TN 412: DIGITAL SIGNAL PROCESSING

LECTURE ONE: Introduction to DSP

1.1: OVERVIEW OF DSP

INTRODUCTION

- A signal is defined as any physical quantity that varies with time, space or another independent variable.

- A signal carries information like speech, music, seismic, image and video.

INTRODUCTION

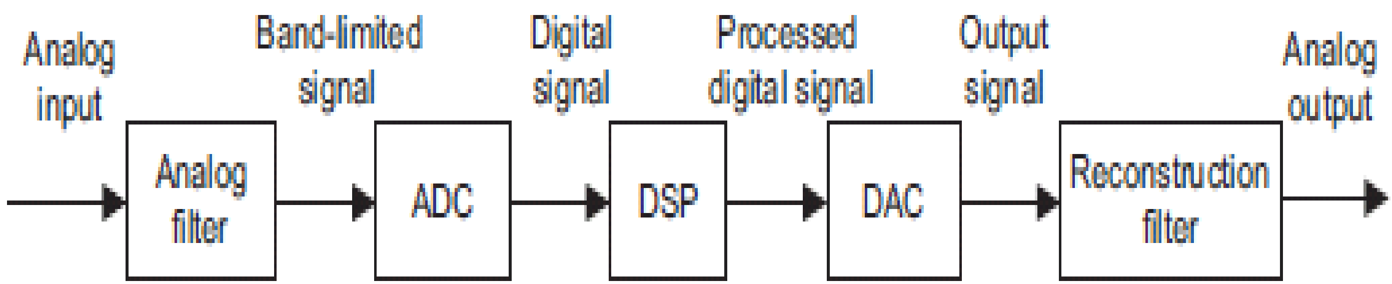
- A system is defined as a physical device that performs an operation on a signal.
- System is characterized by the type of operations that are performed on the signal.
- Such operations are referred to as signal processing.

INTRODUCTION

- Need for Signal Processing
  - When a signal is transmitted from one point to another, there is every possibility of contamination /deformation of the signal by external noise.
  - So to retrieve the original signal at the receiver, suitable filters are to be used. i.e. the signal is processed to obtain the pure signal.

Structure of a DSP system

- BASIC ELEMENTS OF DIGITAL SIGNAL PROCESSING



Cont…

- Analog signal is fed to an analog filter, to limit the frequency range of analog signals prior to the sampling process.

- The band-limited signal at the output of the analog filter is then sampled and converted via the ADC unit into the digital signal, which is discrete both in time and in amplitude.

- The DS processor then accepts the digital signal and processes the digital data according to DSP rules such as lowpass, highpass, and bandpass digital filtering, or other algorithms for different applications.
    - DS processor unit is a special type of digital computer and can be a general-purpose digital computer, a microprocessor, or an advanced microcontroller.

Filters

Any DSP block diagram must contain the filter as the main component of DSP.

Filters have two uses

a) Signal separation
b) Signal restoration

Cont…..

Signal separation is needed when a signal has been contaminated with interference, noise or other signals.

Signal restoration is used when a signal has been distorted in some way or other.

For example an audio recording made with poor equipment may be filtered to get the original sound.

Cont….

Another example of deblurring of an image occurred with an improperly focused lens or a shaky camera.

So these problems can be solved with either analog or digital filters.

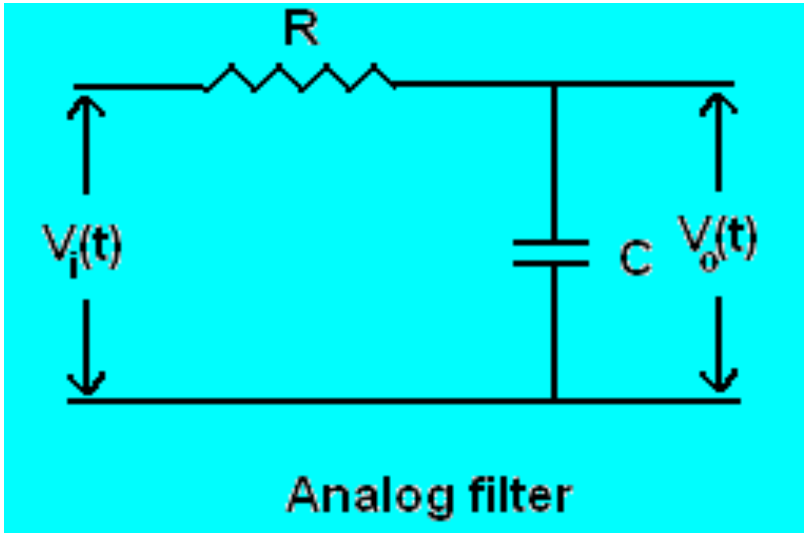
Analog filters

Analog filters take the analog signal as input and process the signal and finally gives the analog output.

An analog filter is constructed using resistors, capacitors, active components etc…

Analog filters

A simple analog low pass filter is shown below



Analog filters

Coming to advantages of Analog filters they are cheap and have a large dynamic range in both amplitude and frequency. But in terms of performance they are not superior to digital filters.

Digital filters

A digital filter processes and generates digital data.

A digital filter constitutes elements like adder, multiplier and delay units.

Digital filters are vastly superior in the level of performance in comparison to analog filters.

