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# A statistical analysis of drop sizes generated from ensembles of randomly corrugated ligaments

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(Received xx; revised xx; accepted xx)

This file contains instructions for authors planning to submit a paper to the *Journal of Fluid Mechanics*. These instructions were generated in L<sup>A</sup>T<sub>E</sub>X using the JFM class file, and the source files for these instructions can be used as a template for submissions. The present paragraph appears in the abstract environment. All papers should feature a single-paragraph abstract of no more than 250 words, which provides a summary of the main aims and results. In addition to the figures in the main article a graphical abstract is now required. It will be used as a small thumbnail in the table of contents and on the abstract page, so multiple panels are not suitable and will be rejected. Please confirm that you have included an image to accompany your abstract, which will be used as the graphical abstract for manuscripts published in 2020. The image must be of aspect ratio 1.2:1 (e.g. 6cm x 5cm) and should be submitted in GIF or high resolution JPEG format (300 dpi). Unless very large, vector graphics are preferred to ensure image sharpness regardless of sizing. If you do not have the copyright to the image, please ensure you have permission to reuse the figure. Captions are not required. Text is actively discouraged, but if it must be used, it should be legible in a small thumbnail (2.4cmx2cm) presented in the table of contents. All graphical abstract images will be considered for a JFM cover selection by the JFM Panel. Please note that we publish 24 covers in a year.

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**Abstract must not spill onto p.2**

## 1. How to submit to the *Journal of Fluid Mechanics*

Authors must submit using the online submission and peer review system [Scholar One](#) (formerly Manuscript Central). If visiting the site for the first time, users must create a new account by clicking on 'register here'. Once logged in, authors should click on the 'Corresponding Author Centre', from which point a new manuscript can be submitted, with step-by-step instructions provided. Authors must at this stage specify whether the submission is a *JFM Paper*, or a *JFM Rapids* paper (see §4 for more details). In addition, authors must specify an editor to whom the paper will be submitted from the drop-down list provided. Note that all editors exclusively deal with either *JFM Paper* or *JFM Rapids* (clearly indicated on the list), so please ensure that you choose an editor accordingly. Corresponding authors must provide a valid ORCID ID in order to submit a manuscript, either by linking an existing ORCID profile to your ScholarOne account or by creating a new ORCID profile. Once your submission is completed you will receive an email confirmation. Book reviews should not be submitted via the online submission site, but should instead be submitted by email to [anne.juel@manchester.ac.uk](mailto:anne.juel@manchester.ac.uk).

## 2. Rules of submission

Submission of a paper implies a declaration by the author that the work has not previously been published, that it is not being considered for publication elsewhere and that it has not already been considered by a different editor of the Journal. If you have uploaded your manuscript via the arXiv function, then please include your E-print Number during the submission process.

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### 3.1. Transparency and Openness Promotion (TOP)

The overarching policy of the *Journal of Fluid Mechanics* is that research articles should contain sufficient information to allow others to understand, replicate and verify findings, and compare them with alternative studies. We therefore require that whenever possible:

**Understanding** - Articles should be written and will be assessed for clarity, both of the execution of the research and for its outcomes and conclusions.

**Replication** - All information required to replicate the study must be provided, within the body of the paper and/or publicly accessible repositories. Examples of what is required include but are not limited by:

- for analytical studies, the mathematically complete set of equations and boundary conditions, any theorems relied upon, appropriately referenced;

• for numerical studies, the mathematically complete set of equations and boundary conditions, sufficient descriptions of the algorithms or packages used to solve them, appropriately referenced, and the resolution used with respect to the independent variables;

• for laboratory experiments, the dimensions and construction of any apparatus, the materials used including their relevant physical properties, the protocol adopted for the running of the experiments, the measurement tools used including their resolution and accuracy, including appropriate calibration standards;

• for field studies, the raw data collected or used, any protocols or tools used to access the data (e.g. data-mining tools) or to process it.

**Verification** - Most studies can be verified or falsified provided that sufficient detail is given for them to be replicated (see above). Where data is manipulated (for example, bringing together multiple data sets by scaling) the raw (dimensional) data relating to the primary measurements (laboratory) or outputs (numerical) should be provided together with the protocols or tools used to process them.

