Linear and Differential Cryptanalysis of DES

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- 2002: The AES standard becomes effective

1. Introduction
   - Data Encryption Standard
   - Linear Cryptanalysis
   - Differential Cryptanalysis
2. Mathematical Framework
   - Substitution-Permutation Network
   - Linear Attack on SPN
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3. Attacks on DES
   - Linear Attack: 4-round DES
   - Differential Attack: 4-round DES
4. Conclusion
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   - Take Home
   - Fun facts

A quick review of DES

DES Function

DES : (Plaintext (P), Key (K)) $\mapsto$ Cipher (C)

DES Unit Blocks

$$\begin{aligned}
\sigma_{i+1} : (0,1)^{64} &\rightarrow (0,1)^{64} \\
(L_i, R_i) &\mapsto (L_i \oplus f(R_i, K_{i+1}), R_i) \\
\tau : (0,1)^{64} &\rightarrow (0,1)^{64} \\
(L_{i+1}, R_{i+1}) &\mapsto (R_{i+1}, L_{i+1})
\end{aligned}$$

- Input for round $i+1$: (L_i, R_i)
- Subkey for round $i+1$: K_{i+1}
- Feistel function (f)
- 16-rounds for $i = 0, 1, ..., 15$

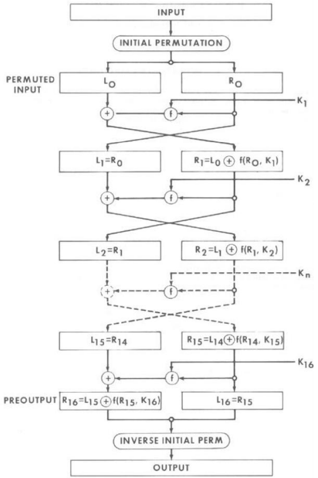

Figure: DES Algorithm

Designers: H Feistel, W Tuchman, D Coppersmith, A Konheim, E Grossman, B Notz, L Smith and B Tuckerman from IBM

Feistel Function

$$f : (0,1)^{32} \times (0,1)^{48} \rightarrow (0,1)^{32}$$
$$(R, K) \mapsto P(S_{Box}(E(R) \oplus K))$$

- Right half of the plaintext (R)
- Expansion function
  $E : (0,1)^{32} \rightarrow (0,1)^{48}$
- Key for the round (K)
- Confusion function (S-Boxes)
  $S_i : (0,1)^6 \rightarrow (0,1)^4$
- Diffusion function
  $P : (0,1)^{32} \rightarrow (0,1)^{32}$

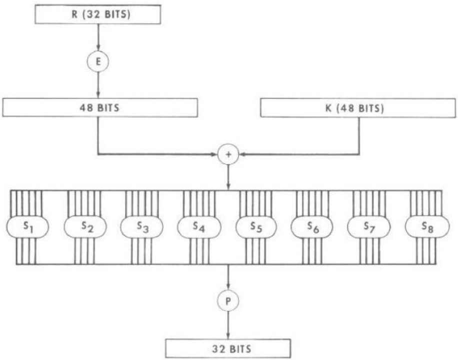

Figure: Feistel Round for DES

Mathematically, the different pieces of DES are:

- Initial Permutation
  $IP : (0,1)^{64} \longrightarrow (0,1)^{64}$
- DES Unit Block functions
  $\sigma_{i+1}(L_i, R_i)$ for $i = 0, 1, ..., 15$
- Transposition
  $\tau : (L_{i+1}, R_{i+1}) \longmapsto (R_{i+1}, L_{i+1})$ for $i = 0, 1, ..., 14$
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Algebraic Representation of DES

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\begin{aligned}
C &= DES_K(P) = IP^{-1}\, \sigma_{16}\, \tau \cdots \tau\, \sigma_1\, IP\,(\,P\,) \\
P &= DES_K^{-1}(C) = IP^{-1}\, \sigma_1\, \tau \cdots \tau\, \sigma_{16}\, IP\,(\,C\,)
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Note: The inverse DES comes from the fact that $\tau^2 = \sigma_i^2 = 1$

Basic Idea of Linear Cryptanalysis

Linear Attack on DES idea:

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- S-Boxes depend on a relatively small number of bits (6) which allows us to write down linear (or affine) expressions that approximate S-Boxes
- The effects of one round do not diffuse quickly over following rounds. Thus linear or affine expressions (as above) that hold per-round can be combined across rounds

Specifically, if P_i are plaintext bits, C_i are ciphertext bits, and K_i are subkey bits, then we wish to find an expression of the form

$$P_{i_1} \oplus P_{i_2} \cdots P_{i_j} \oplus C_{i_1} \oplus C_{i_2} \cdots C_{i_k} = K_{i_1} \oplus K_{i_2} \oplus K_{i_m}$$

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If we find an expression such as above that displays a high *bias*, that is, it holds much more or less frequently than $1/2$ the time, we can exploit this.

Basic Idea of Differential Cryptanalysis

Definition (Differential)

Suppose two plaintext inputs to the system be X and X' with corresponding output ciphertexts Y and Y' respectively. Then the pair of input difference ($\Delta X = X \oplus X'$) and the output difference ($\Delta Y = Y \oplus Y'$) is called a *differential* for the system.

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- The differences do not get diffused fast enough through the permutations
- Differentials are not affected by the round keys as they get XOR-ed out

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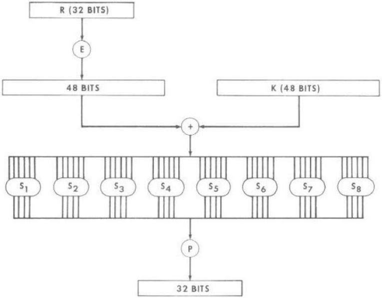
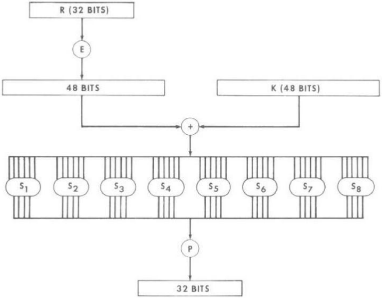
Note: This is essentially a chosen-plaintext attack as we want the specific input difference to occur for every pair.

