**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: Time taken to service a car is 50 minutes.**

**Pr(X > 50) = 1 - Pr(X ≤ 50).**

**Z = (X - 45)/8.0**

**Thus the question can be answered by using the normal table to find**

**Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%**

**Probability that the service manager will not meet his demand will be = 100- 73.4 = 26.6% or 0.2676**

**D) 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans : We have a normal distribution with *μ* = 38 and *σ* = 6. Let X be the number of employees. So according to question**

1. **Probability of employees greater than age of 44= Pr(X>44)**

**Pr(X > 44) = 1 - Pr(X ≤ 44).**

**Z = (X -*μ* )/*σ* = (X - 38)/6**

**Thus the question can be answered by using the normal table to find**

**Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345%**

**Probability that the employee will be greater than age of 44 = 100- 84.1345=15.86%**

**So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%**

**Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

**B) Probability of employees less than age of 30 = Pr(X<30).**

**Z = (X - *μ*)/*σ* = (30 - 38)/6**

**Thus the question can be answered by using the normal table to find**

**Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%**

**So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48 or 36 employees).**

**Therefore the statement B of the question is also TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans: As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ12+σ22). And 2X1 will just scale the normal distribution by 2 times.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

**Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.**

**The Probability of getting value between a and b should be 0.99.**

**So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).**

**The Probability towards left from a = -0.005 (ie. 0.01/2).**

**The Probability towards right from b = +0.005 (ie. 0.01/2).**

**So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.**

**By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.**

**Z=(X- μ) / σ**

**For Probability 0.005 the Z Value is -2.57 (from Z Table).**

**Z \* σ + μ = X**

**Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4**

**Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6**

**So, option D is correct.**

**D) 48.5, 151.4**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:**

**> qnorm(0.025,45\*5,3)**

**[1] 219.1201**

**> qnorm(0.975,45\*5,3)**

**[1] 230.8799**

**> qnorm(0.025,45\*7,3)**

**[1] 309.1201**

**> qnorm(0.975,45\*7,3)**

**[1] 320.8799**

**So the Rupee range with 95% probability for the annual profit of the company is given by,**

**=Profit1 + Profit2**

**= [219.12, 230.87] + [309.12, 320.87]**

**= [528.24, 551.74]**

**So Profit of the company in Rupees (in Million) is between range [528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:**

**> qnorm(0.05,45\*7,3)**

**[1] 310.0654**

**> qnorm(0.05,45\*5,3)**

**[1] 220.0654**

**5th percentile of profit = 310.0654+ 220.0654 = 530.1308 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans: Division 2 with distribution N(7, 42)**