**Comparison** - All graphical information should be supplemented with numerical data or precise algorithms to reproduce it. For example, data points should be provided in a spreadsheet and curves should be defined either explicitly with an equation or as resulting from a precisely defined algorithm.

## 4. Types of paper

### 4.1. *Standard papers*

Regular submissions to JFM are termed ‘standard papers’. Note that such papers must contain original research. Papers should be written in a concise manner; though JFM has no page limit, each paper will be judged on its own merits, and those deemed excessive in length will be rejected or will require significant revision.

### 4.2. *JFM Rapids*

*JFM Rapids* is devoted to the rapid publication of short, high-impact papers across the full range of fluid mechanics. Manuscripts submitted as *JFM Rapids* must be strictly 10 or fewer printed pages, and must be submitted in L<sup>A</sup>T<sub>E</sub>X using the jfm.cls class file, so as to ensure that they meet the page limit and to expedite their production. As with standard papers, the principal and over-riding objective is to publish papers of the highest scientific quality.

Once a paper is submitted, reviewers are asked to provide reports with a short turnaround. In order to be accepted for publication in *JFM Rapids*, such papers must be strongly endorsed by the referees and should require only minor revisions to improve clarity, usually without recourse to a second round of reviewing. In this case, and at the discretion of the editor, some additional pages may be allowed to address specific points raised by the reviewers, such as the addition of an extra figure or some explanatory text.

Papers that are rejected having been submitted to Rapids are rejected on behalf of the whole Journal and may not be submitted for consideration by another associate editor of JFM, whether for Rapids or as a Standard paper.

In cases where the editor, guided by the reviewers, judges that a paper has merit but requires substantial revision that will require significant reviewing, a decision of ‘revise and resubmit’ will be given. On re-submission, such papers will be handled as standard JFM papers and if accepted will not subsequently appear as *JFM Rapids*.

*JFM Rapids* will be published online within one month of final acceptance. They will appear within a designated section on the *Journal of Fluid Mechanics* website. Each *Rapid*

will be cited and indexed as a JFM article but with a distinctive *Rapids* identifier, and will be assigned to a JFM volume.

#### 4.3. *JFM Perspectives*

Review papers are published under *JFM Perspectives* and are by invitation only.

### 5. File types

Authors are strongly encouraged to compose their papers in L<sup>A</sup>T<sub>E</sub>X, using the [jfm.cls](#) style file and supporting files provided, with the [jfm-instructions.tex](#) file serving as a template (please note that this is mandatory for *JFM Rapids*). A PDF of the L<sup>A</sup>T<sub>E</sub>X file should then be generated and submitted via the submission site. For the review process the pdf file should be no more than 10MB. There is no need to submit the L<sup>A</sup>T<sub>E</sub>X source files alongside the PDF, but upon provisional acceptance of the paper, the L<sup>A</sup>T<sub>E</sub>X source files, along with individual figure files and a PDF of the final version, will need to be submitted for typesetting purposes. Authors may also compose standard papers in Word, though this will lead to the paper spending a longer period in production. If using Word, please note that equations must NOT be converted to picture format and the file must be saved with the option ‘make equation editable’. All submitted video abstract files should be formatted as MP4 (H.264). MP4 has full compatibility across commonly used browsers, whereas other video formats will only work on selected browsers. This will ensure the greatest possible dissemination of this work.

### 6. Preparing your manuscript

Authors should write their papers clearly and concisely in English, adhering to JFM’s established style for mathematical notation, as provided in Section 12. We encourage the submission of online supplementary material alongside the manuscript where appropriate (see Section 6.3). Metric units should be used throughout and all abbreviations must be defined at first use, even those deemed to be well known to the readership. British spelling must be used, and should follow the *Shorter Oxford English Dictionary*.

