ECO364: International Trade Theory

Lecture 2b

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- Last Class
    - Gains from Trade, Trade Equilibrium, Relative Wages

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- Today
  - Heckscher-Ohlin (HO) Model
    - Introduction and Definitions
    - Relate Factor Prices to Factor Demand (FF Curve)
    - Relate Good Prices to Factor Prices (SS Curve)

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- Readings
  - KMO Ch. 5 (including appendix)
  - Economist "In the Shadow of Prosperity"
  - Hanson "The Rise of Middle Kingdoms"

Heckscher-Ohlin Model

HO Model

- In the Ricardian model, we assumed that technological differences in production is what gives rise to specialization and trade
- We also assumed that only one factor is used in production
- Although the Ricardian model was able to make predictions about welfare, the worst that could happen to a country is that welfare does not change following trade
  - No welfare loss when moving from autarky to trade

HO Model

- To introduce the possibility of welfare losses, we introduce another factor in production
  - This will allow us to make a prediction on how the income of one factor is affected relative to the income of the other factor following trade
- Another key difference between the Ricardian model and the HO model is the source of trade
  - Ricardian: technological differences
  - HO: factor abundance

Model Environment and Assumptions

- Two countries: North and South
- Two goods: Computers and Textiles
- Two factors: Capital (K) and Labour (L)
  - K: buildings, machinery
  - The supply of K and L in each country is fixed, but it varies across the two countries
- Constant returns to scale in production
  - scale inputs by a factor of 2, output increases by a factor of 2
- Perfect competition
  - firms take prices as given, marginal revenue = marginal cost
- Both factors are mobile across sectors but not across countries
  - this will equalize their returns (wage and rental rate) across sectors within a country

Model Environment and Assumptions

- Assume that North's stock of factors is such that they have a **relative abundance** of capital

$$\frac{K^N}{L^N} > \frac{K^S}{L^S}$$

- We will focus on relative factor abundance (not absolute)
- Similar to the distinction made between comparative and absolute advantage

Model Environment and Assumptions

- The production functions for the two goods are

$$F_C(K_C, L_C) \text{ and } F_T(K_T, L_T)$$

  - In the Ricardian model, we just had $F_C(L) = \frac{1}{a_C} L$
- These production functions will be increasing in each input, but at diminishing rates
  - $\frac{\partial F(K,L)}{\partial K} > 0$ and $\frac{\partial^2 F(K,L)}{\partial^2 K} < 0$
  - $\frac{\partial F(K,L)}{\partial L} > 0$ and $\frac{\partial^2 F(K,L)}{\partial^2 L} < 0$
  - First-order derivatives are just the marginal products of each input
  - e.g. adding the second cook should provide a significant increase in output, but the 10th cook provides much less

Input Intensity of a Good

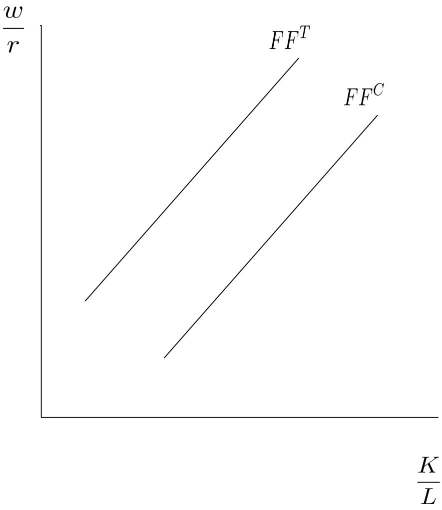
- Since there are now two factors used in production, goods must be different in terms of how dependent they are in each input
  - Computers are more relatively capital intensive than textiles
  - Textiles are more relatively labour intensive than computers

Input Intensity of a Good

- Definition: *A good is **relatively capital intensive** if, for a given relative factor price, the capital-labour ratio used in production is higher than the capital-labour ratio used in the production of the other good*
- Relative factor price is $\frac{w}{r}$, where w is the factor price of labour and r is the factor price of capital
- We can represent differences in relative capital intensity using something called an **FF curve**

