Calculated Risks, Condorcet, Bernoulli, d'Alembert and Inoculation

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Diderot and d'Alembert's *Encyclopédie* contains an entry for "Conjecture." Penned by Diderot, it starts:

> Jugement fondé sur des preuves qui n'ont qu'un certain degré de vraisemblance, c'est-à-dire sur des circonstances dont l'existence n'a pas une liaison assez étroite avec la chose qu'on en conclut, pour qu'on puisse assûrer positivement que les unes étant, l'autre sera ou ne sera pas: mais qu'est-ce qui met en état d'apprétier cette liaison? L'expérience seule.

The article clearly defends the idea that trials should be the basis for scientific research and allow one to go beyond mere conjecture and reach proof. As I would like to show, philosophical, moral and scientific imperatives collided at times, making the transition from conjecture to proof all but impossible.

Smallpox inoculation was an eighteenth-century debate which was to set standards for hypotheses and conclusions. It was a dangerous operation as it involved artificially administering an incurable illness and hoping for the best, in order to guarantee the inoculated patient—assuming he or she survived the operation—immunity from subsequent attacks of the disease. It was the object of numerous debates, in particular in France. In itself it was based on conjecture: nobody was able, at the time, to understand why the same strain of smallpox would be more severe if caught naturally than if inoculated—we now know that subcutaneous administration of the disease allows the body time to create antibodies. In the debate regarding whether or not to

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practice this dangerous method, lobbies and individuals raised different questions: could one usurp God's right to decide who should become ill? If one had one's child inoculated and he or she died as a result, could one be considered to be his or her murderer? Was it not best to let nature take its course? And so on. To make the method seem acceptable, a notion of calculated risk had to be put forward.

As early as 1754, La Condamine had stressed: "ce n'est point ici une question de morale, c'est une affaire de calcul. Ne faisons point un cas de conscience d'un problème d'arithmétique." If the human body can be treated and strengthened, the social body can too, with striking results—at least this was the idea promoted by many thinkers. It implied that individuals could be treated as statistics and led to the recurring use of images: troops in battle with a wise general who manages to spare most of them as against an unwise one whose men are massacred; lotteries in which there were fewer or more winning tickets. It also had a convenient catchphrase which La Condamine probably coined: "La petite vérole nous décime, l'inoculation nous millésime." The method was adopted more readily in Britain than in France, according to a remark by Duboueix in the *Journal encyclopédique* in March 1774 because "les Anglais, nos voisins, [sont] plus sages que nous en ceci, et bien meilleurs calculateurs."[1] I should like to look at what the presentation of calculated risks in favor of inoculation becomes in texts by Condorcet, Bernoulli and d'Alembert, and to argue that the latter two in particular used inoculation to promote developments in mathematics which were to have implications far beyond the field of medicine.

In order to try and make the questions raised scientific, the defenders of inoculation had to wrest the debate from philosophical, moral or theological ones. In his eulogy of the most famous inoculator to practice in France, Théodore Tronchin, a Protestant from Geneva, Condorcet questions the use of nature as an alternative course to medical intervention:

> Ceux surtout qui parlent de médecine font souvent de la nature une espèce d'être moral qui a des volontés, qui supporte impatiemment la contradiction, qui a quelquefois assez de sagacité pour sauver le malade et bien diriger ses efforts, mais qui, malgré les bonnes intentions qu'on lui suppose, est sujette à se tromper presque aussi souvent que les médecins. Il ne faut pas croire que l'art de la médecine puisse consister à s'en rapporter à cet être imaginaire, à ne faire aucun remède, à se contenter d'attendre

[1] *Journal encyclopédique* (March 1774): 491.

> avec tranquillité l'événement, quel qu'il soit, pour se réserver la ressource d'en accuser la nature lorsqu'il est malheureux.[2]