#### 6.1. *Figures*

All authors need to acquire the correct permissions and licences to reproduce figures, which should be submitted with the production files. Further information on applying for permission to reuse figures can be found [here](#). Images should be submitted in EPS or high-resolution TIFF format (1200 dpi for lines, 300 dpi for halftone and colour in RGB format, and 600 dpi for a mixture of lines and halftone) and all labels should be editable. Unless very large, vector graphics are preferred to ensure image sharpness regardless of sizing. The minimum acceptable width of any line is 0.5pt. Each figure should be accompanied by a single caption, to appear beneath, and must be cited in the text. Figures should appear in the order in which they are first mentioned in the text and figure files must be named accordingly (‘Abstract.eps’, ‘Fig1.eps’, ‘Fig2a.tiff’, etc) to assist the production process (and numbering of figures should continue through any appendices). Words *figure 1*, *table 1* and *movie 1* should be lower case. For example see figures ?? and ??. Failure to follow figure guidelines may result in a request for resupply and a subsequent delay in the production process. Note that *all* figures will be relabelled by the typesetter, so please ensure all figure labels are carefully checked against your originals when you receive your proofs.

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$a/d$	$M = 4$	$M = 8$	Callan <i>et al.</i>
0.1	1.56905	1.56	1.56904
0.3	1.50484	1.504	1.50484
0.55	1.39128	1.391	1.39131
0.7	1.32281	10.322	1.32288
0.913	1.34479	100.351	1.35185

---

Table 1: Values of  $kd$  at which trapped modes occur when  $\rho(\theta) = a$ .

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## 6.2. Tables

Tables, however small, must be numbered sequentially in the order in which they are mentioned in the text. Words *table 1*, *table 2* should be lower case throughout. See table 1 for an example.

## 6.3. Online supplementary material

Relevant material which is not suitable for inclusion in the main article, such as movies or numerical simulations/animations, can be uploaded as part of the initial submission. Movies must be submitted in .mp4 format and have the file designation of ‘Movie’. Each movie must be numbered in the order they are mentioned and titled movie 1, movie 2 etc and accompanied by a separate caption. To ensure maths terms display correctly they should be bounded by  $\$$  and written in TeX, e.g. movie 1. Side view of numerical Schlieren contours from case E1N at  $\$z = L_z/2\$$ . Each movie should be no more than 50MB. Upon publication these materials will then be hosted online alongside the final published article. Likewise, should there be detailed mathematical relations, tables or figures which are likely to be useful only to a few specialists or take up excessive space in the article, these should also be published online as supplementary material [designated as ‘Other supplementary material’]. Note that supplementary material is published ‘as is’, with no further intervention made during the Production process, all ‘draft’ information should be removed.

## 7. Editorial decisions

### 7.1. Revision

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### 7.2. Provisional acceptance

If the paper is accepted as suitable for publication you will be sent a provisional acceptance decision. This enables you to upload the final files required for production: (1) the final PDF or word version of the paper, designated as a ‘Main Document’; (2) any source files (see

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### 7.3. *Acceptance*

On receipt of the production files you will be sent an email indicating completion of the acceptance process.

## 8. Publication process

Once a paper has been accepted for publication and the source files have been uploaded, the manuscript will be sent to Cambridge University Press for copyediting and typesetting, and will be assigned a digital object identifier (doi). When the proof is ready, authors will receive an email alert containing a link to the PDF of the proof, and instructions for its correction and return. It is imperative that authors check their proofs closely, particularly the equations and figures, which should be checked against the accepted file, as the production schedule does not allow for corrections at a later stage. Once ready, papers will be published online on [Cambridge Core](#) in the current ‘open’ volume. Each volume will be kept open for approximately two weeks. Note that the PDF published online is the Version of Record and no further alterations/corrections to this document will be allowed. The corresponding author is emailed a link to the published article when it is published online.

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The Journal will publish corrigenda that alter significant conclusions made in a paper. Such corrigenda should be submitted to an associate editor, who will consider the submission similarly to a new paper and may consult referees if appropriate. When published, corrigenda are clearly linked with the original articles to which they refer, and the articles to them.

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## 11. Cambridge Author Services - in partnership with American Journal Experts

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own expense. Use of these services does not guarantee that the manuscript will be accepted for publication nor does it restrict the author to submitting to a Cambridge-published journal.