FF Curve: Factor Prices and Input Ratio

For any fixed value of $\frac{w}{r}$, computers uses more capital relative to labour than textile



Example with Numbers

- Suppose that computers are relatively capital intensive and textiles are relatively labour intensive
- Let's work through a numerical example

$$Q_C = K_C^{2/3} L_C^{1/3} \qquad\qquad Q_T = K_T^{1/3} L_T^{2/3}$$

  - The powers in each input reflect the input intensity for the industry

- To get the optimal values of $K_C,\ K_T,\ L_C,\ L_T$, need to maximize the profit functions

$$P_C Q_C - rK_C - wL_C$$

$$P_T Q_T - rK_T - wL_T$$

- Take the derivatives of these profit functions with respect to each input to get the necessary conditions for optimal inputs

Example with Numbers

$$P_C K_C^{2/3} L_C^{1/3} - rK_C - wL_C$$
$$P_T K_T^{1/3} L_T^{2/3} - rK_T - wL_T$$

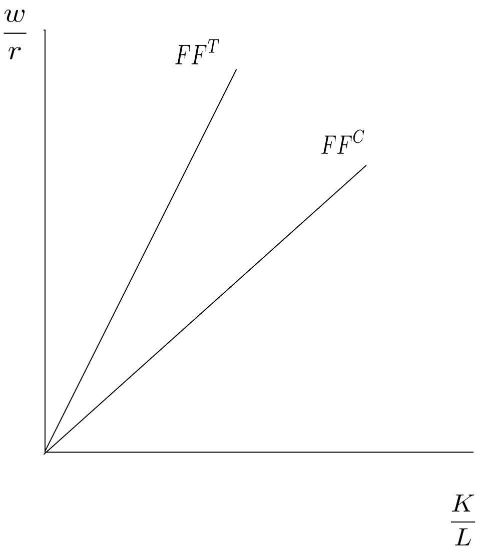
- These conditions show that at the optimal inputs, marginal cost = marginal revenue

$$w = P_T \frac{2}{3} K_T^{1/3} L_T^{-1/3} \qquad w = P_C \frac{1}{3} K_C^{2/3} L_C^{-2/3}$$
$$r = P_T \frac{1}{3} K_T^{-2/3} L_T^{2/3} \qquad r = P_C \frac{2}{3} K_C^{-1/3} L_C^{2/3}$$

- Take the ratio of wage to capital for each good

$$\frac{w}{r} = 2 \frac{K_T}{L_T} \quad \frac{w}{r} = \frac{1}{2} \frac{K_C}{L_C}$$

Example with Numbers



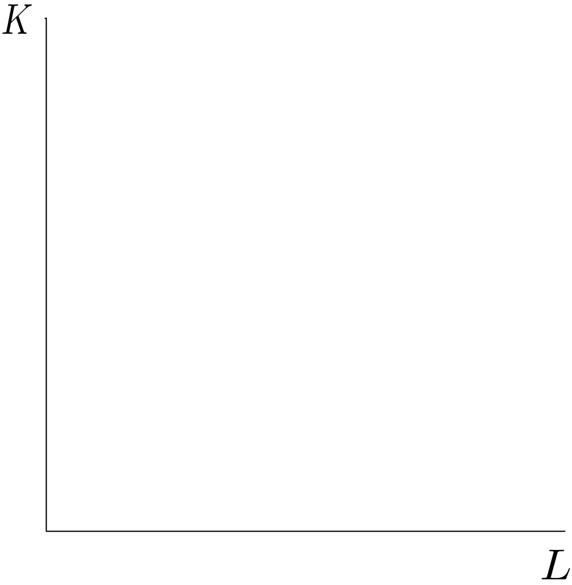
What Determines Factor Prices?