Dive into the Mathematics

Substitution-Permutation Network





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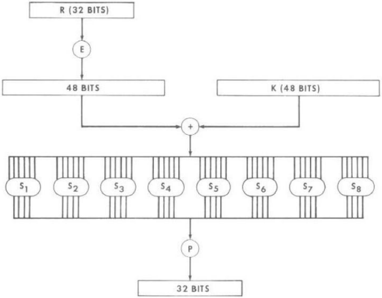
- Reducing the code length from 64 to 16



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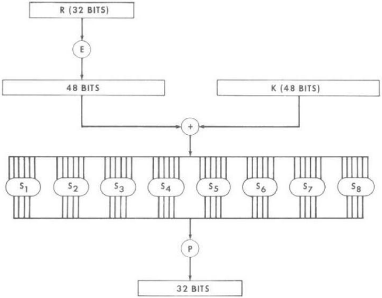

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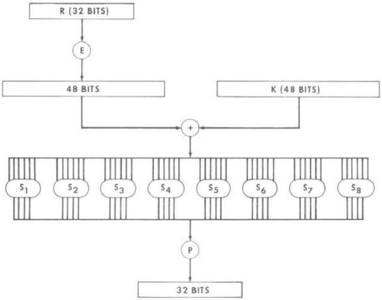

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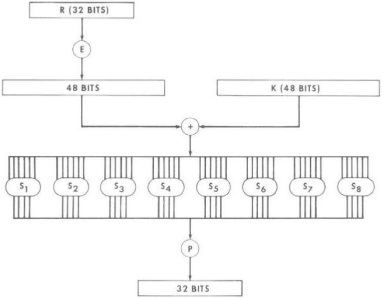

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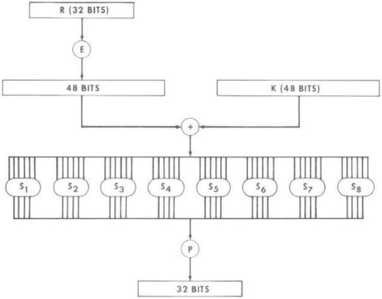

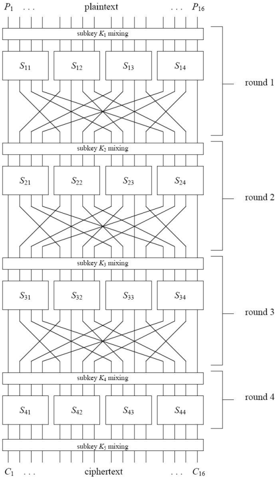

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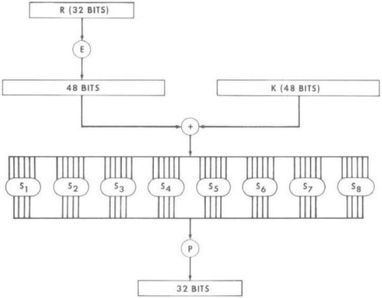

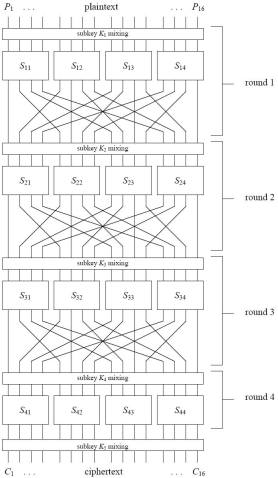

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Disclaimer: This simplification does not affect the discussion of the techniques of Linear/Differential Cryptanalysis.

Simple Substitution-Permutation Network

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- Finally, key-mixing is achieved by XOR-ing the round key with every input block to a round as well as at the end of the last round (so that we can't simply ignore the last round)
- The SPN cipher we will look at will consist of 4 rounds

SPN S-Box and Permutation

SPN uses a single 4-bit S-Box that has the following structure:

input	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
output	E	4	D	1	2	F	B	8	3	A	6	C	5	9	0	7

And the following 16-bit permutation:

input	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
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The S-Box provides the *confusion* function and the permutation applies the *diffusion* operation in SPN, thus making it cryptographically similar to DES.

Mathematics of Linear Cryptanalysis

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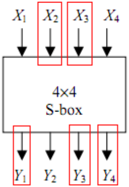
Note that there are 16 possibilities for U and V, hence 256 total possible expressions (for a 4-bit S-box).

S-Box Approximation Example



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$$X_2 \oplus X_3 = Y_1 \oplus Y_3 \oplus Y_4$$

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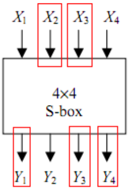

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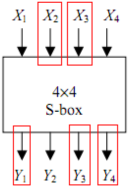

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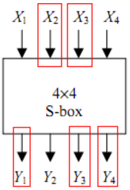

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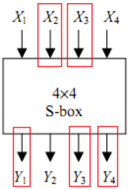

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The number of agreements (minus 8) between the S-Box and every possible expression is summarized in the table below. Thus to get the bias, one must only divide by 16.

