5-6

Determine the maximum deflection in the beam subjected to following loading. The cross-section is given by American Standard Channel (pp. A27-28). Make a choice of the material, as well as (nonzero) values of w_0 and M_0. Determine the maximum deflection. Does it occur in the middle cross-section? You must depict the FBD. Write equations to determine reactions. Depict FBD with loadings in actual numbers. Use singularity functions, not the superposition principle. Using any computer software is not permissible.

Problems 7

For the beam that is depicted below is subjected to a distributed load w_0, concentrated load P and concentrated moment M_0.. The length is specified L= 6 meters. The cross section of the first part and the third part are represented by identical circular beams with radius *c.* The cross-section in the second part is also circular but radius equals 2*c.* Material is aluminum; yield stress= 276 MPa; E= 69.9 GPa; required safety factor is k= 1.5. The allowable displacement $v_{allow=}$ L/400. Design the beam so that it satisfied strength condition, as well as stiffness condition. The moments of inertia of I-beams are given on pages A25-26 of the textbook, the number *s* is your serial number. You must depict the FBD. Write equations to determine reactions. Depict FBD with loadings in actual numbers. Depict equivalent uniform beam and the parts of the original beam segments and their 'tailoring' process. Use singularity functions and equivalent uniform beam method. .Using any computer software is not permissible.

The copy of these assignment page must be included beneath the title page of the project.

The project ought to be presented in the reader-friendly format: For each problem, if the diagrams are not strictly below each other in the same scale as the Include sections of **Abstract** and **Conclusion**: Summarize what you learned from this personal communication project; provide also recommendation(s) to the lecturer on the project assignment and management.

P.S: Entire work must be neatly hand-written. A grade "zero" will be assigned if not doing the project by yourself. No TYPED projects, please!

Students must submit the project to the secretary, Ms. Barbara, office Engineering West 190, and ask her to put a date-stamp. Her office is located in Engineering West building (or an "old" engineering building, in front of the library). Each project submitted later than October 15, 2021, 4 pm leads to 10 credits less, for each work day.

With wishes of success,

Isaac Elishakoff

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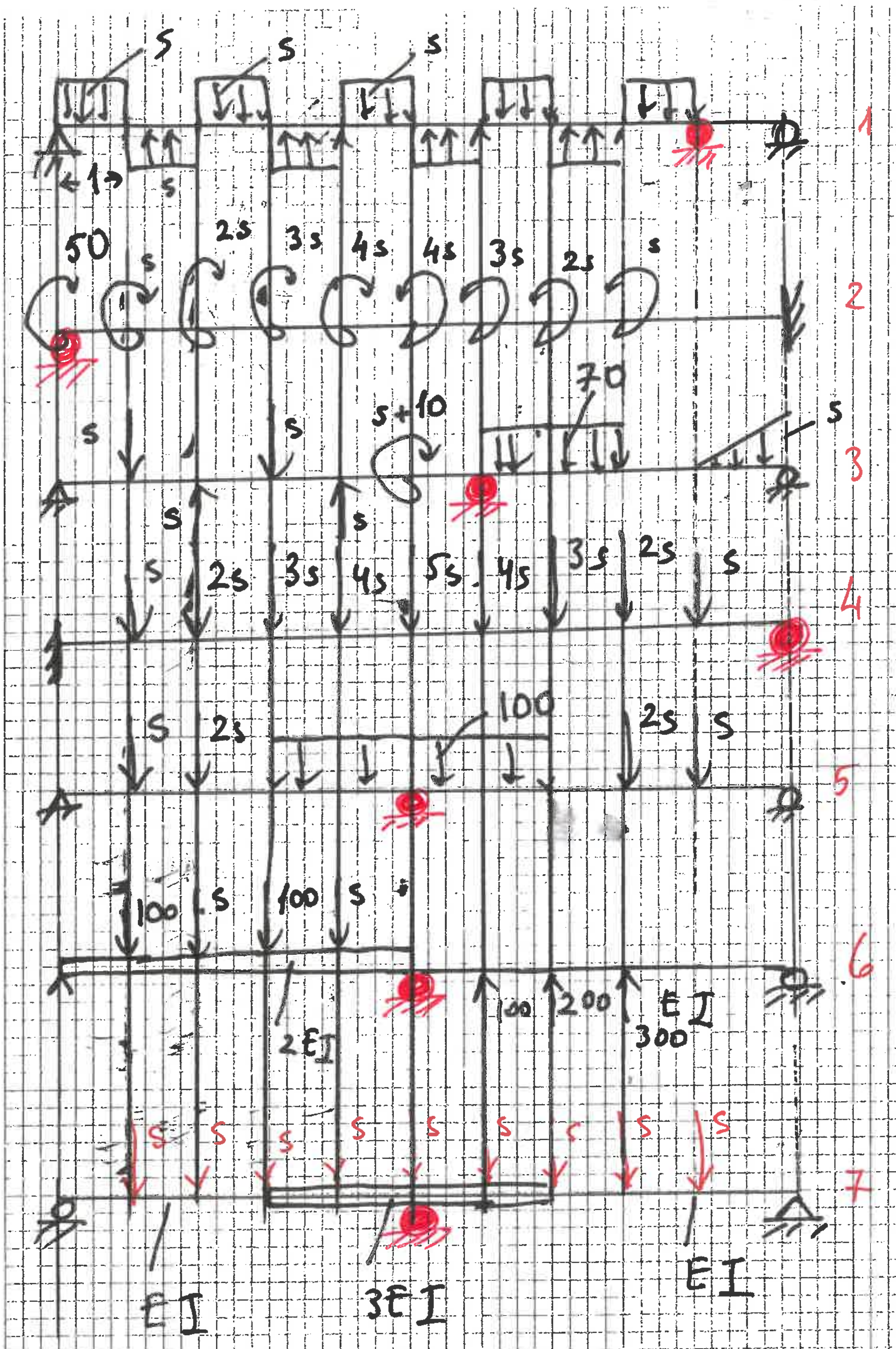
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The report will apply the Direct Integration Method of Singularity Functions to statically determine uniform or stepped beam designs to determine the load's effect on the beam's deflection. The resulting functions following the integration will be plotted using MATLAB software to produce simplified renderings of the beam's shape post loading.

Contents

1 Introduction

One of the most important processes in designing beams is the structural analysis of the part under realistic loading conditions. A proper, realistic, theoretical model allows the engineer or designer to understand the expected stresses and deflections of a beam under expected loading conditions, thus removing the need to conduct experiments for each attempted revision. To properly construct the theoretical models, engineers often rely on extensive sets of equations.

The standard method to solve for beam deflection for beams with continuous cross sectional areas relies on partitions; the designer splits the beam into multiple subsections where loads are applied. The resulting cross sections are then each modeled separately. The singularity equations for beam deflection, by contrast, allow for the reduction of the multiple deflection equations into a single continuous equation. The method will be discussed thoroughly in the Background Theory section.

The report will comprise of the Background Theory section discussing the required equations and methods to solve the problems, the Problem Definition section discussing each of the problems presented in the project, Results where problem solutions are discussed, and the Conclusions section to summarize findings and discuss improvements.

2 Background Theory

To find the deflection equation for a statically determinate beam under loading, there are two principal ways to approach the problem, both of which lead to the same result [1]. The first method, shown in Eq. 1, uses the loading function $w(x)$ to reach the deflection. The form is especially useful for beams under non-linear loading functions. However, due to the required quadruple integration to reach $v(x)$, engineers commonly use a second form.

$$EI\frac{\delta^4 v}{\delta x^4} = w(x) \tag{1}$$

The second form of the deflection equation can be given by Eq. 2. The function utilizes the moment function $W(x)$ and a double integration to model the deflection, generating 2 constants of integration rather than 4. As such, if possible, the second form is preferred. With the form of integration determined, the engineering must then determine the moment function applied onto the beam as a function of the loads, which begins by statically solving the problem.

$$EI\frac{\delta^2 v}{\delta x^2} = M(x) \tag{2}$$

The first step to find the static forces acting on a beam includes the decomposition of the applied and reaction forces onto a Free Body Diagram (FBD). To define the FBD, the report must first determine the type of supports acting on the beam. For a simply supported beam, the pin support creates a reaction x and y directions, while the roller support only creates a reaction in the y direction. For a wall support, the beam must have a reaction in the x and y directions in addition to a moment reaction. Fig. 1 shows an example of a FBD decomposition.

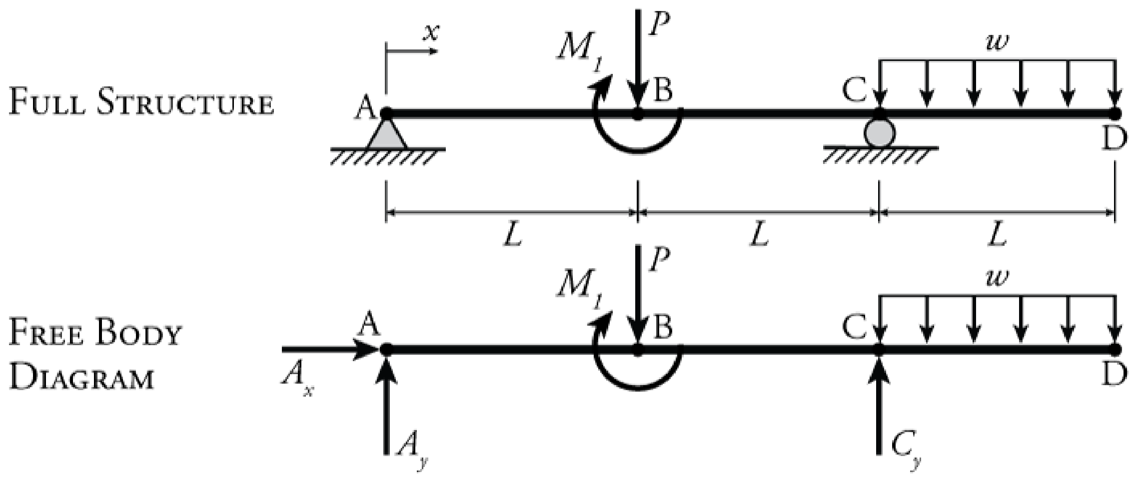


Figure 1: Example Free Body Diagram

With the FBD drawn, the report uses the set of equations shown in Eq. 3 to solve for the reaction forces. In most scenarios, the horizontal force F_x, and therefore the horizontal reaction, will be equal to zero, leaving 2 main equations to be used to solve for the vertical reactions at each of the supports.

$$\begin{aligned}
&\uparrow \sum F_y = 0 \\
&\rightarrow \sum F_x = 0 \\
&\circlearrowleft \sum M_A = 0
\end{aligned} \tag{3}$$

Once the support reactions have been found, the engineer proceeds by selecting Eq. 1 or Eq. 2 to apply. For the purposes of the example, Eq. 2 will be used. For that form, the report sets an x-y coordinate system on the left side of the beam such that the x axis extends along the beam and the y axis extends perpendicular to the length of the beam. With the coordinate system established, the report constructs the Moment equation based on Eq. 4 where the moment M can be found as the product of the force F and the radius of the arm r given as the distance between the force to the axis of rotation.

$$M = F * r \tag{4}$$

For the given example, the moment equation is as shown in Eq. 5. In the equation, any term found containing $< x - x_0 >^n$ conveys the force represented only acts starting from point x_0; that is, if the beam were to be segmented prior to x_0 such that $x < x_0$, the moment generated would be equal to zero. As such, $< x - x_0 >^n$ evaluates to $(x - x_0)^n$ if $x >= x_0$ or 0 if $x < x_0$.

$$M(x) = -P\left\langle x - L\right\rangle^1 - M_I\left\langle x - L\right\rangle^0 + C_y\left\langle x - 2L\right\rangle^1 - \frac{w}{2}\left\langle x - 2L\right\rangle^2 \tag{5}$$

With the moment equation found, the report will integrate it twice as given per Eq. 2 to find $v(x)$, which can be shown in Eq. 6.

$$\begin{aligned}
EI\frac{\delta^2 v}{\delta x^2} &= -P\left\langle x - L\right\rangle^1 - M_I\left\langle x - L\right\rangle^0 + C_y\left\langle x - 2L\right\rangle^1 - \frac{w}{2}\left\langle x - 2L\right\rangle^2 \\
EI\frac{\delta v}{\delta x} &= \frac{-P}{2}\left\langle x - L\right\rangle^2 - M_I\left\langle x - L\right\rangle^1 + \frac{C_y}{2}\left\langle x - 2L\right\rangle^2 - \frac{w}{6}\left\langle x - 2L\right\rangle^3 + C_1 \\
EIv(x) &= \frac{-P}{6}\left\langle x - L\right\rangle^3 - \frac{M_I}{2}\left\langle x - L\right\rangle^2 + \frac{C_y}{6}\left\langle x - 2L\right\rangle^3 - \frac{w}{24}\left\langle x - 2L\right\rangle^4 + C_1 x + C_2
\end{aligned} \tag{6}$$

The last step in finding the displacement equation is to apply the boundary conditions to find the coefficients of integration C_1 and C_2, which use the properties of the supports. As given in Fig. 1, there are two supports that provide vertical reactions to the beam's displacement. In other words, at the point of contact with each of the supports, the vertical displacement must be equal to zero. As such, we can use the conditions shown in Eqs. 7 and 8 to solve for C_1 and C_2.

$$
\begin{aligned}
&v(x=0)=0\\
&\Rightarrow \frac{-P}{6}\langle(0)-L\rangle^3-\frac{M_I}{2}\langle(0)-L\rangle^2+\frac{C_y}{6}\langle(0)-2L\rangle^3-\frac{w}{24}\langle(0)-2L\rangle^4+C_1(0)+C_2\\
&\Rightarrow C_2=0
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&v(x=2L)=0\\
&\Rightarrow \frac{-P}{6}\langle(2L)-L\rangle^3-\frac{M_I}{2}\langle(2L)-L\rangle^2+\frac{C_y}{6}\langle(2L)-2L\rangle^3-\frac{w}{24}\langle(2L)-2L\rangle^4+C_1(2L)+C_2\\
&\Rightarrow C_1=\frac{\frac{PL^3}{6}+\frac{M_IL^2}{2}}{2L}=\frac{PL^2}{12}+\frac{M_IL}{4}
\end{aligned}
\tag{8}
$$

Therefore, the final equation for the deflection of the beam can be found as shown in Eq. 9 where the unknown coefficients have been solved for.

$$
EIv(x)=\frac{-P}{6}\langle x-L\rangle^3-\frac{M_I}{2}\langle x-L\rangle^2+\frac{C_y}{6}\langle x-2L\rangle^3-\frac{w}{24}\langle x-2L\rangle^4+\left(\frac{PL^2}{12}+\frac{M_IL}{4}\right)x \tag{9}
$$

3 Problem Definition

3.1 Problem 1

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads as shown in Fig. 11a. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 11b.

