HW8

FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS

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Section 7.2 Question 11

Exponential Density Expected Value

A company buys 100 lightbulbs, each of which has an expoential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out? (See Exercise 10.)

For reference, exercise 10 says Let X_n be n independent random variables each of which has an exponential density with mean μ. Let M be the minimum value of the X_j. Show that the density for M is exponential with mean $\frac{\mu}{n}$. Hint: Use cumulative distribution functions. With that we use the given relationships:

$$f_1(x) = nf(x)(1 - F(x))^{n-1}$$

$$\lambda` = n\lambda = \frac{n}{\mu} = \frac{1}{\mu`}$$

Where $\lambda = 1/1000$ and $n = 100$ in this case the expected value for is simply μ which is $\frac{\mu}{n}$ or $\frac{1000}{100}$ or about 10 years.

Section 7.2 Question 14

Exponential Density

Assume that X_1 and X_2 are independent random variables, each having an exponential density with parameter λ. Show that $Z = X_1 - X_2$ has density

$$f_Z(z) = (1/2)\lambda e^{-\lambda|z|}$$

With $f(x_1) = \lambda e^{-\lambda_{x_1}}$ and $f(x_2) = \lambda e^{-\lambda_{x_2}}$. For X_1, X_2 we have $f(x_{1,2}) = \lambda^2 e^{-\lambda_{x_1+x_2}}$ $Z = X_1 - X_2$ rewritten this is $X_1 = z + x_2$ which is substituted into $\lambda^2 e^{-\lambda(z+2x_2)}$. Then use convolution and consider z -> $-\infty$ or ∞ with X_1 and X_2.

Such as

$$f_Z(z) = \int_{-\infty}^{\infty}$$

For $-z$:

$$f_Z(z) = \int_{-\infty}^{\infty} z\lambda^2 e^{-\lambda(z+2x_2)}$$

$$X_2 \geq X_1; \quad f_Z(z) = \frac{\lambda}{2} e^{-\lambda z}$$

For z:

$$f_Z(z) = \int_{-\infty}^{\infty} \frac{\lambda}{2} e^{-\lambda(z+2x_2)}$$

$$X_1 \geq X_2; \quad f_Z(z) = \frac{-\lambda}{2} e^{-\lambda z}$$

In which the integral $-\infty$ -> 0 cancels to become

$f(z) = \frac{\lambda}{2} e^{-\lambda|z|}$

Section 8.2 Question 1

Let x be a continuous random variable with mean $\mu = 10$ and variance $\sigma = 100/3$. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

Chebyshev's Inequality

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Where the mean, $\mu = 10$ and standard deviation, $\sigma = 2$.

(a) $P(|x - 10| \geq 2)$

$k\sigma = 2$

$k = \frac{2}{\sqrt{\frac{100}{3}}}$

```
UP <- 1/(2/sqrt(100/3))^2
```

$\frac{1}{\frac{2}{\sqrt{\frac{100}{3}}}} = 8.3333$. However, the highest is 1, thus the answer is approximately 1.

(b) $P(|x - 10| \geq 5)$

$k\sigma = 5$

$k = \frac{2}{\sqrt{\frac{100}{3}}}$

```
UP <- 1/(5/sqrt(100/3))^2
```

$\frac{1}{\frac{2}{\sqrt{\frac{100}{3}}}} = 1.3333$. However, the highest is 1, thus the answer is approximately 1.

(c) $P(|x - 10| \geq 9)$

$k\sigma = 9$

$k = \frac{2}{\sqrt{\frac{100}{3}}}$

```
UP <- 1/(9/sqrt(100/3))^2
```

$\frac{1}{\frac{2}{\sqrt{\frac{100}{3}}}} = 0.4115.$

(d) $P(|x - 10| \geq 20)$

$k\sigma = 20$

$k = \frac{2}{\sqrt{\frac{100}{3}}}$

```
UP <- 1/(20/sqrt(100/3))^2
```

$\frac{1}{\frac{2}{\sqrt{\frac{100}{3}}}} = 0.0833.$