Hindsight allows us to note that there was no even playing field for those who were trying to sort out statistics regarding inoculation, in particular because even when they had figures for natural deaths by smallpox, whether in an epidemic or as isolated cases, these were not always accurate: there was difficulty in distinguishing smallpox from other illnesses like measles or scarlet fever so deaths from one disease could be attributed to another. In addition, there were more or less serious strains of smallpox so some could have higher fatality rates. Also, there was no way of knowing whether people had already acquired immunity to smallpox, for instance by being exposed to it as children, and therefore whether, though counted as survivors in an epidemic, they were not statistics which should have been removed from the pool for the calculation to be exact. Figures were bandied on all sides. D'Alembert stated that the risk of dying from natural rather than artificial smallpox was "à 7 ½, c'est-à-dire quarante fois plus grand." According to Diderot: "l'inoculation prend à peine 1 victime sur 300; donc le risque de mourir de la petite vérole naturelle est, au risque de mourir de l'inoculée, comme 300 à 7, ou 40 à 50 fois plus grand." Because fatality rates from natural smallpox were extremely variable, all statistical evidence was imprecise. Promoters of the method seemed to discredit themselves by offering a wide variety of figures which adversaries could treat as hyperbolic—I quote Diderot once again, who was a staunch supporter of the method but noted the difficulty of obtaining accurate information: "Ils [les défenseurs de l'inoculation] prétendent, sur l'expérience de *M. Tronchin*, que l'inoculation n'enlève pas un malade sur 1500; sur la pratique de *Ramby*, qu'elle n'en enlève pas un sur 1200; sur l'usage des *Orientaux*, qu'à Constantinople elle n'en prend pas un sur 10000."[3] While figures were meant to clarify aspects of the debate, they often confused the common man—or woman. Mme de Maisonneuve signed a "Lettre à Milady T*** sur l'inoculation" which complains about this lack of clarity: "Comme il s'agissait de calculer des probabilités, les Géomètres,

[2]Jean-Antoine Caritat de Condorcet, *Œuvres* 3: 507. The extract starts with the following sentences: "on dit que la médecine doit seconder la nature et non la contrarier. Le mot de *nature* est un de ces mots dont on se sert d'autant plus souvent, que ceux qui les entendent ou les prononcent y attachent plus rarement une idée précise."

[3]Denis Diderot, "De l'inoculation," *Œuvres complètes* (Société encyclopédique française et le Club français du livre, 1970) 5: 37. John Ranby (1703–1773) was George II's sergeant-surgeon before becoming surgeon to the Chelsea hospital.

de leur côté, ont envisagé cette méthode [l'inoculation] comme un problème, et ils en ont donné algébriquement les plus belles solutions du monde. Enfin, grâce à tant de secours, on a embrouillé tous les principes, et l'on est parvenu à ne pouvoir pas même s'accorder sur les faits."[4]

A collateral effect of the search for accurate figures in which to root conjecture was that doctors could become accountable since their success rate was made public—for instance in Monro's tables[5] which were much read during the debate around the authorization of smallpox immunisation in France. There was no possibility of zero risk, but that is what all inoculators were striving for—many sources indicate that the famous Théodore Tronchin took pride from the fact that no one he had inoculated had ever died from the disease, and added that he would give up his practice immediately should anyone fall victim to smallpox inoculated by him. Statements like the following, drawn from an unpublished letter to Grimm, are frequent in his correspondence: "grace au Ciel j'en ai pour temoins tous ceux que j'ai inoculez. il n'est arivé malheur à aucun, & j'espere de pouvoir en dire autant, tant que j'inoculerai."[6] Inoculation had to provide facts and figures to defend its case. The first mortality table in France was published by Deparcieux in 1746 in his *Essai sur les probabilités de la durée de la vie humaine.* An article in *Il Caffè,* the Verri brothers' pro-Enlightenment periodical, attempted to draw up Europe-wide statistics on inoculations and gave the following figures: 24,167 people had been inoculated; 19 of them—that is, 1/1,200—died. Nothing tells us who they were and a pauper or a prince were dealt with equally, in the same way that nothing distinguishes a babe in arms from a senior citizen. When Bernoulli offered his statistical approach to inoculation this lack of distinction between different groups whether by age, by gender etc., was presented by him as a serious disadvantage.

Inoculation's main selling point was based on the fact that no one was certain of steering clear of smallpox during their lifetime, whatever their age, diet, nationality etc., and on the idea that it could prolong your life by eliminating the risk of dying from that most deadly of diseases. In essence, if you survived your inoculation, you had every

[4]Catherine de Maisonneuve, "Lettre à Milady T*** sur l'inoculation," *Journal des Dames* (September 1765): 29–34. Here 30.

[5]Alexander Monro, *An Account of the Inoculation of small Pox in Scotland* (Edinburgh: Drummond and J. Balfour, 1765) 7: 53; French translation: *État de l'inoculation de la petite vérole en Écosse* (Édimbourg et Paris : P. G. Cavelier, 1766).