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 10.

$$
\begin{aligned}
&\uparrow \sum F_y = R_a - 2(1) + 2(1) - 2(1) + 2(1) - 2(1) + 2(1) - 2(1) + 2(1) - 2(1) + R_c + R_b = 0 \\
&\rightarrow \sum F_x = 0 \\
&\circlearrowleft \sum M_A = -2(1)\left(\frac{1}{2}\right) + 2(1)\left(\frac{3}{2}\right) - 2(1)\left(\frac{5}{2}\right) + 2(1)\left(\frac{7}{2}\right) - 2(1)\left(\frac{9}{2}\right) + 2(1)\left(\frac{11}{2}\right) \\
&- 2(1)\left(\frac{13}{2}\right) + 2(1)\left(\frac{15}{2}\right) - 2(1)\left(\frac{17}{2}\right) + R_c(9) + R_b(10) = 0
\end{aligned}
\tag{10}
$$

Given the problem is statically indeterminate, the static equations are insufficient to find the reaction forces for the supports, which are shown in Eq. 11. As such, the report will solve for the reaction forces in terms of other reactions.

$$
\begin{aligned}
R_a &= \frac{9}{10} \\
R_b &= \frac{11}{10} \\
R_c &= R_c
\end{aligned}
\tag{11}
$$

With the reactions determined, the moment and deflection equations can be constructed as shown in Eq. 12.

$$
\begin{aligned}
&EI\frac{\delta^2 v}{\delta x^2} = \frac{11}{10}\langle x-0\rangle^1 - \langle x-0\rangle^2 + 2\langle x-1\rangle^2 - 2\langle x-2\rangle^2 + 2\langle x-3\rangle^2 - 2\langle x-4\rangle^2 \\
&+ 2\langle x-5\rangle^2 - 2\langle x-6\rangle^2 + 2\langle x-7\rangle^2 - 2\langle x-8\rangle^2 + \langle x-9\rangle^2 \\
\\
&EI\frac{\delta v}{\delta x} = \frac{11}{20}\langle x-0\rangle^2 - \frac{1}{3}\langle x-0\rangle^3 + \frac{2}{3}\langle x-1\rangle^3 - \frac{2}{3}\langle x-2\rangle^3 + \frac{2}{3}\langle x-3\rangle^3 - \frac{2}{3}\langle x-4\rangle^3 \\
&+ \frac{2}{3}\langle x-5\rangle^3 - \frac{2}{3}\langle x-6\rangle^3 + \frac{2}{3}\langle x-7\rangle^3 - \frac{2}{3}\langle x-8\rangle^3 + \frac{1}{3}\langle x-9\rangle^3 + C_1 \\
\\
&EIv(x) = \frac{11}{60}\langle x-0\rangle^3 - \frac{1}{12}\langle x-0\rangle^4 + \frac{1}{6}\langle x-1\rangle^4 - \frac{1}{6}\langle x-2\rangle^4 + \frac{1}{6}\langle x-3\rangle^4 - \frac{1}{6}\langle x-4\rangle^4 \\
&+ \frac{1}{6}\langle x-5\rangle^4 - \frac{1}{6}\langle x-6\rangle^4 + \frac{1}{6}\langle x-7\rangle^4 - \frac{1}{6}\langle x-8\rangle^4 + \frac{1}{12}\langle x-9\rangle^4 + C_1 x + C_2
\end{aligned}
\tag{12}
$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the two vertical supports at A and B guarantee the deflection at these points is equal to zero, as shown in Eq. 13

$$\begin{aligned} v(0) &= 0 \\ v(10) &= 0 \end{aligned} \tag{13}$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 14.

$$\begin{aligned} C_2 &= 0 \\ C_1 &= \frac{209}{240} \end{aligned} \tag{14}$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 15.

$$\begin{aligned} EIv(x) = \frac{11}{60}\langle x-0\rangle^3 - \frac{1}{12}\langle x-0\rangle^4 + \frac{1}{6}\langle x-1\rangle^4 - \frac{1}{6}\langle x-2\rangle^4 + \frac{1}{6}\langle x-3\rangle^4 - \frac{1}{6}\langle x-4\rangle^4 \\ + \frac{1}{6}\langle x-5\rangle^4 - \frac{1}{6}\langle x-6\rangle^4 + \frac{1}{6}\langle x-7\rangle^4 - \frac{1}{6}\langle x-8\rangle^4 + \frac{1}{12}\langle x-9\rangle^4 + \frac{209}{240}x \end{aligned} \tag{15}$$

3.2 Problem 2

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 12b.

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 16.

$$\begin{aligned} &\uparrow \sum F_y = R_{ay} = 0 \\ &\rightarrow \sum F_x = R_{ax} = 0 \\ &\circlearrowleft \sum M_A = -50 - 2 - 4 - 6 - 8 + 8 + 6 + 4 + 2 + M_a = 0 \end{aligned} \tag{16}$$

Given the problem is statically determinate, the static equations are sufficient to find the reaction forces for the supports, which are shown in Eq. 17. Since there are no horizontal forces applied, the only non-zero reaction is the reactionary moment, which will be named M_a henceforth.

$$\begin{aligned} M_a &= 50 \\ R_{ax} &= 0 \\ R_{ay} &= 0 \end{aligned} \tag{17}$$

With the reactions determined, the moment and deflection equations can be constructed as shown in Eq. 18.

$$\begin{aligned}
&EI\frac{\delta^2 v}{\delta x^2} = 50\langle x-0\rangle^0 + 2\langle x-1\rangle^0 + 4\langle x-2\rangle^0 + 6\langle x-3\rangle^0 + 8\langle x-4\rangle^0 - 8\langle x-5\rangle^0 \\
&- 6\langle x-6\rangle^0 - 4\langle x-7\rangle^0 - 2\langle x-8\rangle^0 \\
&EI\frac{\delta v}{\delta x} = 50\langle x-0\rangle^1 + 2\langle x-1\rangle^1 + 4\langle x-2\rangle^1 + 6\langle x-3\rangle^1 + 8\langle x-4\rangle^1 - 8\langle x-5\rangle^1 \\
&- 6\langle x-6\rangle^1 - 4\langle x-7\rangle^1 - 2\langle x-8\rangle^1 + C_1 \\
&EIv(x) = 25\langle x-0\rangle^2 + \langle x-1\rangle^2 + 2\langle x-2\rangle^2 + 3\langle x-3\rangle^2 + 4\langle x-4\rangle^2 - 4\langle x-5\rangle^2 \\
&- 3\langle x-6\rangle^2 - 2\langle x-7\rangle^2 - \langle x-8\rangle^2 + C_1 x + C_2
\end{aligned} \tag{18}$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the fixed at A guarantees the deflection and slope at A is equal to zero, as shown in Eq. 19.

$$\begin{aligned} v(10) &= 0 \\ \frac{\delta v(10)}{\delta x} &= 0 \end{aligned} \tag{19}$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 20.

$$
\begin{aligned}
C_2 &= 2770 \\
C_1 &= -560
\end{aligned} \tag{20}
$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 21.

$$
\begin{aligned}
&EIv(x) = 25\langle x-0\rangle^2 + \langle x-1\rangle^2 + 2\langle x-2\rangle^2 + 3\langle x-3\rangle^2 + 4\langle x-4\rangle^2 - 4\langle x-5\rangle^2 \\
&- 3\langle x-6\rangle^2 - 2\langle x-7\rangle^2 - \langle x-8\rangle^2 - 560x + 2770
\end{aligned} \tag{21}
$$

3.3 Problem 3

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 13b.

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 22.

$$
\begin{aligned}
&\uparrow \sum F_y = R_a - 2 + 2 - 2 + 2 - 70(2) - 2(1)\left(\frac{1}{2}\right) + R_b = 0 \\
&\rightarrow \sum F_x = R_{ax} = 0 \\
&\circlearrowleft \sum M_A = -2(1) + 2(2) - 2(3) + 2(4) - 12 - 70(2)(7) - 2(1)\left(\frac{29}{6}\right) + R_{by}(10) = 0
\end{aligned} \tag{22}
$$

Given the problem is statically determinate, the static equations are sufficient to find the reaction forces for the supports, which are shown in Eq. 23. Since there are no horizontal forces applied, the only non-zero reactionary forces are those in the vertical direction, which will be named R_a and R_b henceforth.

$$
\begin{aligned}
R_b &= \frac{2993}{30} \\
R_a &= \frac{1237}{30}
\end{aligned} \tag{23}
$$

With the reactions determined, the moment and deflection equations can be constructed as shown in Eq. 24.

$$
\begin{gathered}
EI\frac{\delta^2 v}{\delta x^2} = \frac{1237}{30}\langle x-0\rangle^1 - 2\langle x-1\rangle^1 + 2\langle x-2\rangle^1 - 2\langle x-3\rangle^1 + 2\langle x-4\rangle^1 \\
+ 12\langle x-5\rangle^0 - 35\langle x-6\rangle^2 + 35\langle x-8\rangle^2 - \frac{1}{3}\langle x-9\rangle^3 \\
EI\frac{\delta v}{\delta x} = \frac{1237}{60}\langle x-0\rangle^2 - \langle x-1\rangle^2 + \langle x-2\rangle^2 - \langle x-3\rangle^2 + \langle x-4\rangle^2 + 12\langle x-5\rangle^1 \\
- \frac{35}{3}\langle x-6\rangle^3 + \frac{35}{3}\langle x-8\rangle^3 - \frac{1}{12}\langle x-9\rangle^4 + C_1 \\
EIv(x) = \frac{1237}{180}\langle x-0\rangle^3 - \frac{1}{3}\langle x-1\rangle^3 + \frac{1}{3}\langle x-2\rangle^3 - \frac{1}{3}\langle x-3\rangle^3 + \frac{1}{3}\langle x-4\rangle^3 \\
+ 6\langle x-5\rangle^2 - \frac{35}{12}\langle x-6\rangle^4 + \frac{35}{12}\langle x-8\rangle^4 - \frac{1}{60}\langle x-9\rangle^5 + C_1 x + C_2
\end{gathered} \tag{24}
$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the two vertical supports at A and B guarantee the deflection at these points is equal to zero, as shown in Eq. 25.

$$
\begin{gathered}
v(0) = 0 \\
v(10) = 0
\end{gathered} \tag{25}
$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 26.

$$
\begin{gathered}
C_2 = 0 \\
C_1 = \frac{-1117357}{1800}
\end{gathered} \tag{26}
$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 27.

$$EIv(x) = \frac{1237}{180}\langle x-0\rangle^3 - \frac{1}{3}\langle x-1\rangle^3 + \frac{1}{3}\langle x-2\rangle^3 - \frac{1}{3}\langle x-3\rangle^3 + \frac{1}{3}\langle x-4\rangle^3$$
$$+ 6\langle x-5\rangle^2 - \frac{35}{12}\langle x-6\rangle^4 + \frac{35}{12}\langle x-8\rangle^4 - \frac{1}{60}\langle x-9\rangle^5 + \frac{-1117357}{1800}x \tag{27}$$

3.4 Problem 4

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 14b.

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 28.

$$\begin{aligned}
&\uparrow \sum F_y = R_{ay} - 2 - 4 - 6 - 8 - 10 - 8 - 6 - 4 - 2 = 0\\
&\rightarrow \sum F_x = R_{ax} = 0\\
&\circlearrowleft \sum M_A = M_a - 2(1) - 4(2) - 6(3) - 8(4) - 10(5) - 8(6) - 6(7) - 4(8) - 2(9) = 0
\end{aligned} \tag{28}$$

Given the problem is statically determinate, the static equations are sufficient to find the reaction forces for the supports, which are shown in Eq. 29. Since there are no horizontal forces applied, the only non-zero reactions are the reactionary moment and vertical reaction, which will be named M_a and R_a henceforth.

$$\begin{aligned}
M_a &= -250\\
R_{ax} &= 0\\
R_{ay} &= 50
\end{aligned} \tag{29}$$

With the reactions determined, the moment and deflection equations can be constructed as shown in Eq. 30.

$$EI\frac{\delta^2 v}{\delta x^2} = 50\left\langle x-0\right\rangle^1 - 250\left\langle x-0\right\rangle^0 - 2\left\langle x-1\right\rangle^1 - 4\left\langle x-2\right\rangle^1 - 6\left\langle x-3\right\rangle^1 - 8\left\langle x-4\right\rangle^1$$
$$- 10\left\langle x-5\right\rangle^1 - 8\left\langle x-6\right\rangle^1 - 6\left\langle x-7\right\rangle^1 - 4\left\langle x-8\right\rangle^1 - 2\left\langle x-9\right\rangle^1$$

$$\begin{gathered} EI\frac{\delta v}{\delta x} = 25\left\langle x-0\right\rangle^2 - 250\left\langle x-0\right\rangle^1 - \left\langle x-1\right\rangle^2 - 2\left\langle x-2\right\rangle^2 - 3\left\langle x-3\right\rangle^2 - 4\left\langle x-4\right\rangle^2 \\ - 5\left\langle x-5\right\rangle^2 - 4\left\langle x-6\right\rangle^2 - 3\left\langle x-7\right\rangle^2 - 2\left\langle x-8\right\rangle^2 - \left\langle x-9\right\rangle^2 + C_1 \end{gathered} \tag{30}$$

$$EIv(x) = \frac{25}{6}\left\langle x-0\right\rangle^3 - 125\left\langle x-0\right\rangle^2 - \frac{1}{3}\left\langle x-1\right\rangle^3 - \frac{2}{3}\left\langle x-2\right\rangle^3 - \left\langle x-3\right\rangle^3 - \frac{4}{3}\left\langle x-4\right\rangle^3$$
$$- \frac{5}{3}\left\langle x-5\right\rangle^3 - \frac{4}{3}\left\langle x-6\right\rangle^3 - \left\langle x-7\right\rangle^3 - \frac{2}{3}\left\langle x-8\right\rangle^3 - \frac{1}{3}\left\langle x-9\right\rangle^3 + C_1 x + C_2$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the fixed at A guarantees the deflection and slope at A is equal to zero, as shown in Eq. 31.

$$\begin{gathered} v(0) = 0 \\ \frac{\delta v(0)}{\delta x} = 0 \end{gathered} \tag{31}$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 32.

$$\begin{gathered} C_2 = 0 \\ C_1 = 0 \end{gathered} \tag{32}$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 33.

$$\begin{gathered} EIv(x) = \frac{25}{6}\left\langle x-0\right\rangle^3 - 125\left\langle x-0\right\rangle^2 - \frac{1}{3}\left\langle x-1\right\rangle^3 - \frac{2}{3}\left\langle x-2\right\rangle^3 - \left\langle x-3\right\rangle^3 - \frac{4}{3}\left\langle x-4\right\rangle^3 \\ - \frac{5}{3}\left\langle x-5\right\rangle^3 - \frac{4}{3}\left\langle x-6\right\rangle^3 - \left\langle x-7\right\rangle^3 - \frac{2}{3}\left\langle x-8\right\rangle^3 - \frac{1}{3}\left\langle x-9\right\rangle^3 \end{gathered} \tag{33}$$

3.5 Problem 5

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 15b.

