DOCTORAL PROGRAM
SWISS FINANCE INSTITUTE

FINANCIAL ECONOMETRICS

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Eric Jondeau

Outline of the Financial Econometrics Course

1. Characteristics of Financial Time Series
2. Modeling Volatility: GARCH Models
3. Modeling Non-Normality
4. Multivariate Models

Readings

Jondeau E., S.-H. Poon, and M. Rockinger (2006), Financial Modeling Under Non-Gaussian Distributions, Springer Finance.

Tsay R. (2002), Analysis of Financial Time Series, Wiley Series in Probability and Statistics.

- **Other books in Econometrics:**

Greene, W. (1993), Econometric Analysis, Macmillan.

Hamilton, J. (1994), Financial Time Series Analysis, Princeton.

FINANCIAL ECONOMETRICS

Lecture 1: Characteristics of Financial Time Series

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Objectives of the lecture

The objectives of this lecture are the following:

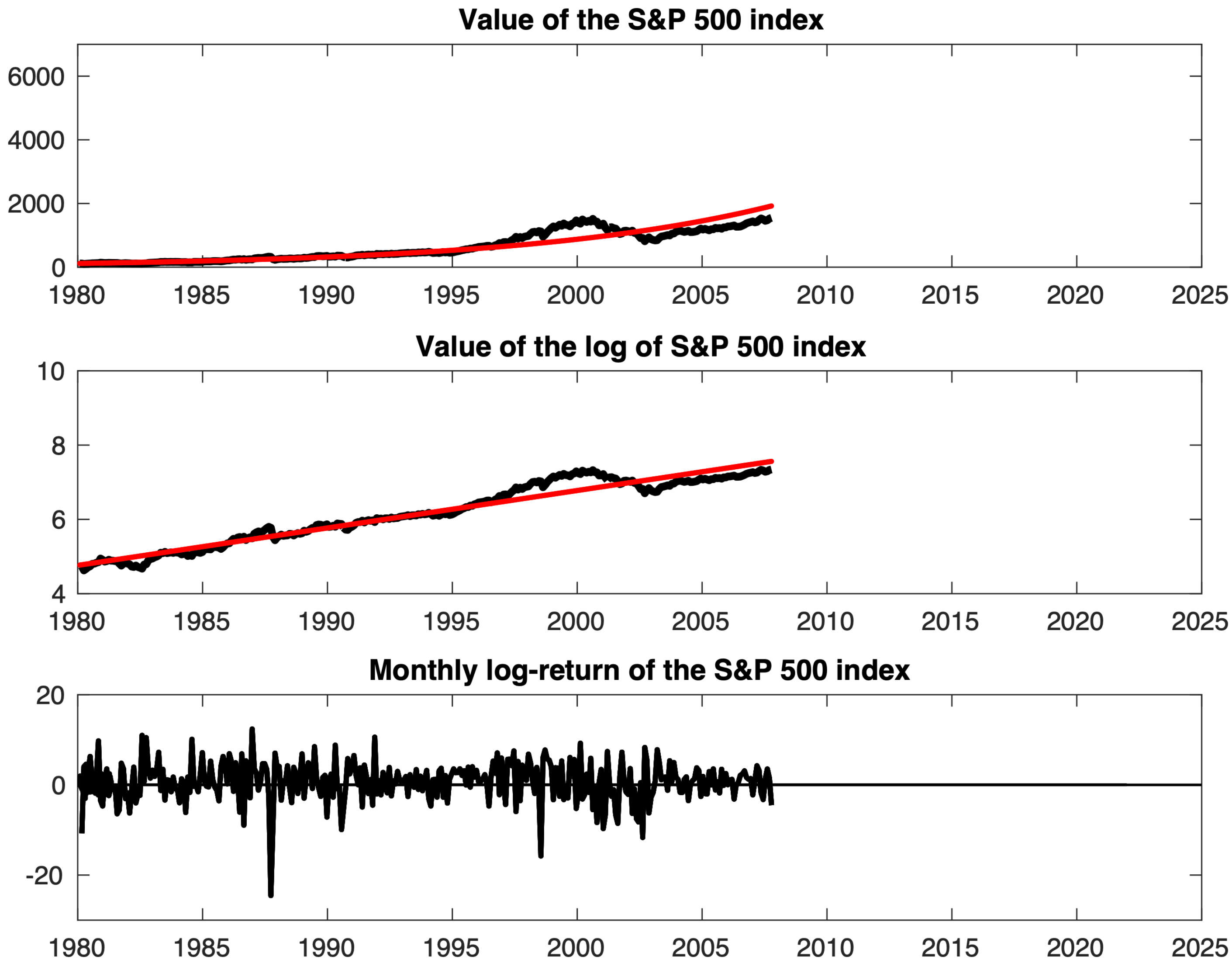
- **Brief description of the variables we are going to use – Asset Returns**
- **Main properties of the distribution of asset returns – Moments**
- **How to test assumptions regarding the distribution of asset returns**
- **Main properties of the time dependency of asset returns – Heteroskedasticity**
- **How to test assumptions regarding the time dependency of asset returns**
- **Main properties of the correlation across asset returns**

Objectives of the lecture

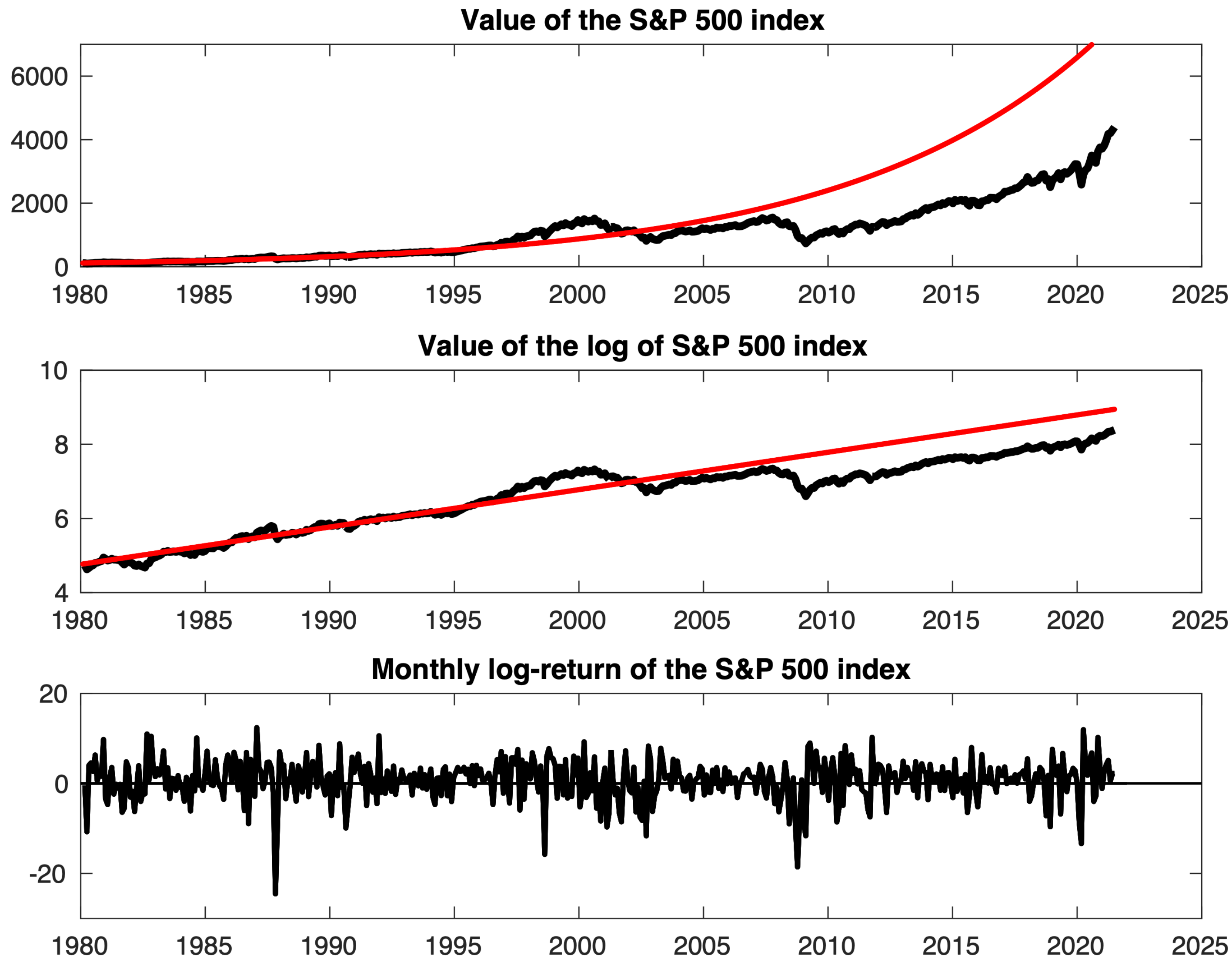
➔ **Asset Returns**

- **Distribution of asset returns**
- **Tests on the distribution of asset returns**
- **Time dependency of asset returns**
- **Correlation across asset returns**

Prices or returns?



Prices or returns?