[6]Bibliothèque Nationale de France MS NAF 6594. Letter to Grimm dated June 20, 1763.

reason to expect to live to a ripe old age. Inoculation thus was a kind of bet. You risked a lot—since you could die from it—but with very favorable odds, and then you reaped huge benefits. Conjecturing had to be rooted in fact, based on some form at least of evidence, rather than on a wait-and-see attitude. But conjecturing also invited one to become active. The shift towards preventive and not merely curative medicine, which is evident nowadays for instance in vaccination programs, was based to a large extent on conjectures about how inoculation could extend human life.

Let us take a look at an essential stage in conjectures regarding inoculation and its success. This is the dialogue or dispute between Bernoulli and d'Alembert.[7] It starts, as so many scientific developments of the time, by a paper read to the Académie des Sciences in Paris, a hotbed for the discussion and promotion of inoculation thanks to La Condamine's efforts and in particular his two *Mémoires* read in 1754 and 1758. He was responsible for Daniel Bernoulli's interest in the question and for the latter's attempts to make the results of inoculation serve statistical studies[8] and demonstrate what he was to call inoculation's *geometrical* truth. The Swiss scientist had already shown an interest in risk analysis, particularly when he published the first version of the Saint-Petersburg paradox (on why gamblers do not generally stake all their money even when potential gains are infinite) in his 1738 work *Specimen theoriae novae de mensura sortis.* He was interested in trying to map extended applications to probabilities in fields beyond mathematics *per se.*

Bernoulli's essay on inoculation and probabilities was read to the Académie des Sciences on March 22, 1760 and in open meetings in April. A detailed reduced version of it was published in June 1760 in the *Mercure de France* for all to read under the telling title *Réflexions sur les avantages de l'inoculation.* The Swiss mathematician started by setting out facts based on data he had managed to collect from various sources and showed the advantage, in terms of mortality, of being inoculated. His main conclusion was that you were 13 times more likely to die from the smallpox than from being inoculated. The figures were sufficient

[7]Some of the elements discussed here figure in my monograph on inoculation: Catriona Seth, *Les rois aussi en mouraient. Les Lumières en lutte contre la petite vérole* (Paris: Desjonquères, 2008). See esp. Chapter VII "Une nouvelle province des mathématiques," 207–32.

[8]Michel Dupont, *Dictionnaire historique des médecins* (Paris : Larousse, 1999) 72. On Bernoulli, see Klaus Dietz and J. A. P. Heesterbeek, "Daniel Bernoulli's epidemiological Model revisited," *Mathematical Biosciences* 180. 1–2 (Nov.–Dec. 2002): 1–21.

to convince readers like Diderot, according to whom Bernoulli had "vu la chose en grand, comme il conviendrait à un souverain qui néglige dans les choses les petits désavantages particuliers pour s'attacher au bien de la masse." Bernoulli was not interested in the specific risks inherent in inoculation, just in the bottom line and in how reflecting on this could provide new applications for scientific reasoning. He thus offered an initial and influential mathematical analysis of mortality rates for smallpox and of the advantages of inoculation as Duvillard, the promoter of life insurance in France was later to remark.[9] His mathematical model of contagion was even used recently to look into computer viruses.

Bernoulli's desire was to calculate the advantage, in terms of life expectancy, if smallpox is no longer potentially a risk. He started from data collected in Breslaw and concerning the death of children, noting that of "20 000 enfants de l'âge de 4 ans, il en meurt environ 700 dans le cours de l'année, ou environ 60 par mois." He estimated that when one undergoes inoculation, one is only at risk of being ill for a month. During that period of a month, at least 60 of the 20,000 four year olds—or one in 333—would die whether or not they were inoculated. If the period at which an inoculated patient is at risk could be reduced from a month to 15 days, the mortality risk would be 1 in 666, "nombre qui diffère peu de 1 sur 593, ou de celui des morts à Londres dans l'hôpital des inoculés de tout âge pendant le cours de quatre années suivant la liste imprimée, publiée par les administrateurs."[10]

To test his theory, Bernoulli then sets up a series of hypotheses. Basing himself on an often quoted figure: that smallpox kills one fourteenth of humanity, he suggests a model in which there are fourteen different types of deadly illnesses, smallpox only being one of them. To make things easier, he supposes that they are all as deadly even if they might strike different age groups. He then conjectures that one might at some point have the means of preventing all fourteen illnesses thanks to "un préservatif de même espèce que l'inoculation." This incidentally shows that he is already thinking outside the box of hypothetical analogies of inoculation like vaccination,[11] but also that

[9]Émile-Étienne Duvillard [de Durand], *Analyse et tableaux de l'influence de la petite vérole sur la mortalité à chaque âge, et de celle qu'un préservatif tel que la vaccine peut avoir sur la population et la longévité* (Paris : Imprimerie impériale, 1806): 5.