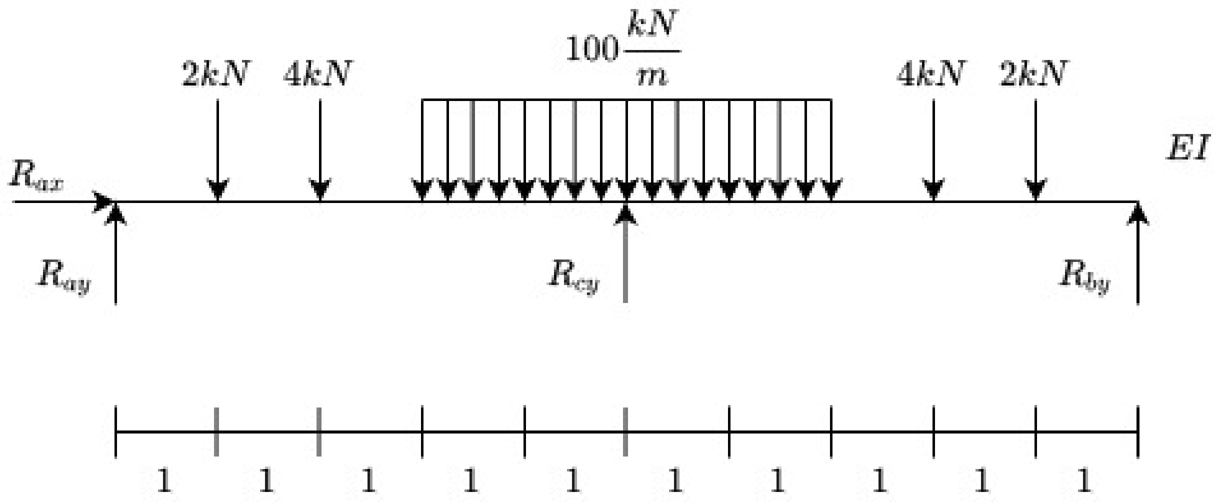


Figure 2: Free Body Diagram for Problem 5

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 34.

$$\begin{aligned}
&\uparrow \sum F_y = R_a - 2 - 4 - 100(4) - 4 - 2 + R_b = 0 \\
&\rightarrow \sum F_x = R_{ax} = 0 \\
&\circlearrowleft \sum M_A = -2(1) - 4(2) - 100(4)(5) - 4(8) - 2(9) + R_{by}(10) = 0
\end{aligned} \tag{34}$$

Given the problem is statically determinate, the static equations are sufficient to find the reaction forces for the supports, which are shown in Eq. 35. Since there are no horizontal forces applied, the only non-zero reactionary forces are those in the vertical direction, which will be named R_a and R_b henceforth.

$$\begin{aligned}
R_b &= 206 \\
R_a &= 206
\end{aligned} \tag{35}$$

With the reactions determined, the moment and deflection equations can be constructed as shown in Eq. 36.

$$EI\frac{\delta^2 v}{\delta x^2} = 206\langle x-0\rangle^1 - 2\langle x-1\rangle^1 - 4\langle x-2\rangle^2 - 50\langle x-3\rangle^2 + 50\langle x-7\rangle^1$$
$$-\,2\langle x-8\rangle^1 - 4\langle x-9\rangle^1$$

$$EI\frac{\delta v}{\delta x} = 103\langle x-0\rangle^2 - \langle x-1\rangle^2 - 2\langle x-2\rangle^2 - \frac{50}{3}\langle x-3\rangle^3 + \frac{50}{3}\langle x-7\rangle^3$$
$$-\,\langle x-8\rangle^2 - 2\langle x-9\rangle^2 + C_1 \tag{36}$$

$$EIv(x) = \frac{103}{3}\langle x-0\rangle^3 - \frac{1}{3}\langle x-1\rangle^3 - \frac{2}{3}\langle x-2\rangle^3 - \frac{25}{6}\langle x-3\rangle^4 + \frac{25}{6}\langle x-7\rangle^4$$
$$-\,\frac{1}{3}\langle x-8\rangle^3 - \frac{2}{3}\langle x-9\rangle^3 + C_1x + C_2$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the two vertical supports at A and B guarantee the deflection at these points is equal to zero, as shown in Eq. 37.

$$\begin{aligned} v(0) &= 0 \\ v(10) &= 0 \end{aligned} \tag{37}$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 38.

$$\begin{aligned} C_2 &= 0 \\ C_1 &= \frac{-7223}{3} \end{aligned} \tag{38}$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 39.

$$EIv(x) = \frac{103}{3}\langle x-0\rangle^3 - \frac{1}{3}\langle x-1\rangle^3 - \frac{2}{3}\langle x-2\rangle^3 - \frac{25}{6}\langle x-3\rangle^4 + \frac{25}{6}\langle x-7\rangle^4$$
$$-\,\frac{1}{3}\langle x-8\rangle^3 - \frac{2}{3}\langle x-9\rangle^3 - \frac{-7223}{3}x \tag{39}$$

3.6 Problem 6

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 16b.

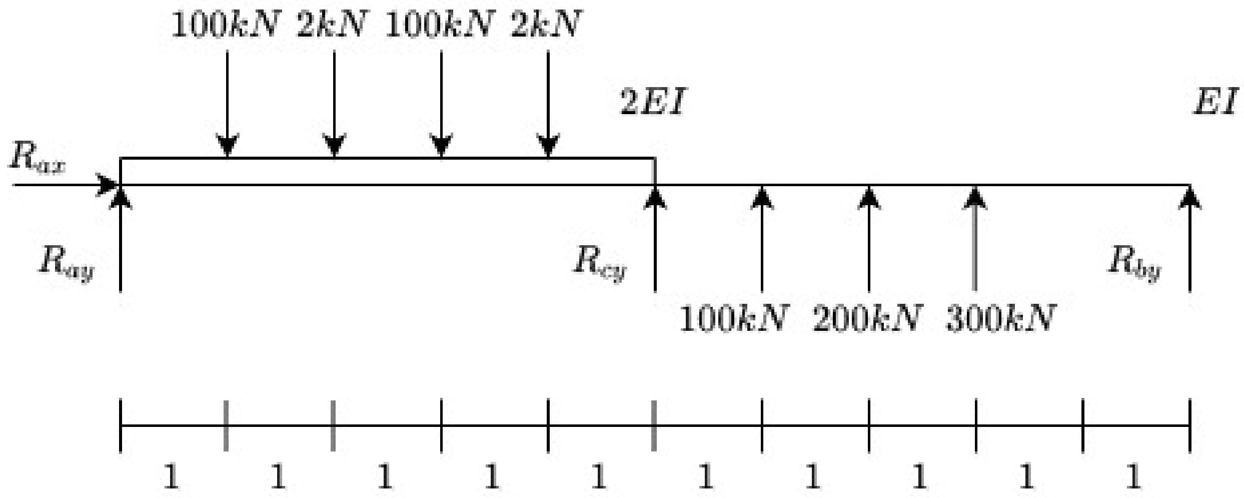


Figure 3: Free Body Diagram for Problem 6

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 40.

$$\begin{aligned}
&\uparrow \sum F_y = R_a - 100 - 2 - 100 - 2 + 100 + 200 + 300 + R_b = 0 \\
&\rightarrow \sum F_x = R_{ax} = 0 \\
&\circlearrowleft \sum M_A = -100(1) - 2(2) - 100(3) - 2(4) + 100(6) + 200(7) + 300(8) + R_{by}(10) = 0
\end{aligned} \tag{40}$$

Given the problem is statically determinate, the static equations are sufficient to find the reaction forces for the supports, which are shown in Eq. 41. Since there are no horizontal forces applied, the only non-zero reactionary forces are those in the vertical direction, which will be named R_a and R_b henceforth.

$$\begin{aligned}
R_b &= -\frac{1994}{5} \\
R_a &= \frac{14}{5}
\end{aligned} \tag{41}$$

Unlike previous problem, the beam's Modulus of Elasticity E and Area Moment of Inertia

I vary throughout the beam. As such, prior to writing the moment and deflection equations, we must first construct an equivalent Free Body Diagram by adding forces and moments. The original shear force and moment diagrams are shown in Fig. 4.

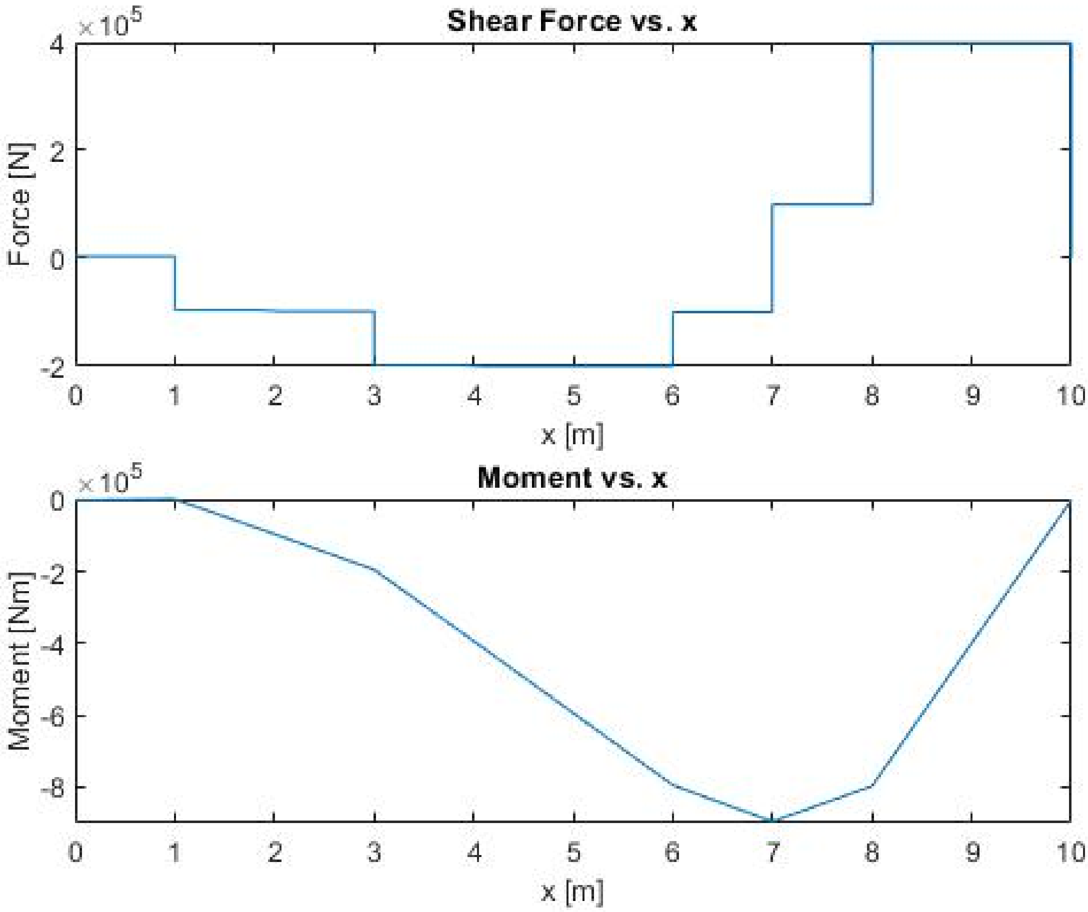


Figure 4: Shear Force and Moment Diagrams assuming EI for non-constant EI.

As given in Fig. 3, at the interval $0 < x < 5$, the beam has the term $2EI$ compared to the interval $5 < x < 10$ with the term EI. To construct the equivalent diagram, the report will multiply all the forces acting on the right side ($x > 5$) by two such that the EI terms match, creating Fig. 5 with constant EI.

With the equivalent Shear Force and Moment Diagrams, the report constructs a new FBD with an equivalent stiffness $2EI$ shown in Fig. 6. The FBD will be used to generate the displacement equations shown in Eq. 42.

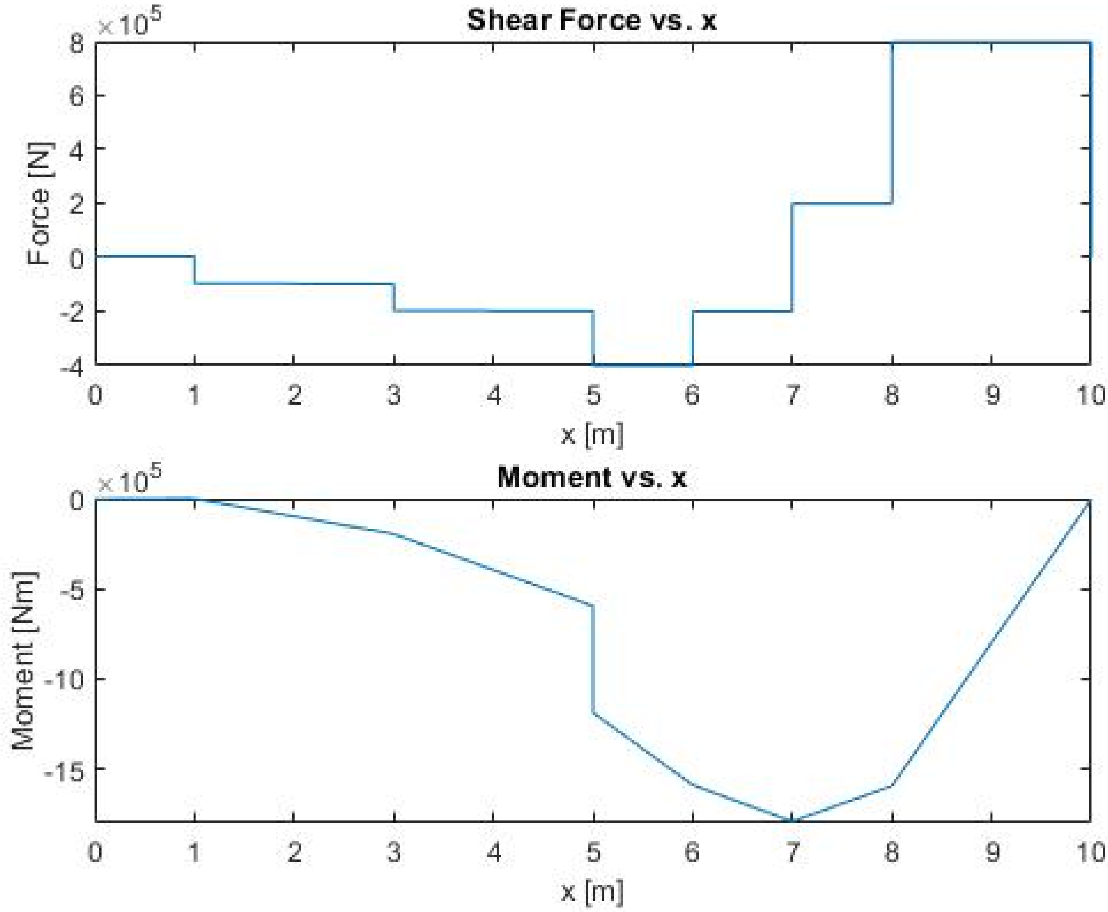


Figure 5: Shear Force and Moment Diagrams for constant EI

$$
\begin{aligned}
2EI\frac{\delta^2 v}{\delta x^2} &= \frac{14}{5}\langle x-0\rangle^1 - 100\langle x-1\rangle^1 - 2\langle x-2\rangle^1 - 100\langle x-3\rangle^1 - 2\langle x-4\rangle^1 \\
&- \frac{1006}{5}\langle x-5\rangle^1 - 594\langle x-5\rangle^0 + 200\langle x-6\rangle^1 + 400\langle x-7\rangle^1 + 600\langle x-8\rangle^1
\end{aligned}
$$

$$
\begin{aligned}
2EI\frac{\delta v}{\delta x} &= \frac{7}{5}\langle x-0\rangle^2 - 50\langle x-1\rangle^2 - 1\langle x-2\rangle^2 - 50\langle x-3\rangle^2 - \langle x-4\rangle^2 \\
&- \frac{503}{5}\langle x-5\rangle^2 - 594\langle x-5\rangle^1 + 100\langle x-6\rangle^2 + 200\langle x-7\rangle^2 + 300\langle x-8\rangle^2 + C_1
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
2EIv(x) &= \frac{7}{15}\langle x-0\rangle^3 - \frac{50}{3}\langle x-1\rangle^3 - \frac{1}{3}\langle x-2\rangle^3 - \frac{50}{3}\langle x-3\rangle^3 - \frac{1}{3}\langle x-4\rangle^3 \\
&- \frac{503}{15}\langle x-5\rangle^3 - 297\langle x-5\rangle^2 + \frac{100}{3}\langle x-6\rangle^3 + \frac{200}{3}\langle x-7\rangle^2 + 100\langle x-8\rangle^3 + C_1 x + C_2
\end{aligned}
$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the two vertical supports at A and B guarantee the deflection at these points is