## 12. Notation and style

Generally any queries concerning notation and journal style can be answered by viewing recent pages in the Journal. However, the following guide provides the key points to note. It is expected that Journal style and mathematical notation will be followed, and authors should take care to define all variables or entities upon first use. Also note that footnotes are not normally accepted. Abbreviations must be defined at first use, glossaries or lists/tables of abbreviations are not permitted.

### 12.1. Mathematical notation

#### 12.1.1. Setting variables, functions, vectors, matrices etc

• **Italic font** should be used for denoting variables, with multiple-letter symbols avoided except in the case of dimensionless numbers such as *Re*, *Pr* and *Pe* (Reynolds, Prandtl, and Péclet numbers respectively, which are defined as `\Rey`, `\Pran` and `\Pen` in the template).

• **Upright Roman font** (or upright Greek where appropriate) should be used for:

(i) (vI) label, e.g. *T*, *t* (transpose)

(ii) Fixed operators: *sin*, *log*, *d*,  $\Delta$ , *exp* etc.

(iii) Constants: *i* ( $\sqrt{-1}$ ),  $\pi$  (defined as `\upi`), etc.

(iv) Special Functions: *Ai*, *Bi* (Airy functions, defined as `\Ai` and `\Bi`), *Re* (real part, defined as `\Real`), *Im* (imaginary part, defined as `\Imag`), etc.

(v) Physical units: *cm*, *s*, etc.

(vi) Abbreviations: *c.c.* (complex conjugate), *h.o.t.* (higher-order terms), *DNS*, etc.

• **Bold italic font** (or bold sloping Greek) should be used for vectors (with the centred dot for a scalar product also in bold):  $\mathbf{i} \cdot \mathbf{j}$

• **Bold sloping sans serif font**, defined by the `\mathsf{bfi}` macro, should be used for tensors and matrices:  $\mathbf{D}$

• **Calligraphic font** (for example  $\mathcal{G}$ ,  $\mathcal{R}$ ) can be used as an alternative to italic when the same letter denotes a different quantity use `\mathcal{a}` in  $\LaTeX$

#### 12.1.2. Other symbols

Large numbers that are not scientific powers should not include commas, but should use a non-breaking space, and use the form 1600 or 16 000 or 160 000. Use *O* to denote ‘of the order of’, not the  $\LaTeX$  *O*.

The product symbol ( $\times$ ) should only be used to denote multiplication where an equation is broken over more than one line, to denote a cross product, or between numbers. The  $\cdot$  symbol should not be used, except to denote a scalar product of vectors specifically.

### 13. Citations and references

All papers included in the References section must be cited in the article, and vice versa. Citations should be included as, for example “It has been shown (Rogallo 1981) that...” (using the `\citep` command, part of the natbib package) “recent work by Dennis (1985)...” (using `\citet`). The natbib package can be used to generate citation variations, as shown below.

`\citet[pp. 2–4]{Hwang70}`:

Hwang & Tuck (1970, pp. 2-4)

`\citep[p. 6]{Worster92}`:

(Worster 1992, p. 6)

`\citep[see][]{Koch83, Lee71, Linton92}`:

(see Koch 1983; Lee 1971; Linton & Evans 1992)

`\citep[see][p. 18]{Martin80}`:

(see Martin 1980, p. 18)

`\citep{Brownell04, Brownell07, Ursell150, Wijngaarden68, Miller91}`:

(Brownell & Su 2004, 2007; Ursell 1950; van Wijngaarden 1968; Miller 1991)

(Briukhanov *et al.* 1967)

Bouguet (2008)

(Joseph & Saut 1990)

The References section can either be built from individual `\bibitem` commands, or can be built using BibTex. The BibTex files used to generate the references in this document can be found in the zip file [jfm-ifcs](#).

Where there are up to ten authors, all authors’ names should be given in the reference list. Where there are more than ten authors, only the first name should appear, followed by *et al.*

JFM discourages citations of manuscript posted on social media sites (such as ResearchGate) or on pre-print servers (e.g. ArXiv), that have not been peer-reviewed or published in journals.