[10]Bernoulli, "Réflexions sur les avantages de l'Inoculation," *Mercure de France* (April 1760): 176.

[11]Vaccination was originally the administration of a relatively mild disease, cowpox, which offered immunity against smallpox.

he is basing his reflections on the age-old image of the panacea. He imagines for argument's sake that all fourteen remedies have the same disadvantages as inoculation. He then poses the question: should one refuse all remedies or accept an overall campaign of immunization? Once again, this seems to foreshadow subsequent medical developments like multiple vaccinations (e.g. Measles, Mumps and Rubella or MMR) and national health services' programs to have all individuals of a certain age inoculated against various diseases. To pursue his case, Bernoulli takes a ballpark figure which is less favorable than reality, suggesting as a basis for calculation a death rate of 1/100 inoculees. If with those figures, you were to decide to use none of the 14 available remedies, you would not impact the mortality rate. If, however, you were to make the decision to immunize everyone, 7/8 of humanity should live long healthy lives with no risk of falling sick.

Bernoulli then changes perspective. He returns to the level of the individual and to the single case of smallpox inoculation: "Si la Providence ne nous a pas accordé ce grand bien [la possibilité d'inoculer contre tout] en entier, faut-il pour cela rejeter la partie qu'elle nous offre?" The problem now is to narrow things down to what is available and assess its value. Bernoulli's calculation is that by extending the practice, it would be possible to save about 3/28 of the land's inhabitants or about 60,000 people per annum in France. In an era when the population was considered to be an indicator of a country's wealth, inoculation would thus allow the kingdom of France to double its inhabitants in a century and, implicitly, to become a stronger, richer nation. Private and public interests would both be served by the medical technique: "L'humanité veut qu'on assure et qu'on conserve la vie à chaque particulier, soit jeune, soit vieux: l'intérêt de l'État demande la population du Royaume." To give an idea of how much would be earned, Bernoulli studies the economic potential of the increased population: "L'augmentation du nombre des sujets produirait dans les revenus du Roi un accroissement que l'on peut évaluer à environ 20 livres par tête chaque année, en supposant le nombre des habitants de 18 millions et les revenus du Roi de 360 millions."[12]

To reach his results, Bernoulli set out a hypothesis of the way in which contagion functions, isolating immune individuals (who have had smallpox or been inoculated) and cannot thus catch the disease from all the others. This gives him a subgroup of non-immune people.

[12]Bernoulli, "Réflexions sur les avantages de l'Inoculation," 178.

Most of them will catch smallpox, and a large proportion of those who suffer from it will die as a result. That means we can calculate a fatality rate. Taking as our starting point the newborn babe's probability of being alive, but not immune, at a given age, we can elaborate differential equations. Bernoulli's take as their basis a fatality rate of 1/8 and an identical risk of infection for each year of life from 0 to 25. We know these are mean figures as death rates were higher for children than for adults. Still, with this rough calculation, Bernoulli shows that life expectancy would rise from about 11.5 to about 25.5—some fourteen years.

Bernoulli's conjectures were more complete and complex than my outline suggests. However, these calculations were important to a wider audience than the academicians to whom the essay was initially read. There were philosophical and economic implications too. In addition, to try and prove the accuracy of his ideas, the *Mercure de France* mentioned the idea of setting up a sort of ancestor of a clinical trial in "un établissement public dans quelque hôpital tel que celui des enfants trouvés où l'on inoculerait tous ceux qui ne donneraient pas d'indication contraire." Apart from the notion of verifying hypotheses, the idea raises the question of consent in medical trials[13]—one which was avoided until recently by some drugs companies who tested remedies on the captive and unknowing poor in developing countries.[14] Bernoulli also suggested that authentic lists of inoculees be kept at once to bear witness to the fact and to publicize the operation and its results, which could potentially lead to improving it.

By taking into account the period in which the newly inoculated subject might be weaker and more at risk of dying than someone who had not been inoculated at all, Bernoulli refined his calculation of risk. He estimated that during such a lapse of time—a fortnight according to him—the effects of the inoculation were no worse than those of catching the smallpox naturally would have been. Therefore the risk would be cancelled out. The demonstration was striking on an immediate level as offering scientific support for inoculation, but also, more widely—though this possibly would have escaped the com-

[13]It is worth recollecting that when inoculation was first propagated in England, trials were carried out on consenting prisoners (who were given their freedom in exchange for submitting to the operation) and on unknowing foundlings. Foundlings were also used as guinea pigs elsewhere in Europe.