- The FF curves gives us the relationship between relative factor price and relative factor demands
  - For a given relative factor price, the FF curve gives us the corresponding relative factor demands in each industry
- But what drives relative factor price?
  - It turns out that in our model, it is the prices of goods (the relative good price, to be precise)

What Determines Factor Prices?

- To show this, we will derive what's called the **SS curve**
  - The SS curve relates relative **good** price $\frac{P_T}{P_C}$ to relative **factor** price $\frac{w}{r}$
- But before we can derive the SS curve, we need to introduce what's called the **Lerner Diagram**
  - The Lerner Diagram will provide the underlying mechanism that will draw out the relationship between relative good price and relative factor price
  - It will contain two curves: the **iso-value** curve and the **iso-cost** curve

Lerner Diagram

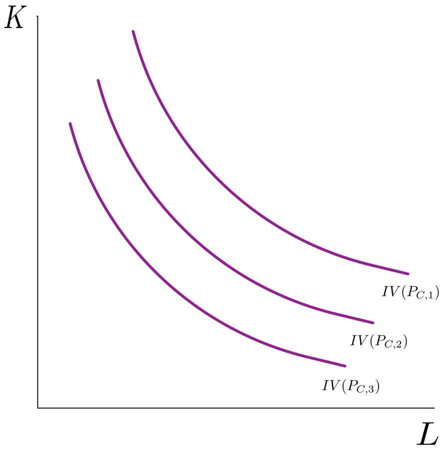


Lerner Diagram: Iso-Value Curves

- The iso-value curves maps the combinations of capital and labour that yield \$1 of revenue
  - Start with $1 = P_C F_C(K, L)$, where $F_C(K, L)$ is the production function of computers (it's also just output of computers for a given K and L)
  - For a given value of P_C, what values of $K$and L produce \$1 of revenue?
  - These look a lot like the indifference curves in consumer theory

Lerner Diagram: Iso-Value Curves

Iso-value curves for prices $P_{C,3} > P_{C,2} > P_{C,1}$



Lerner Diagram: Iso-Cost Curves

- The iso-cost curves maps combinations of capital and labour that (as a bundle) cost \$1
- Given w and r, it is derived from cost function

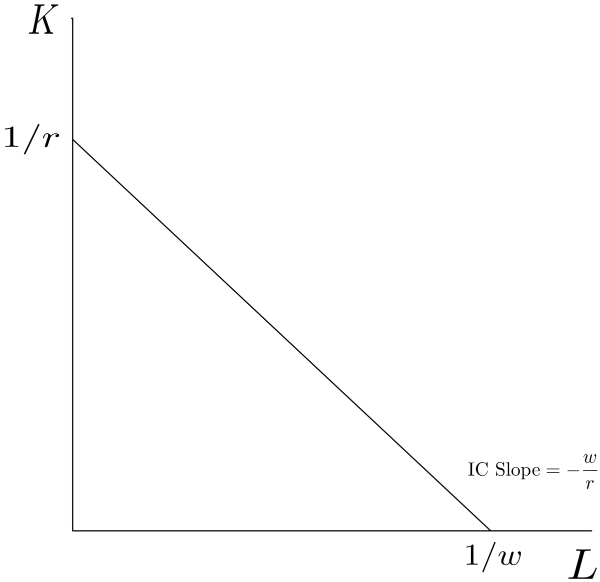
$$wL + rK = 1$$

- Rearranging gives

$$K = \frac{1}{r} - \frac{w}{r}L$$

- We can draw this curve in our Lerner Diagram, noting that the slope of this equation is $-\frac{w}{r}$