Simple return

Two main reasons for investigating return's properties instead of price's properties:

- Investors are mostly interested in returns for their investment decisions
- Properties of returns are in general easier to handle than the properties of price

Holding the asset for one period from $t-1$ to t yields the **one-period simple return**.

$$R_t = \frac{P_t - P_{t-1}}{P_t} \qquad \text{or} \qquad P_t = P_{t-1}(1 + R_t)$$

where

P_t: price of the asset at date t,
R_t: one-period simple return from date $t-1$ to date t.

Continuously compounded (or log-) return

Payment of an annual interest of $R_t^{(m)}$ split into m payments in a year:

- the interest rate for each payment is $R_t^{(m)} / m$
- the net value of the deposit one year later will be $(1 + R_t^{(m)} / m)^m$

In the limit case where the interest is paid continuously ($m \to \infty$):

$$\lim_{m\to\infty}\left(1+\frac{R_t^{(m)}}{m}\right)^m = \exp(r_t) \qquad \text{with} \qquad P_t = P_{t-1} \times \exp(r_t)$$

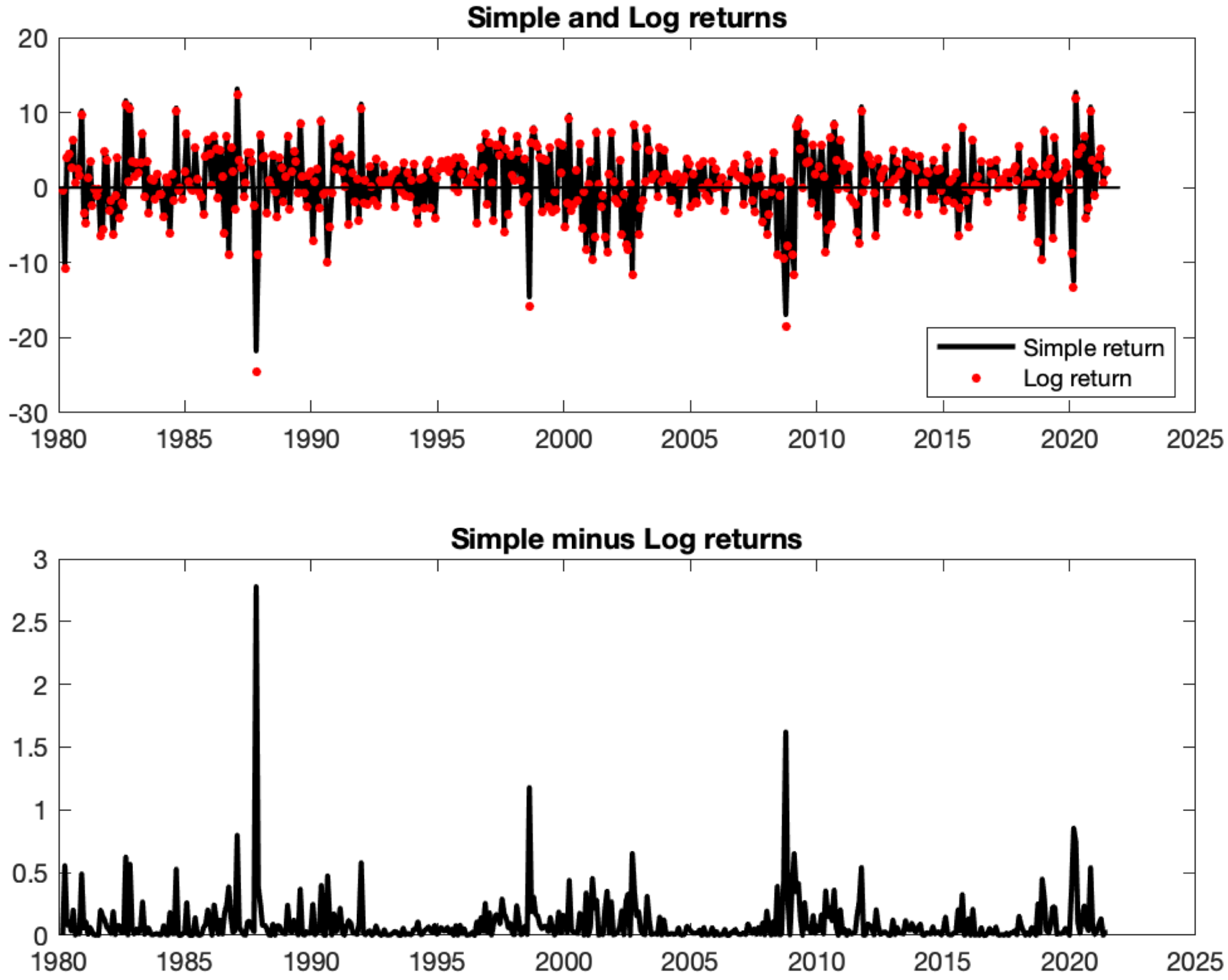
The **continuously compounded (or log-) return** r_t is:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = p_t - p_{t-1}$$

where $p_t = \log(P_t)$ is the log-price

Simple return or Log-return?

S&P 500 monthly returns



Temporal aggregation (multiple-period return)

Holding the asset for *k* periods from t to $t+k$ yields the ***k*-period simple return**:

$$R_t[k] = \frac{P_{t+k} - P_t}{P_t} \qquad \text{or} \qquad P_{t+k} = P_t\left(1 + R_t[k]\right) = P_t\left(1 + R_{t+1}\right)\times\ldots\times\left(1 + R_{t+k}\right)$$

so that

$$R_t[k] = \left(\prod_{j=1}^{k}\left(1 + R_{t+j}\right)\right) - 1$$

The ***k*-period log return** is simply the sum of continuously compounded one-period returns:

$$r_t[k] = \log\left(1 + R_t[k]\right) = \log\left(\prod_{j=1}^{k}\left(1 + R_{t+j}\right)\right) = \sum_{j=1}^{k}\log\left(1 + R_{t+j}\right)$$

so that

$$r_t[k] = \sum_{j=1}^{k} r_{t+j}$$

Contemporaneous aggregation (Portfolio return)

Let p be the portfolio of N assets with weight w_i on the asset i.

The **simple portfolio return** $R_{p,t}$ is the weighted average of the simple returns of the assets:

$$R_{p,t} = \sum_{i=1}^{N} w_i R_{i,t}$$

In contrast, with continuous compounding, the **log portfolio return** $r_{p,t}$ is not the weighted average of the log returns of the assets:

$$r_{p,t} = \log\left(\sum_{i=1}^{N} w_i e^{r_{i,t}}\right) \neq \sum_{i=1}^{N} w_i r_{i,t}$$

Using simple returns or log-returns is an empirical issue, which depends in general on the problem at hand.

Other definitions

Remark 1: Dividend payments

For assets with periodic dividend payments, asset returns must be redefined.

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1 \qquad \text{and} \qquad r_t = \log\left(P_t + D_t\right) - \log\left(P_{t-1}\right)$$

where D_t is the dividend payment of an asset between dates $t-1$ and t
P_t is the price of the asset at the end of period t (dividend not included in P_t).

Most reference indices include dividend payments but not all.

Some reference indices (e.g., MSCI indices) are computed without ("price index") or with dividend payments ("total return index").

Remark 2: Excess returns

Excess return is simply the difference between the asset return and the return on the risk-free asset, in practice the short-term Treasury bill return.

$$Z_{i,t} = R_{i,t} - R_{f,t} \qquad \text{and} \qquad z_{i,t} = r_{i,t} - r_{f,t}$$

with $R_{f,t}$ and $r_{f,t}$ the simple return and log return of the risk-free asset.

Characteristics of Financial Asset Returns

Early work in Finance has made some very strong assumptions for asset returns. The most crucial assumptions are

1. Normality of log-returns

It is a convenient assumption for many applications in Finance (cf. Black-Scholes model for option pricing).

For stock-index returns, it is consistent with the Central Limit Theorem if log-returns are i.i.d.

2. Time independency (i.i.d., or *independent and identically distributed*, process)

It is, to some extent, an implication of the Efficient Market Hypothesis.

In fact, the EMH only imposes unpredictability of returns.

