Homework 5

Directions: Answer the following questions. You are encouraged to work together, join the discussion sessions, use discord, and ask me questions!

1. Let E be a Lebesgue measurable set.

   a) Let N be the non-measurable set given in lecture 2, (one of the so-called Vitali sets). If $E \subset N$, then $m(E) = 0$, where m is the standard Lebesgue measure.

   b) If $m(E) > 0$, then E contains a nonmeasurable set. You may assume $E \subset [0, 1]$. **Hint:** Using N_r as in the Vitali set example, you may write $E = \cup_{r \in R} N_r$.

2. Let $\epsilon \in (0, 1)$ and suppose that A is a Borel measurable subset of $\mathbb{R}$ with $m(A) > 0$. Prove that if

$$m(A \cap I) \leq (1 - \epsilon) m(I)$$

   for every interval I, then $m(A) = 0$.
