IOQM ASSIGNMENTS: BASSIC FACTORIZATION TECHNIQUE

PANKAJ AGARWAL (PAMATHSFAC@GMAIL.COM)

Exercise 1. Find all primes p such that $p+2$ and p^2+2p-8 are primes. (Albania, 2012)

Exercise 2. Let P be a fixed prime number. Find all pairs (x, y) of positive integers satisfying $P(x-y) = xy$. (Estonia, 1996)

Exercise 3. Find all pairs (x, y) of positive integers that satisfy the equation $x^2 - xy + 2x - 3y = 2013$. (Lusophon, 2013)

Exercise 4. Find all positive integer solutions to $2x^2 + 5y^2 = 11(xy - 11)$. (Baltic Way, 1998)

Exercise 5. Let M and N be positive integers satisfying $MN^2 + 876 = 4MN + 217N$. Find the sum of all possible values of M. (Singapore Junior Section, 2012)

Exercise 6. Find all positive integer solutions of the equation $10(M+N) = MN$. (Croatia, 1998)

Exercise 7. How many pairs of integers (x, y) are there such that $2x+5y = xy-1$? (Turkey National Olympiad, 2003 - Round 1)

Exercise 8. Two different positive integers a and b satisfy the equation $a^2 - b^2 = 2018 - 2a$. What is the value of $a + b$? (Australia - Intermediate, 2018)

Exercise 9. Find all the positive integers x and y that satisfy the equation $x(x-y) = 8y-7$. (Junior Balkan Maths Olympiad, Shortlist - 2008)

Exercise 10. Solve $2a^2 + 3a - 44 = 3p^n$ in positive integers where p is a prime. (Turkey - Junior, 2019)

Exercise 11. Find all integers x for which $2x^2 - x - 36$ is the square of a prime number. (Croatia, 2001)

Exercise 12. Determine the set of integers n such that $n^2 + 19n + 92$ is the square of an integer. (RMO, 1992)

Exercise 13. Find all natural numbers n such that $n^2 - 19n + 91$ is a perfect square. (China, 1991)

Exercise 14. Find the sum of all positive integers n for which $n^2 - 19n + 99$ is a perfect square. (AIME, 1999)

Exercise 15. There exist unique positive integers x and y such that $x^2 + 84x + 2008 = y$. Find $x + y$. (AIME, 2008)

Exercise 16. Find all positive integers n such that $n^2 - 4n^3 + 22n^2 - 36n + 18$ is a perfect square. (China Western Maths Olympiad, 2002)

Exercise 17. Determine the pair of positive integers p and q that satisfy the equation $p^2 = q^2 + p + q + 2018$. (Lusophon, 2018)

Exercise 18. Show that the number $n^2 - 2^{2014} \times 2014n + 4^{2013}(2014^2 - 1)$ is not prime, where n is a positive integer. (Conosur, 2014)

Exercise 19. Find all integers (a, b) such that $a^2 + ab - b = 2018$. (New Zealand, 2018)

Exercise 20. How many pairs of positive integers (a, b) satisfy the equation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2004}?$$

(Flanders, Junior - 2004)