QF Group Theory CC2022
By
Zaiku Group

Lecture 11

Delivered by Bambordé Baldé

Friday, 29/7/2022

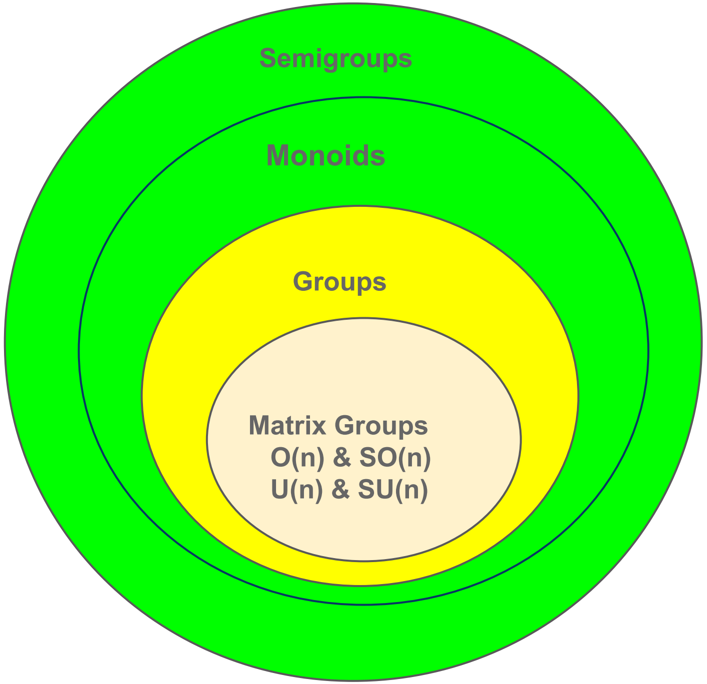
Learning Journey Timeline

Topology & Differential Geometry
9

Group Theory
11/20?

Lie Groups & Rep.
10 - 16?

Completed | Ongoing | TBC (summer) | n is the number of live lectures |



Course Approach Overview

Completed!

We’re here!

Measure Theory & Functional Analysis

$$\int_S f \, \mathrm{d}\mu.$$

Starts on August 29, 2022.

Course Lecturer

James Heffers

Postdoctoral Assistant Professor at University of Michigan who specialises in Several Complex Variables/Pluripotential Theory

Course Structure

Lecture 1	Calculus and Analysis Review (namely integration theory)
Lecture 2 - 5	Sigma algebras, measure axioms/properties, outer measures, borel measures on R.
Lecture 5 - 10	Measurable Functions, integration of non-negative functions, Monotone/Dominated convergence theorem, Fubini-Tonelli, Lebesgue Integrals vs Riemann Integrals
Lecture 11 - 18	Normed Vector Spaces, Linear Functionals, Open Mapping Theorem, Topological Vector Spaces
Lecture 18 - 22	Hilbert Spaces & Operators

k-cycles and disjoint cycles

Definition 1.0

Let $X = \{1, 2, 3, \ldots, n\}$. A permutation $\tilde{\sigma} \in S_n$ is called a $k-$ cycle if there exists a subset $\{a_1, a_2, a_3, \ldots, a_k\}$ of X with exactly $k-$ elements such that $\tilde{\sigma}$ behaves in the following way:

1. $\tilde{\sigma}(a_i) = a_{i+1}$ if $i < k$.
2. $\tilde{\sigma}(a_k) = a_1$.
3. $\tilde{\sigma}(j) = j$ if $j \neq a_i$ for all i.

- The subset $\{a_1, a_2, a_3, \ldots, a_k\}$ is called the support of $\tilde{\sigma}$ and we write $Supp(\tilde{\sigma})$ to denote it.
- When $k = 2$ then a $2-$ cycle is also known as 'transposition'!