Objectives of the lecture

- Asset Returns

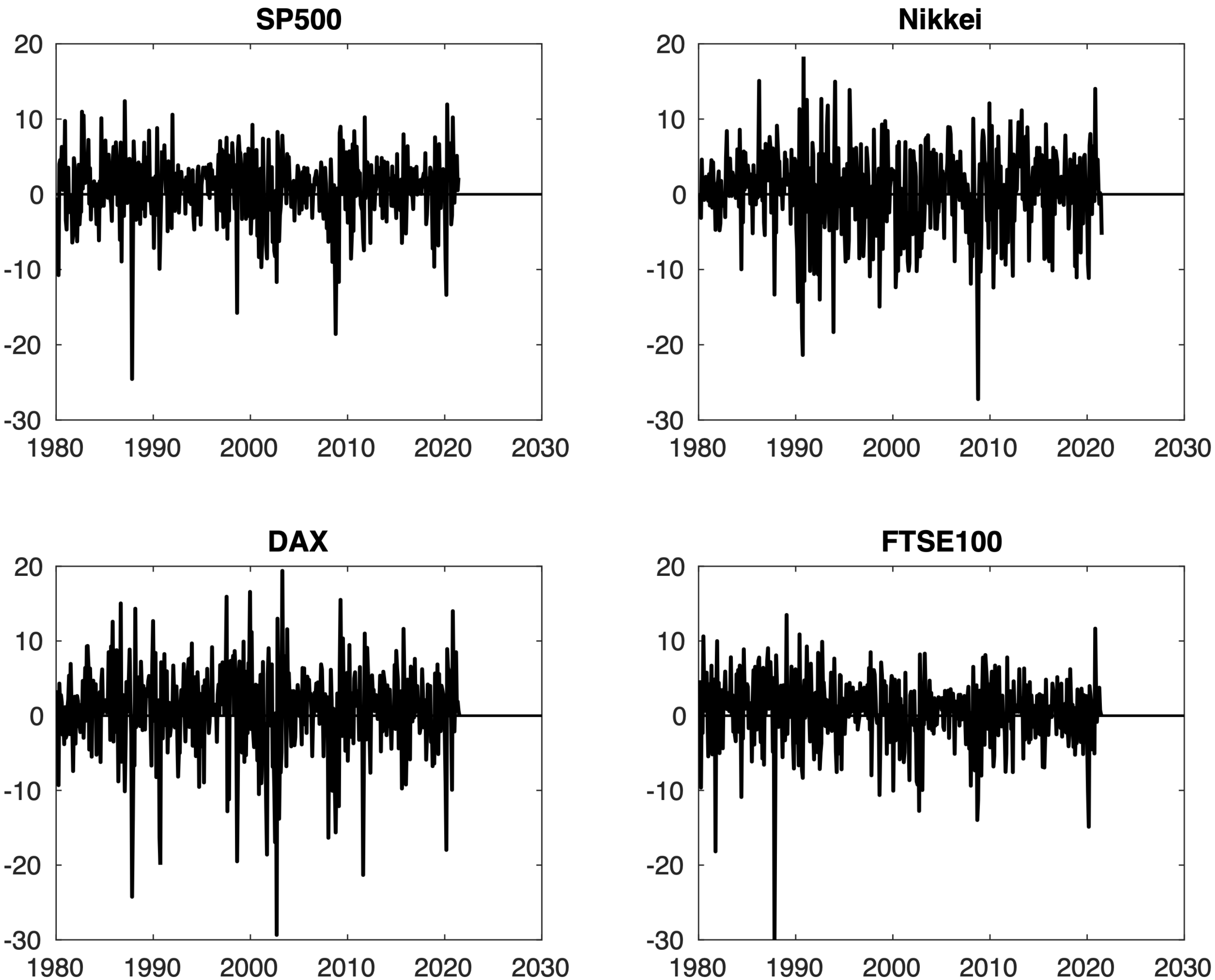
➔ **Distribution of asset returns**

- Tests on the distribution of asset returns

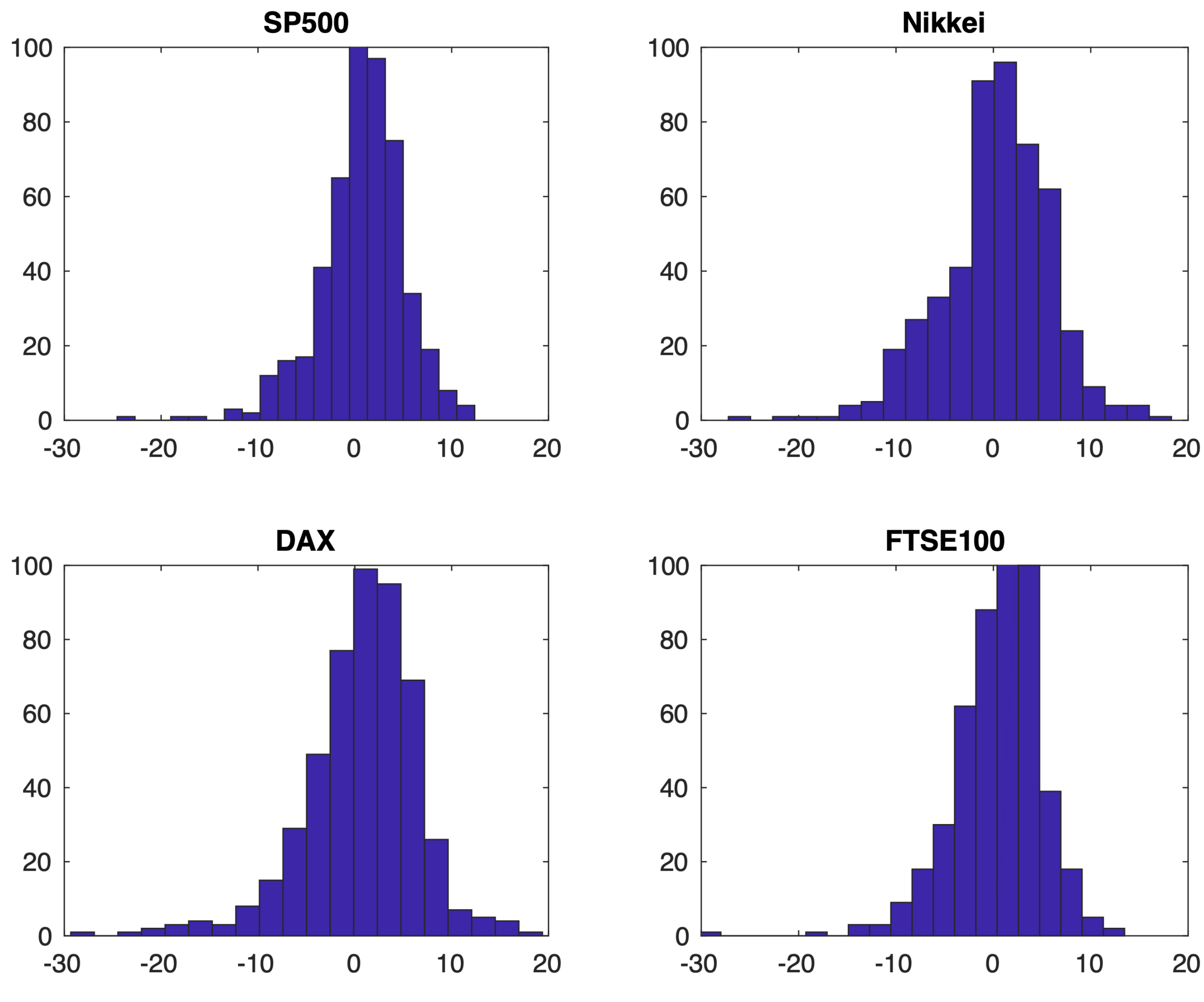
- Time dependency of asset returns

- Correlation across asset returns

Monthly log-returns (Jan. 1980 – July 2021)



Histogram of monthly log-returns (Jan. 1980 – July 2021)



Moments of a random variable

Assume X (log-returns) has the following **cumulative distribution function** (cdf):

$$F_X(x) = \Pr[X \le x] = \int_{-\infty}^{x} f_X(u)\,du .$$

where $f_X(x)$ is the **probability distribution function** (pdf).

Mean or expected value of X:

$$\mu = E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$$

Variance of X:

$$\sigma^2 = V[X] = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)\,dx$$

Moments of a random variable

More generally, we define the ***k*th**

More generally, we define the **k^{th} non-central moment** of X as

$$m_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x)dx \qquad \text{for } k = 1, 2, \ldots$$

The first non-central moment $m_1 = \mu$ is the expected value of X.

We define the **k^{th} central moments** of X as

$$\mu_k = E[(X - m_1)^k] = \int_{-\infty}^{\infty} (x - m_1)^k f_X(x)dx \qquad \text{for } k = 1, 2, \ldots$$

By construction, $\mu_1 = 0$. The second central moment $\mu_2 = \sigma^2 = m_2 - m_1^2$ is the variance

The 3^{rd} central moment $\mu_3 = E[(X - m_1)^3]$ measures the asymmetry of the distribution

The 4^{th} central moment $\mu_4 = E[(X - m_1)^4]$ measures its tail-fatness.