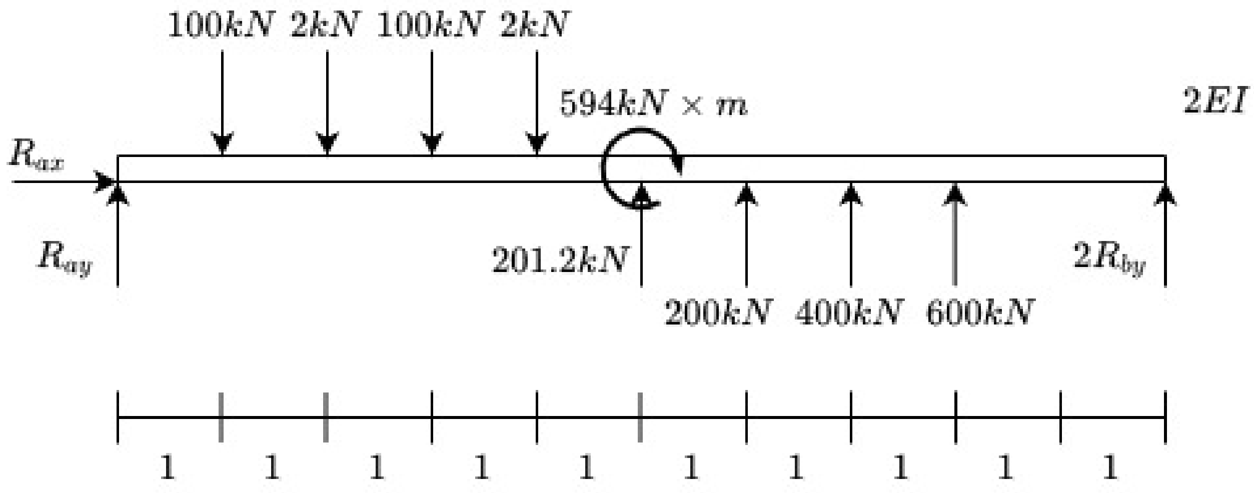


Figure 6: Free Body Diagram for Problem 6 with Equivalent Stiffness

equal to zero, as shown in Eq. 43.

$$\begin{aligned} v(0) &= 0 \\ v(10) &= 0 \end{aligned} \tag{43}$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 44.

$$\begin{aligned} C_2 &= 0 \\ C_1 &= \frac{12263}{5} \end{aligned} \tag{44}$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 45.

$$\begin{aligned} 2EIv(x) = \frac{7}{15}\langle x-0\rangle^3 - \frac{50}{3}\langle x-1\rangle^3 - \frac{1}{3}\langle x-2\rangle^3 - \frac{50}{3}\langle x-3\rangle^3 - \frac{1}{3}\langle x-4\rangle^3 \\ - \frac{503}{15}\langle x-5\rangle^3 - 297\langle x-5\rangle^2 + \frac{100}{3}\langle x-6\rangle^3 + \frac{200}{3}\langle x-7\rangle^2 + 100\langle x-8\rangle^3 + \frac{12263}{5}x \end{aligned} \tag{45}$$

3.7 Problem 7

As given by the project assignment, the first project constitutes of a simply supported beam subject to a series of area loads. By substituting the equivalent reaction forces, the report generates the FBD shown in Fig. 17b.

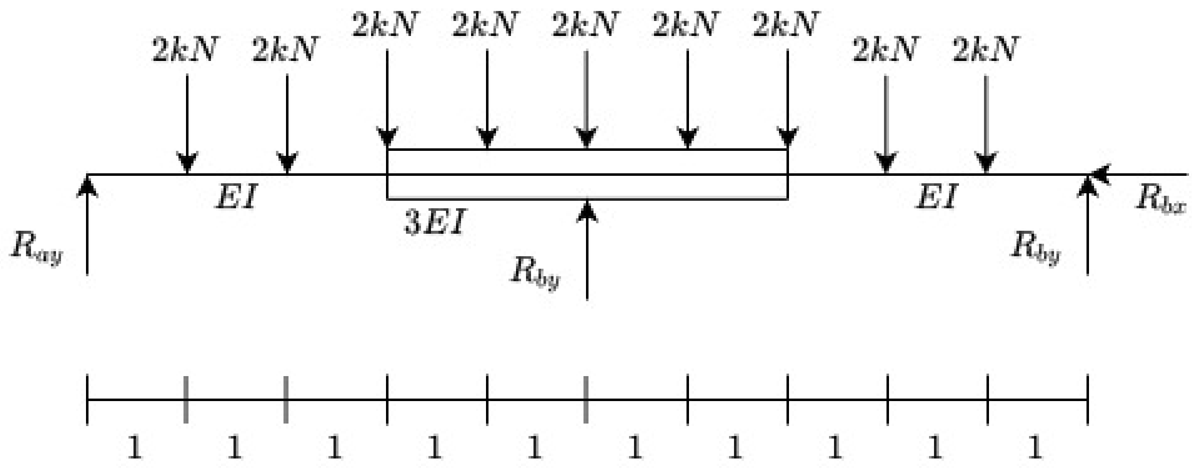


Figure 7: Free Body Diagram for Problem 7

With the Free-Body Diagram completed, the report employs the static equations to find the reaction forces at each of the supports as shown in Eq. 46.

$$\begin{aligned}
&\uparrow \sum F_y = R_a - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 + R_b = 0 \\
&\rightarrow \sum F_x = R_{ax} = 0 \\
&\circlearrowleft \sum M_A = -2(1) - 2(2) - 2(3) - 2(4) - 2(5) - 2(6) - 2(7) - 2(8) - 2(9) + R_{by}(10) = 0
\end{aligned} \tag{46}$$

Given the problem is statically determinate, the static equations are sufficient to find the reaction forces for the supports, which are shown in Eq. 47. Since there are no horizontal forces applied, the only non-zero reactionary forces are those in the vertical direction, which will be named R_a and R_b henceforth.

$$\begin{aligned}
R_b &= 9 \\
R_a &= 9
\end{aligned} \tag{47}$$

Unlike previous problem, the beam's Modulus of Elasticity E and Area Moment of Inertia I vary throughout the beam. As such, prior to writing the moment and deflection equations, we must first construct an equivalent Free Body Diagram by adding forces and moments. The original shear force and moment diagrams are shown in Fig. 8.

As given in Fig. 7, at the interval $0 < x < 3$ and $7 < x < 10$, the beam has the term

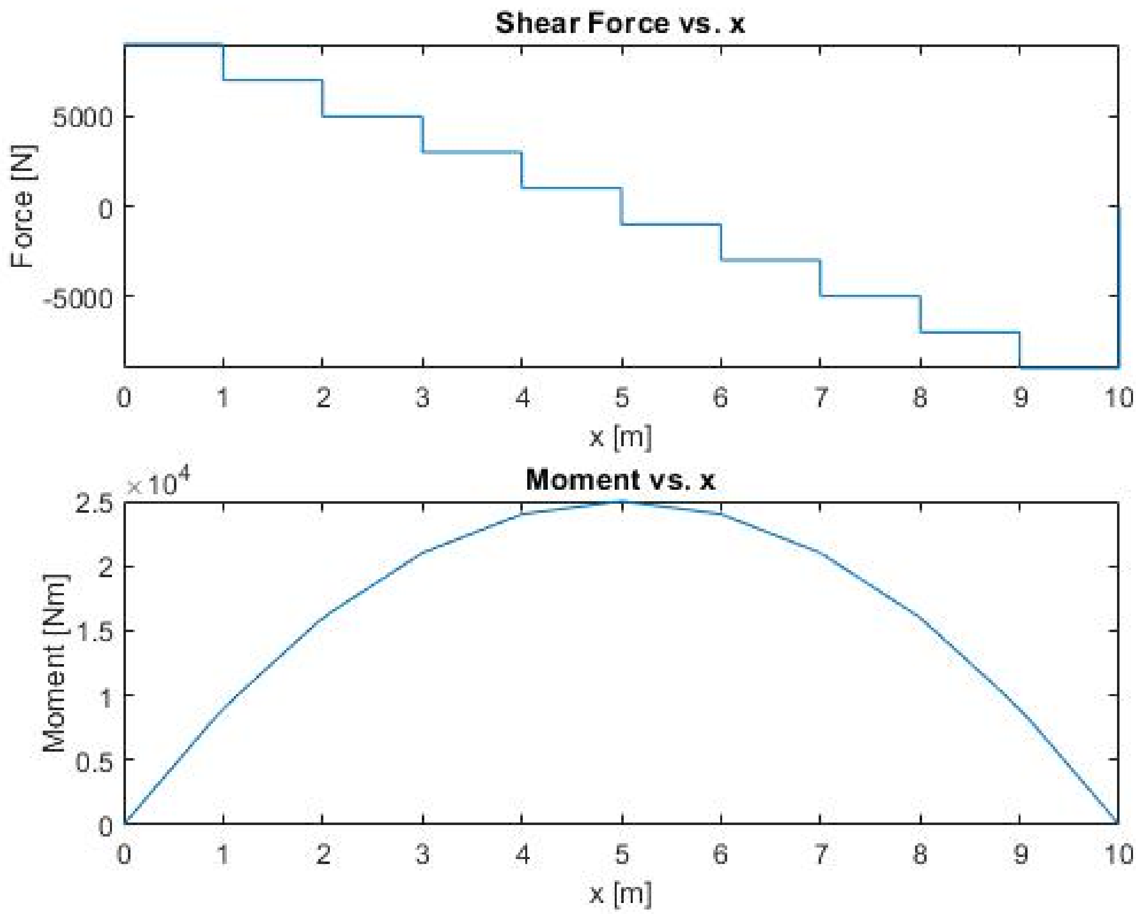


Figure 8: Shear Force and Moment Diagrams assuming EI for non-constant EI.

EI compared to the interval $3 < x < 7$ with term $3EI$. To construct the equivalent diagram, the report will multiply all the where the term EI is present by three such that the EI terms match, creating Fig. 9 with constant EI.

With the equivalent Shear Force and Moment Diagrams, the report constructs a new FBD with an equivalent stiffness $3EI$ shown in Fig. 10. The FBD will be used to generate the displacement equations shown in Eq. 48.

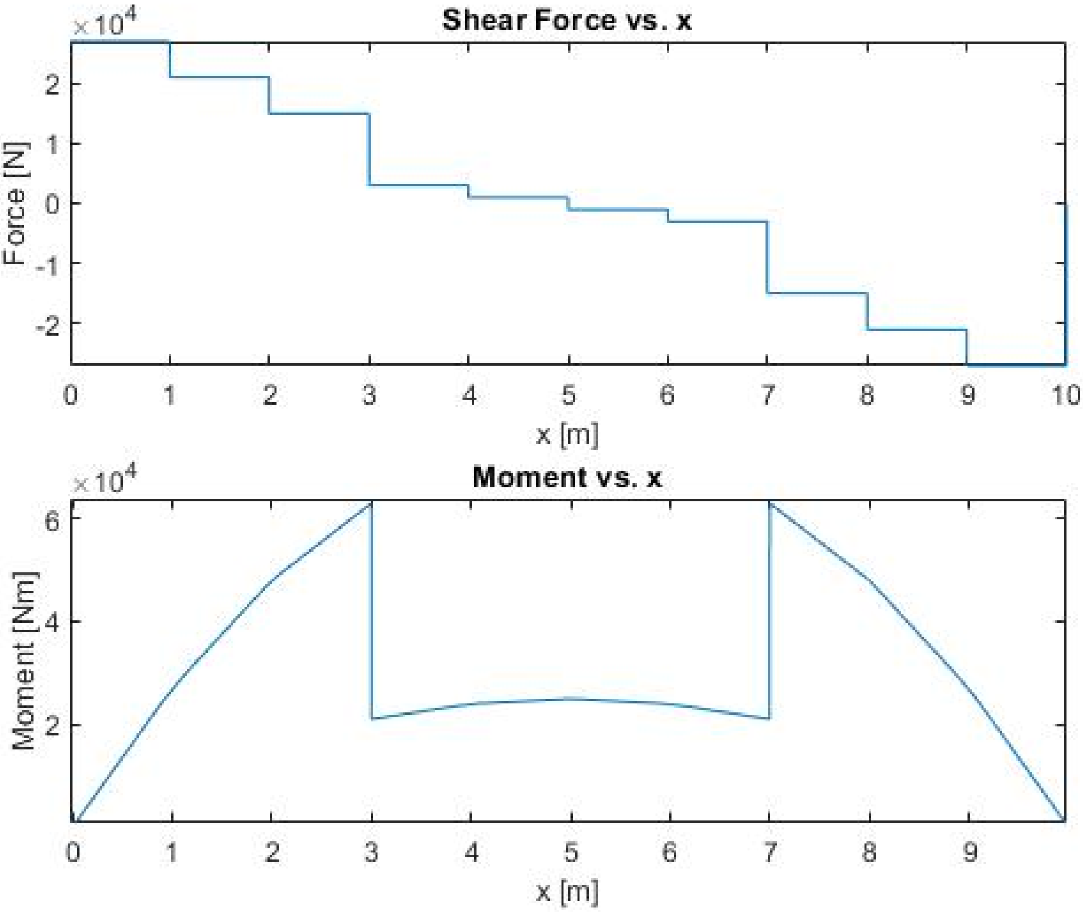


Figure 9: Shear Force and Moment Diagrams for constant EI

$$3EI\frac{\delta^2 v}{\delta x^2} = 27\langle x-0\rangle^1 - 6\langle x-1\rangle^1 - 6\langle x-2\rangle^1 - 12\langle x-3\rangle^1 - 42\langle x-3\rangle^0 - 2\langle x-4\rangle^1$$
$$- 2\langle x-5\rangle^1 - 2\langle x-6\rangle^1 - 12\langle x-7\rangle^1 + 42\langle x-7\rangle^0 - 6\langle x-8\rangle^1 - 6\langle x-9\rangle^1$$

$$3EI\frac{\delta v}{\delta x} = \frac{27}{2}\langle x-0\rangle^2 - 3\langle x-1\rangle^2 - 3\langle x-2\rangle^2 - 6\langle x-3\rangle^2 - 42\langle x-3\rangle^1 - \langle x-4\rangle^2$$
$$- \langle x-5\rangle^2 - \langle x-6\rangle^2 - 6\langle x-7\rangle^2 + 42\langle x-7\rangle^1 - 3\langle x-8\rangle^2 - 3\langle x-9\rangle^2 + C_1 \tag{48}$$

$$3EIv(x) = \frac{9}{2}\langle x-0\rangle^3 - \langle x-1\rangle^3 - \langle x-2\rangle^3 - 2\langle x-3\rangle^3 - 21\langle x-3\rangle^2 - \frac{1}{3}\langle x-4\rangle^3$$
$$- \frac{1}{3}\langle x-5\rangle^3 - \frac{1}{3}\langle x-6\rangle^3 - 2\langle x-7\rangle^3 + 21\langle x-7\rangle^2 - \langle x-8\rangle^3 - \langle x-9\rangle^3 + C_1 x + C_2$$

Following the integration of the singularity moment equation, the report solves for the two constants of integration through the use of boundary conditions. As given in the problem description, the two vertical supports at A and B guarantee the deflection at these points is

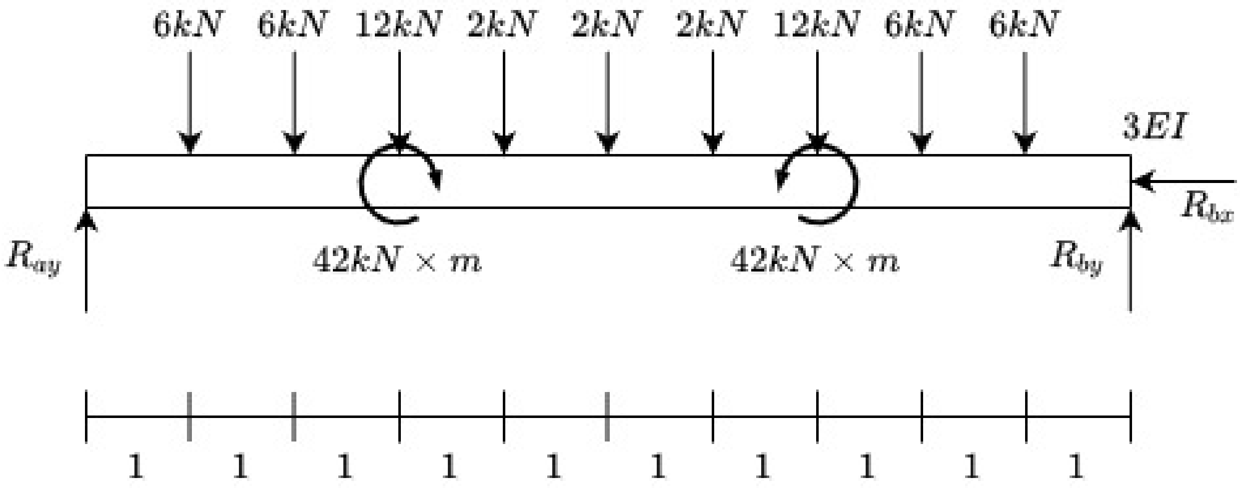


Figure 10: Free Body Diagram for Problem 7 with Equivalent Stiffness

equal to zero, as shown in Eq. 49.

$$\begin{aligned} v(0) &= 0 \\ v(10) &= 0 \end{aligned} \tag{49}$$

Using the boundary conditions shown above, the report finds the constants of integration to the values shown in Eq. 50.

