# HW13

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## Question 1

Use integration by substitution to solve the integral below.

$$\int 4e^{-7x}dx$$

## Answer 1

$$-\frac{4}{7}e^{-7x} + C$$

Steps using substitution:

$U = -7x$, $dU = -7dx$, $dx = \frac{dU}{-7}$ so,

$4\int e^{U}\frac{dU}{-7} = \frac{4}{-7}\int e^{U}dU = -\frac{4}{7}e^{U} + C$ thus, $-\frac{4}{7}e^{-7x} + C$

## Question 2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where $t$ is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

## Answer 2

To find a function, take the integral of the rate:

$$\int (\frac{-3150}{t^4} - 220)dt$$

$$\int (-3150t^{-4} - 220)dt$$

$$\int -3150t^{-4}dt - \int -220dt$$

$$-3150\int t^{-4}dt - 220\int dt$$

$$-3150\frac{-1}{3}t^{-3} - 220t$$

$$N(t) = \frac{1050}{t^3} - 220t + c$$

Solve for $c$ using the initial condition at day 1: $N(1) = 6530$,

$$6530 = \frac{1050}{1^3} - 220(1) + c$$
$$6530 = 1050 - 220 + c$$
$$6530 - 1050 + 220 = c$$
$$5700 = c$$

Result:

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

## Question 3

Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$

## Answer 3

Using the equation of the line and reading from the graph we estimate that the lower bound is 4.5 and upper is 8.5. Thus,

```
library(stats)
# If start and end points are truly 4.5 and 8.5 then
f <-function(x)(2*x-9)
integrate(f, lower = 4.5, upper = 8.5)
```

```
## 16 with absolute error < 1.8e-13
```

The area is 16. This is validated by counting the squares of the image which we assume has an area of 1 and width of 1.

## Question 4

Find the area of the region bounded by the graphs of the given equations $y_1 = x_1^2 - 2x_1 - 2$ and $y_2 = x_2 + 2$.

Enter your answer below.

## Answer 4

Given:

$$y = x^2 - 2x - 2$$
$$y = x + 2$$

Solve for x to find the bounds:

$$x^2 - 2x - 2 = x + 2$$
$$x^2 - 3x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$

Thus, $x = -1$ and $x = 4$.

Using this equation we find the top and bottoms with integration.

$$\int_a^b (top - bottom)dx$$

So far we have $\int_{-1}^{4} (x + 2) - (x^2 - 2x - 2)dx$ which simplified becomes $\int_{-1}^{4} (-x^2 + 3x + 4)dx$.

```
## define the integrated function
integrand <- function(x)
  {
  -x^{2}+3*x+4
  }
## integrate the function from 0 to infinity
integrate(integrand, lower = -1, upper = 4)
```

```
## 20.83333 with absolute error < 2.3e-13
```

Which we can use to calculate the area:

```
area_fun <- function(x) {
  -x^2 + 3 * x + 4
}
integrate(area_fun, -1, 4)
```

```
## 20.83333 with absolute error < 2.3e-13
```

The area is about 20.83.

## Question 5

A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year.There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

## Answer 5

Consider that $x$ is the number of irons to order and we have the following variables:

$$C_{order} - \ Cost\ per\ order$$
$$C_{year} - \ Yearly\ cost\ of\ ordering$$
$$R_{year} - \ Yearly\ cost\ of\ storage$$
$$R_{iron} - Storage\ rate\ of\ one\ flat\ iron\ for\ one\ year$$

$$n_{expected} - \text{ } Expected\ number\ of\ orders\ in\ year$$
$$\bar{x}_{iron} - Average\ number\ of\ irons$$
$$I_{cost} = \text{ } Cost\ of\ inventory$$

Our equations:

$$R_{year} = R_{iron} * \bar{x}_{iron}$$
$$C_{year} = C_{order} \frac{n_{expected}}{x}$$
$$I_{cost} = R_{year} + C_{order}$$

Which, when we enter what we have becomes:

$$3.75 * \frac{x}{2} \quad or\ simply \quad 1.875 * x$$
$$8.25 \times \frac{110}{x} \quad or\ simply \quad \frac{907.5}{x}$$
$$f(x) = 1.875 * x + \frac{907.5}{x}$$

Calculate the minimized value by setting to zero and solving:

$$f'(x) = 1.875 - \frac{907.5}{x^2}$$
$$f'(x) = 0$$
$$1.875 - \frac{907.5}{x^2} = 0$$
$$1.875 = \frac{907.5}{x^2}$$
$$1.875x^2 = 907.5$$
$$x^2 = \frac{907.5}{1.875}$$
$$x = \sqrt{\frac{907.5}{1.875}}$$
$$x = \sqrt{484}$$
$$x = 22$$
$$quantity\ of\ irons\ = 22$$
$$110/22$$
$$quantity\ of\ orders\ = 5$$

The order should have 22 irons with a total of 5 orders.

## Question 6

Use integration by parts to solve the integral below.

$$\int ln(9x) * x^6 dx$$

## Answer 6

Formula: $uv - \int vdu, u = ln(9x), du = \frac{1}{x}dx, dv = x^6, v = \frac{1}{7}x^7$

$$\frac{x^7 ln(9x)}{7} - \int \frac{1}{7}x^7 \frac{1}{x}dx$$
$$\frac{x^7 ln(9x)}{7} - \frac{1}{7}\int \frac{x^7}{x}dx$$
$$\frac{x^7 ln(9x)}{7} - \frac{1}{7}\int x^6 dx$$
$$\frac{x^7 ln(9x)}{7} - \frac{1}{7}(\frac{x^7}{7}) + C$$
$$\frac{x^7 ln(9x)}{7} - \frac{x^7}{49} + C$$

## Question 7

Determine whether $f(x)$ is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

## Answer 7

$$\int_1^{e^6} \frac{1}{6x}dx = \frac{1}{6}ln(x)|_1^{e^6}$$
$$= \frac{1}{6}ln(e^6) - \frac{1}{6}ln(1)$$
$$= \frac{1}{6} \times 6 - \frac{1}{6} \times 0$$
$$= 1$$

The definite integral is 1 on the interval $[1, e^6]$.