Skewness and kurtosis

Standardized skewness and **kurtosis** are defined as

$$S[X] = E\left[\left(\frac{X - m_1}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} \qquad K[X] = E\left[\left(\frac{X - m_1}{\sigma}\right)^4\right] = \frac{\mu_4}{\sigma^4}$$

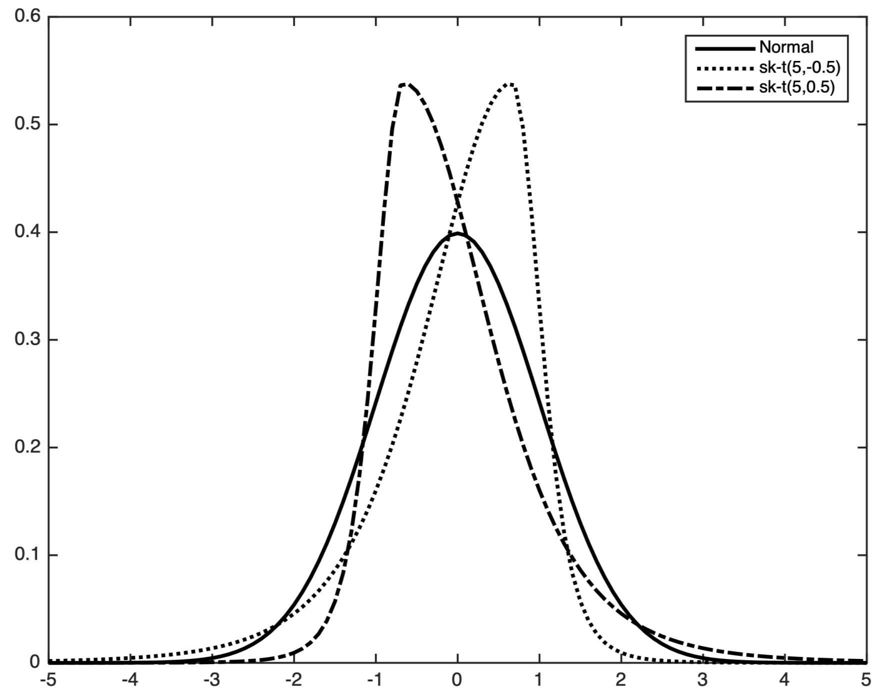
- When $S[X]$ is negative, large realizations of X are more often negative than positive, suggesting that crashes are more likely to occur than booms.

- A large kurtosis $K[X]$ implies that large realizations (either positive or negative) are more likely to occur.

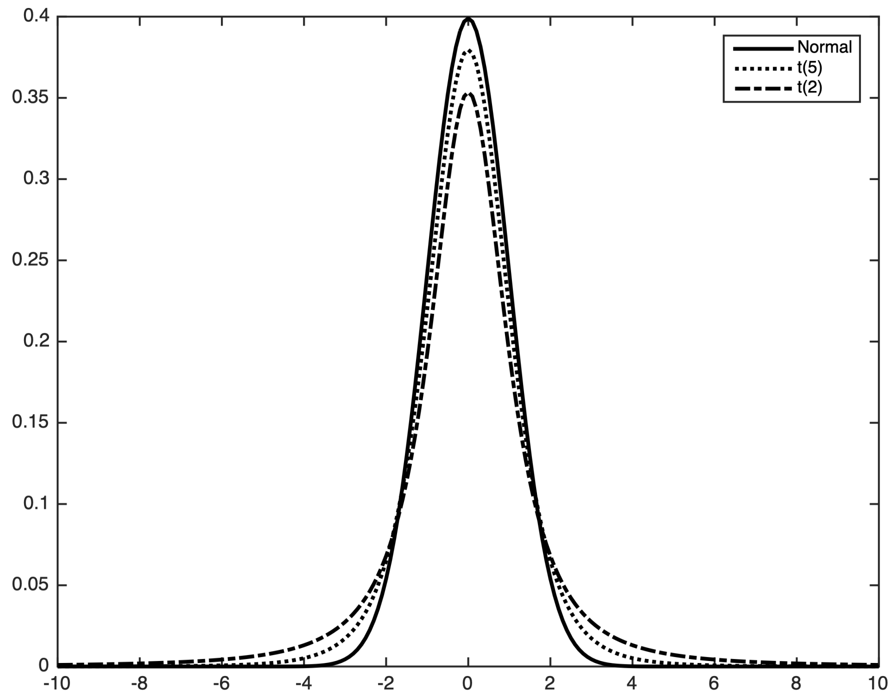
- For the normal distribution, skewness is equal to zero, while standardized kurtosis is equal to 3. We define the excess kurtosis as $K[X] - 3$.

Skewness and kurtosis

Skewness



Kurtosis



Measures of location and dispersion

Consider a log-returns $\{r_t\}$, for $t = 1, \ldots, T$, which are realizations of a random variable.

The **sample mean** (or average) is the natural measure of location:

$$\bar{r} = \hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

If the data sample is drawn from a normal distribution, the sample average is the optimal measure of location.

The **variance** is a popular measure of dispersion. It is the optimal measure for normal returns. The **standard deviation** is the square root of the variance.

$$s^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(r_t - \hat{\mu}\right)^2 \quad \text{or} \quad \hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left(r_t - \hat{\mu}\right)^2$$

(s^2 is the unbiased estimator, while $\hat{\sigma}^2$ is the ML estimator).

The mean and variance are not robust to outliers.

Higher moments

The sample counterpart of the standardized skewness and kurtosis are

$$\hat{S} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^3 \quad \text{and} \quad \hat{K} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^4$$

For a normal distribution, the skewness and the excess kurtosis are zero.

If $\hat{S} < 0$, the distribution has a fatter left tail.
If $\hat{S} > 0$, the distribution has a fatter right tail.

If $\hat{K} < 3$, the distribution has thinner tails than the normal.
If $\hat{K} > 3$, the distribution has fatter tails than the normal.

Remark: If we reject normality, we should probably use robust measures for higher moments.

Objectives of the lecture

- Asset Returns
- Distribution of asset returns

➔ **Tests on the distribution of asset returns**

- Time dependency of asset returns
- Correlation across asset returns

Tests for normality

We consider **unconditional normality** for the moment, i.e., the distribution of the return series itself.

Several tests for the null hypothesis of normality, focusing on different implications of the normality assumption:

- the **moments of the distribution**
    - Jarque-Bera test
- the **properties of the empirical distribution function**
    - Kolmogorov-Smirnov test
    - Anderson-Darling test

We provide some guidelines for the first two tests.

Jarque-Bera Test

The **Jarque-Bera test** is based on the fact that under normality, skewness and excess kurtosis are jointly equal to zero.

Under normality, the first 4 moments have the following asymptotic distributions:

$$\sqrt{T}(\hat{\mu}-\mu) \sim N(0,\sigma^2) \qquad \sqrt{T}(\hat{\sigma}^2-\sigma^2) \sim N(0,2\sigma^4)$$

$$\sqrt{T}(\hat{S}-0) \sim N(0,6) \qquad \sqrt{T}(\hat{K}-3) \sim N(0,24)$$

Remarks:

- Since skewness and kurtosis are already standardized, their distribution does not depend on the mean and/or variance.

- Skewness and kurtosis will be informative only for a large number of observations.

Jarque-Bera Test

Null hypothesis of the test H_0: Skewness = 0 and Kurtosis = 3

Remark 1: Since the normal distribution is defined by the first two moments, there is no constraint on the mean and variance under the null.

Remark 2: Under normality, the sample skewness and kurtosis are mutually independent.

The *JB* test statistic is simply defined as

$$JB = T\left[\frac{\hat{S}^2}{6} + \frac{\left(\widehat{K}-3\right)^2}{24}\right]$$

Under the null hypothesis, it is asymptotically distributed as a $\chi^2(2)$.

If $JB \geq \chi^2_{1-\alpha}(2)$, then the null hypothesis is rejected at level α.

Example: If the *JB* statistic is larger than 6, we reject the normality hypothesis with only 5% of chances of being wrong.

Empirical Distribution Function Goodness-of-Fit Tests

Empirical distribution function goodness-of-fit tests compare the empirical cdf and the assumed theoretical cdf $F^*(x;\theta)$ (here, the normal distribution).

The time series $(r_1, r_2, ..., r_T)$ is associated with some unknown cdf, $F_r(.)$. As the true distribution $F_r(.)$ is unknown, it is approximated by the empirical cdf, $G_T(.)$, defined as

$$G_T(x) = \frac{1}{T} \sum_{t=1}^{T} 1_{\{r_t \leq x\}} \qquad \text{(step function with steps of height } [1/T] \text{ at each observation)}$$

The empirical cdf $G_T(x)$ is compared with the assumed cdf $F^*(x;\theta)$ to see if there is good fit.

Null hypothesis H_0: $F_r(x) = F^*(x;\theta)$ for all x.
Alternative hypothesis H_a: $F_r(x) \neq F^*(x;\theta)$ for at least one value of x.

EDF tests are based on the result that if $F_r(X)$ is the cdf of X, then the variable $F_r(X)$ is uniformly distributed between 0 and 1.