[14]For a recent work on medical trials in postwar Africa, see Guillaume Lachenal, *Le medicament qui devait sauver l'Afrique. Un scandale pharmaceutique aux colonies* (Paris: La Découverte, 2014).

mon man—for offering mathematics an interesting applied role in thinking about how contagion works.

Whilst Bernoulli's paper seems to have impressed many readers, d'Alembert had mixed feelings. He had defended inoculation in the article that scandalized certain readers of the *Encyclopédie*, "Genève," stating: "*Genève* a reçu la première inoculation de la petite vérole, qui a tant de peine à s'établir en France, et qui pourtant s'y établira, quoique plusieurs de nos médecins la combattent encore, comme leurs prédécesseurs ont combattu la circulation du sang, l'émétique et tant d'autres vérités incontestables ou de pratiques utiles."[15] His reactions to Bernoulli were initially read to the same group of individuals as the Swiss mathematician's, the Académie des Sciences. In November 1760 he suggested that purely mathematical conjectures could not be applied to human decisions and he published his text the year after. His comments were interpreted by many as those of a rival disenchanted with the fact of having not been the first to come up with the potential application of statistics to life expectancy. Duvillard suggests that his ideas are "des considérations morales qui lui fournissaient des arguments spécieux contre l'inoculation."[16] Bernoulli considered that the dispute was serious enough for him to stop taking any interest in d'Alembert's works as he explained to Euler in 1768:

> J'ai pris cette résolution à l'occasion d'un mémoire sur l'inoculation, que j'ai envoyé à l'Académie de Paris il y a 8 ans et qui par la nouveauté de l'analyse avait été reçu avec un grand accueil; c'était, si j'ose le dire, comme une nouvelle province incorporée au corps des mathématiques; il semble que le succès de cette nouvelle analyse lui [à d'Alembert] fit mal au cœur; il la critique de mille façons, toutes également ridicules et après l'avoir bien critiquée il se donne pour premier auteur d'une théorie qu'il n'avait pas seulement entendu nommer. Il savait cependant que mon mémoire ne pouvait paraître que dans sept ou huit ans et il ne pouvait en avoir connaissance qu'en qualité d'académicien et à cet égard mon mémoire devait être sacré jusqu'à ce qu'il fût rendu public. *Dolus an virtus quis in hoste requirat!*[17]

[15]"Genève," *Encyclopédie* 7: 577.

[16]Duvillard, *Analyse et tableaux*, 6. V. Hervé Le Bras, "D'Alembert et la querelle de l'inoculation. Le sujet contre la population," *Jean d'Alembert, savant et philosophe: Portrait à plusieurs voix*, éd. M. Emery et P. Monzani (Paris: Éditions des archives contemporaines, 1989) 293–302.

[17]April 1768, letter quoted by Dietz and Heesterbeek. The Latin line from the *Æneid* means: what does it matter whether we vanquish the enemy thanks to valor or to tactics.

Let us take a look at these moral considerations d'Alembert introduced into the mathematical arena.[18] "Mes objections" he states "n'attaquent que les mathématiciens qui pourraient trop se presser de réduire cette matière en équations et en formules, mais je me regarderais comme coupable envers la société si j'avais eu pour but de dissuader mes concitoyens d'une pratique que je crois utile." We are in a gray area here and he is aware of the reactions he could provoke. "Ces réflexions pourraient bien ne pas contenter tout le monde. Les considérations d'après lesquelles je crois qu'on peut se déterminer en leur faveur ne paraîtront peut-être pas concluantes à plusieurs mêmes de ses partisans."[19] D'Alembert stresses that he feels Bernoulli's calculations fail to take into account all the aspects of the question because of one essential fact: you can die from being inoculated. He believes you cannot therefore reduce everything to mere numbers. In his text the reference to a "difficulté de calcul" recurs and he even alludes in several headings to the "Insuffisance du calcul de Bernouilli." He paints inoculation as a gamble and suggests anyone who undergoes it is more or less "dans le cas d'un joueur qui risque 1 contre 200, de perdre tout son bien dans la journée, pour l'expérience d'ajouter à ce bien une somme inconnue, et même assez petite, au bout d'un nombre d'années fort éloigné, et lorsqu'il sera beaucoup moins sensible à la jouissance de cette augmentation de fortune." He is thus indicating that Bernoulli's conjectures have levelled out immediate risks—those of dying or becoming very sick through inoculation—and hypothetical long term advantages: living longer if one has not, in the meantime, succumbed to an accident or another illness. This, for d'Alembert, is an area in which "l'analyse des probabilités ne peut rien nous apprendre." Like Bernoulli, the Frenchman uses rough figures to make his point: "supposons que le joueur auquel nous comparons l'inoculé, se trouve obligé en effet, n'importe par quelle circonstance, ou de risquer 1 contre 200 d'être réduit tout à coup à l'aumône, ou de renoncer à une très médiocre augmentation de fortune qui lui viendra au bout de quelques années, s'il s'expose à ce risque et qu'il y échappe." D'Alembert feels that many a sound man would stick to his current lot rather than risk his future on casting a die. We can easily imagine immediate danger—if, for instance, we are standing at the top of a precipice. Distant and uncertain dangers have less impact

