NNDL2025: Exercises & Assignments
Session 0 (13 Mar)

Exceptionally these exercises are just considered during the Thursday session, no reports are handed in, no points are given.

Computer assignments

In this session we will go through a tutorial covering PyTorch etc.

Mathematical preliminaries

This is background material that you should know, so the exercises should not take too long. (More background on calculus will come in next session)

Mathematical exercices

1. This is a basic exercise using the eigenvalue decomposition to compute the matrix square root.

   Consider a symmetric matrix $\mathbf{C}$. Denote the eigendecomposition of $\mathbf{C}$ as

   $$\mathbf{C} = \mathbf{U}\mathrm{diag}(\lambda_1, \ldots, \lambda_n)\mathbf{U}^T. \tag{1}$$

   Recall what we know about $\mathbf{U}$. Now, consider the matrix $\mathbf{M}$ defined using that EVD as:

   $$\mathbf{M} = \mathbf{U}\mathrm{diag}(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_n})\mathbf{U}^T. \tag{2}$$

   Show that

   $$\mathbf{MM} = \mathbf{C}. \tag{3}$$

   This justifies calling $\mathbf{M}$ the square root of $\mathbf{C}$.

2. Consider the Laplace distribution

   $$p(z|\alpha) = \frac{\alpha}{2}\exp(-\alpha|z|). \tag{4}$$

   The parameter α determines how likely are large values in absolute value. Given a sample $z_1, \ldots, z_N$ of observations, derive the maximum likelihood estimator for α.

3. The multivariate Gaussian distribution is absolutely essential, so let us recall some of its properties.

   Consider a bivariate Gaussian distribution where the two (scalar) variables x, y have zero mean, unit variance, and covariance equal to c.

   (a) Write down the density function (pdf) of the vector (x, y). Hint1: The pdf uses the inverse covariance, and we have

   $$\begin{pmatrix} 1 & c \\ c & 1 \end{pmatrix}^{-1} = \frac{1}{1-c^2} \begin{pmatrix} 1 & -c \\ -c & 1 \end{pmatrix}. \tag{5}$$

   (b) Calculate the marginal distribution of x

   (c) *[Optional since very similar to the above]* Calculate the conditional distribution of x given y

   In (b) and (c) The point is to do the calculations in detail, not use some formulas readily available on the internet. So, please do the detailed derivation.

   Hint2: In (b), use the "completion of square formula" on y

   $$y^2 - 2ay = (y-a)^2 - a^2, \tag{6}$$

   where a is something that contains the other variable (y or x) and the covariance. In (c), you may need a similar formula as well but for x.

   Hint3: In (b) and (c), you may realize at some point that there is a complicated integral to solve. However, you can actually infer its value based on the fact that a probability density must integrate to one, without explicitly calculating the integral.