Kolmogorov-Smirnov and Lilliefors Tests

A measure of the difference between two cdfs is the largest distance between the two functions $G_T(x)$ and $F^*(x;\theta)$. This is the ***KS* test statistic** (Kolmogorov, 1933):

$$KS = \max_{t=1,\cdots,T} \left| G_T(x) - F^*(x;\theta) \right|$$

Recipe for the normal distribution $N(\mu, \sigma^2)$:

0. If μ and σ^2 are unknown, estimate $\hat{\mu}$ and s^2 from the original data (**Lilliefors test**)

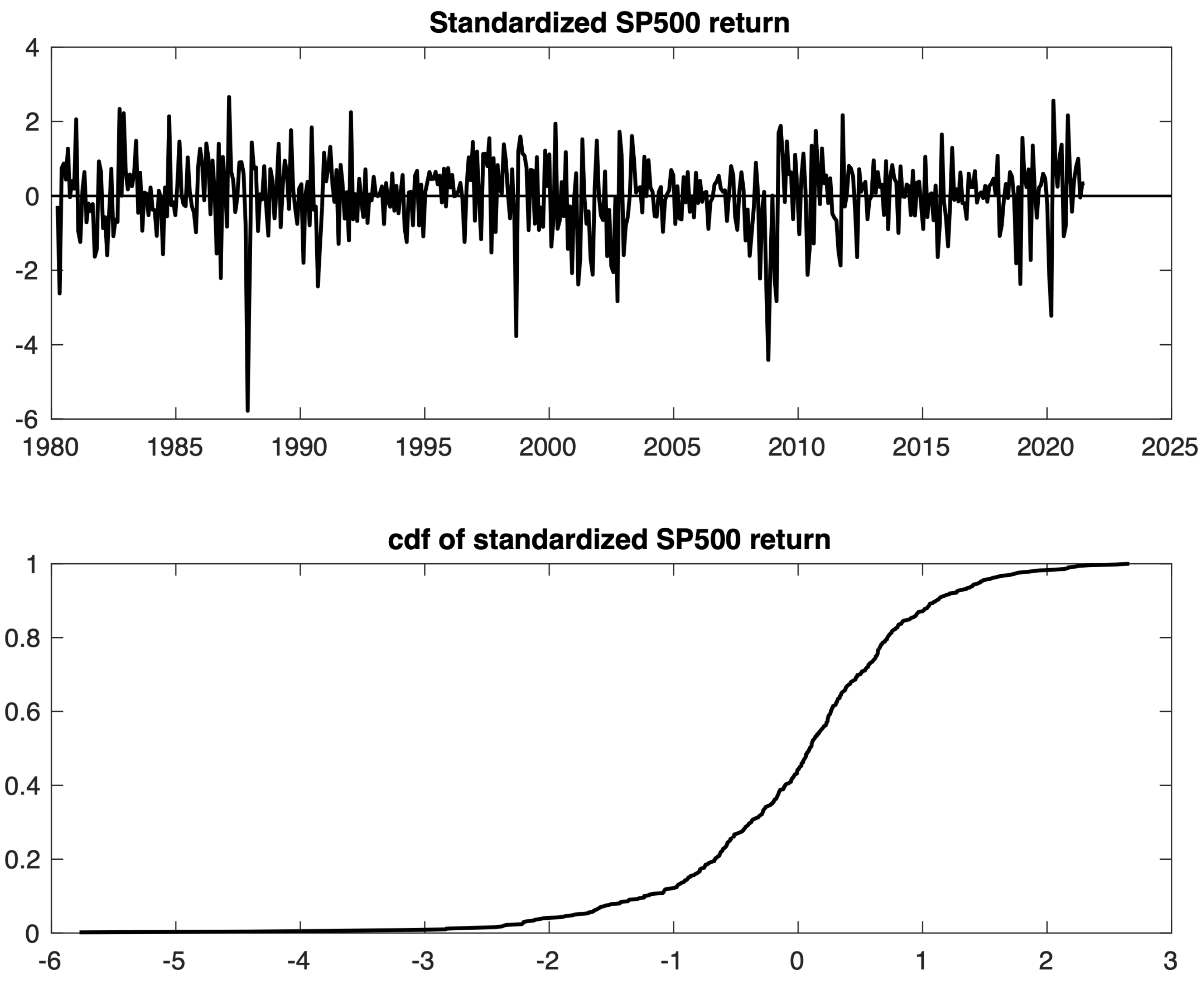
1. Sort the sample data by increasing order and denote the new sample $\{\tilde{r}_t\}_{t=1}^{T}$, with $\tilde{r}_1 \leq \cdots \leq \tilde{r}_T$. By construction, we have $G_T(\tilde{r}_t) = \frac{t}{T}$.

2. Evaluate the assumed theoretical cdf $F^*(\tilde{r}_t;\theta)$ for all values $\{\tilde{r}_t\}_{t=1}^{T}$. Use (μ,σ^2) if they are known or $(\hat{\mu},s^2)$ if they are not

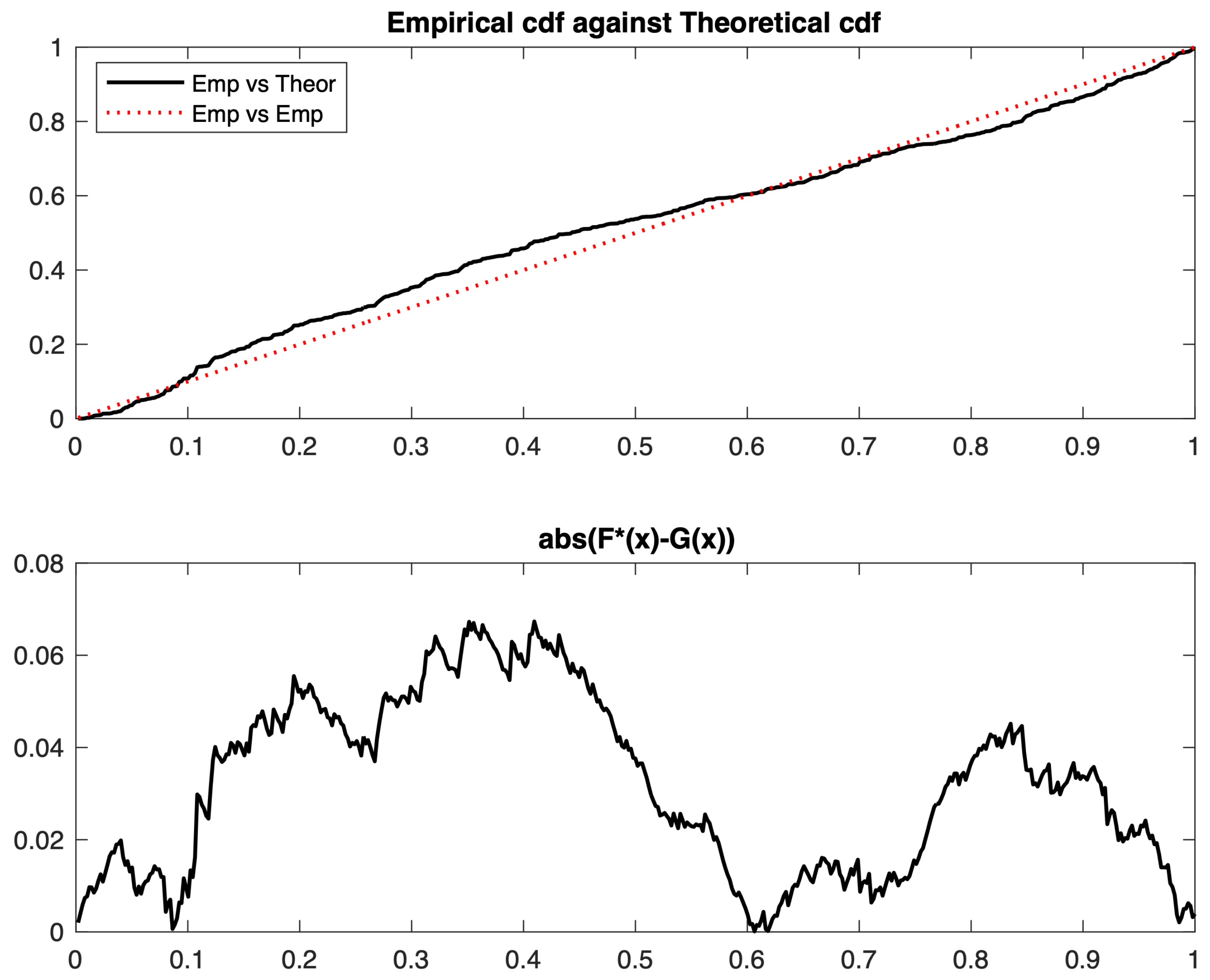
3. Compute the *KS* test statistic: $KS = \max_{t=1,\cdots,T} \left| F^*(\tilde{r}_t) - \frac{t}{T} \right|$.

Critical values: $0.805/\sqrt{T}$, $0.886/\sqrt{T}$, and $1.031/\sqrt{T}$ for a 10%, 5%, and 1% level test

Lilliefors Test



Lilliefors Test



Example

Summary univariate statistics on market returns.
(Jan. 1980 to July 2021, end-of-month data, 499 observations)

	S&P500	**Nikkei**	**DAX**	**FTSE100**
Mean	0.733	0.280	0.689	0.510
Median	1.149	0.771	1.245	0.961
Std err.	4.378	5.680	5.922	4.493
Median of Abs Dev	3.222	4.274	4.343	3.320
IQR	5.158	6.573	6.517	5.006
Minimum	-24.543	-27.216	-29.333	-30.170
Maximum	12.378	18.287	19.374	13.477
Skewness	-0.894	-0.601	-0.859	-1.103
Kurtosis	6.101	4.530	5.813	7.798
JB	265.824	78.585	225.377	578.637
p-value	0.000	0.000	0.000	0.000
KS_L	0.067	0.075	0.063	0.065
critical value	0.036	0.036	0.036	0.036

Objectives of the lecture

- Asset Returns
- Distribution of asset returns
- Tests on the distribution of asset returns

➔ **Time dependency of asset returns**

- Correlation across asset returns

Serial correlation

Are the moments of a given series time dependent? This is an important issue for testing market efficiency.