[18]According to Dietz and Heesterbeek, d'Alembert offers a non-parametric solution whereas Bernoulli's model offered a parametric one.

[19]D'Alembert, *Réflexions sur l'inoculation* (1761), *Œuvres* (Paris, Belin et Bossange, 1821–22)1: 463–514.

on us. In addition, the traditional image of the lottery shows that the apparent rationality of statistics is being applied to events which have their share of hazard.

Another element d'Alembert proposes to take into account is the question of how much one's life expectancy increases through inoculation: "il est aisé d'imaginer une infinité d'hypothèses où l'inoculation augmenterait énormément la vie moyenne, et où néanmoins on serait très imprudent de se soumettre à cette opération." D'Alembert takes a different starting point from Bernoulli and imagines that smallpox is the only deadly disease and that it kills the same number of individuals, year in and year out. Conjectures are clearly rife and are used to give an apparent scientific and objective result whilst they are based on imaginary premises. They simply have, to quote Diderot, a "degré de vraisemblance." If man's allotted span, rather than "three score years and ten" as the Psalms would have it, were to be 100 years, then average life expectancy would be 50 if you did not inoculate, but 100 if you did—that is assuming you were one of the 80% of individuals who survive inoculation. If all newborns were to be inoculated, 20% would die within a fortnight. The remaining 80% would live for a century. Life expectancy from birth would be 4/5 of 100 years, i.e. 80 years, that is 30 years more than if no one were inoculated. But the older you get, the less interesting inoculation becomes. At age 10, inoculation would extend children's life expectancy from 45 to 72 "c'est-à-dire de 27 ans de plus, au moment où on les inoculerait." At 20 you go from 40 to 64, gaining 24 years, and so on. This seems positive but d'Alembert sounds a warning bell: "le risque de mourir de l'inoculation étant un danger instant et présent, et se trouvant d'un contre quatre," it is sufficient to counterbalance the certainty of living to 100 if you survive the operation. This means that the mathematical demonstration is exact *per se*, but not that it offers a sufficiently convincing case for everyone to accept inoculation. This is truer still in that his rough figures, which caricature the results to make them visible, are not reflected in real life: the risks which inoculation entails are fortunately lesser than those suggested in the calculation but, as a result, the increase in life expectancy is too and thus the final reward is less impressive.[20] In addition, d'Alembert indicates that the value of one's life may vary according to one's age—at least for the State: it is

[20]"Dans le cas imaginaire que nous avons pris, le risque de mourir de l'inoculation est très grand, mais la vie moyenne est prodigieusement augmentée; dans le cas réel, le risque est sans doute beaucoup moindre, mais l'augmentation de la vie moyenne est beaucoup moindre aussi."

when one is socially useful that one should be considered especially precious, as opposed to when one is very young or old—and implicitly a burden on society.

