NN and Deep Learning

Ex. 2

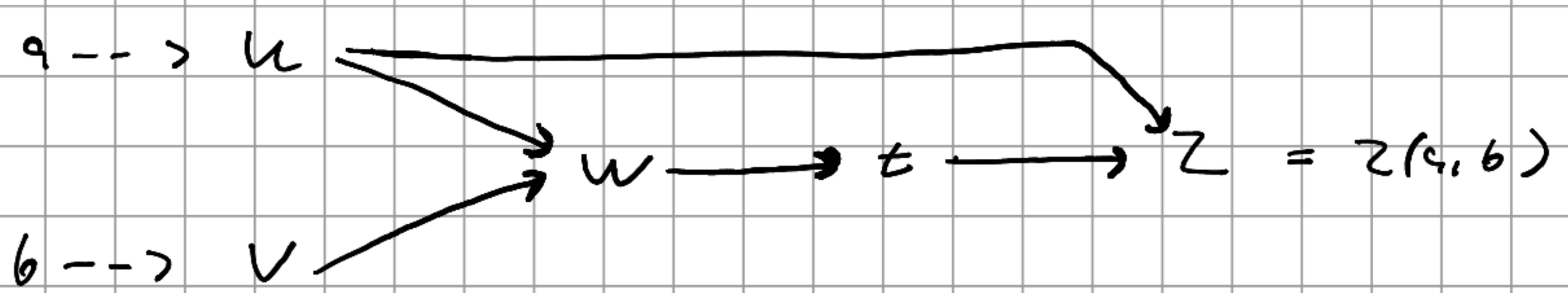
Math 7

a)

Define intermediate variables

$u := a^2$, $v = b^2$, $w = u + v$, $t = w + 1$, $z = \frac{u}{t}$.

Then the graph becomes



b)

Node	Composition	Deriv.
u	a^2	$\frac{\partial u}{\partial a} = 2a$
v	b^2	$\frac{\partial v}{\partial b} = 2b$
w	$u+v$	$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial v} = 1$
t	$w+1$	$\frac{\partial t}{\partial w} = 1$
z	ut^{-1}	$\frac{\partial z}{\partial u} = t^{-1}$, $\frac{\partial z}{\partial t} = -ut^{-2}$

Then the chain rule gives us

$$\frac{\partial z}{\partial a} = \frac{\partial u}{\partial a}\frac{\partial w}{\partial u}\frac{\partial t}{\partial w}\frac{\partial z}{\partial t} + \frac{\partial u}{\partial a}\frac{\partial z}{\partial u} = \frac{2}{9}$$

$$\frac{\partial z}{\partial b} = \frac{\partial v}{\partial b}\frac{\partial w}{\partial v}\frac{\partial t}{\partial w}\frac{\partial z}{\partial t} = -\frac{8}{36} = -\frac{2}{9}$$

c) For the backpropagation

Seen	Var.	Derivative	Evaluate:	Prop:
1	z	$\frac{\partial z}{\partial u} = \frac{1}{t}$, $\frac{u}{t^2}$	$\frac{1}{6}$, $-\frac{1}{9}$	$\tilde{u} = \frac{1}{6}$ $\tilde{t} = -\frac{1}{9}$
2	t	$\frac{\partial t}{\partial w} = 1$	1	$\tilde{w} := 1 \times (-\frac{1}{9}) = -\frac{1}{9}$
3	w	$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial v} = 1$	1	$\tilde{u} := \tilde{u} + (-\frac{1}{9}) = \frac{1}{18}$, $\tilde{v} := -\frac{1}{9}$
4	u	$\frac{\partial u}{\partial a} = 2a$	4	$\tilde{a} := \tilde{u} \times 4 = \frac{2}{9}$
5	v	$\frac{\partial v}{\partial b}$	2	$\tilde{b} := \tilde{v} \times 2 = -\frac{2}{9}$

Giving $\frac{\partial z}{\partial b} = -\frac{2}{9}$, $\frac{\partial z}{\partial a} = \frac{2}{9}$

Math 2