Suppose the time series $(r_1, r_2, ..., r_T)$ is **weakly (or covariance) stationary**:

$$E[r_t] = \mu \quad \forall t$$

$$V[r_t] = \sigma^2 \quad \forall t$$

$$Cov[r_t, r_{t-k}] = \gamma_k \quad \forall t, k \neq 0$$

The **auto-covariance** γ_k measures the dependency of r_t with respect to its own past observation r_{t-k}. We have the following relations

$$\gamma_0 = V[r_t] = E[(r_t - \mu)^2] \qquad \text{and} \qquad \gamma_k = \gamma_{-k}$$

For a **weakly stationary** process, the autocovariance γ_k depends on the horizon k, but not on the date t.

Serial correlation

Since the auto-covariance is an unbounded measure of dependency, we generally prefer the **auto-correlation function**

$$\rho_k = \frac{Cov[r_t, r_{t-k}]}{\sqrt{V[r_t]V[r_{t-k}]}} = \frac{Cov[r_t, r_{t-k}]}{V[r_t]} = \frac{\gamma_k}{\gamma_0} \qquad \text{with } \rho_k = \rho_{-k},\ \rho_0 = 1 \text{ and } -1 \leq \rho_k \leq 1.$$

A consistent estimator of ρ_k is

$$\hat{\rho}_k = \frac{\sum_{t=k+1}^{T} (r_t - \overline{r})(r_{t-k} - \overline{r})}{\sum_{t=1}^{T} (r_t - \overline{r})^2} \qquad \text{for} \quad 0 < k < T-1$$

Since $\hat{\rho}_k$ is a ratio of quadratic functions of r_t, its sample properties are not easily obtained. We must consider large sample properties.

Simple expression for the asymptotic distribution in some special cases.

Serial correlation

→ If r_t is an **i.i.d. process**, then $V[\hat{\rho}_k] = 1/T$, so that

$\sqrt{T}\hat{\rho}_k$ is asymptotically normal $N(0,1)$, for all $k > 0$.

→ If r_t is a **stationary moving average *MA*(*q*) process** with $r_t = \mu + \sum_{i=0}^{q} \theta_i \varepsilon_{t-i}$ with $\theta_0 = 1$ and $\rho_k = 0$ for $k > q$, then

$$V[\hat{\rho}_k] = \frac{1}{T}\left(1 + 2\sum_{i=1}^{q} \rho_i^2\right),$$

so that

$\sqrt{T}\hat{\rho}_k$ is asymptotically normal $N(0,V)$ with $V = 1 + 2\sum_{i=1}^{q} \rho_i^2$ for all $k > q$.

Idea: We test $\rho_k = 0$ by assuming that the process is an $MA(k\text{-}1)$ to estimate $V[\hat{\rho}_k]$.

Box-Pierce / Ljung-Box tests

We now want to test the null hypothesis that the p first serial correlations are equal to 0, $H_0\colon \rho_1 = \cdots = \rho_p = 0$ versus the alternative $H_a\colon \rho_k \neq 0$, for some $k = 1,\ldots, p$.
A simple test can be based on the **Box-Pierce *Q*-statistic**:

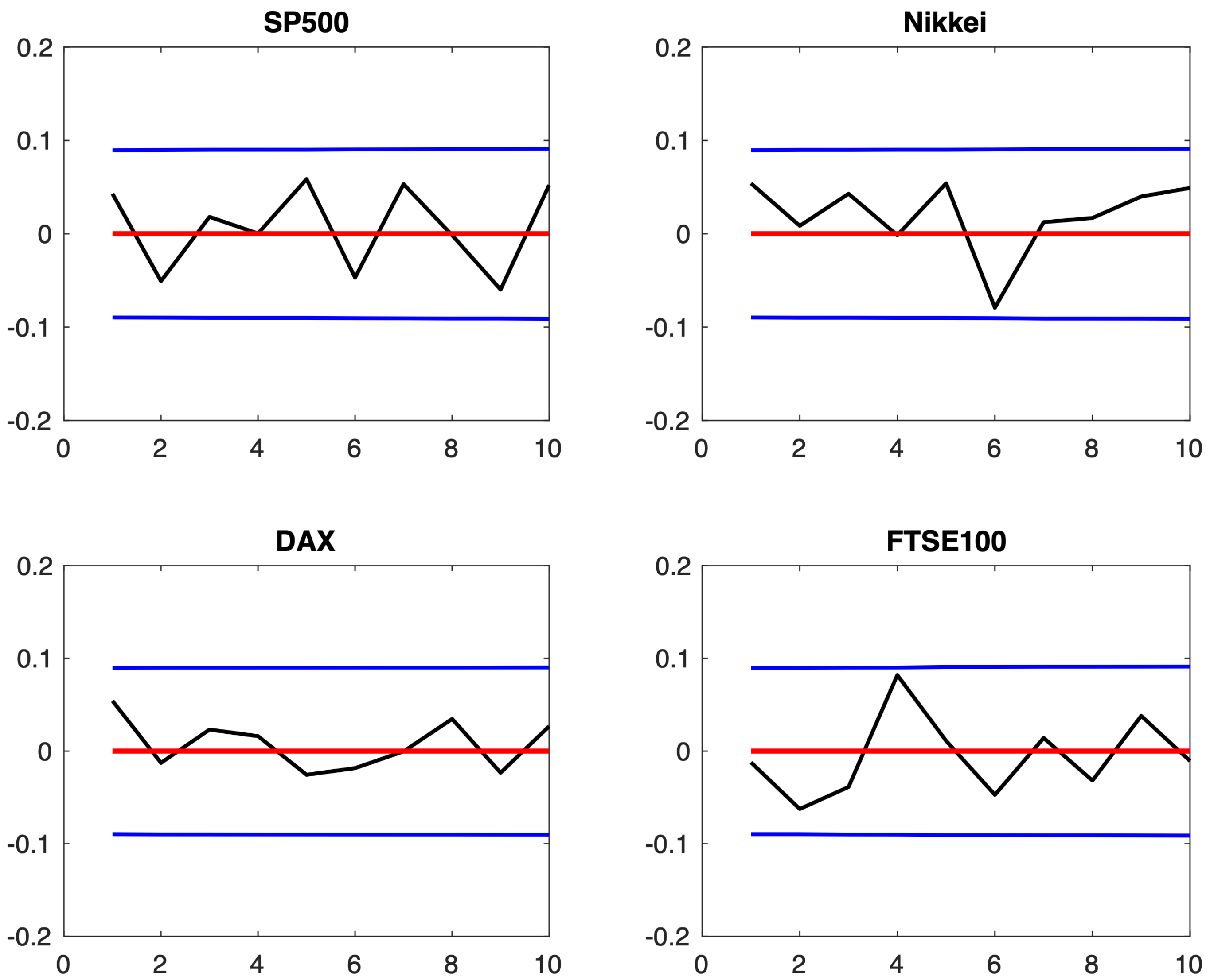
$$Q_p = T\sum_{j=1}^{p}\hat{\rho}_j^2$$

Under the null hypothesis $H_0 : \rho_1 = \ldots = \rho_p = 0$, all the $\hat{\rho}_k$ have $V[\hat{\rho}_k] = 1/T$. Therefore, the Q_p statistic is asymptotically distributed as $\chi^2(p)$.

If $Q_p \geq \chi^2_{1-\alpha}(p)$, then the null hypothesis is rejected at level α. A common practice consists in testing the nullity of the p first serial correlations for different values of p.

The **Ljung-Box *Q'*-statistic**: $Q_p^{'} = T(T+2)\sum_{j=1}^{p}\frac{1}{T-j}\hat{\rho}_j^2$ has better finite-sample properties, with the same asymptotic distribution.

Correlogram of monthly returns



Serial correlation of monthly returns

	S&P500	Nikkei	DAX	FTSE100	Crit. value
ρ_1	0.043	0.054	0.054	-0.012	–
$t(\rho_1)$	0.957	1.206	1.209	-0.272	–
$Q_p^{'}$					
p=1	0.922	1.463	1.471	0.074	3.842
2	2.209	1.500	1.552	2.032	5.992
3	2.375	2.425	1.823	2.786	7.815
4	2.375	2.426	1.953	6.173	9.488
5	4.109	3.904	2.283	6.235	11.071
6	5.221	7.078	2.454	7.362	12.592
7	6.654	7.157	2.454	7.465	14.067
8	6.655	7.303	3.064	7.978	15.507
9	8.471	8.116	3.341	8.711	16.919
10	9.860	9.348	3.707	8.769	18.307

Serial correlation in volatility

To test for time dependency of volatility, we need a time-varying measure of volatility. There are at least two possible measures:

(1) the mean-adjusted squared returns

(2) the absolute returns

Assume that returns have the following dynamics

$$r_t = \mu + \varepsilon_t \qquad \text{with} \qquad \varepsilon_t = \sigma_t z_t,$$

where μ is the constant mean, ε_t the unexpected returns, σ_t the time-varying volatility based on information at date t–1 and z_t is an $N(0,1)$ innovation.