		Output Sum															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Input Sum	0	+8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	−2	−2	0	0	−2	+6	+2	+2	0	0	+2	+2	0	0
	2	0	0	−2	−2	0	0	−2	−2	0	0	+2	+2	0	0	−6	+2
	3	0	0	0	0	0	0	0	0	+2	−6	−2	−2	+2	+2	−2	−2
	4	0	+2	0	−2	−2	−4	−2	0	0	−2	0	+2	+2	−4	+2	0
	5	0	−2	−2	0	−2	0	+4	+2	−2	0	−4	+2	0	−2	−2	0
	6	0	+2	−2	+4	+2	0	0	+2	0	−2	+2	+4	−2	0	0	−2
	7	0	−2	0	+2	+2	−4	+2	0	−2	0	+2	0	+4	+2	0	+2
	8	0	0	0	0	0	0	0	0	−2	+2	+2	−2	+2	−2	−2	−6
	9	0	0	−2	−2	0	0	−2	−2	−4	0	−2	+2	0	+4	+2	−2
	A	0	+4	−2	+2	−4	0	+2	−2	+2	+2	0	0	+2	+2	0	0
	B	0	+4	0	−4	+4	0	+4	0	0	0	0	0	0	0	0	0
	C	0	−2	+4	−2	−2	0	+2	0	+2	0	+2	+4	0	+2	0	−2
	D	0	+2	+2	0	−2	+4	0	+2	−4	−2	+2	0	+2	0	0	+2
	E	0	+2	+2	0	−2	−4	0	+2	−2	0	0	−2	−4	+2	−2	0
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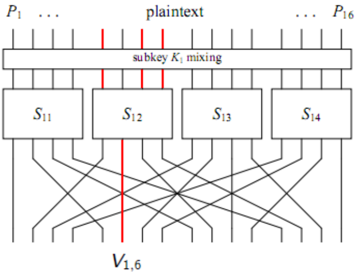
What this means for 1 Round:



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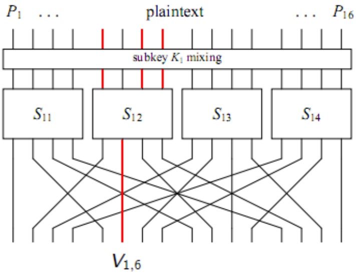

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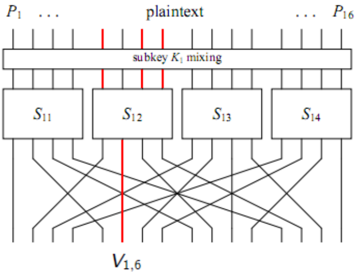

We can now write down the following linear approximation across the 1st round of SPN:

$$\begin{aligned} V_{1,6} &= U_{1,5} \oplus U_{1,7} \oplus U_{1,8} \quad \leftarrow \text{S-Box } S_{12} \text{ approximation above} \\ &= (P_5 \oplus K_{1,5}) \oplus (P_7 \oplus K_{1,7}) \oplus (P_8 \oplus K_{1,8}) \end{aligned}$$

This expression holds with probability of $3/4$ (bias of $+1/4$)

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Assume the following probability distribution

$$Pr(X_1 = i) = \begin{cases} p_1 & i = 0 \\ (1 - p_1) & i = 1. \end{cases}$$

$$Pr(X_2 = i) = \begin{cases} p_2 & i = 0; \\ (1 - p_2) & i = 1. \end{cases}$$

Piling Up Principle (continued):

Assuming that X_1 and X_2 are independent, we get

$$Pr(X_1 = i, X_2 = j) = \begin{cases} p_1 p_2 & i = 0, j = 0; \\ (1 - p_1)p_2 & i = 1, j = 0 \\ p_1(1 - p_2) & i = 0, j = 1 \\ (1 - p_1)(1 - p_2) & i = 1, j = 1. \end{cases}$$

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Hence the bias of $X_1 \oplus X_2 = 0$ is

$$2\epsilon_1\epsilon_2$$

That is, twice the product of the bias of the original expressions.

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Note that if some $\epsilon_k = 0$ (that is, some X_k has no bias) then total bias will be 0. More about the probabilistic analysis later.

Now, come back to per-round approximations. We can write down 4 approximations (S_{ij} represents the j-th S-Box in the i-th round):

$$S_{12} : X_1 \oplus X_3 \oplus X_4 = Y_2$$
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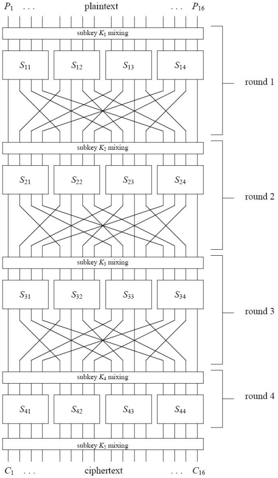

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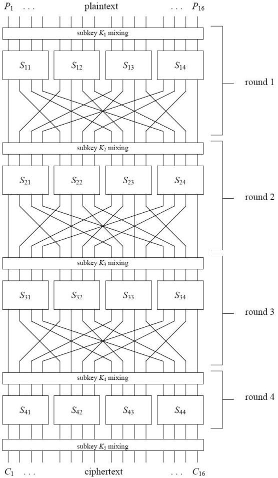
Each of these has a probability bias magnitude of 1/4. We can use the Piling Up Principle to combine them into a single expression relating plaintext bits to ciphertext bits.



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$$\Rightarrow (P_5 \oplus K_{1,5}) \oplus (P_7 \oplus K_{1,7}) \oplus (P_8 \oplus K_{1,8}) = V_{1,6}$$

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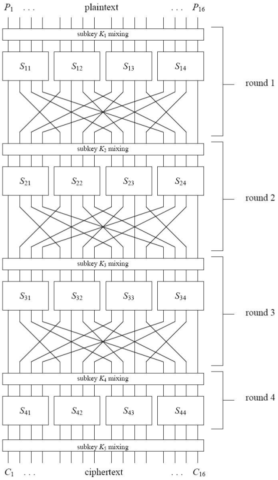

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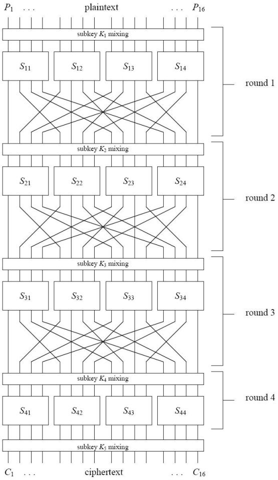

