Project
(Due January 30, 2026)

Characteristics of Financial Time Series

1. Data

You are given a dataset containing 6 indices on the US financial markets, which represent different asset classes:

- stocks: S&P 500 index (total return index) (S&PCOMP(RI))
- government bonds: US Treasury bond index (total return index) (SPUTBIX(RI))
- corporate bonds: US corporate bond index (total return index) (SPUHYBD(RI))
- real-estate securities: US real-estate index (total return index) (WILURET(RI))
- commodities: CRB spot index (total return index) (RJEFCRT(TR))
- currencies: USD against major currencies (price index) (USBINXB)

Indices are total return indices (including dividends or coupons, except for commodities and currencies) at the daily frequency, from January 2000 to November 2025. The objectives of this homework are twofold: (1) investigate the properties of financial returns (2) find a model describing volatility while capturing non-normality.

Choose 3 of the 6 series to work on in the following analysis.

2. Characteristics of Financial Time Series

2.1 Diagnostic for individual assets: exploration

Compute log-returns, and do the following analysis for both daily and weekly returns:

2.1a. Identify the main crashes and booms (say, the five smallest and largest returns) of the S&P 500 and try to identify if they are related to economic or political events.

2.1b. For all indices, test if the magnitude of the crashes and booms is consistent with the hypothesis of normality. To perform this test, assume that returns are normally distributed with the sample mean and variance and compute for this distribution the probability of occurrence of the extreme returns that you observed.

2.1c. Test whether the sample skewness and kurtosis are compatible with the normality hypothesis. That is, test normality using the Jarque-Bera test procedure at the 5% significance level. You are asked to code the Jarque-Bera test yourself.

2.1d. Compute the first 10 auto-correlations of the return series with the confidence interval. Compute the Ljung-Box test statistics and test the null hypothesis that the return series is not serially correlated over the sample at the 5% significance level. Use the same approach to test the null of no serial correlation of squared returns.

2.1e. Are your conclusions of points (a) to (d) altered by the change of frequency? Elaborate on the *temporal aggregate normality* feature.

2.2 Diagnostic for a portfolio

The idea is to construct a portfolio composed of all asset classes, using equal weights. To define the daily (weekly) portfolio return, you compute the portfolio return as the average of the daily (weekly) simple return of the six asset classes. Use simple returns.

2.2a. Compute the summary statistics of point 2 (sample mean, variance, skewness, kurtosis, minimum, and maximum) at daily frequency. Compare the statistics with those of the individual stocks (point 2a.). Elaborate on the contemporaneous aggregate normality feature.

2.2b. Re-do the same exercise at weekly frequency. Elaborate on the relative effect of temporal and contemporaneous aggregate normality features.

2.2c. How would interpret these results from an asset allocation and risk management perspective?

3. **Modeling Volatility and Non-normality**

We now consider a portfolio of corporate stocks and bonds.

Before estimating the GARCH model with daily data, it is important to remove those days when the market was closed. Identify them with $r_t = 0$. (Remark: These corrections do not affect the estimation of the GARCH models but have important consequences for the test of adequacy of the assumed distribution to the empirical distribution.)

3.1 Estimation of a GARCH model

3.1a. For both stocks and bonds, provide evidence on the non-normality (Jarque-Bera test) and the auto-correlation (Ljung-Box test, with 4 lags) of the excess log-returns.

3.1b. Estimate an AR(1) model on stocks and bonds to filter out autocorrelation. We now denote $\hat{\varepsilon}_t$ the residuals of the AR(1) model.

3.1c. For $\hat{\varepsilon}_t$, test the ARCH effect using the LM test of Engle (1982), using 4 lags.

3.1d. Estimate a GARCH(1,1) model for $\hat{\varepsilon}_t$, using the conditional ML technique. Comment the parameter estimates (in particular, the sum $\alpha_1 + \beta_1$).

3.2 MLE or QMLE?

3.2a. Estimate a GARCH(1,1) model for the series at hand (corporate stocks and bonds), assuming normality (ML estimation) at the daily frequency. Report the standard errors. Is there evidence of non-normality in standardized residuals?

3.2b. Report the QML estimation of the GARCH(1,1) model. The normal pdf is used to construct the likelihood, but the normal distribution is not assumed to be the true distribution of returns. Comment the parameter estimates and standard errors.

3.2c. Test the adequation of the normal distribution to the standardized residuals, using the Diebold-Gunther-Tay test (with $N = 40$ cells). Run the Ljung-Box tests for the serial correlation in the moments of the probability integral transform with $K = 10$ lags. What does the test procedure suggest as a source of rejection of the null hypothesis?

3.3 Estimation assuming a Student *t* distribution

3.3a. Estimate the GARCH(1,1) model under the assumption that the true distribution is a Student *t* distribution. The degree-of-freedom parameter is denoted v. Using the delta method, test the null hypothesis of normality $H_0: 1/v = 0$.

3.3b. Test the adequation of the Student *t* distribution to the standardized residuals, using the Diebold-Gunther-Tay test. Once again, what does the test procedure suggest as a source of rejection of the null hypothesis?

3.4 Estimation assuming a skewed Student *t* distribution

3.4a. Estimate the GARCH(1,1) model under the assumption that the true distribution is a skewed Student *t* distribution. The degree-of-freedom parameter is denoted v and the asymmetry parameter is denoted λ. Test the null hypothesis of normality $H_0: 1/v = \lambda = 0$.

3.4b. Test the adequation of the skewed Student *t* distribution to the standardized residuals, using the Diebold-Gunther-Tay test.

3.4c. Compare the sample moments of the innovations of the GARCH model estimated in question 2 with the moments implied by the estimates of v and λ.

3.4d. Plot the time series of volatility obtained in points 3.2, 3.3, and 3.4. Comment.