Remark: The next session is devoted to the modeling of the conditional volatility.

Serial correlation in volatility

Conditional on the information set I_{t-1}, we have

$$E\left[\varepsilon_t^2 \mid I_{t-1}\right] = E\left[\sigma_t^2 z_t^2 \mid I_{t-1}\right] = \sigma_t^2 E\left[z_t^2 \mid I_{t-1}\right] = \sigma_t^2$$

because z_t^2 is distributed as a $\chi^2(1)$. Therefore, ε_t^2 can be viewed as a proxy for the volatility at time *t*.

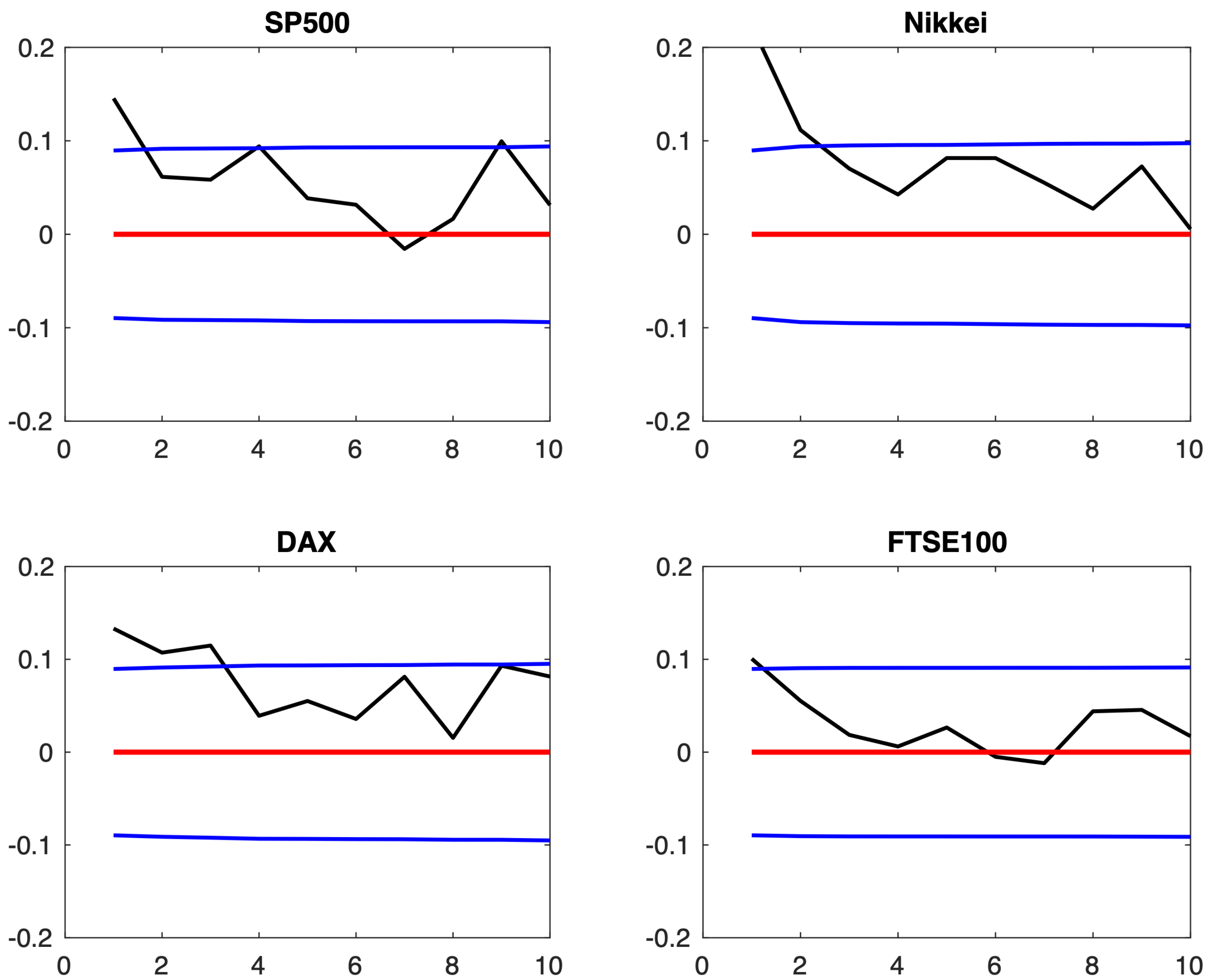
Alternatively, we have $\varepsilon_t \sim N(0, \sigma_t^2)$. Then

$$E\left[|\varepsilon_t| \mid I_{t-1}\right] = \sigma_t \sqrt{2/\pi}.$$

Consequently, $|\varepsilon_t| / \sqrt{2/\pi}$ can be used as a proxy for σ_t.

These two measures are noisy estimators of conditional volatility.

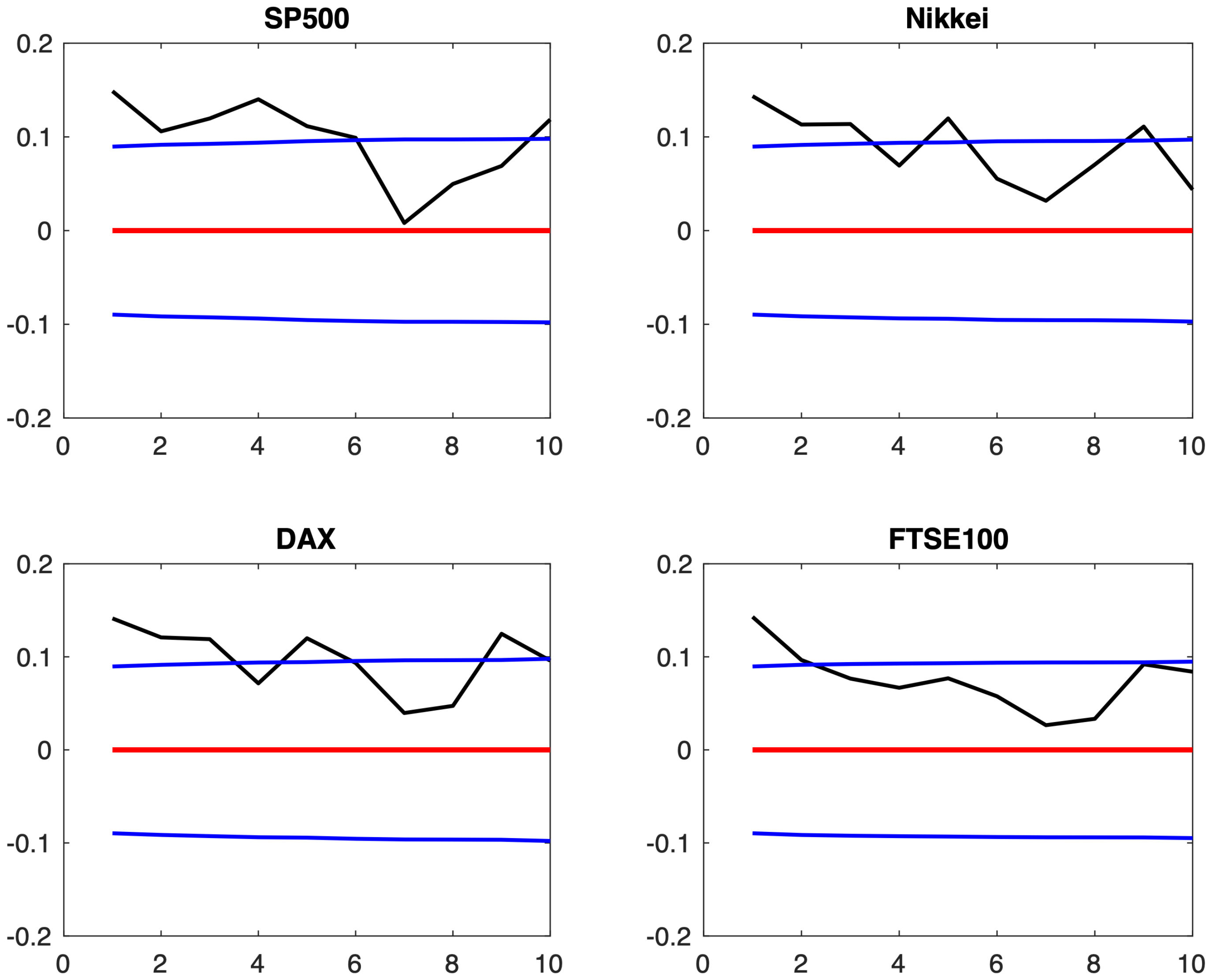
Correlogram of squared returns



Serial correlation of squared returns

	S&P500	Nikkei	DAX	FTSE100	Crit. value
ρ_1	0.145	0.224	0.133	0.101	−
$t(\rho_1)$	3.245	4.987	2.972	2.246	−
$Q_p^{'}$					
p=1	10.592	25.017	8.885	5.075	3.842
2	12.490	31.275	14.657	6.606	5.992
3	14.205	33.760	21.291	6.780	7.815
4	18.667	34.675	22.065	6.799	9.488
5	19.416	38.037	23.599	7.155	11.071
6	19.921	41.400	24.245	7.168	12.592
7	20.045	42.943	27.590	7.239	14.067
8	20.182	43.324	27.711	8.223	15.507
9	25.224	46.007	32.128	9.277	16.919
10	25.721	46.023	35.516	9.429	18.307