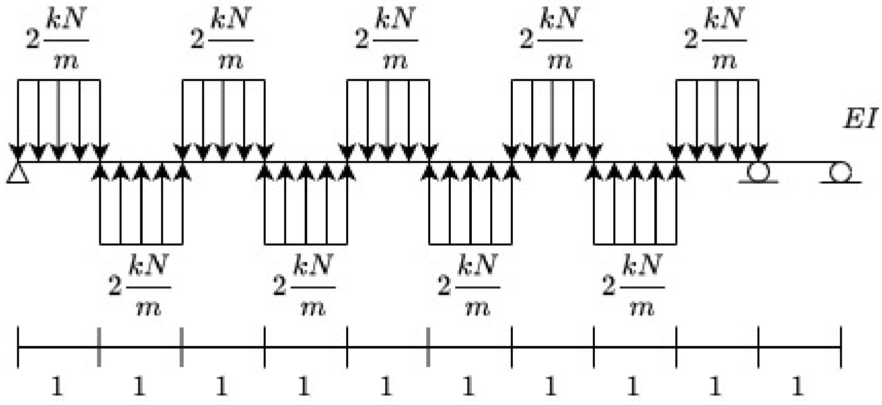
$$\begin{aligned} C_2 &= 0 \\ C_1 &= \frac{-307}{2} \end{aligned} \tag{50}$$

After plugging in the boundary conditions, the report finds the deflection equation shown in Eq. 51.

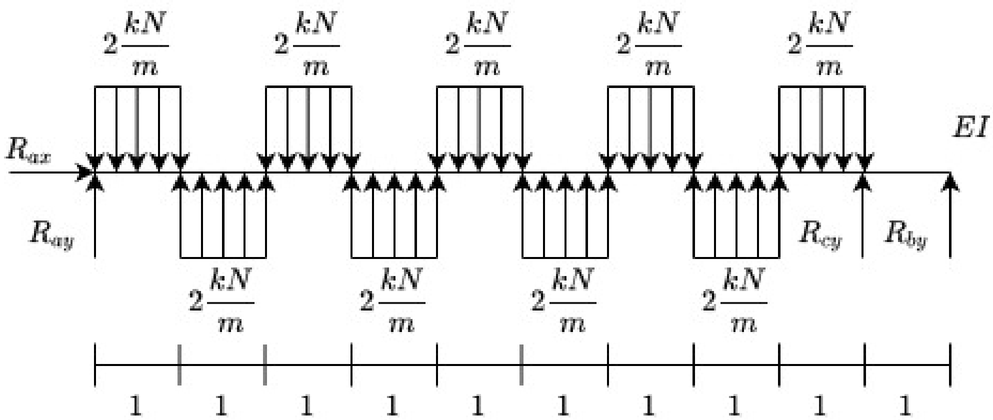
$$\begin{aligned} 3EIv(x) = \frac{3}{2}\langle x-0\rangle^3 - \langle x-1\rangle^3 - \langle x-2\rangle^3 - 2\langle x-3\rangle^3 - 21\langle x-3\rangle^2 - \frac{1}{3}\langle x-4\rangle^3 \\ - \frac{1}{3}\langle x-5\rangle^3 - \frac{1}{3}\langle x-6\rangle^3 - 2\langle x-7\rangle^3 + 21\langle x-7\rangle^2 - \langle x-8\rangle^3 - \langle x-9\rangle^3 - \frac{307}{2}x \end{aligned} \tag{51}$$

4 Results

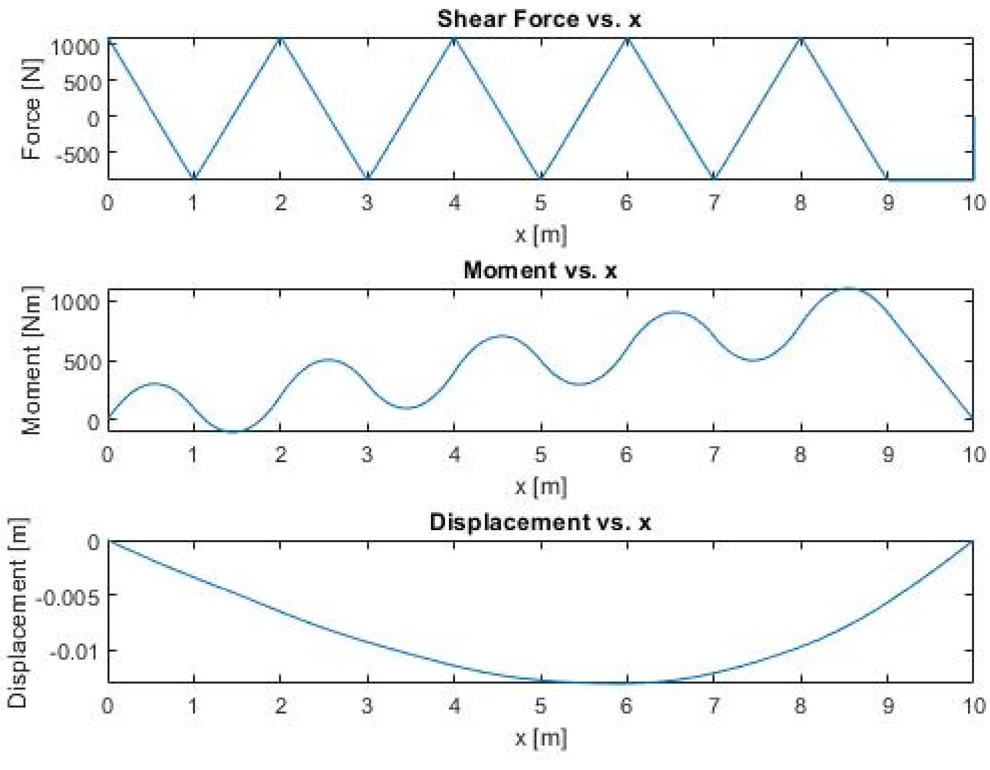
4.1 Problem 1



(a) Given Diagram for Problem 1



(b) Free Body Diagram for Problem 1

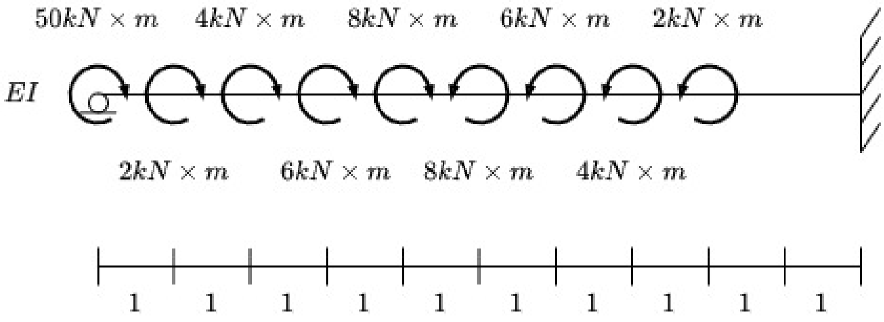


(c) Shear Force, Moment, and Displacement Diagram for Problem 1

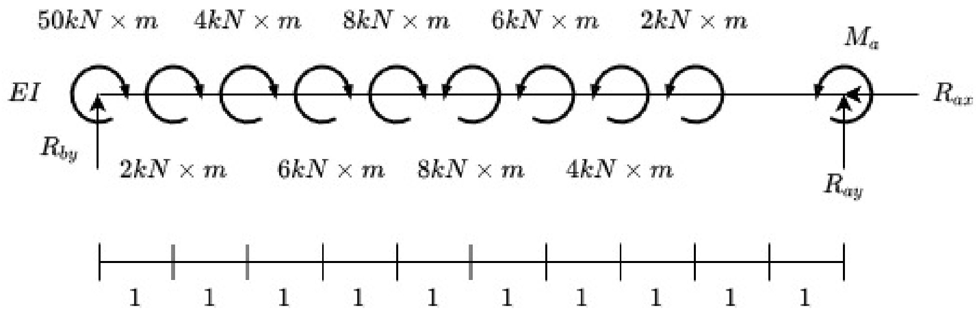
Using the deflection equation, the report finds the maximum displacement occurs at $x = 5.8959m$, which has a downward displacement of $6353.0/EI$ as shown in Fig. 11c. Given the maximum displacement allowed $v_a = L/500$, or 0.02 meters, and a maximum yield stress of 276 MPa, the report uses Eq. 52 and a safety factor $k = 1.5$ to find the required Moment of Inertia I of the beam. As such, the W410 X 85 I-Beam would satisfy both the strength and stiffness requirements with a Moment of Inertia $I = 17.9 * 10^6\, mm^4$.

$$
\begin{aligned}
&I > \frac{v_{max}}{Ev_a} \\
&I > 4.544 * 10^6\, mm^2 \\
&I > \frac{M_{max}}{\frac{1}{2}\sigma_{allowed}} = \frac{M_{max}}{\frac{1}{2}\sigma_Y/k} \\
&I > 11.984 * 10^6\, mm^2
\end{aligned}
\tag{52}
$$

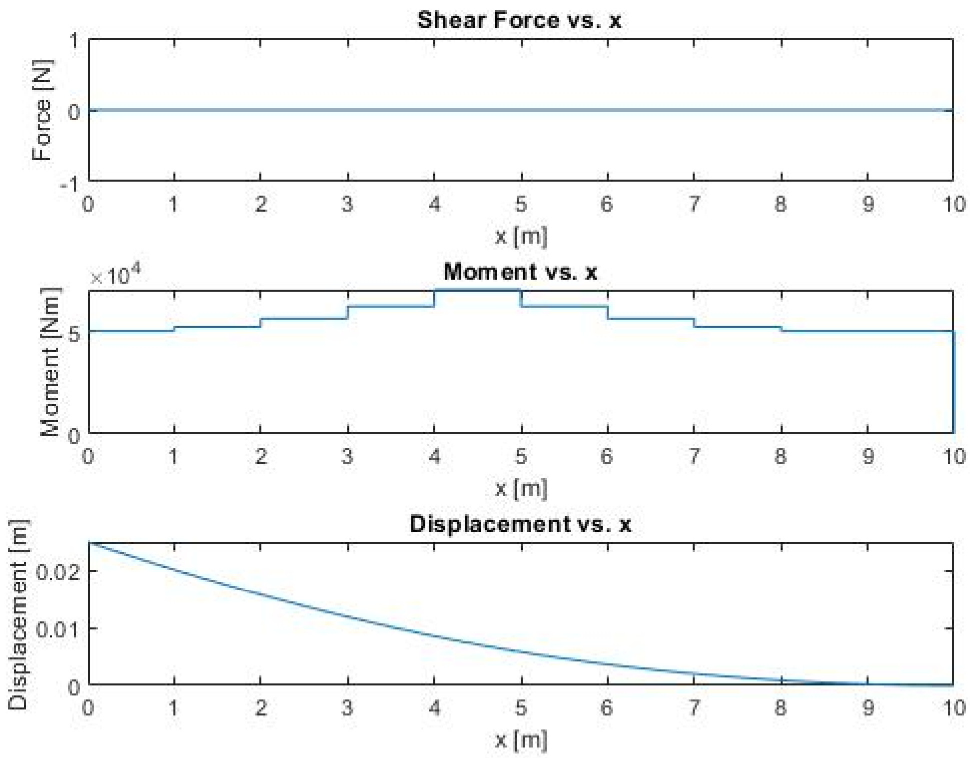
4.2 Problem 2



(a) Given Diagram for Problem 2



(b) Free Body Diagram for Problem 2

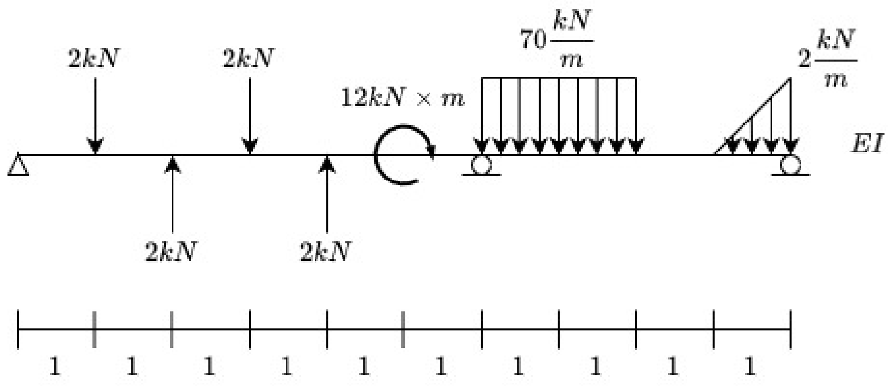


(c) Shear Force, Moment, and Displacement Diagram for Problem 2

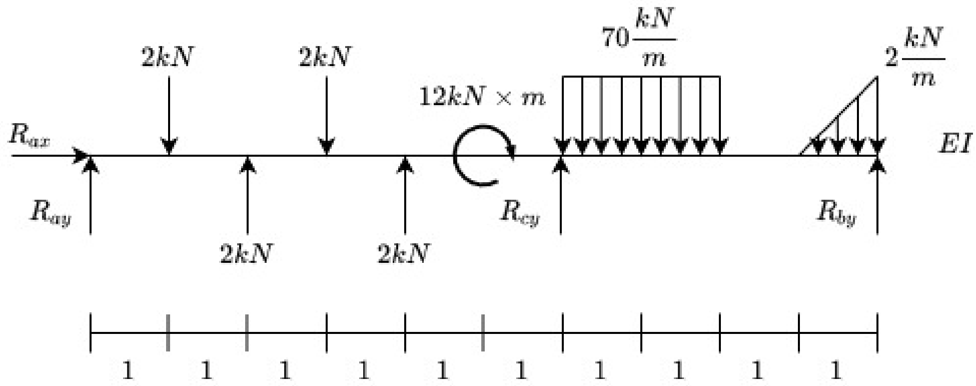
Using the deflection equation, the report finds the maximum displacement occurs at $x = 0m$, which has a downward displacement of $2770000/EI$ as shown in Fig. 12c. Given the maximum displacement allowed $v_a = L/400$, or 0.025 meters, and a maximum yield stress of 276 MPa, the report uses Eq. 53 and a safety factor $k = 1.5$ to find the required Moment of Inertia I of the beam. As such, any radius c greater than 0.2120 meters will suffice.