D'Alembert suggests a different way of approaching the question of risk analysis for inoculation. He describes it as a "manière nouvelle et plus convaincante de calculer les avantages de l'*inoculation*, dans l'hypothèse que l'*inoculation* puisse causer la mort." The initial premises are simple: "1°. que l'inoculation préserve de la petite vérole naturelle; 2°. qu'elle augmente en effet la vie moyenne des hommes." If we stick to the figures mentioned by Bernoulli and which d'Alembert also uses, about 1 in 14 deaths can be attributed to the smallpox. About 20,000 people die in Paris every year; approximately 1,400 deaths are due to the illness. One can reason on the level of the whole country: "supposons 700 000 habitants dans Paris, il y a donc une personne sur 500, qui meurt de la petite vérole par an, et par conséquent une sur 6 000 par mois." But amongst the 6,000 dead, some would have developed immunity against smallpox, which means the fatality rate for those who have not had the disease must be higher than previously stated. The older you get, the more likely it is you have developed immunity and therefore the smaller the number of potential smallpox victims becomes. So if half of the 6,000 monthly dead were immune, the non-immune individuals' risk rises to 1/3,000. D'Alembert imagines that one could gradually reach the stage where "la partie du genre humain que la petite vérole enlève chaque mois, ne serait pas plus petite, ou même serait plus grande que celle qui succomberait à l'inoculation." If that were to be the case, there would be no reason to hesitate as inoculation offers lifelong protection from smallpox. This goes to show, according to d'Alembert, that there are different chronological scales at work here. If you have a 1/3,000 chance of dying from smallpox within 3 months, the risk inoculation entails of dying within a month is probably lesser. An ordinary individual will probably not make too much of a difference between one and three months. It is therefore preferable to choose the risk which will deliver you *ad vitam* from smallpox and to consider the danger factor as practically non-existent: "Voilà, ce me semble, ce qu'on peut dire de plus fort en faveur de l'inoculation; cette manière d'en calculer l'avantage, quoiqu'elle ait échappé à ses plus zélés partisans, est, si je ne me trompe, la moins sujette aux objections qu'il est possible." And yet, this result's advantages cannot be evaluated in a "précise, mathématique et rigoureuse" manner. One can just state that they are considerable—as close as possible to zero risk.

D'Alembert adds another twist to his revision of risk calculation:

> Dès qu'on accordera qu'on peut mourir de l'inoculation, je n'oserai plus blâmer un père qui craindra de faire inoculer son fils. Car si ce fils par malheur en est la victime, son père aura éternellement à se faire le reproche affreux d'avoir avancé la mort de ce qu'il avait de plus cher; et je ne connais rien à mettre dans la balance vis-à-vis d'un pareil malheur.

The image of the scales reminds us of precise objective calculations, something which is impossible here. If a son catches the smallpox, his father will mourn him, "mais quelle différence entre le *désespoir* d'avoir *hâté la mort* de ce fils, et *le malheur* de la lui avoir *laissé subir*, parce qu'il n'a pas osé courir le risque de la lui donner?" The father will not be comforted by the fact that statistics were in his favor when he had the child inoculated: "Quand il y aurait dix mille à parier contre un qu'on aura le second reproche à se faire plutôt que le premier, je ne sais si cette différence de probabilité serait suffisante pour justifier à ses propres yeux un père qui aurait perdu son fils par l'inoculation; je doute encore plus que cette raison pût consoler une mère." Women are more sentimental, particularly about their children, according to d'Alembert, but fathers are granted feelings too. It is noteworthy that at this stage d'Alembert includes a reference to one of the few documented cases of fatalities apparently due to inoculation, that of Mme Châtelain's daughter: the child asked to be inoculated like her elder sister, the mother reluctantly agreed and the young girl died.[21] Just as the essay indicates that feelings interfere with mathematics, a sad anecdote which would have been familiar to most contemporaries interrupts the scientific reasoning, letting emotion take over.

The conclusion is that each individual parent should choose for his or her children. One must not blame those who have "le courage ou la prudence de courir ce risque et de le préférer à celui d'attendre la petite vérole naturelle" and are convinced by rational arguments and calculated risks to have their infant inoculated, but nor must one criticise those who refuse a dangerous operation for their offspring. Bernoulli adds that since inoculation is potentially fatal, woe betide

[21] "[. . .] mère infortunée, qui a eu la douleur cruelle de voir périr par l'inoculation une de ses filles, quoiqu'elle n'eût pas à se reprocher de l'y avoir livrée sans son consentement, et qu'elle eût même cédé avec beaucoup de peine aux instances que cette jeune et malheureuse personne lui avait faites à ce sujet." Mme Chastelain had agreed to have her elder daughter inoculated. The younger one pleaded to undergo the operation too and died, apparently as a result of it. The case was something of a "cause célèbre" in eighteenth-century France.

the man who tries to influence anyone either way. Each man should assess his position according to how he rates tenderness and duty, whether he considers his child "comme son bien, ou comme le bien de l'État." Mathematics cannot offer the answer: "ce rapport est inappréciable, chacun peut l'estimer à son gré, suivant le degré et l'espèce de sentiment dont il est pourvu, et se déterminer en conséquence."

