

# Abstract Mathematics 101 Bootcamp

## Lecture 7 (PART C) Introduction to Metric Topology

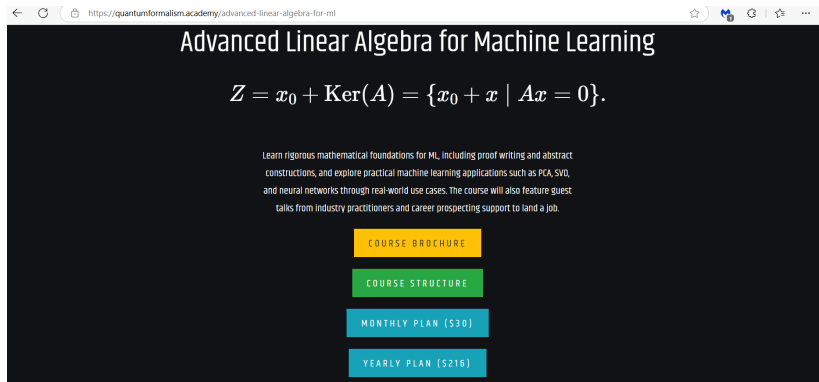
Bambordé Baldé

Quantum Formalism (QF) Free Bootcamp  
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QF's CORE MISSION:  
Make Abstract Mathematics Accessible.



# QF Academy Career-focused Course



A screenshot of a web browser displaying the QF Academy website. The browser's address bar shows the URL <https://quantumformalism.academy/advanced-linear-algebra-for-ml>. The website has a dark background with white text. At the top, the title "Advanced Linear Algebra for Machine Learning" is displayed. Below it, the mathematical equation  $Z = x_0 + \text{Ker}(A) = \{x_0 + x \mid Ax = 0\}.$  is shown. A paragraph of text describes the course content, mentioning rigorous mathematical foundations, proof writing, abstract constructions, and practical machine learning applications like PCA, SVD, and neural networks. At the bottom, there are four colored buttons: "COURSE BROCHURE" (yellow), "COURSE STRUCTURE" (green), "MONTHLY PLAN (\$30)" (teal), and "YEARLY PLAN (\$216)" (blue).

Advanced Linear Algebra for Machine Learning

$$Z = x_0 + \text{Ker}(A) = \{x_0 + x \mid Ax = 0\}.$$

Learn rigorous mathematical foundations for ML, including proof writing and abstract constructions, and explore practical machine learning applications such as PCA, SVD, and neural networks through real-world use cases. The course will also feature guest talks from industry practitioners and career prospecting support to land a job.

COURSE BROCHURE

COURSE STRUCTURE

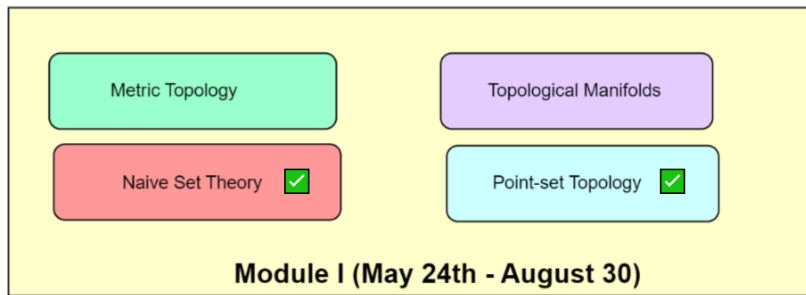
MONTHLY PLAN (\$30)

YEARLY PLAN (\$216)

Visit [quantumformalism.academy](https://quantumformalism.academy) to learn more.



# Bootcamp Overview



# Lecture 7B Recap

- Closed Sets in Metric Spaces
- The Metric Topology
- The Standard Topology



# Equivalent Metrics

## Definition 1.0

Let  $d_1$  and  $d_2$  be two metrics on  $M$  with corresponding topologies  $\mathcal{T}_{d_1}$  and  $\mathcal{T}_{d_2}$ . The metrics  $d_1$  and  $d_2$  are said to be equivalent if  $\mathcal{T}_{d_1} = \mathcal{T}_{d_2}$ , i.e., if the two metrics generate the same open sets.

- We write  $d_1 \simeq d_2$  to indicate the equivalence between the two.

**Curiosity question (homework):** Let  $M = \mathbb{R}^n$  and  $d$  the Euclidean metric with corresponding standard topology  $\mathcal{T}_d$ . Can you find other metrics defined in  $\mathbb{R}^n$  that are equivalent to the Euclidean metric  $d$ ?



# The Metric Topology's Hausdorffness

## Theorem 1.0

For any metric space  $(M, d)$ , the induced topological space  $(M, \mathcal{T}_d)$  is a Hausdorff space.

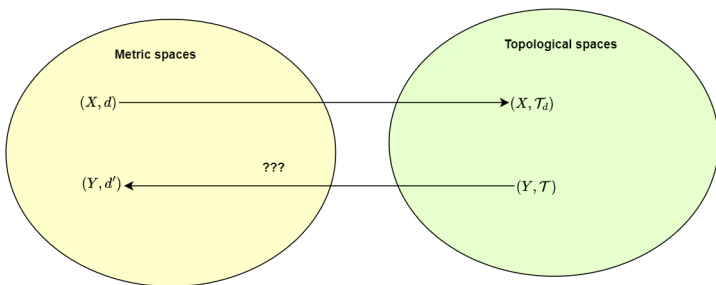
- Recall that  $(M, \mathcal{T}_d)$  to be Hausdorff, it needs to satisfy the following: For all  $p_1, p_2 \in M$  with  $p_1 \neq p_2$ , there exist some neighbourhoods  $N_{p_1}$  and  $N_{p_2}$  of the respective points such that  $N_{p_1} \cap N_{p_2} = \emptyset$ .

**Question:** Does the converse hold i.e. is every Hausdorff space a metric space? In other words, is being Hausdorff a minimum and necessary condition for a topological space to be metrisable?



# The Metrisability Question

- Can we construct metric spaces from topological spaces? If yes, what are the minimum and necessary conditions for doing so?



# Metrisable Topological Spaces

## Definition 1.1

A topological space  $(X, \mathcal{T})$  is *metrisable* if there exists a metric  $d$  on  $X$  such that  $\mathcal{T} = \mathcal{T}_d$ .

- A famous metrisation theorem by soviet mathematician Pavel Samuilovich Urysohn states that: Every second-countable, regular Hausdorff space  $X$  is metrisable.
- Nagata-Smirnov metrisation theorem: A topological space is metrisable if and only if it is regular, Hausdorff and has a countably locally finite ( $\sigma$ -locally finite) basis.





# Homework Exercises (i)

- Let  $X$  be any set and  $\mathcal{T} = \mathcal{P}(X)$ . Is  $(X, \mathcal{T})$  metrisable?
- What if we now have  $\mathcal{T} = \{\emptyset, X\}$ ?
- Now consider  $X = \{0, 1\}$  and  $\mathcal{T} = \{\emptyset, X, \{1\}\}$  (Sierpinski topology). Is  $(X, \mathcal{T})$  metrisable?

**Natural curiosity (homework):** Suppose now that  $X = \mathbb{R}$ , can you construct a non-trivial example of a non-metrisable topology on  $\mathbb{R}$ ?



# Congratulations for making it this far!



# The Upcoming Lectures (8A,8B)

- Topological Bases
- Product Topology
- Projection Maps
- Topological Invariants
- Connected Spaces
- Connected Subsets

