Post14

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Question

In Exercises 13 – 16, show that the Taylor series for $f(x)$, as given in Key Idea 8.8.1, is equal to $f(x)$ by applying Theorem 8.8.1; that is, show $\lim_{n\to\infty} R_n(x) = 0$.

In this case, we have $f(x) = e^x$.

Answer

From 8.8.1, we have

$$f(x) = e^x \ and \ f'(x) = e^x$$

And since;

$$d/dx(e^x) = (e^x)$$

$$|R_n(x)| \leq \frac{max|f^{n+1}(z)|}{(n+1)!}|x^{n+1}|$$

$$|R_n(x)| \leq \frac{e^z}{(n+1)!}|x^{n+1}|$$

$$\lim_{n\to\infty} \frac{e^z x^{n+1}}{(n+1)!} = 0$$

$$\lim_{n\to\infty} R_n(x) = 0$$

Set to 0 we get;

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$$f(x) = \sum_{n=0}^{\infty} \frac{e^0}{n!}(x-0)^n = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$