No one will contest d'Alembert's mathematical genius. What is fascinating here is that this single childless man was prepared to see that non-mathematical elements which cannot be factored precisely may become essential parameters in an equation. It is interesting that this inclusion of ethical dilemmas in a scientific debate was considered to have been a moral fault by other "philosophes" and by Diderot, in particular. Paradoxically, d'Alembert wants to tell the whole truth whether or not it is good for the cause he supports while Diderot would have had him hide his lamp under a bushel and come out clearly in favor of an operation which many considered to be one of the foremost campaigns for progress at the time: "On a trop confondu, dit M. d'Alembert, l'intérêt public avec l'intérêt particulier. —*Cela se peut, mais celui qui apprend aux hommes à séparer ces deux intérêts est un bon géomètre, à la bonne heure, mais un très mauvais citoyen.*"

In this debate, human lives were transformed into statistics—which were to form the basis for actuarial science as we know it. The mathematical implications were huge. D'Alembert went beyond discussing the value of the different parameters applied by Bernoulli and tried to set out the necessary tension between individual and collective risks. What is paradoxical is that Diderot himself in the "Conjecture" article in the *Encyclopédie* seemed to allow for such doubts:

> Il y a un certain point indiscernable où nous cessons de conjecturer, & où nous assûrons positivement; ce point, tout étant égal d'ailleurs, varie d'un homme à un autre, & d'un instant à un autre dans le même homme, selon l'intérêt qu'on prend à l'évenement, le caractere, & une infinité de choses dont il est impossible de rendre compte. Un exemple jettera quelque jour sur ceci. Nous savons par expérience, que quand nous nous exposons dans les rues par un grand vent, il peut nous arriver d'être tués par la chûte de quelque corps; cependant nous n'avons pas le moindre soupçon que cet accident nous arrivera: le rapport des évenemens connus pour, aux évenemens connus contre, n'est pas assez grand pour former le doute & la *conjecture*. Remarquez cependant qu'il s'agit ici de l'objet le plus important à l'homme, la conservation de sa vie. Il y a dans toutes les choses une unité qui devroit être la même pour tous les hommes, puisqu'elle est fondée sur les expériences, & qui n'est peut-être la même ni pour deux hommes, ni pour deux actions de la vie, ni pour deux instans: cette unité réelle seroit

> celle qui résulteroit d'un calcul fait par le philosophe Stoïcien parfait, qui se comptant lui-même & tout ce qui l'environne pour rien, n'auroit d'égard qu'au cours naturel des choses.

An interesting case which shows how conjectures based on suppositions can go wrong is that of reactions to unfortunate deaths attributable to inoculation or its sequels. When Monsieur and Madame de Pernan, acquaintances of the *philosophes*, lost their son who had just been inoculated, the question was whether to treat this unfortunate death as a negligible statistic or not. The Enlightenment tension between reason and sensibility is evidenced in exchanges of letters. Morellet wrote to Suard: "Le chevalier m'avait expliqué le malheur arrive à Mad de Pernan. Il n'en faut plus parler. Je lui réponds que ces cas sont arrivés ici."[22] The mother is allowed to grieve, but the case must not be discussed openly. In modern parlance, young Pernan was collateral damage. Condorcet, who was probably more attuned to public opinion wondered, in a letter to Turgot, about the effects on the family and on the public: "C'est un malheur affreux pour cette famille et une chose très fâcheuse pour l'inoculation en elle-même."[23] Private grief comes first and "malheur affreux" shows the intensity of the feeling he can empathize with, while he is concerned about the impact of the sad death of a child on the pro-inoculation campaign and the "chose très fâcheuse" which this negative statistic represents. Conjecture is not proof. Probability is not certainty. Deaths like young Pernan's were striking confirmation of this. In the same way, Condorcet rejoiced in the inoculation of prince Ferdinand of Parma, a grandson of Louis XV, considering this was a huge victory for the method as Italy was seen on the whole to be opposed to progress. Each event was thus taken into account as part of a narrative in which inoculation became one of the touchstones of the brave new world to come.

In a nutshell, we have one of the Enlightenment's paradoxes: an attempt to rationalize everything while allowing for personal feelings, a realization that, despite what one would wish to believe, the common good is not always the obvious path for the individual. By reasoning on reason's failure or on its limits in the inoculation debate, d'Alembert certainly displeased a number of "philosophes" and gave fodder to their enemies, but he also indicated that the human factor

[22]André Morellet, *Lettres* (Oxford: Voltaire Foundation, 1991) vol. I. Letter dated July 2, 1772.

[23]*Correspondence inédite de Condorcet et de Turgot*, Charles Henry, ed. (Paris: Charavay, 1883): 84. Letter dated May 24, 1772.

would always be an imponderable in mathematical equations, or at least that it should not be neglected, least of all by the scientists who were trying to use conjecture to engineer human behavior.

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