

# Abstract Mathematics 101 Bootcamp

## Lecture 7 (PART B) Introduction to Metric Topology

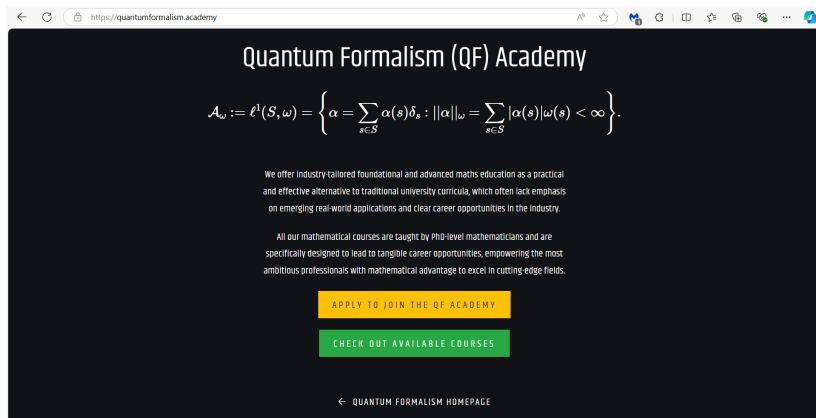
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Quantum Formalism (QF) Academy

$$\mathcal{A}_\omega := \ell^1(S, \omega) = \left\{ \alpha = \sum_{s \in S} \alpha(s) \delta_s : \|\alpha\|_\omega = \sum_{s \in S} |\alpha(s)| \omega(s) < \infty \right\}.$$

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# Bootcamp Overview

Metric Topology

Topological Manifolds

Naive Set Theory



Point-set Topology



**Module I (May 24th - August 30)**



# Lecture 7A Recap

- Metric spaces
- Open sets in metric spaces

## Proposition (1.1 in Lecture 7A)

Given any metric space  $(M, d)$ , the following properties hold:

- 1  $\emptyset$  and  $M$  are open.
- 2 If  $U_1, U_2, \dots, U_k$  are open subsets of  $M$ , then their intersection  $U_1 \cap U_2 \cap \dots \cap U_k$  is open.
- 3 If  $\{U_i\}_{i \in I}$  is an arbitrary collection of open subsets of  $M$ , then the union over the collection  $\bigcup_{i \in I} U_i$  is open.



# Closed Sets in Metric Spaces

## Definition 1.0

Let  $(M, d)$  be a metric space. We say that a subset  $A \subseteq M$  is closed in respect to the metric  $d$  if  $A^c$  is open.

**Curiosity question (homework):** Let  $M = \mathbb{R}$  and  $d$  the Euclidean metric. Is it true that the intervals  $[a, b]$  and  $(-\infty, 0)$  are closed?



# Closed Sets in Metric Spaces

## Proposition 1.0

Given any metric space  $(M, d)$ , the following properties hold:

- 1  $\emptyset$  and  $M$  are closed.
- 2 If  $C_1, C_2, \dots, C_k$  are closed subsets of  $M$ , then their union  $C_1 \cup C_2 \cup \dots \cup C_k$  is closed.
- 3 If  $\{C_i\}_{i \in I}$  is an arbitrary collection of closed subsets of  $M$ , then the intersection over the collection  $\bigcap_{i \in I} C_i$  is closed.

**Question:** The properties above are the same as the closed sets in point-set topology right?



# The Metric Topology

## Proposition 1.1

Given a metric space  $(M, d)$ , the following collection of subsets is a topology on  $M$ :

$$\mathcal{T}_d = \{U \subseteq M \mid U \text{ is open with respect to } d\}.$$

*Proof* : Homework (trivial if you already proved the properties of open subsets)!

- $\mathcal{T}_d$  is called a metric topology on  $M$ , i.e. the topology induced by the metric  $d$ . Hence, we'll write  $(M, \mathcal{T}_d)$  to denote the topological space induced by  $d$ .



# The Standard Topology

## Definition 1.1

When  $M = \mathbb{R}^n$  and  $d$  is the Euclidean metric, then  $\mathcal{T}_d$  is called the 'standard topology' on  $\mathbb{R}^n$ .

- Most topological spaces of interest in applied subjects such as physics will be subspaces of  $(\mathbb{R}^n, \mathcal{T}_d)$ .
- From now on, always assume  $\mathcal{T}_d$  to be the standard topology whenever  $M \subseteq \mathbb{R}^n$ .
- When the metric is understood from the context, we'll also just write  $M$  for the underlying topology instead  $(M, \mathcal{T}_d)$ .





# Homework Exercises (i)

- Consider the circle  $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ . Show that the subspace topology induced by the standard topology on  $\mathbb{R}^2$  makes  $S^1$  into a topological space.
- Show that the real line interval  $[0, 1]$  equipped with the subspace topology from the standard topology on  $\mathbb{R}$  is a topological space.

**Important note:**  $[0, 1]$  is a very important topological space as it will help construct 'Path-Connected' topological spaces. Not to be mistaken with 'Connected' topological spaces!



# Homework Exercises (ii)

- 1 Consider the topological spaces  $X = (-1, 1)$  and  $Y = (0, 5)$  constructed from the standard topology on  $\mathbb{R}$ . Let  $f : X \longrightarrow Y$  be defined as  $f(x) = \frac{5}{2}(x + 1)$ . Is  $f$  a homeomorphism?
- 2 Consider now the topological spaces  $X = (-1, 1)$  and  $Y = \mathbb{R}$  again constructed from the standard topology on  $\mathbb{R}$ . Let  $f : X \longrightarrow Y$  be defined as  $f(x) = \tan\left(\frac{\pi x}{2}\right)$ . Is  $f$  a homeomorphism?
- 3 Prove or disprove whether the circle  $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$  is homeomorphic to  $X = [0, 1]$ .



# Congratulations for making it this far!