**Supplementary data.** Supplementary material and movies are available at <https://doi.org/10.1017/jfm.2019...>

**Acknowledgements.** Acknowledgements may be included at the end of the paper, before the References section or any appendices. Several anonymous individuals are thanked for contributions to these instructions.

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 331 manuscript, for example, “A.G. and T.F. derived the theory and T.F. and T.D. performed the simulations. All  
 332 authors contributed equally to analysing data and reaching conclusions, and in writing the paper.”

## 333 14. Appeals process

334 The *Journal of Fluid Mechanics* has an appeal procedure which provides authors with the  
 335 opportunity to respond to the editorial decision on their manuscript, should they think that  
 336 their manuscript was treated in an unfair manner during the peer-review process. Authors  
 337 have the right to appeal to the Editor or Editor-in-Chief against any decision taken on their  
 338 manuscript at any stage. An appeal will be considered at the direction of the Editorial Board  
 339 of the Journal.

### 340 14.1. How do I appeal?

341 **Step 1.** Requests to have the decision on a submission re-considered should be made in  
 342 the first instance to the Associate Editor who handled the submission and made the decision.  
 343 Send a rebuttal letter to the Associate Editor, explaining clearly why you disagree with the  
 344 decision on your manuscript and including a detailed response to any points of contention in  
 345 the referees’ reports. The Associate Editor will consider your appeal and either invite you to  
 346 submit a revised paper or confirm the original decision.

347 **Step 2.** In case you remain unsatisfied with the Associate Editor’s response after Step 1 or  
 348 at any stage should you consider that your submission was treated unfairly, you should send a  
 349 letter of appeal to the Editor-in-Chief via the Journal email ([JFMEditorial@cambridge.org](mailto:JFMEditorial@cambridge.org)).  
 350 Your letter should explain clearly the grounds for your appeal.

351 **Step 3.** The Editor-in-Chief will consider the grounds of your appeal and if he considers  
 352 there to be a *prima facie* case to consider may assign one of the Deputy Editors to consider  
 353 the appeal in detail. All appeal requests are handled on a case by case basis and the Deputy  
 354 Editor’s or Editor-in-Chief’s decision is final. Appeals are normally considered on the basis  
 355 of whether or not the process of review was conducted appropriately. Papers will not routinely  
 356 be sent for further review.

## 357 Appendix A.

358 This appendix contains sample equations in the JFM style. Please refer to the  $\LaTeX$  source  
 359 file for examples of how to display such equations in your manuscript.

$$360 \quad (\nabla^2 + k^2)G_s = (\nabla^2 + k^2)G_a = 0 \quad (\text{A } 1)$$

$$361 \quad \nabla \cdot \mathbf{v} = 0, \quad \nabla^2 P = \nabla \cdot (\mathbf{v} \times \mathbf{w}). \quad (\text{A } 2)$$

$$362 \quad G_s, G_a \sim 1/(2\pi) \ln r \quad \text{as} \quad r \equiv |P - Q| \rightarrow 0, \quad (\text{A } 3)$$

$$363 \quad \left. \begin{array}{l} \frac{\partial G_s}{\partial y} = 0 \quad \text{on} \quad y = 0, \\ G_a = 0 \quad \text{on} \quad y = 0, \end{array} \right\} \quad (\text{A } 4)$$

$$-\frac{1}{2\pi} \int_0^\infty \gamma^{-1} [\exp(-k\gamma|y-\eta|) + \exp(-k\gamma(2d-y-\eta))] \cos k(x-\xi)t \, dt, \quad 0 < y, \quad \eta < d, \quad (\text{A } 5)$$

$$\gamma(t) = \begin{cases} -i(1-t^2)^{1/2}, & t \leq 1 \\ (t^2-1)^{1/2}, & t > 1. \end{cases} \quad (\text{A } 6)$$

$$-\frac{1}{2\pi} \int_0^\infty B(t) \frac{\cosh k\gamma(d-y)}{\gamma \sinh k\gamma d} \cos k(x-\xi)t \, dt$$

$$G = -\frac{1}{4}i(H_0(kr) + H_0(kr_1)) - \frac{1}{\pi} \int_0^\infty \frac{e^{-k\gamma d}}{\gamma \sinh k\gamma d} \cosh k\gamma(d-y) \cosh k\gamma(d-\eta) \quad (\text{A } 7)$$

