Discovering Mathematical Optimization with Python

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About me

- Research Internship, INOCS team
  INRIA Lille-Nord, Francia
- PhD(c) in Engineering Sciences,
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- Master in Industrial Engineering,
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About me

- My favorites programming languages:
  - Python
  - Julia (https://introajulia.org/)
  - C++
- Today's presentation (Python)
  - `https://github.com/pambus/or_gametheory`

Outline for section 1

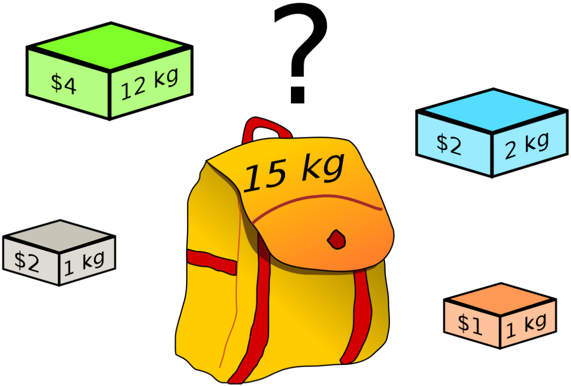
Operations Research

Scientific approach to decision making, that optimizes the operation of a system, generally assuming that resources are scarce. (Winston, 2005).

Example 1: Knapsack

We have n items and a backpack of capacity C. Each item i has a given utility u_i and weight w_i. We want to choose which items to carry in the backpack, maximizing the obtained utility and without exceeding the capacity of the backpack.

We will define an optimization model to solve this problem.



Parts of an optimization model

- The **decision variables** that we want to determine.
- The **objective function** (our goal) that we want to optimize (maximize or minimize).
- The **constraints** that the solution must satisfy.

Model for the Knapsack example

Variables

$$x_i = \begin{cases} 1 & \text{If item i is placed in the backpack} \\ 0 & \text{In other case} \end{cases} \quad \forall i \in [0, ..., n]$$

Model

$$\max \sum_i \text{utility}_i * x_i$$

s.a

$$\sum_i \text{weight}_i * x_i \leq \text{Capacity} \qquad \forall i \in [0, ..., n]$$

$$x_i \in [0, 1] \qquad \forall i \in [0, ..., n]$$

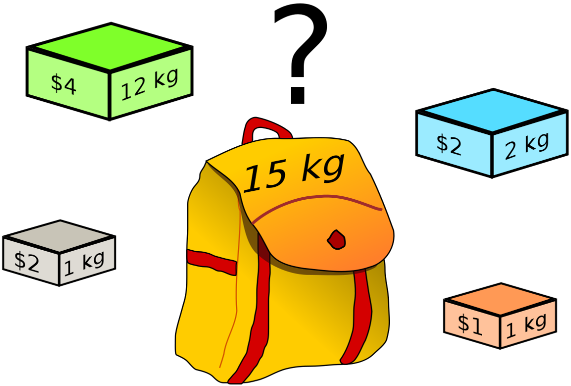
Model

$$\max\ 4x_1 + 2x_2 + 2x_3 + 1x_4$$

s.a

$$\sum_i 12x_1 + 2x_2 + 1x_3 + 2x_4 \leq 15$$

$$x_i \in [0, 1] \qquad \forall i \in [0, ..., n]$$



Ways to obtain exact solution

- **Manual**
  - Visual
  - Simplex (or others)
- **Solvers**
  - SCIP
  - Cplex
  - Gurobi
  - others

Example: Paint

Example 2.1-1, Taha 2012

A company produces interior and exterior paints with two raw materials: Pigments and Binders. The table below provides the basic data for the problem.

	Gallon of exterior paint	Gallon of interior paint	Maximum daily availability
Pigments	6	4	24
Binders	1	2	6
Utility per gallon	5	4	

This company wants to determine the optimal combination of interior and exterior painting that maximizes total daily utility.

Additional conditions:

- A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than one ton.
- Also, the maximum daily demand for interior paint is two kilograms.

Model for Paint example

Variables

x_1 = Gallons of exterior paint produced daily.

x_2 = Gallons of interior paint produced daily.

Objective Function

Maximum profit: $\max\ 5000x_1 + 4000x_2$

Constraints

Raw material pigments: $6x_1 + 4x_2 \leq 24$

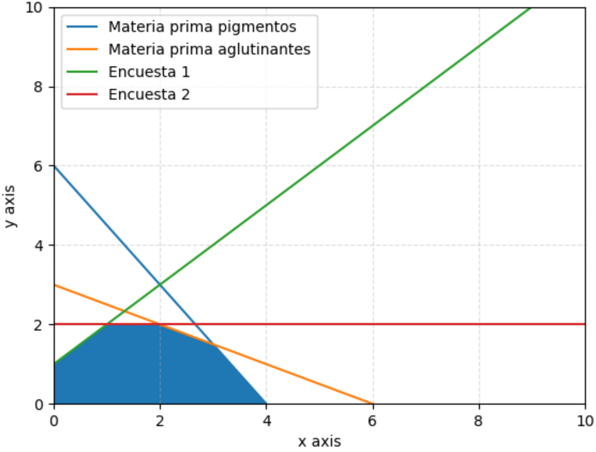
Raw material binders: $1x_1 + 2x_2 \leq 6$

Survey 1: $x_2 \leq x_1 + 1$

Survey 2: $x_2 \leq 2$

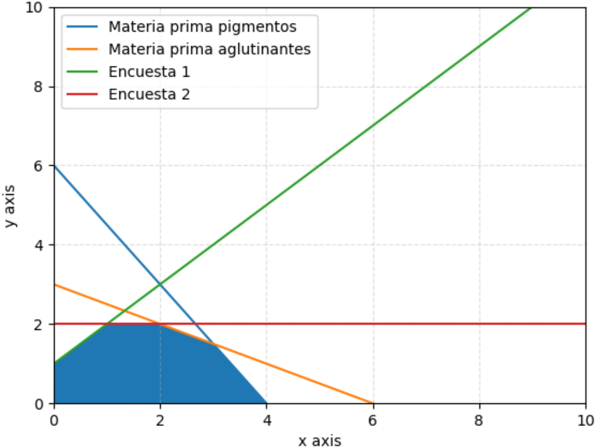
Non-negativity: $x_1, x_2 \geq 0$

Model for painting problem, visual solution



Important concepts for Operations Research

- A feasible solution: A solution that meets all constraints.
- Optimal solution: Best feasible solution.
- Feasible Region: Set of feasible solutions.



Heuristics

A form of problem solving that occupies practical methods that do not guarantee an optimal or perfect result.

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