Lerner Diagram: Iso-Cost Curves

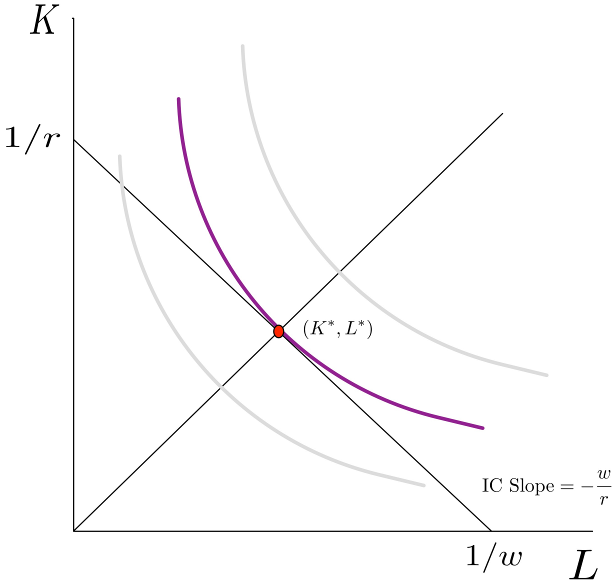


Lerner Diagram: Equilibrium

- Two conditions for an equilibrium

1. The equilibrium bundle (K^*, L^*) must lie on both the iso-value and iso-cost curve
    - This is because perfect competition requires that revenue $=$ cost
    - If profits were positive, firms enter driving down prices
    - If profits were negative, firms exit driving up prices
2. Both the iso-value and iso-cost curve have the same slope
    - The slope of the iso-cost curve is just $-\frac{w}{r}$
    - We still need to derive the iso-value slope

Lerner Diagram: Equilibrium



Lerner Diagram: Slope of the Iso-Value Curve

- The slope of iso-value curve is just the marginal rate of technical substitution
  - It is an idea similar to the marginal rate of substitution from consumer theory
- To derive this, start with

$$1 = P_C F_C(K, L)$$

- Move price to the left-hand side

$$\frac{1}{P_C} = F_C(K, L)$$

  - Since the 1 here is the \$1 revenue, $1/P_C$ is just the output (as you might expect since it's equal to F)

Lerner Diagram: Slope of the Iso-Value Curve

- In the Lerner Diagram we show the relationship between the input K *as a function* of L
- Keeping this in mind, take the total derivative of the equation $\frac{1}{P_C} = F_C(K, L)$ with respect to L
- In other words, take the derivative of both sides of the equation with respect to L

$$\begin{aligned}
\frac{d(1/P_C)}{dL} &= \frac{dF_C(K,L)}{dL} \\
&= \frac{\partial F_C(K,L)}{\partial K}\frac{dK}{dL} + \frac{\partial F_C(K,L)}{\partial L}\frac{dL}{dL} \\
&= \frac{\partial F_C(K,L)}{\partial K}\frac{dK}{dL} + \frac{\partial F_C(K,L)}{\partial L}
\end{aligned}$$

Lerner Diagram: Slope of the Iso-Value Curve

- Multiply through by dL and note the fact that $d(1/P_C) = 0$ because $1/P_C$ is a constant

$$0 = \frac{\partial F_C(K,L)}{\partial K} dK + \frac{\partial F_C(K,L)}{\partial L} dL$$

- Rearranging the expression above

$$\frac{dK}{dL} = -\frac{\frac{\partial F_C(K,L)}{\partial L}}{\frac{\partial F_C(K,L)}{\partial K}}$$

- The ratios in the right side are the marginal products of labour and capital
  - They are a decreasing function of the inputs themselves based on our assumption on the production functions (e.g. $\frac{\partial F(K,L)}{\partial K} > 0$ and $\frac{\partial^2 F(K,L)}{\partial^2 K} < 0$, $\frac{\partial F(K,L)}{\partial L} > 0$ and $\frac{\partial^2 F(K,L)}{\partial^2 L} < 0$)