Note that when equations are included in definitions, it may be suitable to render them in line, rather than in the equation environment:  $\mathbf{n}_q = (-y'(\theta), x'(\theta))/w(\theta)$ . Now  $G_a = \frac{1}{4}Y_0(kr) + \widetilde{G}_a$  where  $r = \{[x(\theta) - x(\psi)]^2 + [y(\theta) - y(\psi)]^2\}^{1/2}$  and  $\widetilde{G}_a$  is regular as  $kr \rightarrow 0$ . However, any fractions displayed like this, other than  $\frac{1}{2}$  or  $\frac{1}{4}$ , must be written on the line, and not stacked (ie 1/3).

$$\begin{aligned} \frac{\partial}{\partial n_q} \left( \frac{1}{4}Y_0(kr) \right) &\sim \frac{1}{4\pi w^3(\theta)} [x''(\theta)y'(\theta) - y''(\theta)x'(\theta)] \\ &= \frac{1}{4\pi w^3(\theta)} [\rho'(\theta)\rho''(\theta) - \rho^2(\theta) - 2\rho'^2(\theta)] \quad \text{as } kr \rightarrow 0. \end{aligned} \quad (\text{A } 8)$$

$$\frac{1}{2}\phi_i = \frac{\pi}{M} \sum_{j=1}^M \phi_j K_{ij}^a w_j, \quad i = 1, \dots, M, \quad (\text{A } 9)$$

where

$$K_{ij}^a = \begin{cases} \partial G_a(\theta_i, \theta_j)/\partial n_q, & i \neq j \\ \partial \widetilde{G}_a(\theta_i, \theta_i)/\partial n_q + [\rho'_i \rho''_i - \rho_i^2 - 2\rho_i'^2]/4\pi w_i^3, & i = j. \end{cases} \quad (\text{A } 10)$$

$$\rho_l = \lim_{\zeta \rightarrow Z_l^-(x)} \rho(x, \zeta), \quad \rho_u = \lim_{\zeta \rightarrow Z_u^+(x)} \rho(x, \zeta) \quad (\text{A } 11a, b)$$

$$(\rho(x, \zeta), \phi_{\zeta\zeta}(x, \zeta)) = (\rho_0, N_0) \quad \text{for } Z_l(x) < \zeta < Z_u(x). \quad (\text{A } 12)$$

$$\tau_{ij} = (\overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j) + (\overline{\bar{u}_i u_j^{SGS}} + \overline{u_i^{SGS} \bar{u}_j}) + \overline{u_i^{SGS} u_j^{SGS}}, \quad (\text{A } 13a)$$

$$\tau_j^\theta = (\overline{\bar{u}_j \bar{\theta}} - \bar{u}_j \bar{\theta}) + (\overline{\bar{u}_j \theta^{SGS}} + \overline{u_j^{SGS} \bar{\theta}}) + \overline{u_j^{SGS} \theta^{SGS}}. \quad (\text{A } 13b)$$

$$\mathbf{Q}_C = \begin{bmatrix} -\omega^{-2}V'_w & -(\alpha^t\omega)^{-1} & 0 & 0 & 0 \\ \frac{\beta}{\alpha\omega^2}V'_w & 0 & 0 & 0 & i\omega^{-1} \\ i\omega^{-1} & 0 & 0 & 0 & 0 \\ iR_\delta^{-1}(\alpha^t + \omega^{-1}V''_w) & 0 & -(i\alpha^t R_\delta)^{-1} & 0 & 0 \\ \frac{i\beta}{\alpha\omega}R_\delta^{-1}V''_w & 0 & 0 & 0 & 0 \\ (i\alpha^t)^{-1}V'_w & (3R_\delta^{-1} + c^t(i\alpha^t)^{-1}) & 0 & -(\alpha^t)^{-2}R_\delta^{-1} & 0 \end{bmatrix}. \quad (\text{A } 14)$$

$$\boldsymbol{\eta}^t = \hat{\boldsymbol{\eta}}^t \exp[i(\alpha^t x_1^t - \omega t)], \quad (\text{A } 15)$$

where  $\hat{\boldsymbol{\eta}}^t = \mathbf{b} \exp(i\gamma x_3^t)$ .

