Hospital at Home: Joint Patient Selection and Supply Chain Coordination

1 Envelope Assumptions

A1. **Fixed Service Cadence:** Each HaH patient is visited by a nurse once a day, the treatment bundle(medicines/equipment etc.) is delivered once a day.

A2. **Patient/Demand Aggregation by Condition:** The HaH eligible patients are grouped by conditions, each group requires a distinct treatment bundle to be delivered to home (i.e. a patient in group 2 only has demand for bundle 2).

A3. **Patient HaH Recovery/Demand Curve (from Prediction Model):** Each patient has a unique recovery/demand curve: x-axis : duration of treatment at home; y-axis : improving condition and decreasing demand for the treatment bundle.

2 Model

Sets & Indices

$\mathcal{P}$ The Set of all patients, indexed by i.

$\mathcal{E} \subseteq \mathcal{P}$ HaH-eligible patients.

$\mathcal{H} = \mathcal{P} \setminus \mathcal{E}$ High-risk patients (must stay in hospital).

$\mathcal{K} = \{1, \ldots, K\}$ Condition groups, indexed by k.

$\mathcal{J} = \{1, \cdots, J\}$ Hospitals/depots, indexed by j.

$\mathcal{T} = \{1, \ldots, T\}$ Planning horizon, indexed by t.

Decision Variables

HaH Patient Selection:

$x_i \in \{0, 1\}$ $= 1$ if patient i is selected HaH; $= 0$ if patient i is treated in-hospital.

HaH Supply Chain Coordination:

$y^0_{kj} \geq 0$ Initial inventory level of bundle k at depot j.

$y_{kjt} \geq 0$ Inventory of bundle k at depot j at start of day t.

$z_{ijkt} \geq 0$ Units of bundle k delivered from j to patient i on day t.

$a_{ijt} = \text{If}(\sum_k z_{ijkt} > 0)$ If a delivery needs made from depot j to HaH patient i on day t.

Parameters

Demand side - Patient:

l_i^{Hosp} (Predicted) Length of stay of patient i if treated in hospital

l_i^{Home} (Predicted) Length of stay of patient i if treated at home

d_{it} Demand curve: units of treatment bundle demanded by patient i on day t.

Supply side - Treatment at Hospital:

B Total hospital bed capacity.

c^{Hosp} In-hospital treatment cost (bed/staff/pharmacy/equipment) per patient per day.

Supply side - HaH:

c^S Clinical staff home visit cost per patient per day.

c_k^P Unit procurement cost of bundle k.

c^V Per unit inventory cost.

c^D Delivery cost per unit distance.

Model Formulation

$$\begin{aligned}
\min \quad & \underbrace{\sum_{i\in\mathcal{P}} c^{\text{Hosp}} l_i^{\text{Hosp}}(1-x_i)}_{\text{In-hospital operation cost}} + \underbrace{\sum_{i\in\mathcal{E}} c^S l_i^{\text{Home}} x_i}_{\text{HaH staff visit cost}} + \underbrace{\sum_{(i,j,k,t)\in\mathcal{E}\times\mathcal{J}\times\mathcal{K}\times\mathcal{T}} c_k^P z_{ikjt}}_{\text{HaH procurement cost}} + \underbrace{\sum_{(j,t)\in\mathcal{J}\times\mathcal{T}} c^D \text{TSP}_{jt}}_{\text{HaH transportation cost}} \\
& + \underbrace{\sum_{(j,k,t)\in\mathcal{J}\times\mathcal{T}} c^V y_{kjt}}_{\text{HaH Inventory hold cost}} \\
\text{s.t.} \quad & x_i = 0, \quad \forall\, i \in \mathcal{H} && \text{(In-eligibility)} \\
& \sum_{i\in P}(1-x_i) \ \le \ B && \text{(Bed capacity)} \\
& \sum_{j\in J}\sum_{k\in K} z_{ijkt} \ = \ d_{it}x_i \quad \forall\, i\in E,\ t=1,\dots,L_i && \text{(Demand fulfillment)} \\
& y_{k,j,t-1} = y_{kjt} + \sum_{i\in E} z_{ijkt} \quad \forall\, k\in K,\ j\in J,\ t=1,\dots,T && \text{(Inventory)} \\
& \text{TSP}_{jt} = \text{LinearFunction}\left(\sqrt{\sum_{i\in\mathcal{E}} a_{ijt}}\right) && \text{(TSP tour length approx. formula)} \\
& x_i \in \{0,1\},\ w_{jt}\in\{0,1\},\ y_{kjt},\ z_{ijkt},\ n_{jt} \ge 0
\end{aligned}$$

3 Sensitivity Analysis: When is HaH cost effective?

Compare the total cost of two operation modes:

- Total cost of treatment in-hospital only
- Total cost of treatment in-hospital + HaH

Parameters to sweep:

- Percentage of HaH-eligible patients: $|\mathcal{E}|/|\mathcal{P}|$

- In-hospital Costs: (c^{Hosp}) and HaH Costs: ((c^S, c^P, c^D, c^V))

- Number of supply chain depots: J.

- Hospital Capacity: B (Need to add abandonment cost to model the revenue lost from limited capacity and the recuperation of patients via HaH)