A simple digital filter

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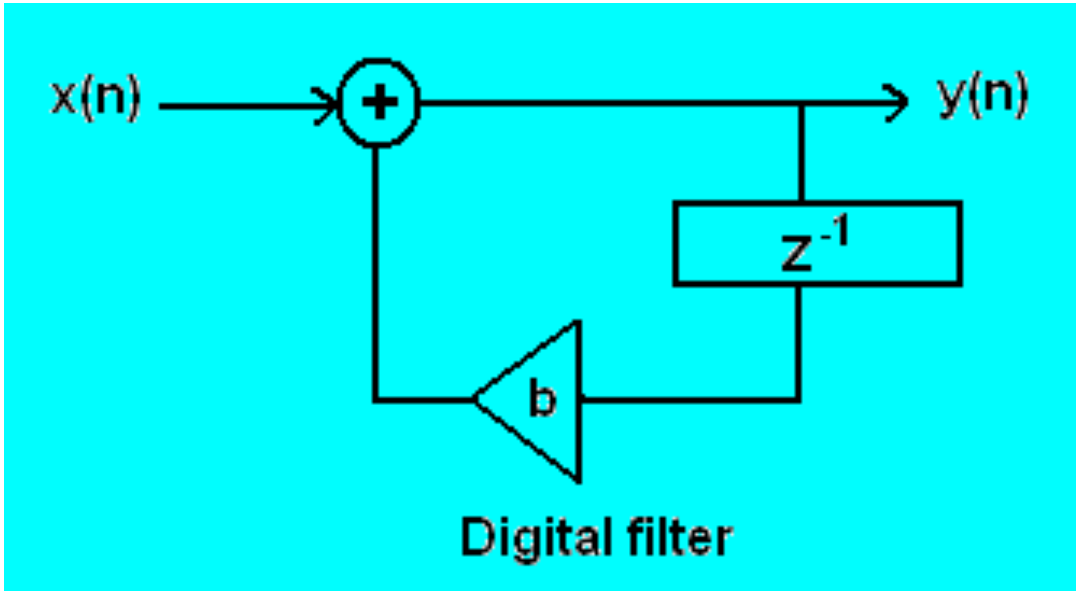


Advantages :

There are many advantages with digital filters.

- Unlike analog filters ,the digital filter performance is not influenced by component ageing, temperature and power variations.

Advantages :

- A digital filter is highly immune to noise and relatively stable.
- Digital filters afford a wide variety of shapes for the amplitude and phase responses.
- Impedance matching problems are minimum.

Advantages :

- Transportation and reconfiguration is very easy ,which is not true in the case of analog filters.
- Multiple filtering is possible only in digital filters.
- Computational problems are minimum.

Disadvantages:

There are few disadvantages also.

- They are expensive
- Quantization error occurs due to finite word length in the representation of signals and parameters.
- The accuracy of the digital filter depends on the word length used to encode them in binary form.
- The signal bandwidth of the input signal is limited by ADC and DAC.

Differences between analog and digital filters :

- An analog filter is constructed using active, passive components like resistors, capacitors and opamps etc..

  A digital filter constitutes adder, multiplier and delay elements
- An analog filter is denoted by a differential equation.

  A digital filter is denoted by a difference equation.

- Laplace transform is used for the analysis of analog filter.

  Z transforms are used for the analysis of digital filters.
- The frequency response of an analog filter can be modified by changing the components.

  The frequency response can be changed by changing the filter coefficients.

Types of Digital filters

Broadly speaking ,two types of digital filters exists.

- FIR Filters(Finite impulse response filters)
- IIR Filters (Infinite Impulse response filters)

ADVANTAGES OF DSP

- **Accuracy**: The analog circuits are prone to temperature and external effects, but the digital filters have no such problems.

- **Flexibility**: Reconfiguration of analog filters is very complex whereas the digital filters can be reconfigured easily by changing the program coefficients.

Cont…

- Digital signals can be easily **stored on any magnetic** media or optical media.

- **Easy operation:** Even complex mathematical operations can be performed easily using computers, which is not the case for analog signal processing.

Cont….

- **Multiplexing:** Digital signal processing provides the way for Integrated service digital network (ISDN) where digitized signals can be multiplexed with other digital data and transmitted through the same channel.

DISADVANTAGES OF DSP

- When analog signal is changing very fast, it is difficult to convert to digital form .(beyond 100KHz range)
- When the signal is weak, within a few tenths of millivolts, we cannot amplify the signal after it is digitized.
- DSP hardware is more expensive than general purpose microprocessors & micro controllers.

Cont….

- DSP techniques are limited to signals with relatively low bandwidths (5 MHz video bandwidth).

- The need for an ADC and DAC makes DSP uneconomical for simple applications(e.g. simple filters).

- Higher power consumption and size of a *DSP* implementation may make it unsuitable for small-size applications.

APPLICATION OF DSP

Digital signal processing has variety of applications in diverse fields like:

- ✓Digital filtering
- ✓Spectral analysis
- ✓Radar processing
- ✓Biomedical engineering
- ✓Military applications
- ✓ Speech compression for increased storage space
- ✓Speech processing
- ✓Image processing
- ✓Disk and robot control
- ✓Telecommunication
- ✓Consumer electronics

Signals

1.2 CLASSIFICATION OF SIGNALS

- Multiplechannel and Multidimensional signals
  - Multiplechannel signals are those signals generated by multiple sources or multiple sensors. And they can be represented in vector form.

$$\mathbf{S}_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

Cont…

- Multi-dimensional is the signal which is a functional of M independent variables.

$$\mathbf{I}(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t) \end{bmatrix}$$

1.2. Classification of Signals

Continuous-Time versus Discrete-Time Signals:

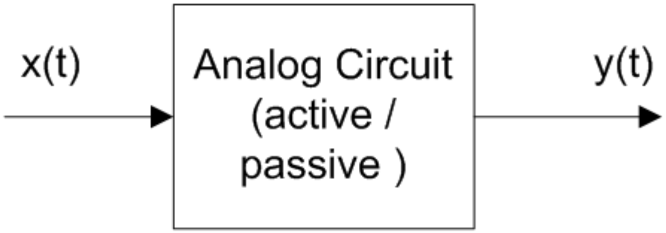
Continuous-Time or analog signal are defined for every value of time.

$$x_1(t) = \cos \pi t$$

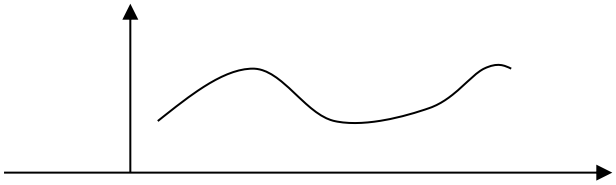
$$x_2(t) = e^{-|t|}$$

are examples of analog signals

- **Analog Signal**
  - **Continuous in time.**
  - **Amplitude may take on any value in the continuous range of (-∞, ∞).**



- **Analog Processing**
  - **Differentiation, Integration, Filtering, Amplification.**
  - **Implemented via passive or active electronic circuitry.**