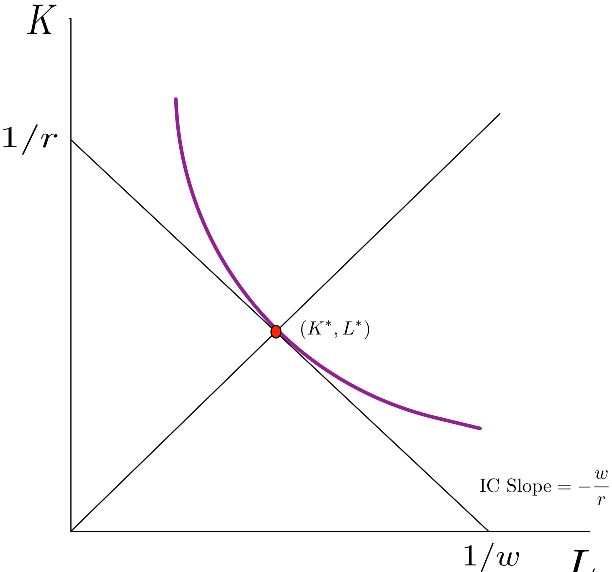
Lerner Diagram: Equilibrium

- Now that we have the slope of both the iso-cost and iso-value curves, we can solve for the equilibrium input bundle
  - Iso-cost slope
$$-\frac{w}{r}$$
  - Iso-value slope
$$\frac{dK}{dL} = -\frac{\frac{\partial F_C(K,L)}{\partial L}}{\frac{\partial F_C(K,L)}{\partial K}}$$

Lerner Diagram: Equilibrium

- Recall that the equilibrium conditions are
  1. the equilibrium bundle (K^*, L^*) must lie on both the iso-cost and iso-value curve
  2. the slopes of the iso-cost and iso-value curves must be equal
     - In consumer theory, the equilibrium bundle is such that the slope of the indifference curve is equal to the slope of the budget constraint
- Thus, the equilibrium set of inputs must be such that the iso-cost curve and the iso-value curves are **tangent** to each other

Lerner Diagram: Equilibrium



Lerner Diagram: Two Industries

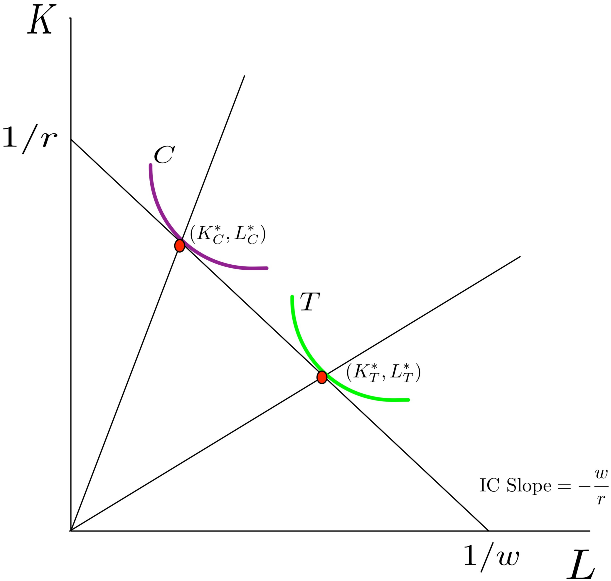
- So far, we've been considering just one industry: computers
- With two industries, both industries share the same iso-cost curve
  - wage and rental rates are the same across industries due to perfect labour/capital mobility

$$1 = rK + wL$$

- Their iso-value curves are different because each industry has its own production function and face different prices

$$\begin{aligned} 1 &= P_C F_C(K, L) \\ 1 &= P_T F_T(K, L) \end{aligned}$$

Lerner Diagram: Two Industries



Lerner Diagram: Two Industries

- The straight line from the origin is the **Output Expansion Path (OEP)**
  - They reflect the factor intensity of a good
  - Steeper OEP means the good is more capital intensive