$$\begin{aligned}
&\frac{\pi r^4}{4} > \frac{v_{max}}{Ev_a} \\
&r > 0.2120\,m \\
&\frac{\pi r^4}{4} > \frac{M_{max}}{\frac{1}{2}\sigma_Y/k} \\
&r > 0.1764\,m
\end{aligned} \tag{53}$$

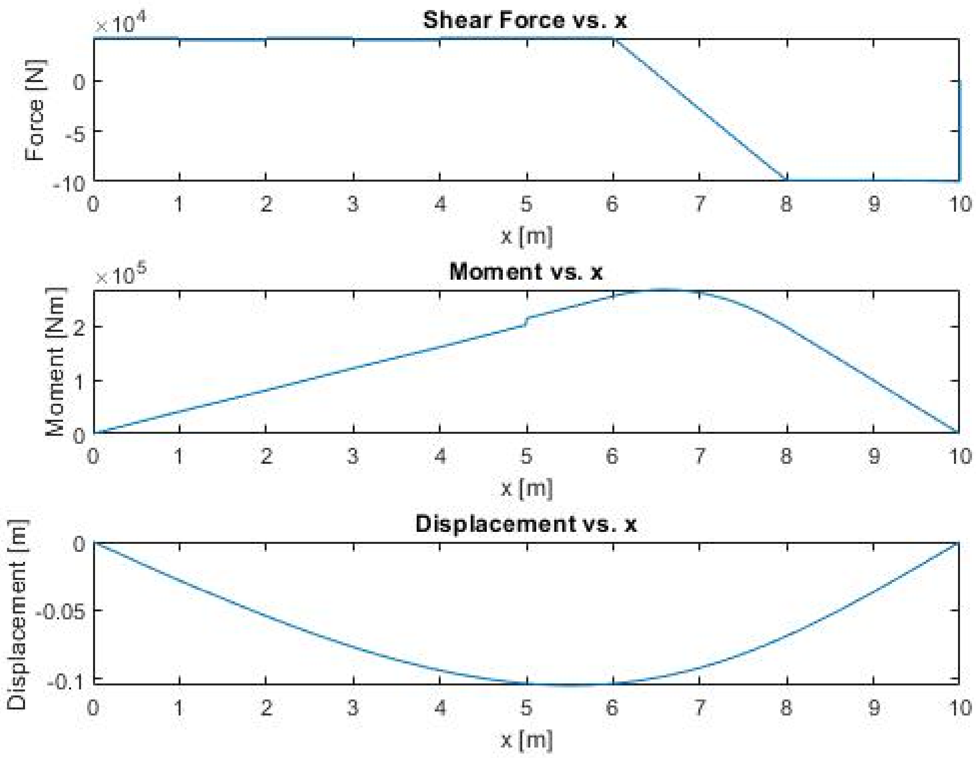
4.3 Problem 3



(a) Given Diagram for Problem 3



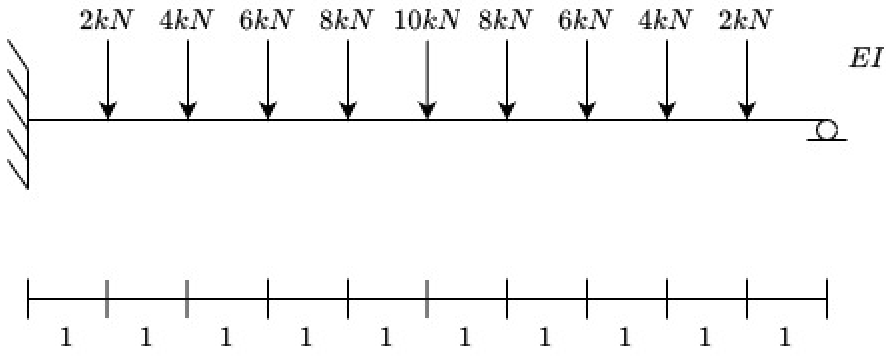
(b) Free Body Diagram for Problem 3



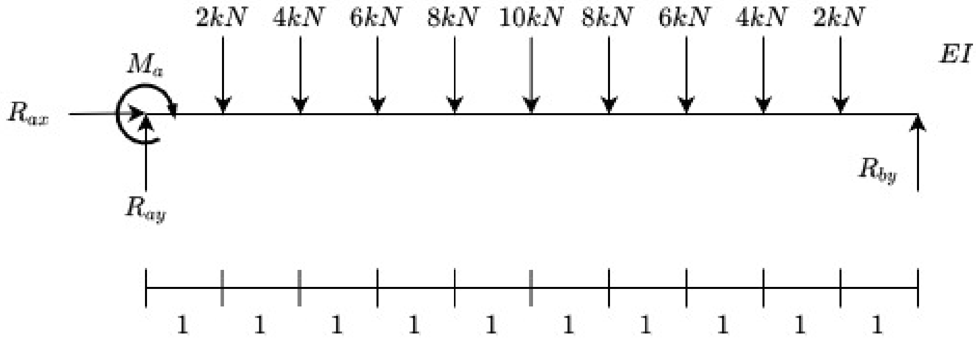
(c) Shear Force, Moment, and Displacement Diagram for Problem 3

Using the deflection equation, the report finds the maximum displacement occurs at $x = 5.5155m$, which has a downward displacement of $-2289467/EI$ as shown in Fig. 13c. Assuming the beam chosen is a circular beam with a radius of $c = 0.2467$ Moment of Inertia $I = 312 * 10^6\, mm^4$, the shear force, moment, and displacement can be seen in Fig. 13c.

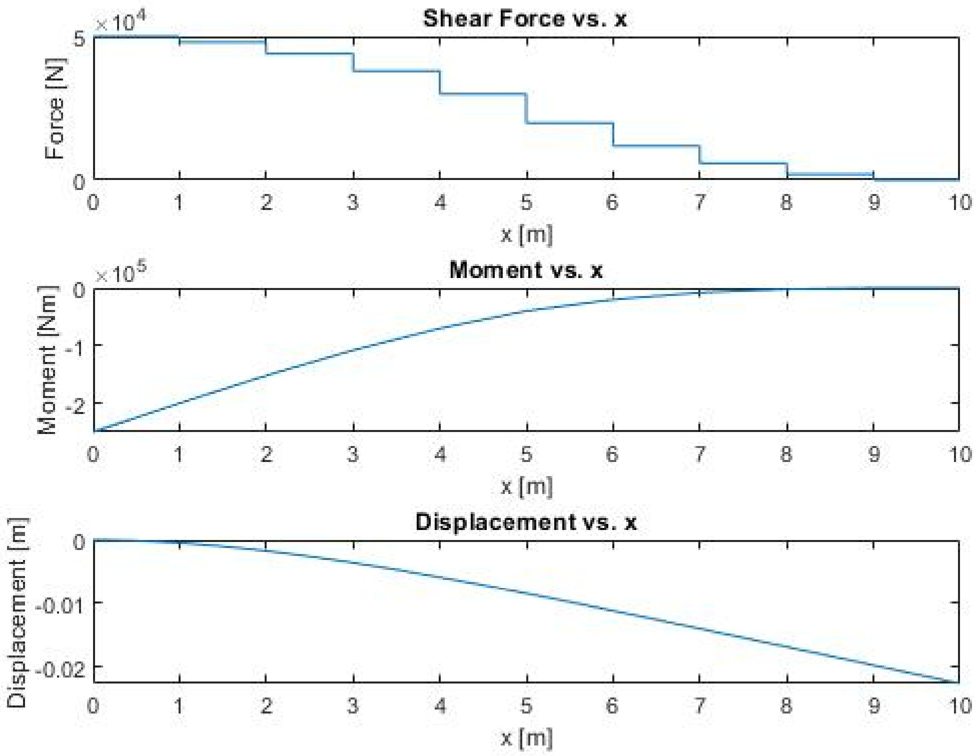
4.4 Problem 4



(a) Given Diagram for Problem 4



(b) Free Body Diagram for Problem 4

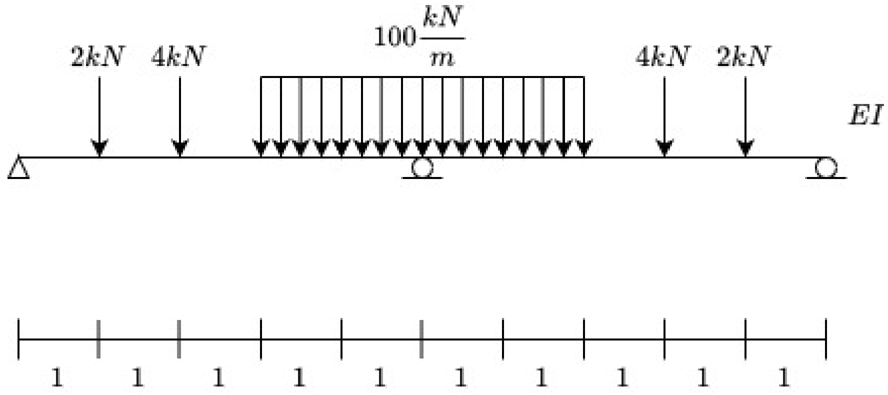


(c) Shear Force, Moment, and Displacement Diagram for Problem 4

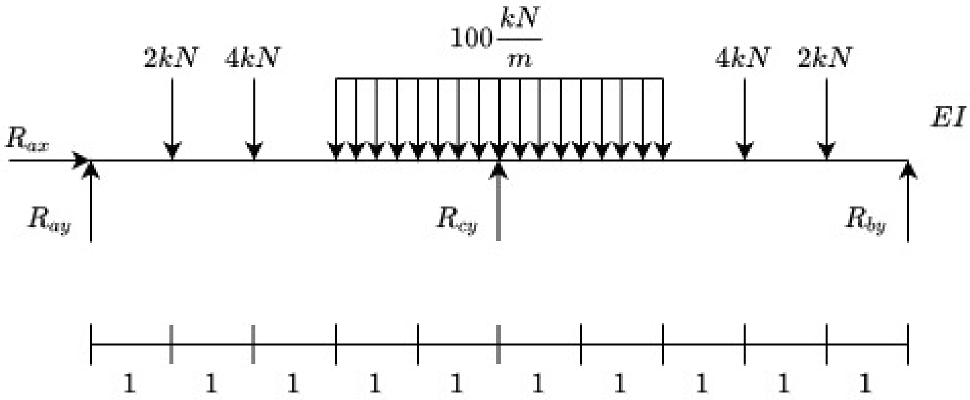
Using the deflection equation, the report finds the maximum displacement occurs at $x = 10m$, which has a downward displacement of $-5708330/EI$ as shown in Fig. 12c. Given the maximum displacement allowed $v_a = L/400$, or 0.025 meters, and a maximum yield stress of 276 MPa, the report uses Eq. 54 and a safety factor $k = 1.5$ to find the required Moment of Inertia I of the beam. As such, a rectangular beam with a base b of 10 centimeters requires a height h¿43.48 centimeters, or a Moment of Inertia $I > 3623.19 * 10^6\, mm^4$.

$$
\begin{aligned}
&\frac{1}{12}bh^3 > \frac{v_{max}}{Ev_a} \\
&h > 0.4348\, m \\
&\frac{1}{12}bh^3 > \frac{M_{max}}{\frac{1}{2}\sigma_Y/k} \\
&h > 0.3920\, m
\end{aligned}
\tag{54}
$$

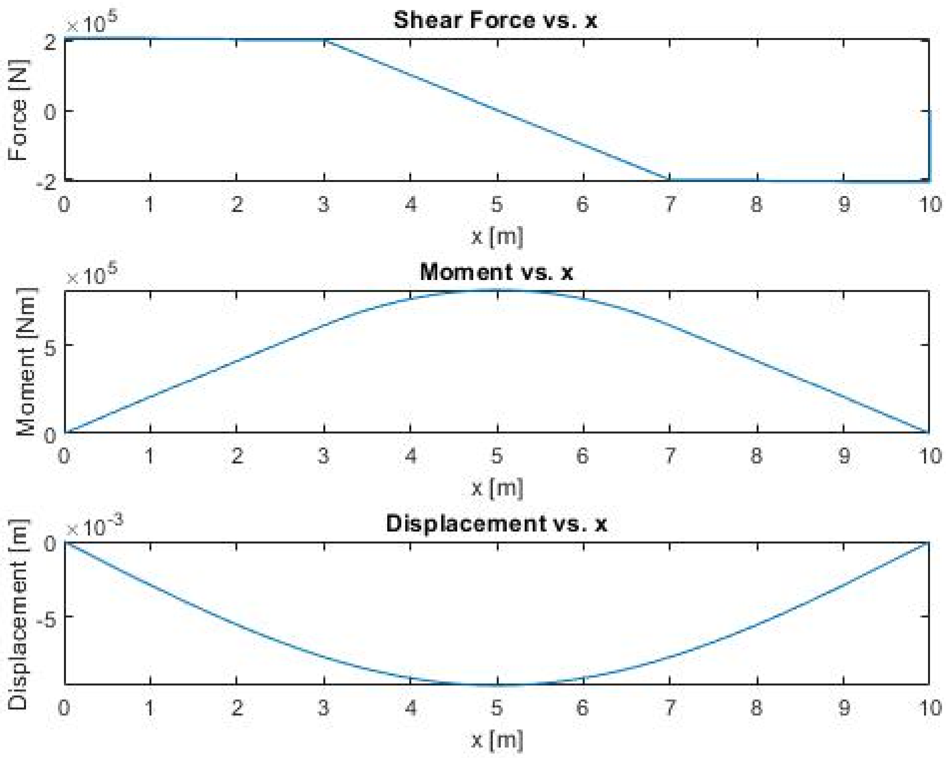
4.5 Problem 5



(a) Given Diagram for Problem 5



(b) Free Body Diagram for Problem 5



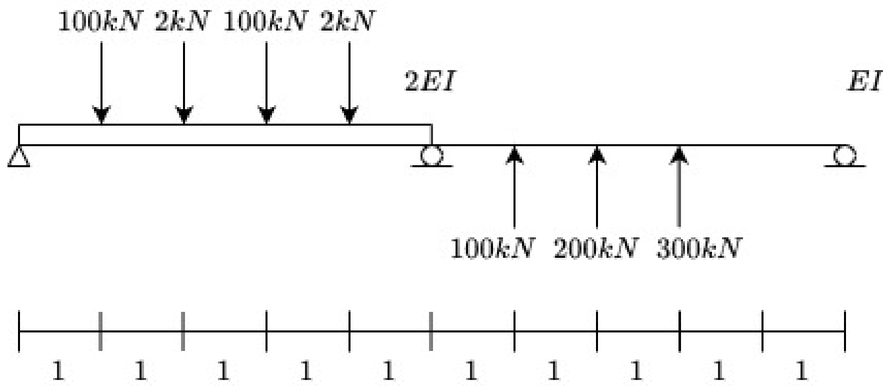
(c) Shear Force, Moment, and Displacement Diagram for Problem 5

Using the deflection equation, the report finds the maximum displacement occurs at $x = 5.0m$, which has a downward displacement of $-7852656/EI$ as shown in Fig. 15c. Given the maximum displacement allowed $v_a = L/400$, or 0.025 meters, and a maximum yield stress of 276 MPa as that of an aluminum beam , the report uses Eq. 55 and a safety factor $k = 1.5$ to find the required Moment of Inertia I of the beam. As such, a circular beam with a radius c greater than 0.3497 meters will suffice, or a Moment of Inertia $I > 11739.11 * 10^6\, mm^4$.