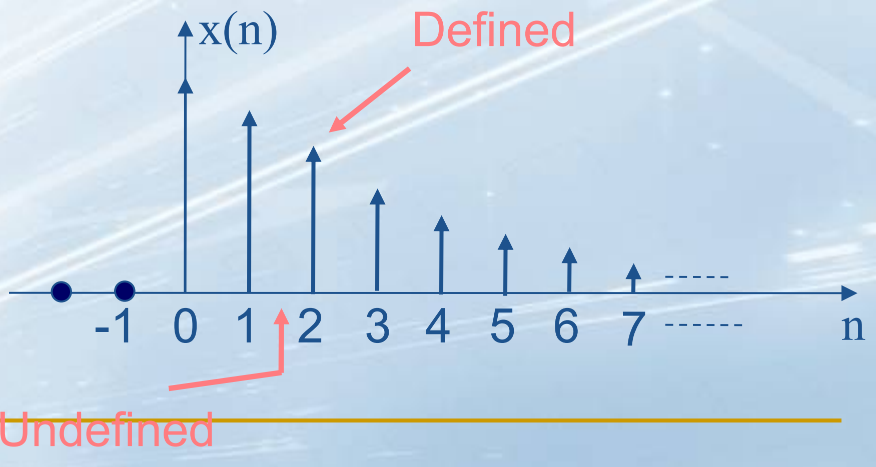
1.2. Classification of Signals

Continuous-Time versus Discrete-Time Signals:

Discrete-Time signals are defined only at certain specific value of time.

- **Continuous Amplitude.**
- **Only defined for certain time instances.**
- **Can be obtained from analog signals via sampling.**

$$x(n) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



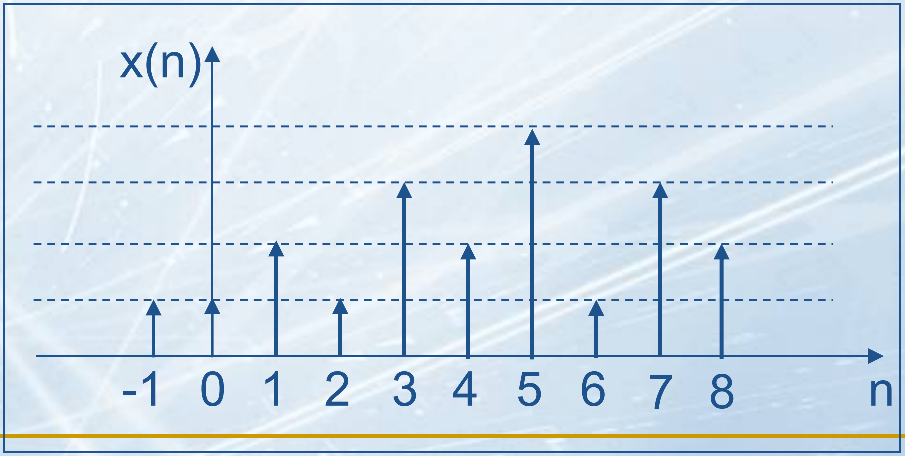
The function provide an example of a discrete-time signal.

1.2. Classification of Signals

Continuous-Valued versus Discrete-Valued Signals:

The values of a CT or DT Signal can be continuous or discrete.

- **If a signal takes on all possible values of a finite or an infinite range, it is CONTINUOUS-VALUED Signal.**
- **If the signal takes on values from a finite set of possible values, it is DISCRETE-VALUED Signal. Also called *Digital Signal* because of the discrete values.**



Digital Signal with 4 different amplitude values

1.2. Classification of Signals

Deterministic versus Random Signals:

Deterministic Signal

Any signal whose past, present and future values are precisely known without any uncertainty. It can be expressed by formula

$$x(n) = 7 \cdot \cos(5 \cdot n)$$

Random Signal

A signal in which cannot be approximated by a formula to a reasonable degree of accuracy (i.e. noise).

1.3. Concept of Frequency in CT & DT Signals

The concept of frequency is directly related to the concept of time. It has the dimension of inverse time.

1.3.1. Continuous-Time Sinusoidal Signals:

A simple harmonic oscillation is mathematically described by the following CT sinusoidal signal:

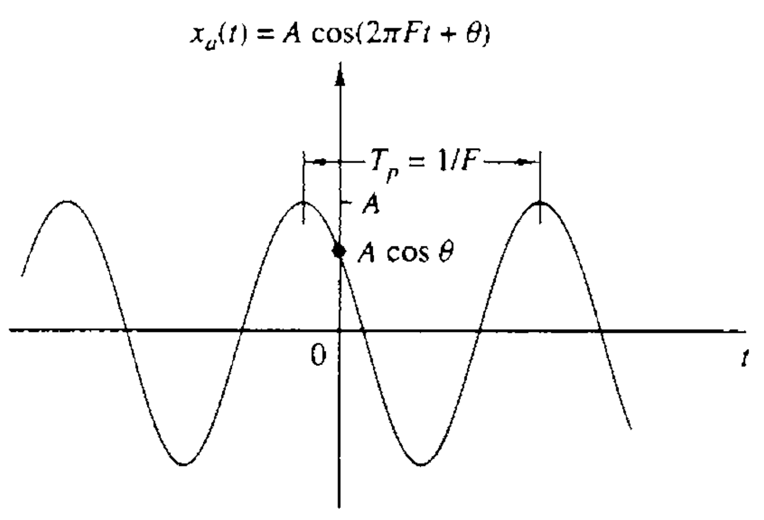
θ phase in rad

Amplitude

$$x_a(t) = A\cos(\Omega t + \theta), \quad -\infty < t < \infty$$

Analog Signal

Ω is frequency in rad/s



Instead of Ω the frequency F in Hz is used

$$\Omega = 2\pi F$$

$$x_a(t) = A\cos(2\pi Ft + \theta), \quad -\infty < t < \infty$$

1.3. Concept of Frequency in CT & DT Signals

Analog Sinusoidal Signal Properties :

- **For every fixed value of the frequency *F*, $x_a(t)$ is periodic.**

$x_a(t+T_P) = x_a(t)$ where $T_P = 1/F$ is the **fundamental period** of the sinusoidal signal.