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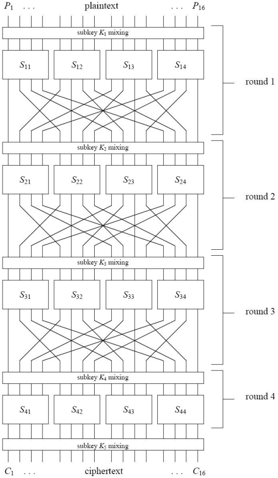
By the Piling Up Principle this holds with bias

$$2 \times 1/4 \times 1/4 = 1/8$$



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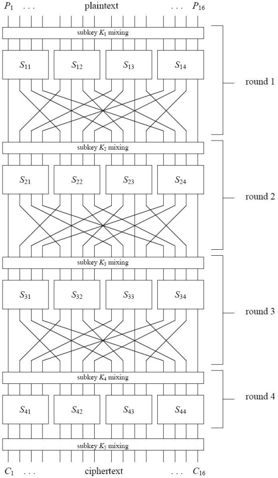

Note the implicit assumption that S-Boxes are independent. More on this later.

Using this principle we can write the following equation over 3 rounds of SPN:

$$U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 = \Sigma K$$

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Next we show how we can extract key bits using this information.

Attack Idea:

We have an expression that links plaintext bits to input bits to the 4th round of SPN that holds with high bias (1/32).

$$U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 = \Sigma K$$

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For each value of the guessed *target partial subkey* we can undo the last round and determine the bias of the equation. Highest bias indicates likely correct guess.

SPN Linear Approximation

$$U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 = 0$$

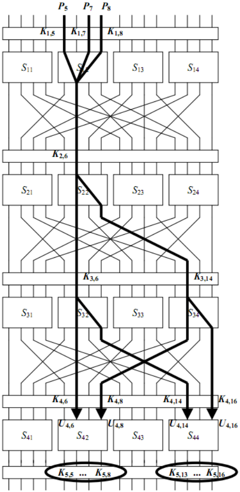

Figure: Linear Attack on SPN

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For the SPN cipher approximation with bias $1/32$ we need about 1000 pairs to perform the attack with near full probability of success.

DEMO: Attack on 4-round SPN

Mathematics of Differential Cryptanalysis

Differential: $(\Delta P, \Delta C) = (P \oplus P', C \oplus C')$

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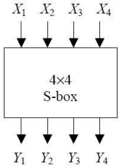
- Choose the characteristic with maximum probability and use that to retrieve subkey bits of the cipher

Analyzing the S-Box

Let us consider the S-Box of the SPN cipher as we constructed it:

- 4×4 S-Box
- Input: $X = [X_1\ X_2\ X_3\ X_4]$
- Output: $Y = [Y_1\ Y_2\ Y_3\ Y_4]$
- Difference pair: $(\Delta X, \Delta Y)$



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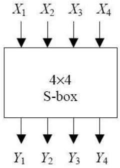

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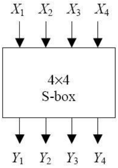

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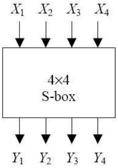

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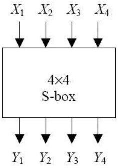

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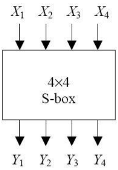

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5. Iterate through steps 2 to 4 for $\Delta X = \{0000, ..., 1111\}$

Sample Difference Pairs for the S-Box

X	Y	ΔY		
		$\Delta X = 1011$	$\Delta X = 1000$	$\Delta X = 0100$
0000	1110	0010	1101	1100
0001	0100	0010	1110	1011
0010	1101	0111	0101	0110
0011	0001	0010	1011	1001
0100	0010	0101	0111	1100
0101	1111	1111	0110	1011
0110	1011	0010	1011	0110
0111	1000	1101	1111	1001
1000	0011	0010	1101	0110
1001	1010	0111	1110	0011
1010	0110	0010	0101	0110
1011	1100	0010	1011	1011
1100	0101	1101	0111	0110
1101	1001	0010	0110	0011
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0110	1011	0010	1011	0110
0111	1000	1101	1111	1001
1000	0011	0010	1101	0110
1001	1010	0111	1110	0011
1010	0110	0010	0101	0110
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Note that, for $\Delta X = 1011$, $\Delta Y = 0010$ occurs 8 times, out of the possible 16 times. So, the pair $(1011, 0010)$ has a probability of occurrence $8/16 = 1/2$.

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0111	1000	1101	1111	1001
1000	0011	0010	1101	0110
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We can tabulate the complete data for the S-Box to check this probabilities.

Difference Distribution Table

Input Difference	Output Difference 0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	0	0	0	2	0	2	4	0	4	2	0	0
2	0	0	0	2	0	6	2	2	0	2	0	0	0	0	2	0
3	0	0	2	0	2	0	0	0	0	4	2	0	2	0	0	4
4	0	0	0	2	0	0	6	0	0	2	0	4	2	0	0	0
5	0	4	0	0	0	2	2	0	0	0	4	0	2	0	0	2
6	0	0	0	4	0	4	0	0	0	0	0	0	2	2	2	2
7	0	0	2	2	2	0	2	0	0	2	2	0	0	0	0	4
8	0	0	0	0	0	0	2	2	0	0	0	4	0	4	2	2
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B	0	0	8	0	0	2	0	2	0	0	0	0	0	2	0	2
C	0	2	0	0	2	2	2	0	0	0	0	2	0	6	0	0
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E	0	0	2	4	2	0	0	0	6	0	0	0	0	0	2	0
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2	0	0	0	2	0	6	2	2	0	2	0	0	0	0	2	0
3	0	0	2	0	2	0	0	0	0	4	2	0	2	0	0	4
4	0	0	0	2	0	0	6	0	0	2	0	4	2	0	0	0
5	0	4	0	0	0	2	2	0	0	0	4	0	2	0	0	2
6	0	0	0	4	0	4	0	0	0	0	0	0	2	2	2	2
7	0	0	2	2	2	0	2	0	0	2	2	0	0	0	0	4
8	0	0	0	0	0	0	2	2	0	0	0	4	0	4	2	2
9	0	2	0	0	2	0	0	4	2	0	2	2	2	0	0	0
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B	0	0	8	0	0	2	0	2	0	0	0	0	0	2	0	2
C	0	2	0	0	2	2	2	0	0	0	0	2	0	6	0	0
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3	0	0	2	0	2	0	0	0	0	4	2	0	2	0	0	4
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B	0	0	**8**	0	0	2	0	2	0	0	0	0	0	2	0	2
C	0	2	0	0	2	2	2	0	0	0	0	2	0	6	0	0
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- In an ideal S-Box, we would like all entries to be 1
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- Just divide the entries by $2^4 = 16$ to get the probabilities, and exploit the scenario of highest probability of occurrence ($Pr(2|B) = 8/16 = 1/2$ here)

