Univariate Gaussian Classifier

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Some notation; We have a feature x and a set $\{w_1, w_2, ..., w_j\}$ of j classes. In the univariate case, x is one dimensional, $x \in \mathbb{R}$.

According to Bayes rule, the posterior probability for class j can be computed as

$$P(w_j|x) = \frac{P(x|w_j)P(w_j)}{P(x)} \tag{1}$$

where

$$P(x) = \sum_{j=1} P(x|w_j)P(w_j) \tag{2}$$

and $P(w_j)$ is the a priori probability. A priori probability means the probability given in advance. For example, if we have two classes w_1 and w_2 and we know that it has a 75% chance of getting class w_1 and 25% chance of getting class w_2 we have $P(w_1) = 0.75$ and $P(w_2) = 0.25$.

We want to classify a feature x to the class with the highest posterior probability. That is decide w_i if $P(w_i|x) \geq P(w_j|x)$ for all $j \neq i$.

This can be written as I classify x as w_i if $g_i(x) \geq g_j(x)$, where possible discriminant functions are:

$$g_i(x) = P(w_i|x) = \frac{P(x|w_i)P(w_i)}{P(x)} \tag{3}$$

or

$$g_i(x) = P(x|w_i)P(w_i) \tag{4}$$

or

$$g_i(x) = \ln(P(x|w_i)P(w_i)) \tag{5}$$

All these discriminant functions will have the maximum for the same x.

Let's choose to use

$$g_i(x) = P(x|w_i)P(w_i) \tag{6}$$

Figure 1: This is what you need to implement

and let's use the univariate gaussian distribution

$$P(x|w_i) = \frac{1}{\sigma_i\sqrt{2\pi}}e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}} \tag{7}$$

where μ_i is the mean for class i and σ_i is the standard deviation for class i. So, every class will have a mean value μ_i and a variance σ_i^2 estimated from the training data, giving every class a probability distribution $P(x|w_i)$. To classify a feature value x, choose the the class with the highest posterior probability $g_i(x) = P(x|w_i)P(w_i)$.