- **CT sinusoidal signal with different frequencies are themselves different.**

$$\cos(20\cdot\pi\cdot t) \neq \cos(40\cdot\pi\cdot t)$$

- **Increasing the frequency *F* results in an increase in the rate of oscillation of the signal.**

1.3. Concept of Frequency in CT & DT Signals

Analog Sinusoidal Signal Periodicity:

- if $x_a(t+T_p) = x_a(t)$ then $x_a(t) \equiv periodic$
- T_P is the smallest value to satisfy the above property.
- **Fundamental Period:** $T_p = \frac{1}{F} = \frac{2\cdot\pi}{\Omega}$
- **Proof:**

$$\cos(a \pm b) = \cos(a)\cdot\cos(b) \mp \sin(a)\cdot\sin(b$$

$$x_a(t+T_p) = A\cdot\cos[\Omega\cdot(t+T)+\theta]$$
$$\Rightarrow A\cdot\cos\left[\Omega\cdot t+\Omega\cdot\frac{2\cdot\pi}{\Omega}+\theta\right] = \cdot\cos(\Omega\cdot t+\theta+2\cdot\pi)$$
$$\Rightarrow A\cdot\cos(\Omega\cdot t+\theta) = x_a(t)$$

1.3. Concept of Frequency in CT & DT Signals

Complex Exponential Signal:

$$x_a(t) = Ae^{j(\Omega t+\theta)}$$

Euler Manipulations:

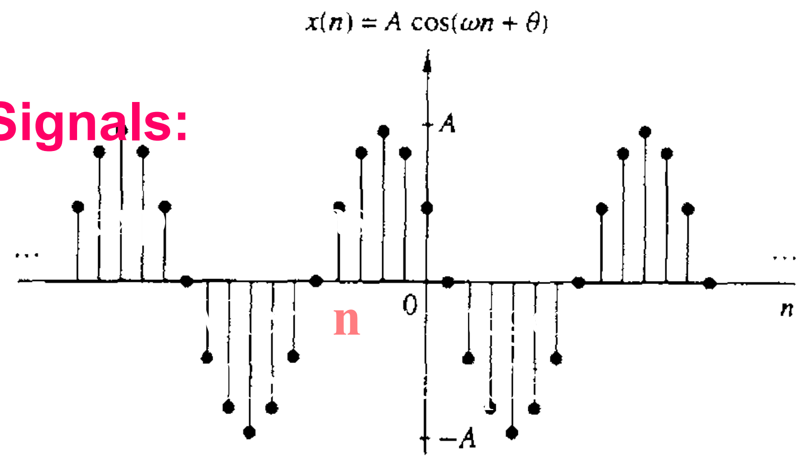
$$\cos(x)=\frac{1}{2}\cdot\left(e^{j\cdot x}+e^{-j\cdot x}\right) \quad , \quad \sin(x)=\frac{1}{2\cdot j}\cdot\left(e^{j\cdot x}-e^{-j\cdot x}\right)$$

$$e^{j\cdot x}=\cos(x)+j\cdot\sin(x) \quad , \quad e^{-j\cdot x}=\cos(x)-j\cdot\sin(x)$$

$$A\cdot\cos\left(\Omega\cdot t+\theta\right)=\frac{A}{2}\cdot e^{j\cdot(\Omega\cdot t+\theta)}+\frac{A}{2}\cdot e^{-j\cdot(\Omega\cdot t+\theta)}$$

1.3. Concept of Frequency in CT & DT Signals

1.3.2 Discrete-Time Sinusoidal Signals:



Example of a discrete-time sinusoidal signal ($\omega = \pi/6$ ($f = 1/12$) and $\theta = \pi/3$)

Amplitude

$$x(n) = A\cos(\omega n + \theta), -\infty < n$$

Discrete Signal

ω is frequency in rad/sample

θ phase in rad

Instead of the ω frequency f in cycle per sample is used

$$\omega = 2\pi f$$

$$x(n) = A\cos(2\pi f n + \theta), -\infty < n < \infty$$

1.3. Concept of Frequency in CT & DT Signals

Discrete-Time Sinusoidal Signal Properties:

- **A discrete-time sinusoid signal is periodic only if its frequency *f* is a rational number.**

$$f = \frac{N_1}{N_2} \text{ where } N_1, N_2 = rational \quad number$$

- **The period <u>N</u> MUST be an integer > 0.**
- **Discrete Signals whose frequencies are separated by a multiple of 2πk are identical. (k = integer)**

$$\cos[(\omega_0 + 2\pi)n + \theta] = \cos(\omega_0 n + 2\pi n + \theta) = \cos(\omega_0 n + \theta)$$

$$\cos(a \pm b) = \cos(a) \cdot \cos(b) \mp \sin(a) \cdot \sin(b)$$

Proof:

$$\cos\left[\left(\omega_0 + 2 \cdot \pi \cdot k\right) \cdot n + \theta\right] = \cos\left[\omega_0 \cdot n + 2 \cdot \pi \cdot k \cdot n + \theta\right]$$

$$Let \qquad a = \omega_0 \cdot n + \theta \qquad b = 2 \cdot \pi \cdot k \cdot n$$

$$= \cos(\omega_0 \cdot n + \theta) \cdot \cos(2 \cdot \pi \cdot k \cdot n) \mp \sin(\omega_0 \cdot n + \theta) \cdot \sin(2 \cdot \pi \cdot k \cdot n)$$

$$\cos(2 \cdot \pi \cdot k \cdot n) = 1 \forall k \qquad \sin(2 \cdot \pi \cdot k \cdot n) = 0 \forall k$$

$$\therefore \qquad \cos\left[\left(\omega_0 + 2 \cdot \pi \cdot k\right) \cdot n + \theta\right] = \cos(\omega_0 \cdot n + \theta)$$

1.3. Concept of Frequency in CT & DT Signals

Discrete-Time Sinusoidal Signal Periodicity:

- $x(n+N) = x(n)$ **for all n**
- **Proof:**

$$\Rightarrow A \cdot \cos\left[2 \cdot \pi \cdot f_0 \cdot (n+N) + \theta\right] =$$

$$A \cdot \cos[2 \cdot \pi \cdot f_0 \cdot n + 2 \cdot \pi \cdot f_0 \cdot N + \theta] =$$

$$A \cdot \cos[2 \cdot \pi \cdot f_0 \cdot n + \theta] \qquad IFF \;\; 2 \cdot \pi \cdot f_0 \cdot N = 2 \cdot \pi \cdot k$$

