Homework 8

Directions: Answer the following questions. You are encouraged to work together, join the discussion sessions, use discord, and ask me questions!

1. Suppose f is measurable, non-negative, and $\int f \, d\mu = 0$. Then $f = 0$ a.e.

2. Suppose f is real-valued, measurable, and for every measurable set A, we have $\int_A f \, d\mu = 0$. Then $f = 0$ a.e.

3. Find an example of a sequence of functions $\{f_n\}$ on $[0, 1]$ such that each f_n is Riemann integrable, $f_n \leq f_{n+1}$, and $f_n \to f$, but f is not Riemann integrable.

   Remark: This further elucidates the strength of the Lebesgue integral compared to the Riemann integral by showing that MCT and DCT do not hold for Riemann integrals. Take some time to reflect upon what the difference is between a Riemann sum is compared to a simple function!
