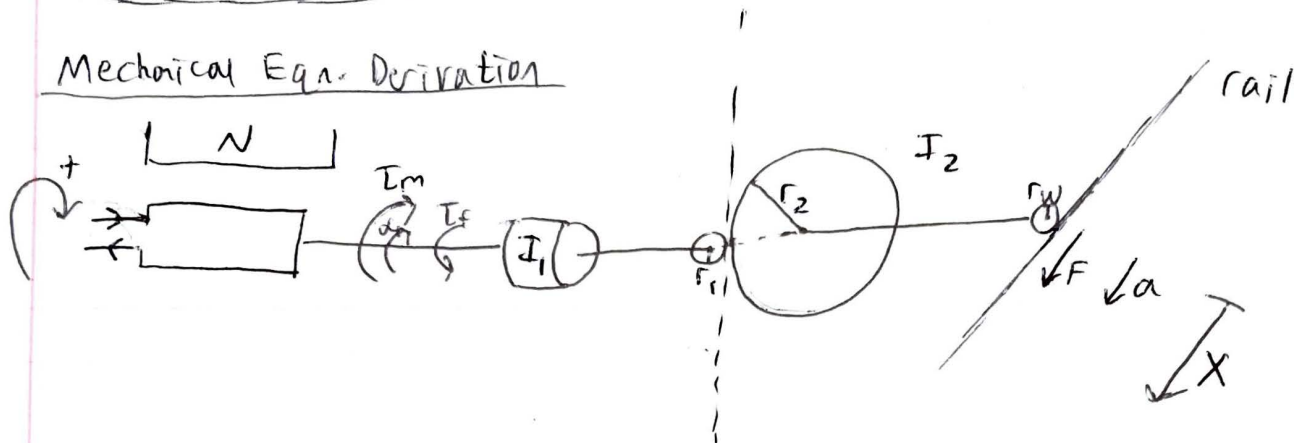## Mechanical Eqn. Derivation

T_m = torque of motor

I_1 = Rotational Inertia DC, FW, Gear 1

T_f = frictional torque = KV

I_2 = Rotational Inertia of Gearbox + wheel

$|\vec{P_1}| = |\vec{P_2}|$ = Normal Force

F = friction/force of train moving

g = gravity

Equations

$$\sum \vec{T} = I\vec{\alpha} \qquad \vec{L} = \vec{r} \times \vec{F} = rF\sin\theta$$

Motor

$$T_m - T_f - r_1 P_1 = I_1 \alpha_1$$

Wheels

$$r_2 P_2 - r_w F = I_2 \alpha_2$$

Train

$$\sum \vec{F} = m\vec{a}$$

$$F - \mu mg = ma$$

Notes

$P_1 = P_2$

$S = r_w \theta$

$v = r_w \omega$

$a = r_w \alpha$

S = arc length

r_w = radius (wheel)

Gear Ratio → γ (Gamma) $= \dfrac{r_2}{r_1}$
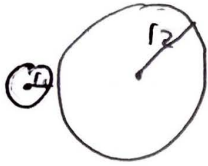
$$\alpha_1 = \gamma \, \alpha_2$$

$$s_1 = s_2$$

$$a = r_w \, \alpha_2$$

$$r_1 \theta_1 = r_2 \theta_2$$

$$r_1 \omega_1 = r_2 \omega_2$$

$$\alpha_1 = \dfrac{\gamma \, a}{r_w}$$

$$r_1 \alpha_1 = r_2 \alpha_2$$

$$\alpha_1 = \dfrac{r_2}{r_1} \alpha_2$$

$$a = \dfrac{\alpha_1 \, r_w}{\gamma}$$

$\uparrow$
Gamma (γ

Goal: Go from $\tau_m \rightarrow a$

$\uparrow$ $\nearrow$

Torque of   Linear acceleration along track

motor

• Solve for a torque equation that looks like

$$\tau_m - \tau_f - \boxed{} = \bigcirc \dfrac{a}{r_w}$$

$\uparrow$ $\uparrow I_{eq}$

Nm   units of I ($kg \, m^2$)

Sol<u>n</u>

$$\tau_m - \tau_f + \dfrac{\mu m g \, r_w}{\gamma} = \left(\dfrac{I_2}{\gamma} - \dfrac{r_w^2 \, m}{\gamma} + \dfrac{I_1}{\gamma} \right) \dfrac{a}{r_w}$$

$$F = Ma + \mu mg$$

$$\gamma \alpha_2 = \alpha_1$$

$$P = \frac{I_2 \alpha_2 + r_\omega F}{r_2}$$

$$\frac{a}{r_\omega} = \alpha_2 \qquad \frac{a\gamma}{r_\omega} = \alpha_1$$

$$\tau_m - \tau_f - r_1 \left(\frac{I_2 \alpha_2 + r_\omega (ma + \mu mg)}{r_2} \right) = I_1 \alpha_1$$

$$\tau_m - \tau_f - \frac{r_1 I_2 \alpha_2}{r_2} + \frac{r_1 r_\omega ma}{r_2} + \frac{\mu mg r_1 r_\omega}{r_2} = I_1 \alpha_1$$

$$\tau_m - \tau_f - \frac{r_1 I_2 \alpha_2}{r_2} + \frac{\mu mg r_1 r_\omega}{r_2} - I_1 \alpha_1 = -\frac{r_1 r_\omega ma}{r_2}$$

$$\frac{r_2 \tau_m}{Mr_1 r_\omega} + \frac{\tau_f r_2}{Mr_1 r_\omega} + \frac{I_2 \alpha_2}{M} - \mu g + \frac{I_1 \alpha_1 r_2}{r_1 M} = a$$

$$\tau_m - \tau_f - \gamma I_2 \alpha_2 + \gamma \mu mg r_\omega - I_1 \alpha_1 = -\gamma r_\omega ma$$

$$\tau_m - \tau_f - \gamma I_2 \frac{a}{r_\omega} + \gamma \mu mg r_\omega - I_1 \frac{a\gamma}{r_\omega} = -\gamma r_\omega ma$$

$$\tau_m - \tau_f + \gamma \mu mg r_\omega = \gamma I_2 \frac{a}{r_\omega} - \gamma r_\omega ma + I_1 \frac{a\gamma}{r_\omega}$$

$$\tau_m - \tau_f + \frac{\mu mg r_\omega}{\gamma} = \left(\frac{I_2}{\gamma} - \frac{r_\omega^2 m + I_1}{\gamma} \right) \frac{a}{r_\omega}$$