- **Because k and N are integers, f_0 is rational.**

$$f_0 = \frac{k}{N}$$

1.3. Concept of Frequency in CT & DT Signals

Example:

Is the signal periodic, If periodic, what is fundamental period (N)?

$$\cos\left(2\cdot\pi\cdot f\cdot n\right) \Rightarrow f = frequency$$

$$\cos\left(\frac{2\cdot\pi}{3}\cdot n\right) \quad \Rightarrow \quad \cos\left(\frac{2\cdot\pi}{3}\cdot n\right)=\cos\left(2\cdot\pi\cdot\frac{1}{3}\cdot n\right)\Rightarrow f=\frac{1}{3}\Rightarrow N=3$$

$$\cos\left(\frac{\pi}{7}\cdot n\right) \quad \Rightarrow \quad \cos\left(\frac{\pi}{7}\cdot n\right)=\cos\left(2\cdot\pi\cdot\frac{1}{14}\cdot n\right)\Rightarrow f=\frac{1}{14}\Rightarrow N=14$$

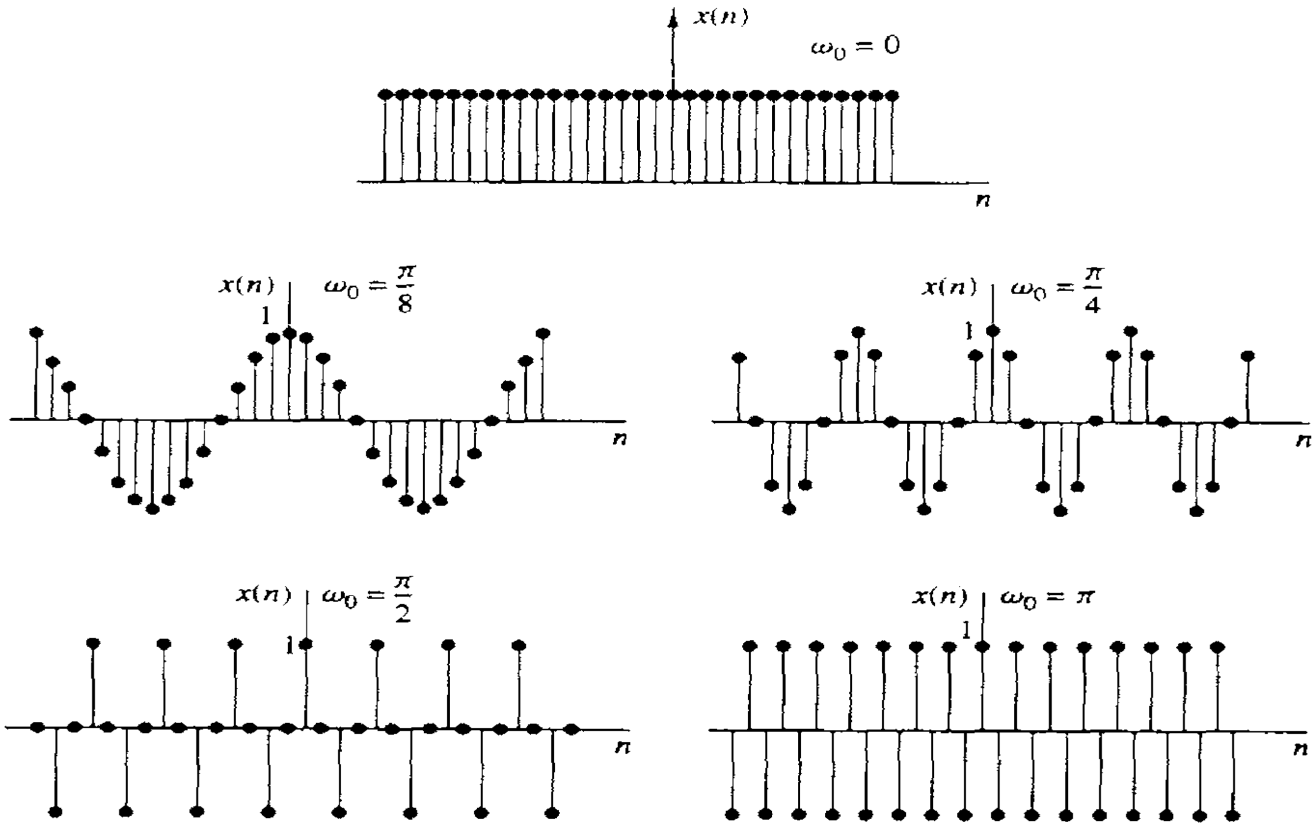
$$\cos\left(\sqrt{2}\cdot\pi\cdot n\right) \quad \Rightarrow \quad \cos\left(\sqrt{2}\cdot\pi\cdot n\right)=\cos\left(2\cdot\pi\cdot\frac{\sqrt{2}}{2}\cdot n\right)\Rightarrow f=\frac{\sqrt{2}}{2}\Rightarrow Not\ Periodic$$

$$\cos\left(\frac{5}{7}\cdot n\right) \quad \Rightarrow \quad \cos\left(\frac{5}{7}\cdot n\right)=\cos\left(2\cdot\pi\cdot\frac{5}{14\cdot\pi}\cdot n\right)\Rightarrow f=\frac{5}{14\cdot\pi}\Rightarrow Not\ Periodic$$