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But, from our discussion above, this is infeasible, and we hope to exploit this.

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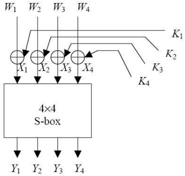
- Difference pair: $(\Delta W, \Delta Y)$



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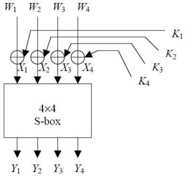

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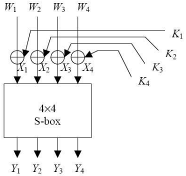

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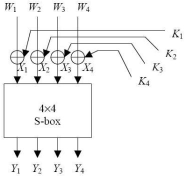

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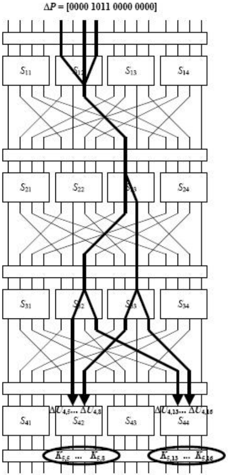
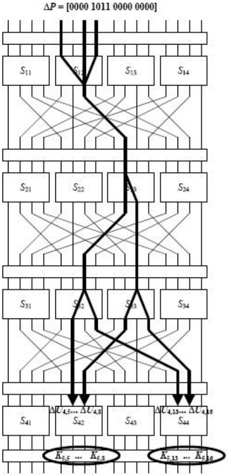
Thus, the keyed S-Box has the same difference distribution table.

Constructing a Differential Characteristic





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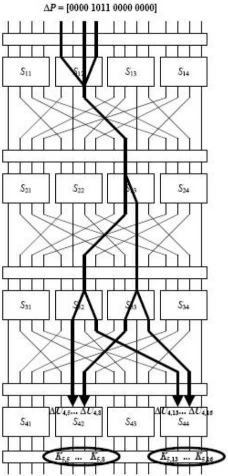

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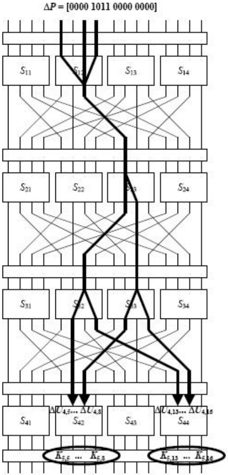

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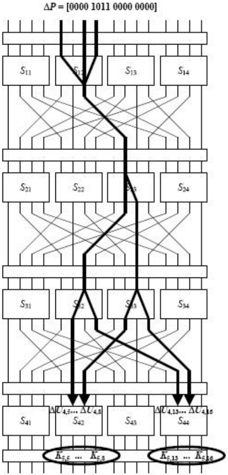

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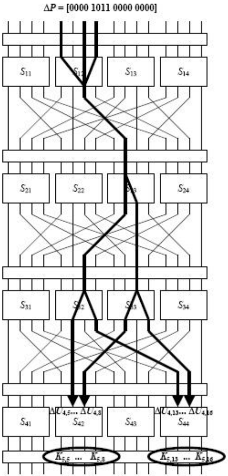

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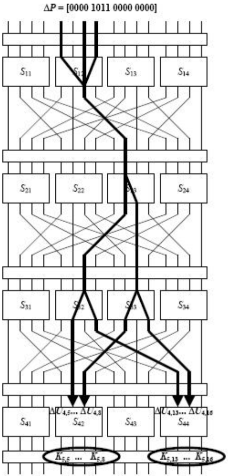

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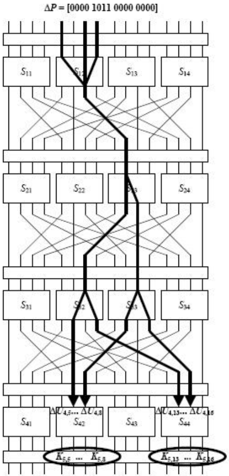

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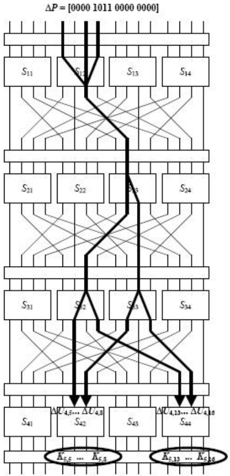

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$$0B00 \rightarrow 0040 \rightarrow 0220 \rightarrow 0606$$

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Note: The probability of this characteristic $Pr = 27/1024 \gg 1/2^{16}$