Correlogram of absolute returns



Serial correlation of absolute returns

	S&P500	Nikkei	DAX	FTSE100	Crit. value
ρ_1	0.149	0.144	0.141	0.143	–
$t(\rho_1)$	3.321	3.203	3.153	3.191	–
$Q_p^{'}$					
p=1	11.093	10.324	10.001	10.243	3.842
2	16.726	16.756	17.334	14.925	5.992
3	23.929	23.264	24.455	17.879	7.815
4	33.826	25.691	27.033	20.124	9.488
5	40.103	32.930	34.300	23.112	11.071
6	45.067	34.483	38.714	24.792	12.592
7	45.101	34.997	39.510	25.149	14.067
8	46.360	37.521	40.647	25.715	15.507
9	48.783	43.795	48.570	30.021	16.919
10	55.955	44.768	53.263	33.622	18.307

Volatility asymmetry

Another important feature of financial markets: volatility is more affected by negative news than positive news.

To illustrate the point, we perform the following regressions

$$\varepsilon_t^2 = \omega + \beta \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \times 1_{\{\varepsilon_{t-1}<0\}}$$

or

$$\left|\varepsilon_t\right| = \omega + \beta \left|\varepsilon_{t-1}\right| + \gamma \left|\varepsilon_{t-1}\right| \times 1_{\{\varepsilon_{t-1}<0\}}$$

where β measures the direct effect of past returns, and
γ captures the additional impact of negative return shocks.

Volatility asymmetry

	S&P500	Nikkei	DAX	FTSE100
		Squared returns		
ω	17.682	26.711	32.031	18.490
(std err.)	(2.170)	(3.047)	(3.853)	(2.665)
β	-0.119	0.027	-0.056	0.035
(std err.)	(0.109)	(0.084)	(0.099)	(0.134)
γ	0.330	0.253	0.239	0.078
(std err.)	(0.113)	(0.090)	(0.104)	(0.137)
		Absolute returns		
ω	2.799	3.756	3.800	2.884
(std err.)	(0.194)	(0.254)	(0.265)	(0.205)
β	-0.008	0.036	0.010	0.060
(std err.)	(0.061)	(0.060)	(0.062)	(0.064)
γ	0.281	0.175	0.231	0.141
(std err.)	(0.061)	(0.060)	(0.062)	(0.063)

Objectives of the lecture

- Asset Returns
- Distribution of asset returns
- Tests on the distribution of asset returns
- Time dependency of asset returns

➔ **Correlation across asset returns**

Cross-correlation

How to measure the dependence between two series?

We first define the **covariance** between two series $r_{1,t}$ and $r_{2,t}$

$$Cov[r_{1,t}, r_{2,t}] = \frac{1}{T}\sum_{t=1}^{T}\left(r_{1,t} - \mu_1\right)\left(r_{2,t} - \mu_2\right)$$

where μ_1 and μ_2 denote the sample means of $r_{1,t}$ and $r_{2,t}$.

The interpretation of the covariance is not easy, because it depends on the level of the variances. Then, we define the **correlation** between $r_{1,t}$ and $r_{2,t}$

$$\rho[r_{1,t}, r_{2,t}] = \frac{Cov[r_{1,t}, r_{2,t}]}{\sqrt{V[r_{1,t}]V[r_{2,t}]}}$$

By construction, $\rho[r_{1,t}, r_{1,t}] = 1$, $\rho[r_{1,t}, r_{2,t}] = \rho[r_{2,t}, r_{1,t}]$ and $-1 \leq \rho[r_{1,t}, r_{2,t}] \leq 1$.

Remark: The correlation is only a measure of **linear association**. We may find that two series are uncorrelated, while they are not independent.

Cross-correlation

The correlation between $r_{1,t}$ and $r_{2,t}$ is estimated as

$$\hat{\rho}[r_{1,t}, r_{2,t}] = \frac{\sum_{t=1}^{T}(r_{1,t} - \hat{\mu}_1)(r_{2,t} - \hat{\mu}_2)}{\sqrt{\sum_{t=1}^{T}(r_{1,t} - \hat{\mu}_1)^2 \sum_{t=1}^{T}(r_{2,t} - \hat{\mu}_2)^2}}$$

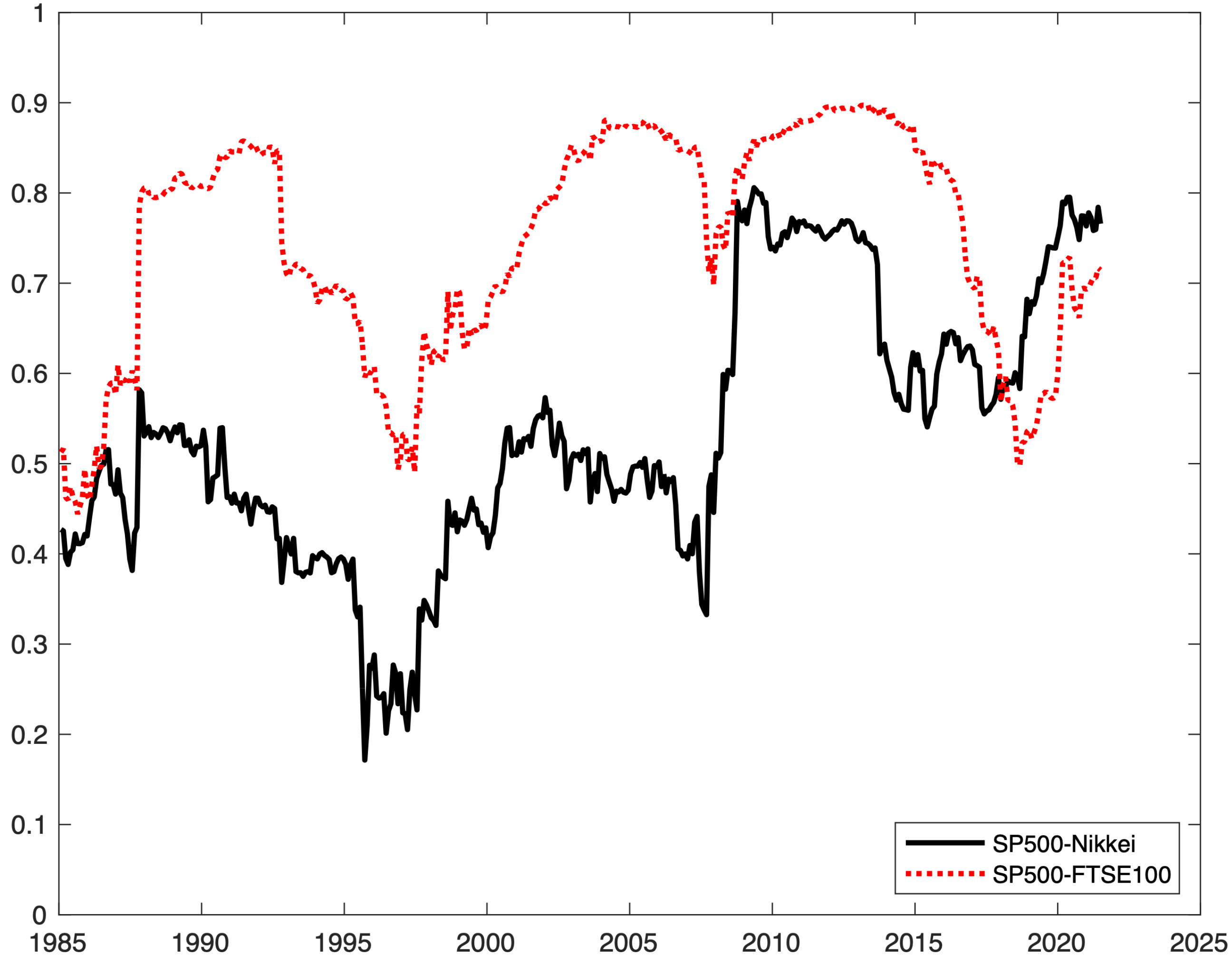
Correlation matrix between the four stock market returns

	S&P500	**Nikkei**	**DAX**	**FTSE100**
SP500	1	0.526	0.672	0.744
Nikkei	0.526	1	0.498	0.473
DAX	0.672	0.498	1	0.670
FTSE100	0.744	0.473	0.670	1

All correlations are positive.

Cross-correlation

Correlation of monthly returns computed using rolling windows of 60 months.



Characteristics of Financial Asset Returns

For actual returns, these assumptions have been rejected in several ways:

1. Normality

- Asymmetry
- Fat tails
- Aggregated normality

2. Time independency

- No serial correlation
- Volatility clustering
- Asymmetric volatility

3. Multivariate dependency

- Time-varying correlation