1.3. Concept of Frequency in CT & DT Signals

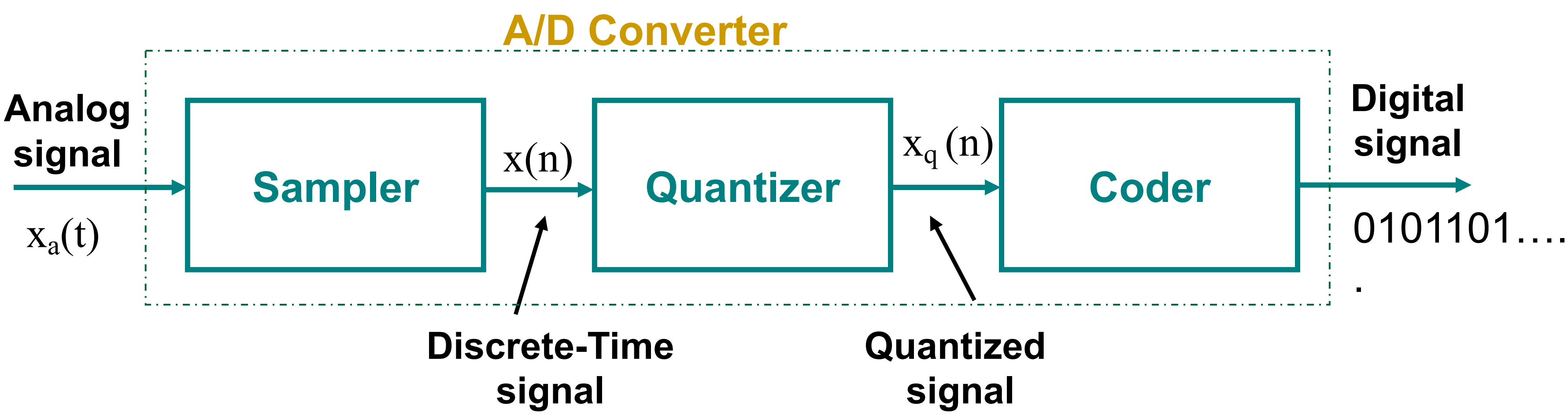
Example:

$$x(n) = \cos \omega_0 n$$



Analog to Digital Converter (A/D):

Conceptually, the A/D comprise 3 step process as in the following figure.



Analog to Digital Converter (A/D):

Sampling:

It is the conversion of a CT signal into DT signal obtained by taking "Samples" of the CT signal at DT instants.

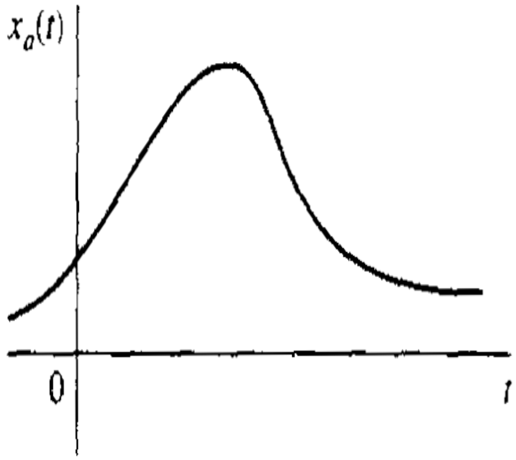
$$t = nT = \frac{n}{F_s}$$

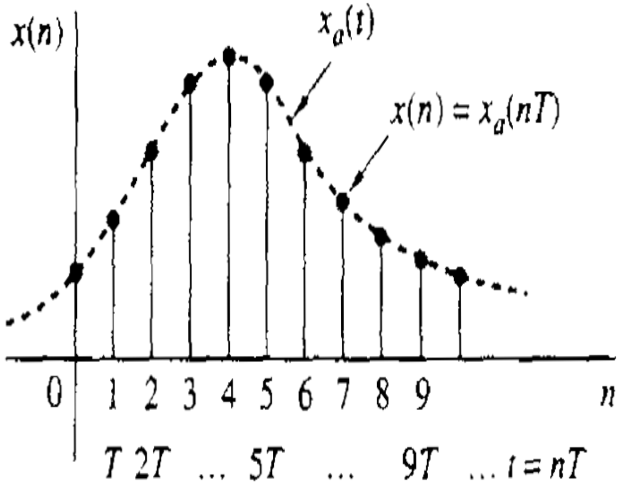
Periodic or Uniform Sampling:

This type of sampling is used most often in practice, described by the relation:

$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$

where x(n) is the DT signal obtained by taking samples of the analog signal $x_a(t)$ every T seconds.





The rate at which the signal is sampled is F_s: $F_s = 1/T$

F_s is called the SAMPLING RATE or SAMPLING FREQUENCY (Hz)

Analog to Digital Converter (A/D):

Sampling:

Consider an analog sinusoidal signal of the form:

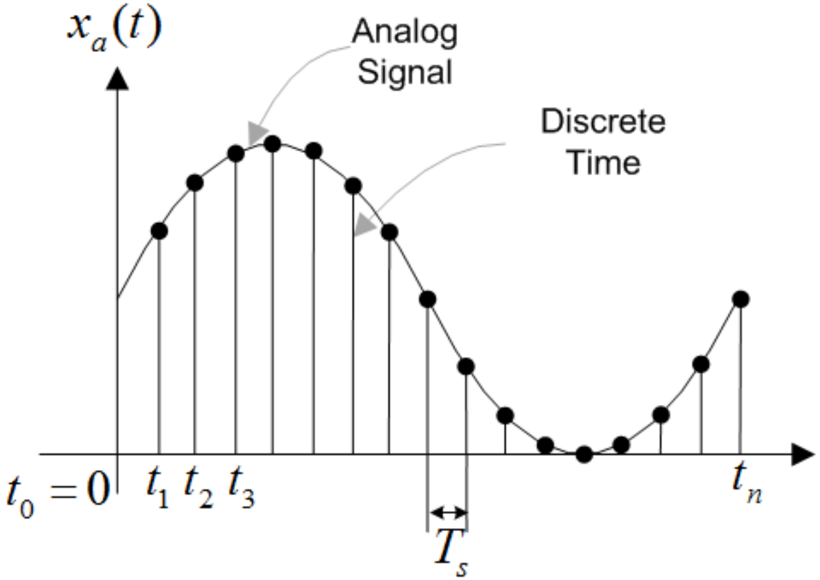
$$x_a(t) = A \cdot \cos(2 \cdot \pi \cdot F \cdot t)$$

Sampling Frequency: $F_s = \frac{1}{T_s}$

Normalized frequency: $f = \frac{F}{F_s}$

Sampled Signal:

$$x(n) = A \cdot \cos\left(2 \cdot \pi \cdot F \cdot \frac{n}{F_s}\right) = A \cdot \cos\left(2 \cdot \pi \cdot \frac{F}{F_s} \cdot n\right) = A \cdot \cos(2 \cdot \pi \cdot f \cdot n)$$



Relation among frequency variable:

Continuous-time signals		Discrete-time signals
$\Omega = 2\pi F$		$\omega = 2\pi f$
$\frac{\text{radians}}{\text{sec}}$ Hz		$\frac{\text{radians}}{\text{sample}}$ $\frac{\text{cycles}}{\text{sample}}$
	$\xrightarrow{\omega = \Omega T,\ f = F/F_s}$	$-\pi \le \omega \le \pi$ $-\frac{1}{2} \le f \le \frac{1}{2}$
	$\xleftarrow{\Omega = \omega/T,\ F = f \cdot F_s}$	
$-\infty < \Omega < \infty$ $-\infty < F < \infty$		$-\pi/T \le \Omega \le \pi/T$ $-F_s/2 \le F \le F_s/2$

Analog to Digital Converter (A/D):

Sampling:

We observe that the fundamental difference between CT and DT signals in their range of values of the frequency variables F and *f* or Ω and ω.

