Question One:

A)

Corresponding lean proof:

```
theorem q1a (A B C : Prop) :
  (A ∧ C) ∨ (B ∧ C) → (A ∨ B) ∧ C  :=
assume h : (A ∧ C) ∨ (B ∧ C),
and.intro
  (or.elim h
    (assume k : A ∧ C, or.intro_left B (and.elim_left(k)))
    (assume l : B ∧ C, or.intro_right A (and.elim_left(l))))
  (or.elim h
    (assume k : A ∧ C, and.elim_right(k))
    (assume l : B ∧ C, and.elim_right(l)))
```

B)

Corresponding proof tree:

$$
\dfrac{
  \dfrac{}{(A \wedge C) \vee (B \wedge C)}\,\text{h}
  \qquad
  \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \wedge C}\,\text{k}}{A, C}\,[\wedge\text{E}]}{A \vee B, A, C}\,[\vee\text{I}]}{(A \vee B) \wedge C}\,[\wedge\text{I}]}{A \to (A \vee B) \wedge C}\,\text{k}[\to\text{I}]
  \qquad
  \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \wedge C}\,\text{k}}{B, C}\,[\wedge\text{E}]}{A \vee B, B, C}\,[\vee\text{I}]}{(A \vee B) \wedge C}\,[\wedge\text{I}]}{A \to (A \vee B) \wedge C}\,\text{k}[\to\text{I}]
}{
  \dfrac{(A \vee B) \wedge C}{(A \wedge C) \vee (B \wedge C) \to (A \vee B) \wedge C}\,\text{h}[\to\text{I}]
}\,[\vee E]
$$

Question Two:

Truth table to check validity of the logical equivalence $(A \wedge C) \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge C$:

A	B	C	$A \wedge C$	$B \wedge C$	$(A \wedge C) \wedge (B \wedge C)$	$(A \vee B)$	$(A \vee B) \vee C$
F	F	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	T	F	F	F	F	T	F
F	T	T	F	T	T	T	T
T	F	F	F	F	F	T	F
T	F	T	T	F	T	T	T
T	T	F	F	F	F	T	F
T	T	T	T	T	T	T	T

Question Three:

A)

Proof tree for proof of $A \to (A \land B) \lor (A \land \neg B)$ by natural deduction:

$$\dfrac{\dfrac{}{B \lor \neg B}\,\text{EM} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A}\,\text{x} \quad \dfrac{}{B}\,\text{y}}{A \land B}\,[\land\text{I}]}{(A \land B) \lor (A \land \neg B)}\,[\lor\text{I}]}{B \to (A \land B) \lor (A \land \neg B)}\,\text{y}[\to\text{I}] \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A}\,\text{x} \quad \dfrac{}{\neg B}\,\text{z}}{A \land \neg B}\,[\land\text{I}]}{(A \land B) \lor (A \land \neg B)}\,[\lor\text{I}]}{\neg B \to (A \land B) \lor (A \land \neg B)}\,\text{z}[\to\text{I}]}{\dfrac{(A \land B) \lor (A \land \neg B)}{A \to (A \land B) \lor (A \land \neg B)}\,\text{x}[\to\text{I}]}\,[\lor E]$$

B)

Proof implemented in lean:

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open classical

theorem q3b (A B : Prop) :
  A → (A ∧ B) ∨ (A ∧ ¬B) :=
assume x : A,
or.elim (em B)
(assume y : B, or.intro_left (A ∧ ¬B) (and.intro(x)(y)))
(assume z : ¬B, or.intro_right (A ∧ B) (and.intro(x)(z)))
```

Question 4:

A)

Expressing $P \land (Q \to R)$ in CNF form:
First we must create the corresponding truth table.

P	Q	R	P	$Q \to R$	$P \land (Q \to R)$
F	F	F	F	T	F
F	F	T	F	T	F
F	T	F	F	F	F
F	T	T	F	T	F
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	T	F	F
T	T	T	T	T	T

Enumerating all the rows from the conclusion column.

1. $\neg P \wedge \neg Q \wedge \neg R$
2. $\neg P \wedge \neg Q \wedge R$
3. $\neg P \wedge Q \wedge \neg R$
4. $\neg P \wedge Q \wedge R$
5. $P \wedge Q \wedge R$

After negating these formulae (from only the T rows) then joining them with an AND we get:

$$(P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)$$

B)

Expressing $P \wedge (Q \rightarrow R)$ in DNF form: First we must create the corresponding truth table.

P	Q	R	P	$Q \rightarrow R$	$P \wedge (Q \rightarrow R)$
F	F	F	F	T	F
F	F	T	F	T	F
F	T	F	F	F	F
F	T	T	F	T	F
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	T	F	F
T	T	T	T	T	T

Enumerating all the rows from the conclusion column.

1. $\neg P \wedge \neg Q \wedge \neg R$
2. $\neg P \wedge \neg Q \wedge R$
3. $\neg P \wedge Q \wedge \neg R$
4. $\neg P \wedge Q \wedge R$
5. $P \wedge Q \wedge R$

After joining these formulae (from only the T rows) with an OR we get:

$$(P \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R)$$