HW7

FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS

Zachary Palmore

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1.

Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

$$Y\ over\ the\ interval\ 1 \leq v \leq k,\ \ f(v)\ = \frac{(k-v+1)^n - (k-v)^n}{k^n}$$

Where k is the number of possibilities in n mutually independent variables uniformly distributed from 1 to k, v the variables of at least one X, and f(v) the distribution function.

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

Modeled as an geometric distribution…

```
y <- 8 # eighth year
p <- 1/10 # fails once every 10 years
pgeom(y, p, lower.tail = FALSE) # geometric probability of no failure in n years
```

```
## [1] 0.3874205
```

```
pgeom(y, p, lower.tail = TRUE) # geometric probability of failure after n years
```

```
## [1] 0.6125795
```

```
# expected value
1/p
```

```
## [1] 10
```

```r
# standard deviation
round(sqrt((1-p)/(p^2)),4)
```

```
## [1] 9.4868
```

The probability that the machine will fail after 8 years is 0.3874.

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

Modeled as an exponential distribution…

```r
y <- 8 # number of years
p = 1/10 # probability of failure
pexp(y, p, lower.tail = FALSE) # exponential probability of no failure in n years
```

```
## [1] 0.449329
```

```r
pexp(y, p, lower.tail = TRUE) # exponential probability of failure after n years
```

```
## [1] 0.550671
```

```r
e <- 1/p # expected value
e
```

```
## [1] 10
```

```r
e # standard deviation
```

```
## [1] 10
```

The probability that the machine will fail after 8 years is 0.4493.

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

Modeled as a binomial distribution…

```r
y <- 8 # number of years
p = 1/10 # probability of failure
pbinom(0, y, p, lower.tail = FALSE)    # binomial probability of no failure in n years
```

```
## [1] 0.5695328
```

```r
pbinom(0, y, p, lower.tail = TRUE)  # binomial probability of failure after n years
```

```
## [1] 0.4304672
```

```r
# expected value
u <- y * p
u
```

```
## [1] 0.8
```

```r
# standard deviation
sqrt(u * (1-p))
```

```
## [1] 0.8485281
```

The probability that the machine will fail after 8 years is 0.4305.

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

Modeled as a poisson distribution…

```r
y <- 8 # number of years
p = 1/10 # probability of failure
lam <- y * p
ppois(0, lam, lower.tail = FALSE)    # poisson probability of no failure in n years
```

```
## [1] 0.550671
```

```r
ppois(0, lam, lower.tail = TRUE)   # poisson probability of failure after n years
```

```
## [1] 0.449329
```

```r
# expected value
lam
```

```
## [1] 0.8
```

```r
# standard deviation
sqrt(lam)
```

```
## [1] 0.8944272
```

The probability that the machine will fail after 8 years is 0.4493.