Sampling means mapping from infinite frequency range for F (or Ω) into a finite frequency range for *f* (or ω).

Since the highest frequency in a DT signal is ω = π or *f* = 1/2.

With sampling rate F_s the corresponding highest values of F and Ω are:

$$F_{\max} = \frac{F_s}{2} = \frac{1}{2T} \qquad \Omega_{\max} = \pi F_s = \frac{\pi}{T}$$

Analog to Digital Converter (A/D):

$$x_1(t) = \cos 2\pi(10)t$$

Sampling:

$$x_2(t) = \cos 2\pi(50)t$$

Examples:

I. Two analog sinusoidal signals:

Which are sampled at a rate F_s = 40 Hz.

Discrete-time signals:

$$x_1(n) = \cos 2\pi\left(\frac{10}{40}\right)n = \cos\frac{\pi}{2}n$$

However, $\cos 5\pi n/2 = \cos(2\pi n + \pi n/2) = \cos \pi n/2$

$$x_2(n) = \cos 2\pi\left(\frac{50}{40}\right)n = \cos\frac{5\pi}{2}n$$

This mean $x_2(n) = x_1(n)$

The frequency F_2 = 50 Hz is an alias of the frequency F_1 = 10 Hz at the sampling rate of 40 samples per second.

F_2 is not the only alias of F_1

Analog to Digital Converter (A/D):

Aliasing

- **Aliasing** **occurs when input frequencies (again greater than half the sampling rate) are folded and superimposed onto other existing frequencies.**

In order to prevent alias

$$F_s \geq 2 \cdot F_{max}$$

where F_{max} is the highest input frequency

Nyquist Rate:

Minimum sampling rate to prevent alias.

$$F_N = 2 \cdot F_{max}$$

Analog to Digital Converter (A/D):

Sampling:

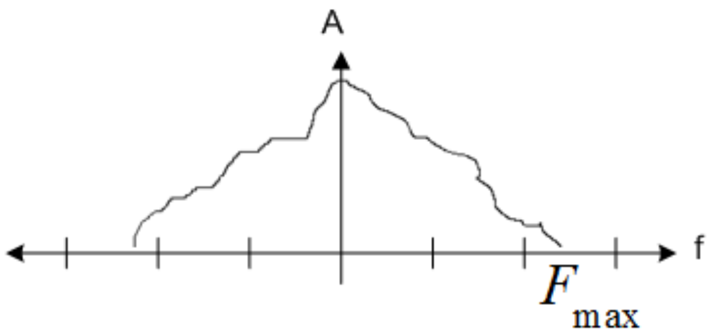
Sampling Theorem:

Given Band Limited (Frequency Limited Signal) with highest frequency F_{max}:

The signal can be exactly reconstructed provided the following is satisfied:

Sampling Frequency: $F_s \geq 2 \cdot F_{max}$

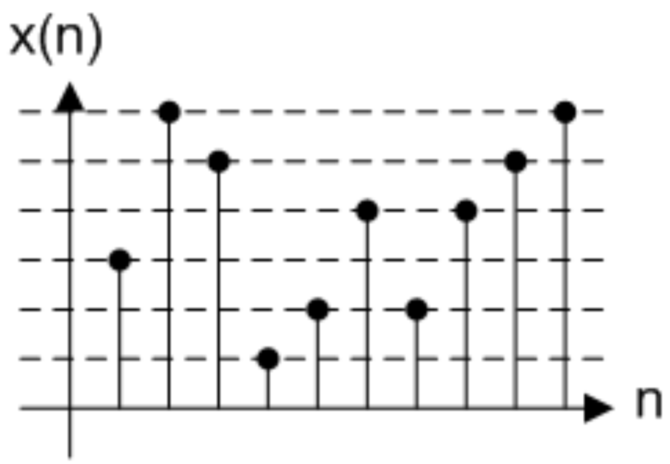
The samples are not quantized (analog amplitudes)



Analog to Digital Converter (A/D):

Quantization:

The process of converting a DT continuous amplitude signal into digital signal by expressing each sample value as a finite number of digits is called QUANTIZATION.