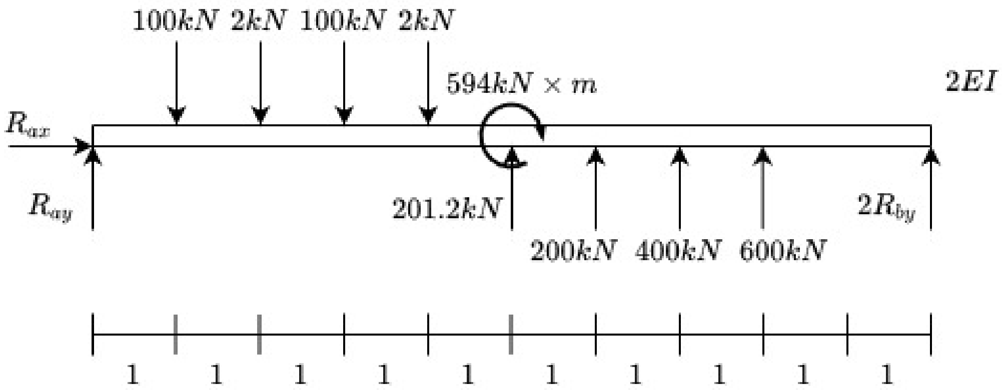
$$\begin{aligned}
&\frac{\pi r^4}{4} > \frac{v_{max}}{E v_a} \\
&r > 0.2472\, m \\
&\frac{\pi r^4}{4} > \frac{M_{max}}{\frac{1}{2}\sigma_Y / k} \\
&r > 0.3497\, m
\end{aligned} \tag{55}$$

Assuming the beam chosen has a Moment of Inertia $I = 4493.65 * 10^6\, mm^4$, the shear force, moment, and displacement can be seen in Fig. 15c where the maximum displacement occurs exactly at the center of the beam due to symmetry about its midpoint.

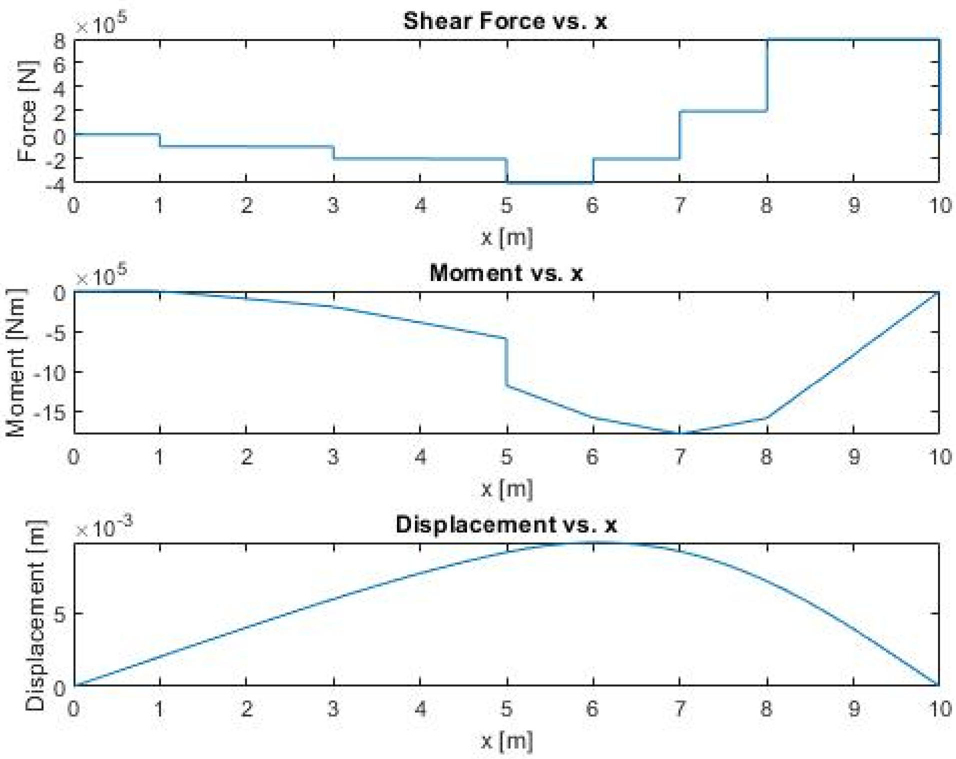
4.6 Problem 6



(a) Given Diagram for Problem 6



(b) Free Body Diagram for Problem 6

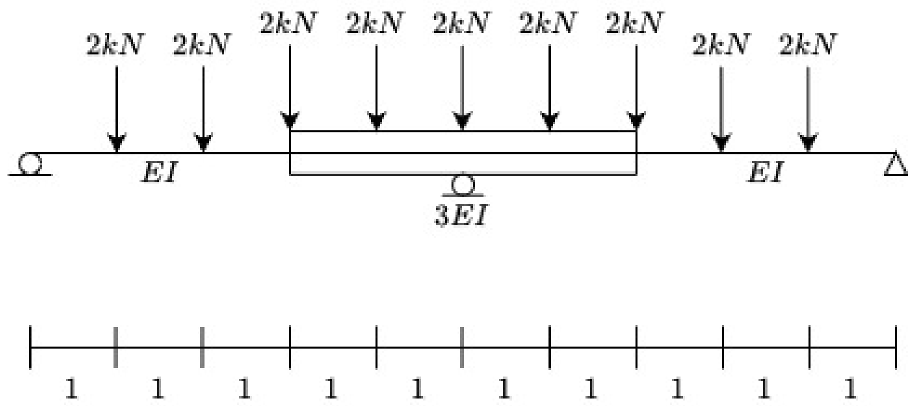


(c) Shear Force, Moment, and Displacement Diagram for Problem 6

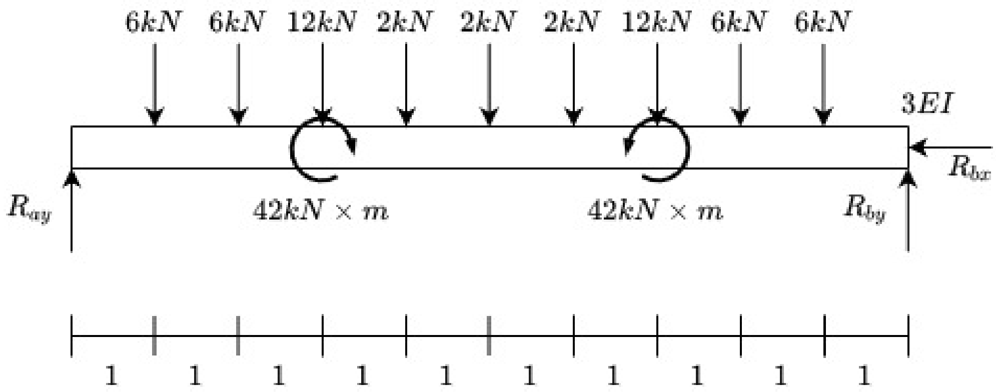
Using the deflection equation, the report finds the maximum displacement occurs at $x = 5.0m$, which has a downward displacement of $11930984/EI$ as shown in Fig. 16c. Assuming an aluminum beam with a maximum yield stress of 276 MPa, the report uses Eq. 56 and a safety factor $k = 1.5$ to find the required Moment of Inertia I of the beam. As such, a circular beam with a radius c greater than 0.3969 meters will suffice, or a Moment of Inertia $I > 19480.35 * 10^6\, mm^4$.

$$
\begin{aligned}
&\frac{\pi r^4}{4} > \frac{v_{max}}{E v_a} \\
&r > 0.3053\, m \\
&\frac{\pi r^4}{4} > \frac{M_{max}}{\frac{1}{2}\sigma_Y / k} \\
&r > 0.3969\, m
\end{aligned}
\tag{56}
$$

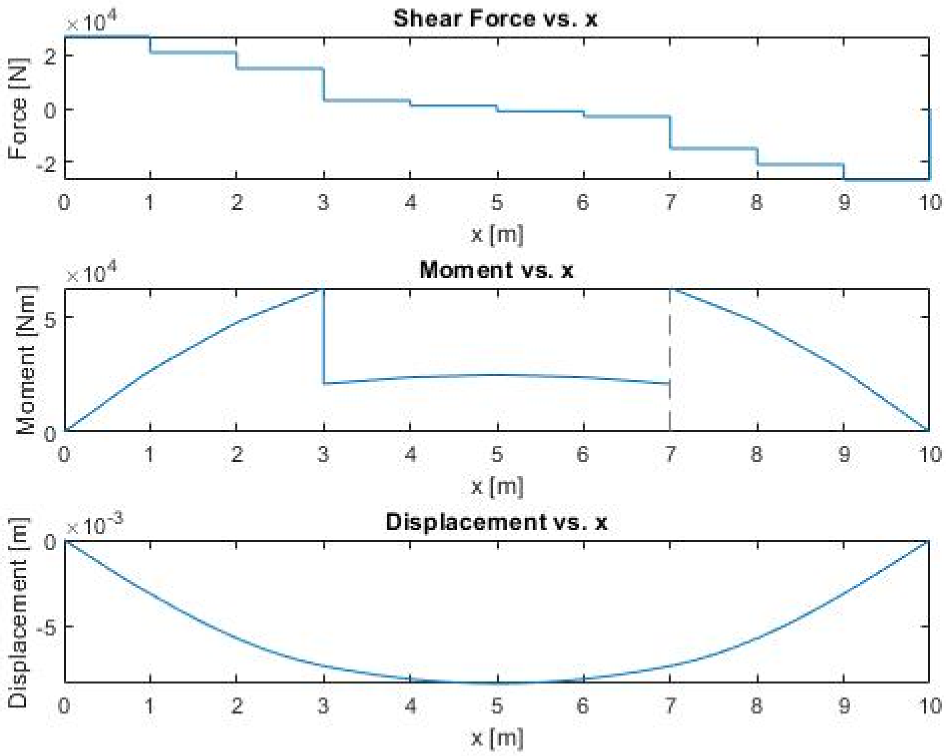
4.7 Problem 7



(a) Given Diagram for Problem 7



(b) Free Body Diagram for Problem 7



(c) Shear Force, Moment, and Displacement Diagram for Problem 7

Using the deflection equation, the report finds the maximum displacement occurs at $x = 5.0m$, which has a downward displacement of $396333/EI$ as shown in Fig. 17c. Assuming an aluminum beam with a maximum yield stress of 276 MPa, the report uses Eq. 57 and a safety factor $k = 1.5$ to find the required Moment of Inertia I of the beam. As such, a circular beam with a radius c greater than 0.1718 meters will suffice, or a Moment of Inertia $I > 683.64 * 10^6\, mm^4$.

$$\begin{aligned} &\frac{\pi r^4}{4} > \frac{v_{max}}{E v_a} \\ &r > 0.1304\, m \\ &\frac{\pi r^4}{4} > \frac{M_{max}}{\frac{1}{2}\sigma_Y / k} \\ &r > 0.1718\, m \end{aligned} \tag{57}$$

5 Conclusions

The project provided excellent practice for real-life working conditions in beam deflection. It is often the case in large projects that communication stands as the greatest barrier towards completion, which was perfectly demonstrated through the project. From discussions of misunderstandings with the professor to design choices based on abstract loads, the project greatly improved my understanding of beam deflection and how to troubleshoot all issues that may arise from it. In addition to the theoretical modeling, the modeling of the beam on MATLAB allowed for the improvement of the ability to convey ideas clearly and objectively.

Future projects should consider decreasing the workload on students by substituting a large number of problems with a complex central problem that incorporates all the required skills; in addition to removing some of the repetition, the problem would more closely model real life's working conditions.