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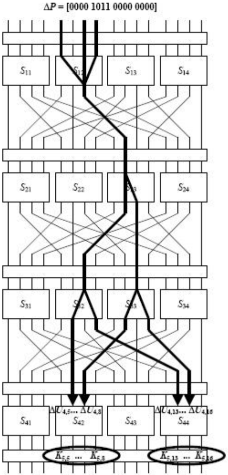
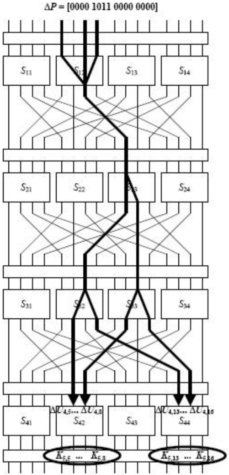
- Highest probability indicates likely correct guess



Differential Characteristic flow



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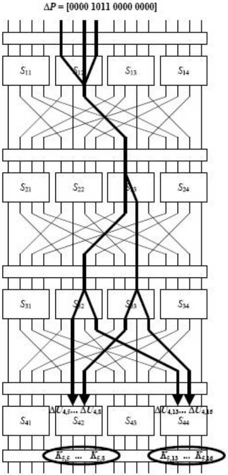

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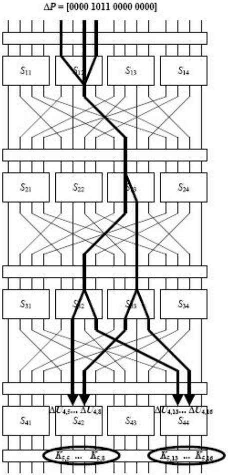

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Partial Subkey guess only for

$$[K_{5,5} \ldots K_{5,8}, K_{5,13} \ldots K_{5,16}]$$

DEMO: Attack on 4-round SPN

It's time for a Break!

Do these work for DES?

Linear Cryptanalysis of 4 Round DES

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- We will adapt the linear attack on SPN to a linear attack on a 4-round DES algorithm and mount this attack.
- We will also show how to extend this attack to a full 16-round DES algorithm and show it is faster than an exhaustive search.

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For a given S-Box $a \in \{1, 2, \ldots 8\}$, $1 \leq \alpha \leq 63$, $1 \leq \beta \leq 15$, we define $NS_a(\alpha, \beta)$ as the number of times (out of 64) for S-Box a that for all the input patterns masked by α the output pattern masked by β agrees with the value of S-Box a.

$$NS_a(\alpha, \beta) = |\{X | 0 \leq X < 64, (\bigoplus_{i=0}^{5} X[i] \cdot \alpha[i]) = (\bigoplus_{j=0}^{3} S_a(X)[j] \cdot \beta[j])\}|$$

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This looks more complicated, but is in fact the same expression as for SPN, except it takes into account all possible S-Boxes.

Using *NS* to find a linear approximation for the S-Boxes.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	4	-2	2	-2	2	-4	0	4	0	2	-2	2	-2	0	-4
3	0	-2	6	-2	-2	4	-4	0	0	-2	6	-2	-2	4	-4
4	2	-2	0	0	2	-2	0	0	2	2	4	-4	-2	-2	0
5	2	2	-4	0	10	-6	-4	0	2	-10	0	4	-2	2	4
6	-2	-4	-6	-2	-4	2	0	0	-2	0	-2	-6	-8	2	0
7	2	0	2	-2	8	6	0	-4	6	0	-6	-2	0	-6	-4
8	0	2	6	0	0	-2	-6	-2	2	4	-12	2	6	-4	4
9	-4	6	-2	0	-4	-6	-6	6	-2	0	-4	2	-6	-8	-4
10	4	0	0	-2	-6	2	2	2	2	-2	2	4	-4	-4	0
11	4	4	4	6	2	-2	-2	-2	-2	-2	2	0	-8	-4	0
12	2	0	-2	0	2	4	10	-2	4	-2	-8	-2	4	-6	-4
13	6	0	2	0	-2	4	-10	-2	0	-2	4	-2	8	-6	0
14	-2	-2	0	-2	4	0	2	-2	0	4	2	-4	6	-2	-4
15	-2	-2	8	6	4	0	2	2	4	8	-2	8	-6	2	0
16	2	-2	0	0	-2	-6	-8	0	-2	-2	-4	0	2	10	**-20**
17	2	-2	0	4	2	-2	-4	4	2	2	0	-8	-6	2	4
18	-2	0	-2	2	-4	-2	-8	4	6	4	6	-2	4	-6	0
19	-6	0	2	-2	4	2	0	4	-6	4	2	-6	4	-2	0
20	4	-4	0	0	0	0	0	-4	-4	4	4	0	4	-4	0
21	4	0	-4	-4	4	-8	-8	0	0	-4	4	8	4	0	4
22	0	6	6	2	-2	4	0	4	0	6	2	2	2	0	0
23	4	-6	-2	6	-2	-4	4	4	-4	-6	2	-2	2	0	4
24	6	0	2	4	-10	-4	2	2	0	-2	0	2	4	-2	-4
25	2	4	-6	0	-2	4	-2	6	8	6	4	10	0	2	-4

Figure: $NS_5(\alpha, \beta) - 32$

It turns out that the highest bias is for S-Box 5 with $\alpha = 16$ and $\beta = 15$

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holds with bias of absolute value ~ 0.31

Tracing through the Feistel function to the right input bits R, round key bits K, and Feistel function output bits F we get

$$R_{17} \oplus F_{25} \oplus F_{14} \oplus F_8 \oplus F_3 = K_{26}$$

Let P_L and P_R be the left and right plaintext bits. Let C_L and C_R be the left and right ciphertext bits. Let K_i be the round key for round i and U_{4L} and U_{4R} the left and right halves of the input to round 4.

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The bias magnitude for this expression is

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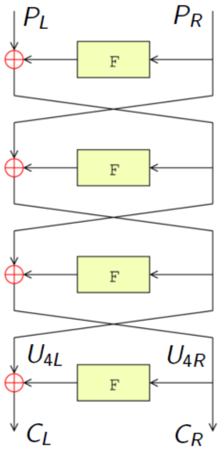
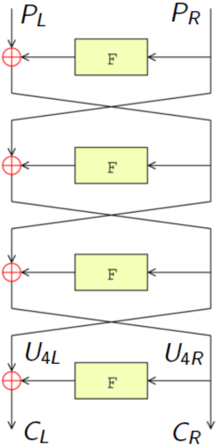
This is the best expression for the 3-round DES.