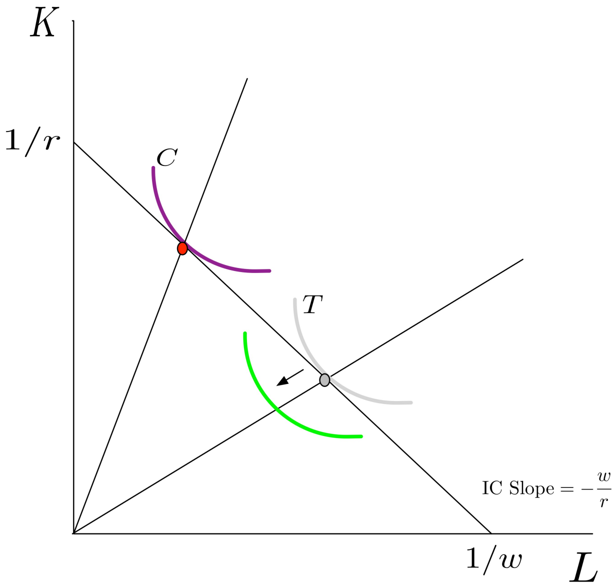
Drawing the SS Curve

- So how does any of this relate back to the SS curve
    - The curve that show the relationship between $\frac{P_T}{P_C}$ and $\frac{w}{r}$
- Let's conduct a thought experiment

Thought Experiment

- Suppose the price of textile, P_T, increases
- To produce \$1 worth of revenue for textiles, we now need *fewer* inputs (both capital or labour) than before
  - $1 = P_T F_T(K, L)$
- Thus, the iso-value curve for textiles shifts *in*
- The iso-value curve for computers does not change

Thought Experiment



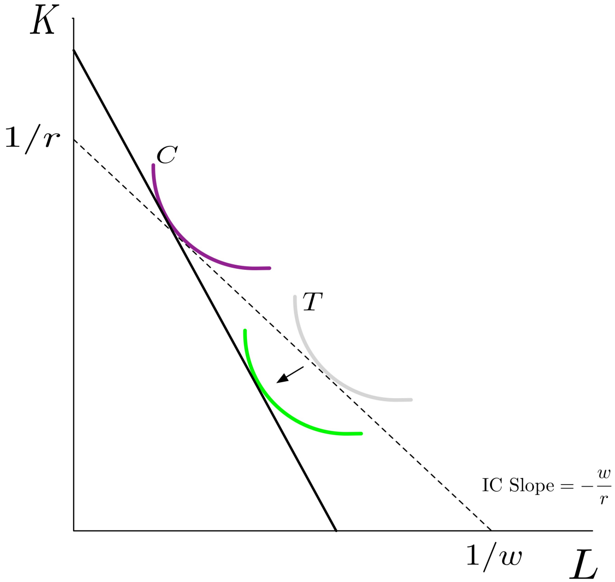
Thought Experiment

- Following this increase in textile price, there are now profits to be earned in the textile industry
  - wages and rental of capital have not changed
- This will induce new firms to enter (or incumbent firms to expand their production)
  - as a result, capital and labour will flow into the textile industry because of the demand of these new firms
- The economy produces more textiles and fewer computers
- Since textile is relatively labour-intensive, the increase demand for labourers will cause an increase in wages
- On the other hand, return to capital will decrease
- Thus, following an increase in $\frac{P_T}{P_C}$, $\frac{w}{r}$ rises

Thought Experiment

- The rise in $\frac{w}{r}$ is a combination of an increase in w due to greater labour demand *and* a decrease in r
  - For every unit of labour that leaves the computer industry, firms manufacturing computers must give up more capital than what's demanded by textile firms
  - Thus, r decreases
- r must decrease in order to avoid negative profits in the computer industry
  - negative profits entails an iso-value curve *above* the iso-cost line

Thought Experiment



Conclusion

- This thought experiment allows us to map how changes in relative prices affects relative returns to factors
- The mapping is called the SS curve
- In the next class, we will put this together with the FF curve
  - This will cleanly relate relative goods prices, relative factor prices, and the resulting optimal capital-labour ratio in each industry
  - We will then introduce the Stolper-Samuelson Theorem, which will confirms our theoretical prediction about how changes in goods prices affects the real returns of the factors used in the production of those goods
  - This will enable us to say something about how the income distribution in a country is affected by a price shock arising from trade