6 Appendix

6.1 Main MATLAB Plotting

```
%% Header
% Name: Pedro Almeida
```

```
% Date: October 1st, 2021
% Course: EGM 4523C { Intermediate Strength of Materials
% Serial Number: 2

%% Preparation
clc; clear; close all;
s = 2;
range_x = [0 10];
run_problems = [72];



%% Problem 1
if any(run_problems(:) == 1)
    figure(1)

    %Constants
    L = 10; %Beam Length
    E = 69.9e9; %Young's Modulus
    v_allowed = L/500; %Allowed Displacement
    k = 1.5; %Safety Factor
    sY = 276e6; %Yield Stress
%     I = 6.95e-6;
    I = 1/E;

    %Problem Solution
    syms x

    %Shear Force Plot
    subplot(3,1,1)
    shear = 1000*(.55*s*pw(x,0,0) -s*pw(x,0,1) + 2*s*pw(x,1,1) - 2*s*pw(x,2,1) + 2*s*pw(
    fplot(shear, range_x);
```

```
␣␣␣␣title('Shear␣Force␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Force␣[N]')

␣␣␣␣%Moment␣Plot
␣␣␣␣subplot(3,1,2)
␣␣␣␣moment␣=␣1000*(0.55*s*pw(x,0,1)␣-(s/2)*pw(x,0,2)␣+␣2*(s/2)*pw(x,1,2)␣-␣2*(s/2)*pw(x,
␣␣␣␣fplot(moment,␣range_x);
␣␣␣␣title('Moment␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Moment␣[Nm]')

␣␣␣␣%Displacement␣Plot
␣␣␣␣subplot(3,1,3)
␣␣␣␣displacement␣=␣1000*(0.55*(s/6)*pw(x,0,3)␣-(s/24)*pw(x,0,4)␣+␣2*(s/24)*pw(x,1,4)␣-␣2
␣␣␣␣fplot(displacement,␣range_x);
␣␣␣␣title('Displacement␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Displacement␣[m]')



␣␣␣␣[I1,␣I2]␣=␣getMoment(displacement,␣range_x,␣moment,␣v_allowed,␣E,␣sY,␣k);
␣␣␣␣fprintf('Min␣Area␣=␣%f\n',I1);
␣␣␣␣fprintf('Min␣I␣=␣%f\n',I2);
end




%%␣Problem␣2
if␣any(run_problems(:)␣==␣2)
␣␣␣␣figure(2)
```

```
␣␣␣␣%Constants

␣␣␣␣L␣=␣10;␣%Beam␣Length

␣␣␣␣E␣=␣69.9e9;␣%Young’s␣Modulus

␣␣␣␣v_allowed␣=␣L/400;␣%Allowed␣Displacement

␣␣␣␣k␣=␣1.5;␣%Safety␣Factor

␣␣␣␣sY␣=␣276e6;␣%Yield␣Stress

␣␣␣␣I=1/E;

%␣␣␣␣␣I␣=␣1585.121602e-6;␣%


␣␣␣␣%Problem␣Solution

␣␣␣␣syms␣x


␣␣␣␣%Shear␣Force␣Plot

␣␣␣␣subplot(3,1,1)

␣␣␣␣shear␣=␣0;

␣␣␣␣fplot(shear,␣range_x);

␣␣␣␣title(’Shear␣Force␣vs.␣x’);

␣␣␣␣xlabel(’x␣[m]’)

␣␣␣␣ylabel(’Force␣[N]’)


␣␣␣␣%Moment␣Plot

␣␣␣␣subplot(3,1,2)

␣␣␣␣moment␣=␣1000*(50*pw(x,0,0)␣+␣s*pw(x,1,0)␣+␣2*s*pw(x,2,0)␣+␣3*s*pw(x,3,0)␣+␣4*s*pw(x

␣␣␣␣fplot(moment,␣range_x);

␣␣␣␣title(’Moment␣vs.␣x’);

␣␣␣␣xlabel(’x␣[m]’)

␣␣␣␣ylabel(’Moment␣[Nm]’)


␣␣␣␣%Displacement␣Plot

␣␣␣␣subplot(3,1,3)
```

```
    displacement = (1000*(25*pw(x,0,2) + (s/2)*pw(x,1,2) + 2*(s/2)*pw(x,2,2) + 3*(s/2)*p
    fplot(displacement, range_x);
    title('Displacement vs. x');
    xlabel('x [m]')
    ylabel('Displacement [m]')

    [I1, I2] = getMoment(displacement, range_x, moment, v_allowed, E, sY, k);
    fprintf('Min Area = %f\n',I1);
    fprintf('Min I = %f\n',I2);

    %Min Radius = 0.2120 meters
end

%% Problem 3
if any(run_problems(:) == 3)
    figure(3)

    %Constants
    L = 10; %Beam Length
    E = 69.9e9; %Young's Modulus
    v_allowed = L/400; %Allowed Displacement
    k = 1.5; %Safety Factor
    sY = 276e6; %Yield Stress
    I = 1/E;
    I = 2908.086809e-6; %

    %Problem Solution
    syms x

    %Shear Force Plot
    subplot(3,1,1)
```

```
␣␣␣␣shear␣=␣1000*(((7*s)/60␣+␣41)*pw(x,0,0)␣-␣s*pw(x,1,0)␣+␣s*pw(x,2,0)␣-␣s*pw(x,3,0)␣+␣
␣␣␣␣fplot(shear,␣range_x);
␣␣␣␣title('Shear␣Force␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Force␣[N]')

␣␣␣␣%Moment␣Plot
␣␣␣␣subplot(3,1,2)
␣␣␣␣moment␣=␣1000*(((7*s)/60␣+␣41)*pw(x,0,1)␣-␣s*pw(x,1,1)␣+␣s*pw(x,2,1)␣-␣s*pw(x,3,1)␣+
␣␣␣␣fplot(moment,␣range_x);
␣␣␣␣title('Moment␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Moment␣[Nm]')

␣␣␣␣%Displacement␣Plot
␣␣␣␣subplot(3,1,3)
␣␣␣␣displacement␣=␣(1000*((((7*s)/60␣+␣41)/6)*pw(x,0,3)␣-␣(s/6)*pw(x,1,3)␣+␣(s/6)*pw(x,2
␣␣␣␣fplot(displacement,␣range_x);
␣␣␣␣title('Displacement␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Displacement␣[m]')

␣␣␣␣[A,␣I]␣=␣getMoment(displacement,␣range_x,␣moment,␣v_allowed,␣E,␣sY,␣k);
␣␣␣␣fprintf('Min␣Area␣=␣%f\n',A);
␣␣␣␣fprintf('Min␣I␣=␣%f\n',I);
end



%%␣Problem␣4
if␣any(run_problems(:)␣==␣4)
␣␣␣␣figure(4)
```

```
␣␣␣␣%Constants
␣␣␣␣L␣=␣10;␣%Beam␣Length
␣␣␣␣E␣=␣69.9e9;␣%Young's␣Modulus
␣␣␣␣v_allowed␣=␣L/400;␣%Allowed␣Displacement
␣␣␣␣k␣=␣2;␣%Safety␣Factor
␣␣␣␣sY␣=␣276e6;␣%Yield␣Stress
␣␣␣␣I␣=␣1/E;
␣␣␣␣I␣=␣3623.188406e-6;␣%

␣␣␣␣%Problem␣Solution
␣␣␣␣syms␣x

␣␣␣␣%Shear␣Force␣Plot
␣␣␣␣subplot(3,1,1)
␣␣␣␣shear␣=␣1000*(25*s*pw(x,0,0)␣-␣s*pw(x,1,0)␣-␣2*s*pw(x,2,0)␣-␣3*s*pw(x,3,0)␣-␣4*s*pw(
␣␣␣␣fplot(shear,␣range_x);
␣␣␣␣title('Shear␣Force␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Force␣[N]')

␣␣␣␣%Moment␣Plot
␣␣␣␣subplot(3,1,2)
␣␣␣␣moment␣=␣1000*(25*s*pw(x,0,1)␣-␣125*s*pw(x,0,0)␣-␣s*pw(x,1,1)␣-␣2*s*pw(x,2,1)␣-␣3*s*
␣␣␣␣fplot(moment,␣range_x);
␣␣␣␣title('Moment␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Moment␣[Nm]')

␣␣␣␣%Displacement␣Plot
␣␣␣␣subplot(3,1,3)
```

```
␣␣␣␣displacement␣=␣(1000*((25/6)*s*pw(x,0,3)␣-␣(125/2)*s*pw(x,0,2)␣-␣(s/6)*pw(x,1,3)␣-␣2
␣␣␣␣fplot(displacement,␣range_x);
␣␣␣␣title('Displacement␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Displacement␣[m]')

␣␣␣␣[A,␣I]␣=␣getMoment(displacement,␣range_x,␣moment,␣v_allowed,␣E,␣sY,␣k);
␣␣␣␣fprintf('Min␣Area␣=␣%f\n',A);
␣␣␣␣fprintf('Min␣I␣=␣%f\n',I);
end



%%␣Problem␣5
if␣any(run_problems(:)␣==␣5)
␣␣␣␣figure(5)

␣␣␣␣%Constants
␣␣␣␣L␣=␣10;␣%Beam␣Length
␣␣␣␣E␣=␣69.9e9;␣%Young's␣Modulus
␣␣␣␣v_allowed␣=␣L/400;␣%Allowed␣Displacement
␣␣␣␣k␣=␣2;␣%Safety␣Factor
␣␣␣␣sY␣=␣276e6;␣%Yield␣Stress
␣␣␣␣I␣=␣1/E;␣%
␣␣␣␣I␣=␣11739.112283e-6;

␣␣␣␣%Problem␣Solution
␣␣␣␣syms␣x

␣␣␣␣%Shear␣Force␣Plot
␣␣␣␣subplot(3,1,1)
␣␣␣␣shear␣=␣1000*((3*s␣+␣200)*pw(x,0,0)␣-␣s*pw(x,1,0)␣-␣2*s*pw(x,2,0)␣-␣100*pw(x,3,1)␣+␣
```

```
    fplot(shear, range_x);
    title('Shear Force vs. x');
    xlabel('x [m]')
    ylabel('Force [N]')

    %Moment Plot
    subplot(3,1,2)
    moment = 1000*((3*s + 200)*pw(x,0,1) - s*pw(x,1,1) - 2*s*pw(x,2,1) - 50*pw(x,3,2) +
    fplot(moment, range_x);
    title('Moment vs. x');
    xlabel('x [m]')
    ylabel('Moment [Nm]')

    %Displacement Plot
    subplot(3,1,3)
    displacement = (1000*(((3*s + 200)/6)*pw(x,0,3) - (s/6)*pw(x,1,3) - 2*(s/6)*pw(x,2,3
    fplot(displacement, range_x);
    title('Displacement vs. x');
    xlabel('x [m]')
    ylabel('Displacement [m]')

    [A, I] = getMoment(displacement, range_x, moment, v_allowed, E, sY, k);
    fprintf('Min Area = %f\n',A);
    fprintf('Min I = %f\n',I);
end


%% Problem 6_1
if any(run_problems(:) == 61)
    figure(6)
```

```
    %Constants
    L = 10; %Beam Length
    E = 69.9e9; %Young's Modulus
    v_allowed = L/400; %Allowed Displacement
    k = 1.5; %Safety Factor
    sY = 276e6; %Yield Stress
    I = 0; %

    %Problem Solution
    syms x

    %Shear Force Plot
    subplot(2,1,1)
    shear = 1000*(((7*s)/5)*pw(x,0,0) - 100*pw(x,1,0) - s*pw(x,2,0) - 100*pw(x,3,0) - s*
    fplot(shear, range_x);
    title('Shear Force vs. x');
    xlabel('x [m]')
    ylabel('Force [N]')

    %Moment Plot
    subplot(2,1,2)
    moment = 1000*(((7*s)/5)*pw(x,0,1) - 100*pw(x,1,1) - s*pw(x,2,1) - 100*pw(x,3,1) - s
    fplot(moment, range_x);
    title('Moment vs. x');
    xlabel('x [m]')
    ylabel('Moment [Nm]')
end

%% Problem 6_2
if any(run_problems(:) == 62)
    figure(7)
```

```
␣␣␣␣%Constants
␣␣␣␣L␣=␣10;␣%Beam␣Length
␣␣␣␣E␣=␣69.9e9;␣%Young's␣Modulus
␣␣␣␣v_allowed␣=␣L/400;␣%Allowed␣Displacement
␣␣␣␣k␣=␣1.5;␣%Safety␣Factor
␣␣␣␣sY␣=␣276e6;␣%Yield␣Stress
%␣␣␣␣␣I␣=␣1/E;
␣␣␣␣I␣=␣19480.349915e-6;

␣␣␣␣%Problem␣Solution
␣␣␣␣syms␣x

␣␣␣␣%Shear␣Force␣Plot
␣␣␣␣subplot(3,1,1)
␣␣␣␣shear␣=␣1000*(((7*s)/5)*pw(x,0,0)␣-␣100*pw(x,1,0)␣-␣s*pw(x,2,0)␣-␣100*pw(x,3,0)␣-␣s*
␣␣␣␣fplot(shear,␣range_x);
␣␣␣␣title('Shear␣Force␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Force␣[N]')

␣␣␣␣%Moment␣Plot
␣␣␣␣subplot(3,1,2)
␣␣␣␣moment␣=␣1000*(((7*s)/5)*pw(x,0,1)␣-␣100*pw(x,1,1)␣-␣s*pw(x,2,1)␣-␣100*pw(x,3,1)␣-␣s
␣␣␣␣fplot(moment,␣range_x);
␣␣␣␣title('Moment␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Moment␣[Nm]')

␣␣␣␣%Displacement␣Plot
␣␣␣␣subplot(3,1,3)
```

```
␣␣␣␣displacement␣=␣(1000*(((7*s)/30)*pw(x,0,3)␣-␣(100/6)*pw(x,1,3)␣-␣(s/6)*pw(x,2,3)␣-␣(
␣␣␣␣fplot(displacement,␣range_x);
␣␣␣␣title('Displacement␣vs.␣x');
␣␣␣␣xlabel('x␣[m]')
␣␣␣␣ylabel('Displacement␣[m]')

␣␣␣␣[A,␣I]␣=␣getMoment(displacement,␣range_x,␣moment,␣v_allowed,␣E,␣sY,␣k);
␣␣␣␣fprintf('Min␣Area␣=␣%f\n',A);
␣␣␣␣fprintf('Min␣I␣=␣%f\n',I);
end



%%␣Problem␣7_1
if␣any(run_problems(:)␣==␣71)
␣␣␣␣figure(8)

␣␣␣␣%Constants
␣␣␣␣L␣=␣10;␣%Beam␣Length
␣␣␣␣E␣=␣69.9e9;␣%Young's␣Modulus
␣␣␣␣v_allowed␣=␣L/400;␣%Allowed␣Displacement
␣␣␣␣k␣=␣1.5;␣%Safety␣Factor
␣␣␣␣sY␣=␣276e6;␣%Yield␣Stress
␣␣␣␣I␣=␣0;␣%

␣␣␣␣%Problem␣Solution
␣␣␣␣syms␣x

␣␣␣␣%Shear␣Force␣Plot
␣␣␣␣subplot(2,1,1)
␣␣␣␣shear␣=␣1000*(4.5*s*pw(x,0,0)␣-␣s*pw(x,1,0)␣-␣s*pw(x,2,0)␣-␣s*pw(x,3,0)␣-␣s*pw(x,4,(
␣␣␣␣fplot(shear,␣range_x);
```

```
    title('Shear Force vs. x');
    xlabel('x [m]')
    ylabel('Force [N]')

    %Moment Plot
    subplot(2,1,2)
    moment = 1000*(4.5*s*pw(x,0,1) - s*pw(x,1,1) - s*pw(x,2,1) - s*pw(x,3,1) - s*pw(x,4,
    fplot(moment, range_x);
    title('Moment vs. x');
    xlabel('x [m]')
    ylabel('Moment [Nm]')
end

%% Problem 7_2
if any(run_problems(:) == 72)
    figure(9)

    %Constants
    L = 10; %Beam Length
    E = 69.9e9; %Young's Modulus
    v_allowed = L/400; %Allowed Displacement
    k = 1.5; %Safety Factor
    sY = 276e6; %Yield Stress
%     I = 1/(3*E);
    I = 683.640162e-6/3;

    %Problem Solution
    syms x

    %Shear Force Plot
    subplot(3,1,1)
```

```
    shear = 1000*(3*4.5*s*pw(x,0,0) - 3*s*pw(x,1,0) - 3*s*pw(x,2,0) - s*pw(x,3,0) - 5*s*
    fplot(shear, range_x);
    title('Shear Force vs. x');
    xlabel('x [m]')
    ylabel('Force [N]')

    %Moment Plot
    subplot(3,1,2)
    moment = 1000*(3*4.5*s*pw(x,0,1) - 3*s*pw(x,1,1) - 3*s*pw(x,2,1) - s*pw(x,3,1) - 5*s
    fplot(moment, range_x);
    title('Moment vs. x');
    xlabel('x [m]')
    ylabel('Moment [Nm]')

    %Displacement Plot
    subplot(3,1,3)
    displacement = (1000*(3*4.5*(1/6)*s*pw(x,0,3) - 3*(1/6)*s*pw(x,1,3) - 3*(1/6)*s*pw(x
    fplot(displacement, range_x);
    title('Displacement vs. x');
    xlabel('x [m]')
    ylabel('Displacement [m]')

    [A, I] = getMoment(displacement, range_x, moment, v_allowed, E, sY, k);
    fprintf('Min Area = %f\n',A);
    fprintf('Min I = %f\n',I);
end
```

6.2 Piecewise Function

```
function out = pw(x,x0,n)
    out = piecewise(x>=x0, (x-x0)^n, 0);
end
```

6.3 getMoment Function

```
function␣[I1,␣I2]␣=␣getMoment(displacement,␣range_x,␣moment,␣v_allowed,␣E,␣sY,␣k)


␣␣␣␣Sa␣=␣sY␣/␣k;


␣␣␣␣[~,␣a,␣~,␣b]␣=␣getMax(moment,␣range_x);
␣␣␣␣M␣=␣max(abs(a),␣abs(b));


␣␣␣␣I1␣=␣M/((1/2)␣*␣Sa)␣*␣10^6;


␣␣␣␣disp(M)


␣␣␣␣[~,␣c,␣~,␣d]␣=␣getMax(displacement,␣range_x);
␣␣␣␣v␣=␣max(abs(c),␣abs(d));


␣␣␣␣disp(v)


␣␣␣␣I2␣=␣v/(E*v_allowed)␣*␣10^6;
end
```

6.4 getRadius Function

```
function␣r␣=␣getRadius(I)
␣␣␣␣r␣=␣((I␣*␣10^-6␣*␣4)/pi)^(1/4);
end
```

6.5 getMax Function

```
function␣[x1,␣mins,␣x2,␣maxs]␣=␣getMax(f,␣range)
␣␣␣␣domain␣=␣linspace(range(1),␣range(2),␣1000);


␣␣␣␣disps␣=␣subs(f,␣domain);
```

```
    [mins, i1] = min(disps);
    [maxs, i2] = max(disps);

    x1 = domain(i1);
    x2 = domain(i2);

end
```

References

[1] F. P. Beer, J. T. DeWolf, E. R. Johnston, and D. F. Mazurek, *Mechanics of materials*. McGraw-Hill Education, 2020.