We perform the attack on the 4-round DES similarly to how we performed the attack on 4-round SPN, by "undoing" the last round.



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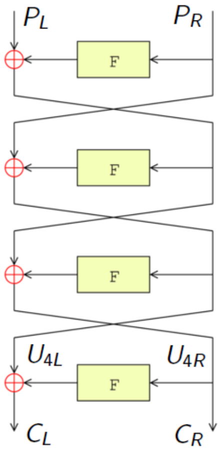

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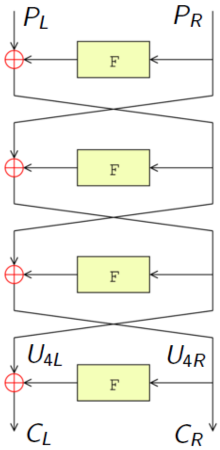
This bit, $U_{4L}[17]$, is affected by the result of S-Box 1 in round 4. If we guess correctly for the 6 partial subkey bits $K_{4,1}$ to $K_{4,6}$, the 3-round equation will hold with high bias, otherwise it will likely hold with close to 0 bias.



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Thus, this attack will give us 6 of the 56 key bits. If it takes a year to find a key through exhaustive search of 2^{56} bits, it only takes 5 days to exhaustively search 2^{50} bits.

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Then if $q^{(i)}$'s are independent, the success rate of the attack is

$$\int_{x=-2\sqrt{N}\epsilon}^{\infty} \left(\prod_{K_n^{(i)} \neq K_n} \int_{-x-4\sqrt{N}\epsilon q^{(i)}}^{x+4\sqrt{N}\epsilon(1-q^{(i)})} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \right) \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

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Let $d = 1$ and $l_1 = 17$ in the above expression - this corresponds to our attack (distribution over guessing for one bit, $U_{4R}[17]$ in the equation). The following table summarizes the success rates:

N	$2\epsilon^{-2}$	$4\epsilon^{-2}$	$8\epsilon^{-2}$	$16\epsilon^{-2}$
Success Rate	0.486	0.785	0.967	0.999

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For a full 16-round DES, the best approximation has bias $\sim 1.19 \times 10^{-22}$, thus we would need on the order of 2^{47} plaintext-ciphertext pairs.

DEMO: Attack on 4-round DES

Differential Cryptanalysis of 4 Round DES

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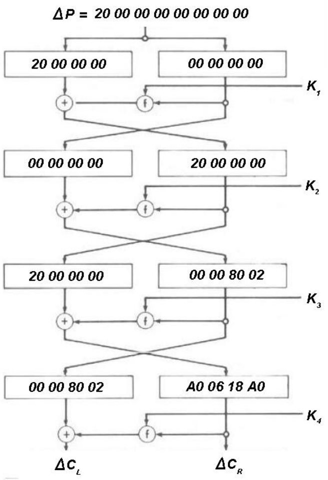
- The guess approving the differential characteristic with highest probability is correct



Differential Characteristic of DES

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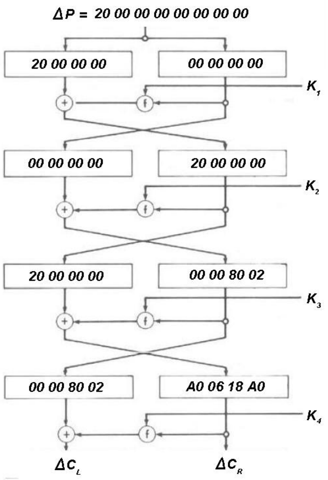

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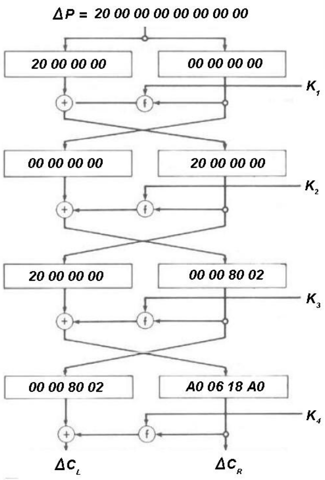

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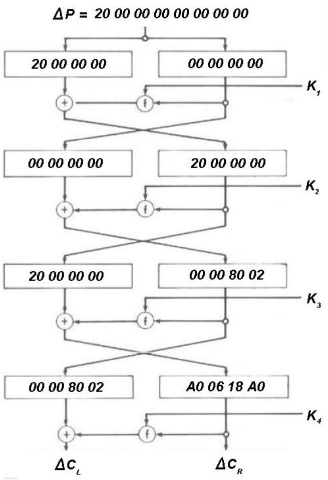

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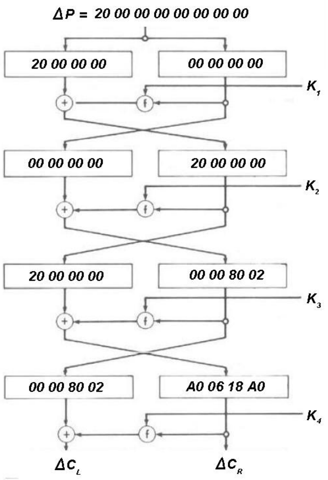

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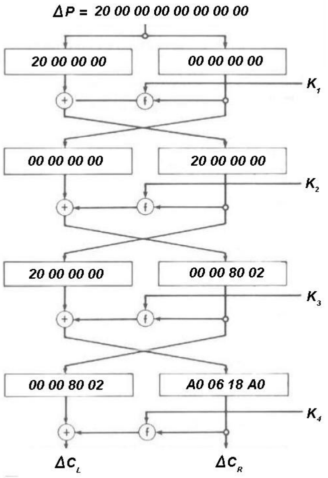