Definition 1.1

Let $\tilde{\sigma_1}$ be a k_1- cycle and $\tilde{\sigma_2}$ be a k_2-cycle. Then we say the cycles $\tilde{\sigma_1}$ and $\tilde{\sigma_2}$ are disjoint if $Supp_{(}\tilde{\sigma_1}) \bigcap Supp(\tilde{\sigma_2}) = \emptyset$.

Concrete Examples (2- cycles in S_4)

Let $X = \{1, 2, 3, 4\}$ and $Sym(X) = S_4$ be its symmetric group of $4! = 24$ permutations on the set X.

1. Consider the following permutation given by the 2-line notation and an intuitive diagrammatic picture:

$$\tilde{\sigma_1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 3 & 4 \end{pmatrix}$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[l, bend right] \end{tikzcd}$$

Then $\tilde{\sigma_1}$ is a $2-cycle$ with $Supp(\tilde{\sigma_1}) = \{1, 2\}$ right?

2. Consider now the permutation $\tilde{\sigma_2}$

$$\tilde{\sigma_2} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 4 & 3 \end{pmatrix}$$

$$\begin{tikzcd} 3 \arrow[r] & 4 \arrow[l, bend right] \end{tikzcd}$$

Is $\tilde{\sigma_2}$ also a $2-cycle$ and $Supp(\tilde{\sigma_2}) = \{3, 4\}$?

Challenge 1

Are there other $2 - \mathit{cycles}$ on S_4 other than the examples above? If yes, you're encouraged to:

1. Identify all the $2 - \mathit{cycles}$ of S_4.
2. How many disjoint $2 - \mathit{cycles}$ are in S_4?

Concrete Examples (3- cycles in S_4)

Let $X = \{1, 2, 3, 4\}$ and $Sym(X) = S_4$ be its symmetric group of $4! = 24$ permutations on the set X.

1. Consider the following permutation given by the 2-line notation and an intuitive diagrammatic picture:

$$\tilde{\sigma_1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 3 & 1 & 4 \end{pmatrix}$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[r] & 3 \arrow[ll, bend right] \end{tikzcd}$$

Then $\tilde{\sigma_1}$ is a $3 - \mathit{cycle}$ with $Supp(\tilde{\sigma_1}) = \{1, 2, 3\}$ right?

2. Consider now the permutation $\tilde{\sigma_2}$

$$\tilde{\sigma_2} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[r] & 4 \arrow[ll, bend right] \end{tikzcd}$$

Is $\tilde{\sigma_2}$ also a $3 - \mathit{cycle}$ and $Supp(\tilde{\sigma_2}) = \{1, 2, 4\}$?

Challenge 2

Are there other $3-cycles$ on S_4 other than the examples above? If yes, you're encouraged to:

1. Identify all the $3-cycles$ of S_4.
2. How many disjoint $3-cycles$ are in S_4?
3. Identify all the $4-cycles$ in S_4. Also, how many are disjoint?
4. Are there $5-$ cycles in S_4?

Cycle Notation Convention

Definition 1.2

Let $\tilde{\sigma} \in S_n$ be a $k-$ cycle with support $\{a_1, a_2, a_3, \ldots, a_k\}$. It is a standard convention to use the so-called cycle notation as follows:

1. We write $(a_1 a_2 a_3 \ldots a_k)$ to denote the permutation $\tilde{\sigma}$!
2. Consequently, the support is denoted $Supp(a_1 a_2 a_3 \ldots a_k)$.

Concrete Examples:

1. Consider the following S_4 $2-cycle$ encountered previously:

$$\tilde{\sigma_1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 3 & 4 \end{pmatrix}$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[l, bend right] \end{tikzcd}$$

In cycle notation we'll rewrite it as (12).

2. Similarly, consider the following S_4 $3-cycle$ encountered previously:

$$\tilde{\sigma_1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 3 & 1 & 4 \end{pmatrix}$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[r] & 3 \arrow[ll, bend right] \end{tikzcd}$$

In cycle notation we'll rewrite it as (123).

Challenge 3

Identify all the cycles of S_2 and S_3. Then identify all the disjoint cycles of S_2 and S_3.

QF

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