$$\text{Det}[\rho\omega^2\delta_{ps} - C_{pqr}^t k_q^t k_r^t] = 0, \quad (\text{A } 16)$$

$$\langle k_1^t, k_2^t, k_3^t \rangle = \langle \alpha^t, 0, \gamma \rangle \quad (\text{A } 17)$$

$$\mathbf{f}(\theta, \psi) = (g(\psi) \cos \theta, g(\psi) \sin \theta, f(\psi)). \quad (\text{A } 18)$$

$$f(\psi_1) = \frac{3b}{\pi[2(a+b\cos\psi_1)]^{3/2}} \int_0^{2\pi} \frac{(\sin\psi_1 - \sin\psi)(a+b\cos\psi)^{1/2}}{[1-\cos(\psi_1-\psi)](2+\alpha)^{1/2}} dx, \quad (\text{A } 19)$$

$$\begin{aligned} g(\psi_1) = & \frac{3}{\pi[2(a+b\cos\psi_1)]^{3/2}} \int_0^{2\pi} \left( \frac{a+b\cos\psi}{2+\alpha} \right)^{1/2} \left\{ f(\psi)[(\cos\psi_1 - b\beta_1)S + \beta_1 P] \right. \\ & \times \frac{\sin\psi_1 - \sin\psi}{1-\cos(\psi_1-\psi)} + g(\psi) \left[ \left( 2+\alpha - \frac{(\sin\psi_1 - \sin\psi)^2}{1-\cos(\psi-\psi_1)} - b^2\gamma \right) S \right. \\ & \left. \left. + \left( b^2\cos\psi_1\gamma - \frac{a}{b}\alpha \right) F\left(\frac{1}{2}\pi, \delta\right) - (2+\alpha)\cos\psi_1 E\left(\frac{1}{2}\pi, \delta\right) \right] \right\} d\psi, \end{aligned} \quad (\text{A } 20)$$

$$\alpha = \alpha(\psi, \psi_1) = \frac{b^2[1-\cos(\psi-\psi_1)]}{(a+b\cos\psi)(a+b\cos\psi_1)}, \quad \beta - \beta(\psi, \psi_1) = \frac{1-\cos(\psi-\psi_1)}{a+b\cos\psi}. \quad (\text{A } 21)$$

$$\left. \begin{aligned} H(0) &= \frac{\epsilon \bar{C}_v}{\tilde{v}_T^{1/2}(1-\beta)}, & H'(0) &= -1 + \epsilon^{2/3} \bar{C}_u + \epsilon \hat{C}'_u; \\ H''(0) &= \frac{\epsilon u_*^2}{\tilde{v}_T^{1/2} u_P^2}, & H'(\infty) &= 0. \end{aligned} \right\} \quad (\text{A } 22)$$

LEMMA 1. Let  $f(z)$  be a trial Batchelor (1971, pp. 231–232) function defined on  $[0, 1]$ . Let  $\Lambda_1$  denote the ground-state eigenvalue for  $-\text{d}^2 g/\text{d}z^2 = \Lambda g$ , where  $g$  must satisfy  $\pm \text{d}g/\text{d}z + \alpha g = 0$  at  $z = 0, 1$  for some non-negative constant  $\alpha$ . Then for any  $f$  that is not identically

396 zero we have

$$397 \quad \frac{\alpha(f^2(0) + f^2(1)) + \int_0^1 \left(\frac{df}{dz}\right)^2 dz}{\int_0^1 f^2 dz} \geq \Lambda_1 \geq \left(\frac{-\alpha + (\alpha^2 + 8\pi^2\alpha)^{1/2}}{4\pi}\right)^2. \quad (\text{A } 23)$$

398 COROLLARY 1. Any non-zero trial function  $f$  which satisfies the boundary condition  
399  $f(0) = f(1) = 0$  always satisfies

$$400 \quad \int_0^1 \left(\frac{df}{dz}\right)^2 dz. \quad (\text{A } 24)$$

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