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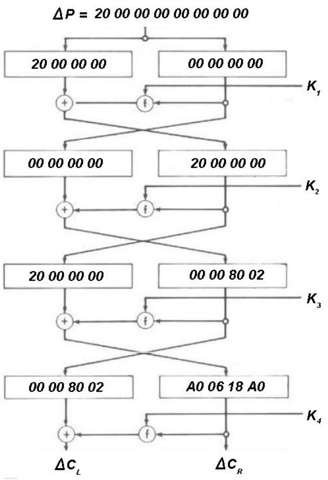
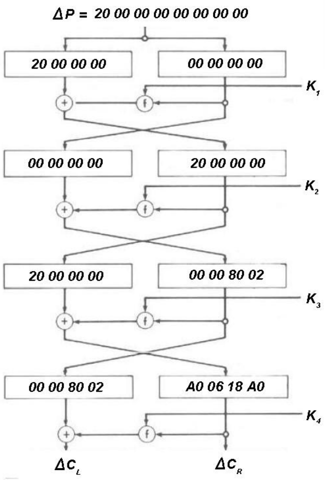
Differential Characteristic

$20\,00\,00\,00\,00\,00\,00\,00 \rightarrow A0\,06\,18\,A0\,00\,00\,80\,02$

Differential Attack: 4-round DES





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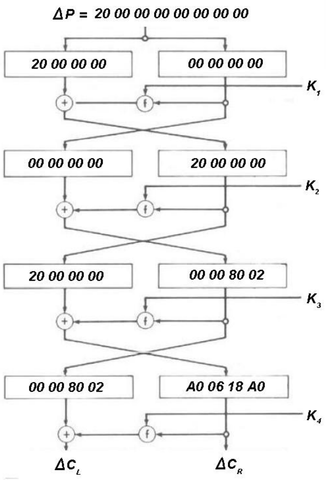

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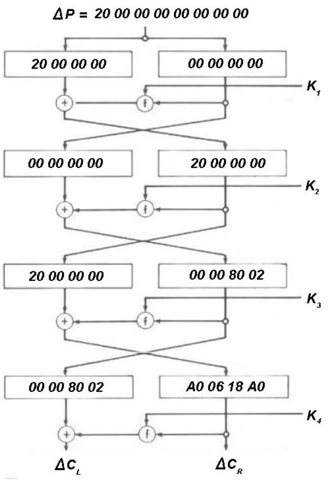

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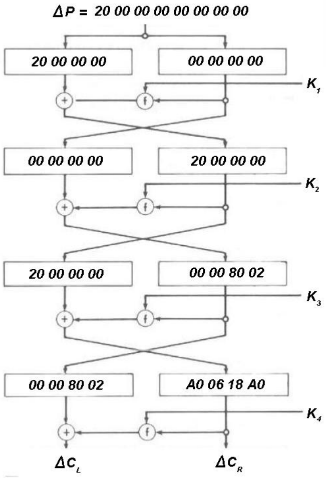
So, just guess the *target partial subkey* for $S_2, ...S_8$ individually,



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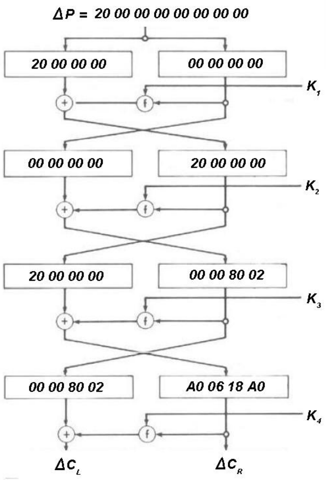

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I used $2^4 = 16$ pairs for 100% success in this case. For 16 round DES, 2^{47} pairs are needed, out of which 2^{36} are "good pairs".

DEMO: Attack on 4-round DES

Concluding Remarks

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This assumption worked well for us in practice, but is not necessarily true. We could have proceeded differently without this assumption - [see John Manferdelli's Boolean Functions slides]

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Defn: A function F is bent if its Walsh transform $\mathcal{W}\{F\}$ is $\pm 2^{p/2}$.

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To sumarize: When $p \geq 2q$ and p even, differential-resistant is equivalent to linear resistant, and to vectorial Bentness.

‘Moral of the story’

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- Verify if this is better than any other existing attack

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AES, which replaced DES, provides an 8-Bit S-Box in the SubBytes step and higher diffusion through the ShiftRows and MixColumns steps.

Just for Fun!

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- Matsui prescribed another rearrangement of the S-Boxes, which is good enough to prevent both linear and differential attacks.
- Biham and Shamir illustrated a rearrangement where differential attack becomes way more efficient.

IBM Knew in 1974

Date: Wed, 19 Feb 92 09:43:31 EST
From: "Don Coppersmith" <copper@watson.ibm.com>

Adi,

We have kept quiet about the following for 18 years, and decided it's time to break the silence.

We (IBM crypto group) knew about differential cryptanalysis in 1974. This is why DES stood up to this line of attack; we designed the S-boxes and the permutation in such a way as to defeat it.

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- Each output difference bit permuted to 1 each to S-boxes of next round

Another Email with Don Coppersmith

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Date: Mon, 14 Oct 91 09:12:35 EDT
From: "Don Coppersmith" <copper@watson.ibm.com>
To: biham@cs.technion.ac.il
Subject: press reports

Eli,
The press reports (NY Times, Oct 4 and again Oct 13) both imply a very
small amount of chosen plaintext
...
Yet my understanding of DES and differential cryptanalysis is that
massive amounts of chosen plaintext is required, more like 10^15,
give or take an order of magnitude.

I understand you don't want to publicize the actual parameters.
But please confirm my suspicion that the reporters have misunderstood
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Thanks.   Don
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Don was wrong. 2^{47} is closer to 10 than to 10^{15}.

- Eli Biham

References:

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J. Manferdelli, UW Cryptography Lecture Notes

The End.

Questions?