

Abstract Mathematics 101 Bootcamp

Lecture 8 (PART A) Introduction to Metric Topology

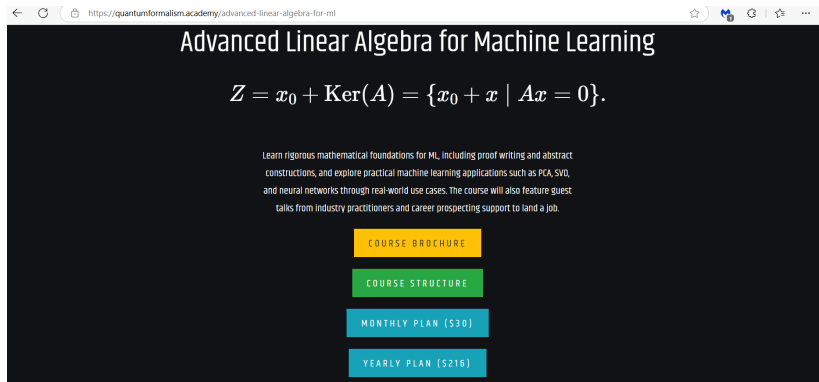
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Quantum Formalism (QF) Free Bootcamp
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QF's CORE MISSION:
Make Abstract Mathematics Accessible.



QF Academy Career-focused Course



A screenshot of a web browser displaying the course page for 'Advanced Linear Algebra for Machine Learning' on the quantumformalism.academy website. The browser's address bar shows the URL 'https://quantumformalism.academy/advanced-linear-algebra-for-ml'. The page has a dark background with white text. At the top, the course title is displayed in a large font. Below it, a mathematical equation is shown: $Z = x_0 + \text{Ker}(A) = \{x_0 + x \mid Ax = 0\}.$ Further down, a paragraph describes the course content, mentioning rigorous mathematical foundations, proof writing, abstract constructions, and practical machine learning applications like PCA, SVD, and neural networks. At the bottom of the page, there are four colored buttons: 'COURSE BROCHURE' (yellow), 'COURSE STRUCTURE' (green), 'MONTHLY PLAN (\$30)' (teal), and 'YEARLY PLAN (\$216)' (light blue).

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Advanced Linear Algebra for Machine Learning

$$Z = x_0 + \text{Ker}(A) = \{x_0 + x \mid Ax = 0\}.$$

Learn rigorous mathematical foundations for ML, including proof writing and abstract constructions, and explore practical machine learning applications such as PCA, SVD, and neural networks through real-world use cases. The course will also feature guest talks from industry practitioners and career prospecting support to land a job.

COURSE BROCHURE

COURSE STRUCTURE

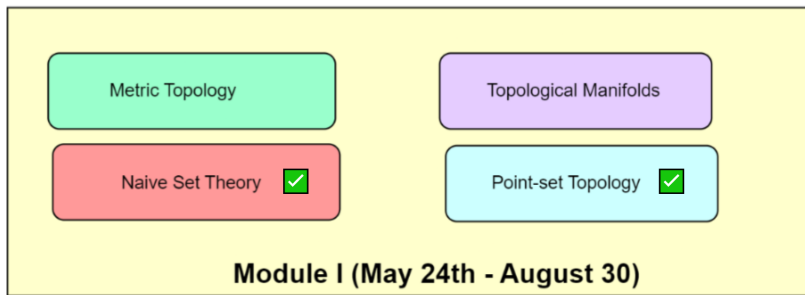
MONTHLY PLAN (\$30)

YEARLY PLAN (\$216)

Visit quantumformalism.academy/courses to learn more.



Bootcamp Overview



Lecture 7C Recap

- Equivalent Metrics
- The Metric Topology's Hausdorffness
- Metrisable Topological Spaces



Topological Bases

Definition 1.0

Let (X, \mathcal{T}) be any topological space. A collection of open sets $\mathcal{B} \subseteq \mathcal{T}$ is called a **basis** (or just **base**) if for any open set $O \in \mathcal{T}$ there is a $\mathcal{B}' = \{B_i\}_{i \in I} \subseteq \mathcal{B}$ such that $O = \bigcup_{i \in I} B_i$.

- \mathcal{T} itself is a basis, right? For any open set $O \in \mathcal{T}$, we can make $\mathcal{B}' = \{\emptyset, O\}$ and so $O = \emptyset \cup O$?
- If $X = \mathbb{R}^2$ with the standard topology \mathcal{T}_d given by the Euclidean metric d , then $\mathcal{B} = \{B_r(p) \mid p \in \mathbb{R}^2, r > 0\}$ forms a basis.



The Metric Topology's Base

Proposition 1.0

If X is a topological space induced by a metric topology \mathcal{T}_d , then the collection of all the open balls $\mathcal{B} = \{B_r(p) \mid p \in X, r > 0\}$ forms a basis.

Proof : Homework!

- This shows the importance of open balls in metric spaces!

Natural curiosity: Can topological bases be uncountable? In the definition of basis, we didn't mention anything about cardinality right?



Second-Countable Spaces

Definition 1.1

A topological space (X, \mathcal{T}) is called 'second-countable' if it has a countable basis $\mathcal{B} \subseteq \mathcal{T}$.

- This notion of being second-countable is very important. Most practical topological spaces i.e. the ones used in applied topics such as theoretical physics, are second-countable.
- Indeed, the standard definition of a topological manifold requires the underlying topological to be second-countable and Hausdorff!



Homework Exercises

Exercise (i)

If $X = \{\beta_1, \beta_2, \beta_3\}$ and $\mathcal{T} = \{\emptyset, X, \{\beta_1\}, \{\beta_2\}, \{\beta_1, \beta_2\}\}$. Which of the following (if any) forms a basis:

- ① $\mathcal{B}_1 = \{X, \{\beta_1\}, \{\beta_2\}\}$.
- ② $\mathcal{B}_2 = \{\emptyset, X, \{\beta_1\}, \{\beta_2\}\}$.
- ③ $\mathcal{B}_3 = \{\emptyset, X, \{\beta_1\}\}$.
- ④ $\mathcal{B}_4 = \{\emptyset, X, \{\beta_2\}\}$.



Homework Exercises

Exercise (ii)

Which of the following statements are true (you need to prove):

- ① The collection $\mathcal{B} = \{(a, b) \subseteq \mathbb{R} : a < b\}$ is a basis on \mathbb{R} in respect to the standard topology.
- ② The collection $\mathcal{B} = \{[a, b] \subseteq \mathbb{R} : a < b\}$ is a basis on \mathbb{R} in respect to the standard topology.
- ③ The collection $\mathcal{B} = \{[a, b) \subseteq \mathbb{R} : a < b\}$ is a basis on \mathbb{R} in respect to the standard topology.



Congratulations for making it this far!



The Upcoming Lectures (8B)

- Product Topology
- Projection Maps