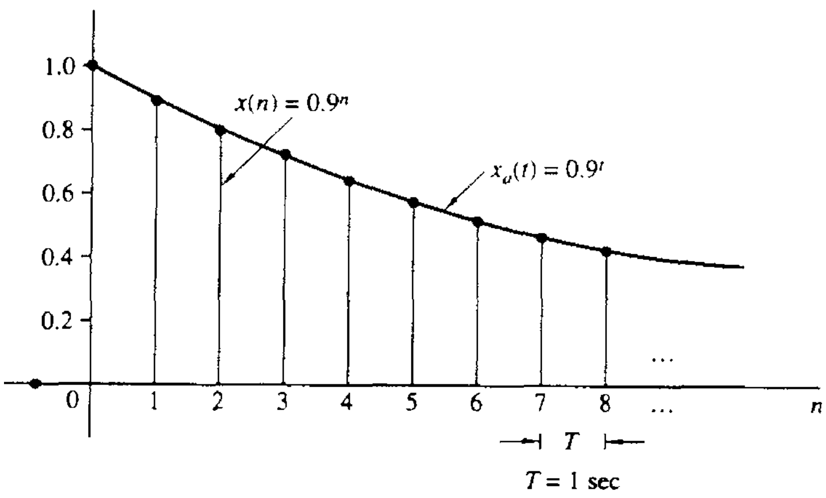


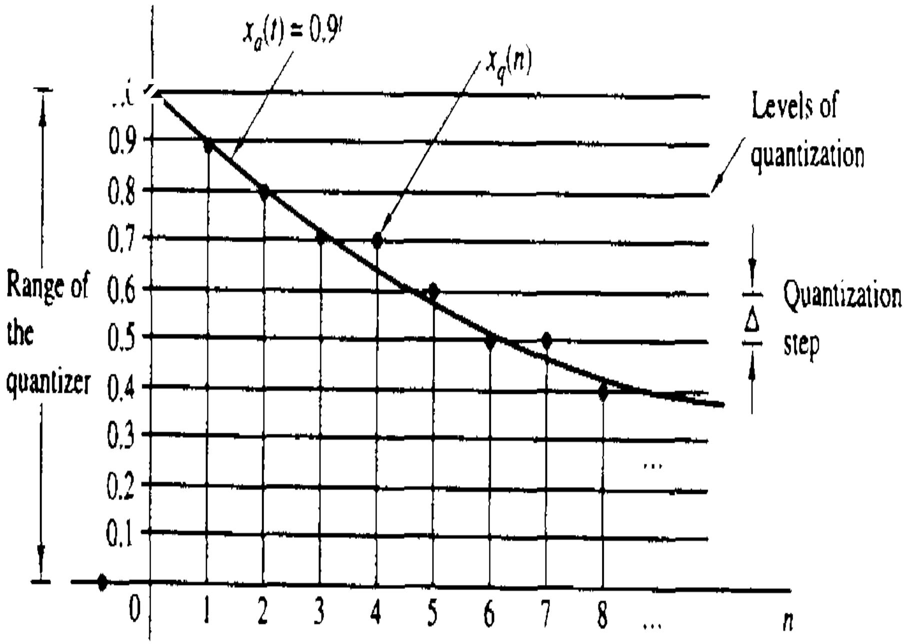
Analog to Digital Converter (A/D):

Quantization: quantization can be illustrated with the following example

$$x_a(t) = 0.9^t, \quad t \geq 0$$

$$x(n) = \begin{cases} 0.9^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$





Analog to Digital Converter (A/D):

n	$x(n)$ Discrete-time signal	$x_q(n)$ (Truncation)	$x_q(n)$ (Rounding)	$e_q(n) = x_q(n) - x(n)$ (Rounding)
0	1	1.0	1.0	0.0
1	0.9	0.9	0.9	0.0
2	0.81	0.8	0.8	−0.01
3	0.729	0.7	0.7	−0.029
4	0.6561	0.6	0.7	0.0439
5	0.59049	0.5	0.6	0.00951
6	0.531441	0.5	0.5	−0.031441
7	0.4782969	0.4	0.5	0.0217031
8	0.43046721	0.4	0.4	−0.03046721
9	0.387420489	0.3	0.4	0.012579511

Analog to Digital Converter (A/D):

Coding:

The coding process in an A/D converter assigns a unique binary number to each quantization level. If we have L levels we need at least L different binary numbers. With a word length of b bits we can create 2^b different binary numbers. Hence we have $2^b \geq L$, or equivalently, $b \geq \log_2 L$. Thus the number of bits required in the coder is the smallest integer greater than or equal to $\log_2 L$. In our example it can easily be seen that we need a coder with $b = 4$ bits. Commercially available A/D converters may be obtained with finite precision of $b = 16$ or less. Generally, the higher the sampling speed and the finer the quantization, the more expensive the device becomes.

Cont….

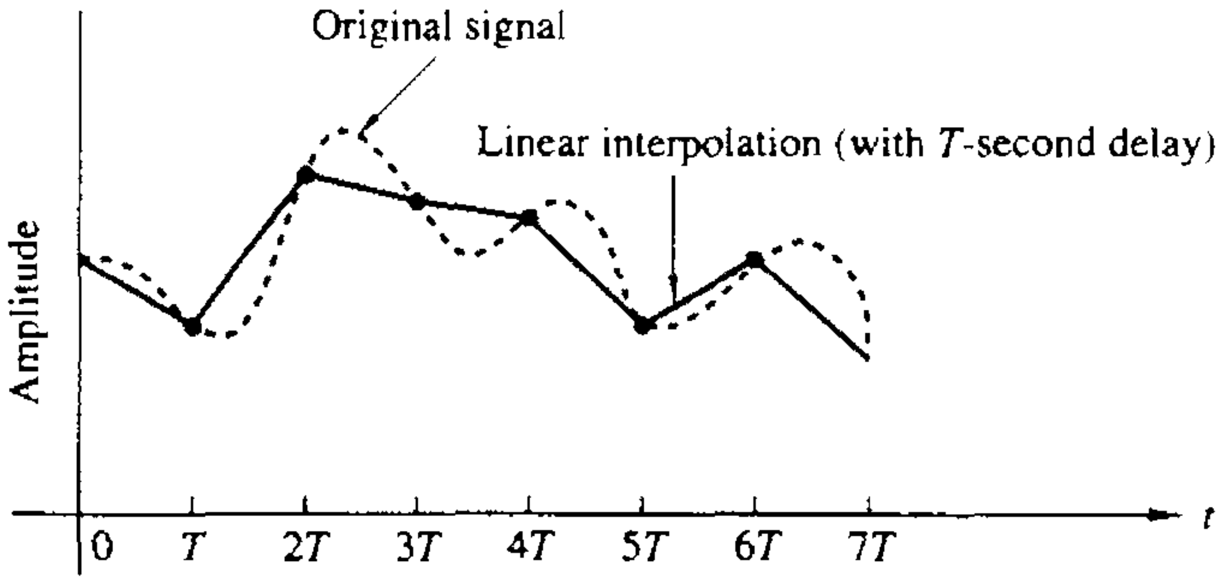
D/A converter is used to interpolate the signal between samples. It can be done by

- ✓ Sampling theorem
- ✓ Zero order hold Method
- ✓ Linear interpolation

✓ **Describe how these methods works !!!!!**

Digital Signal Processing

Digital to Analog Converter (A/D):



Tutorial Questions for Upcoming Tutorial Session

- **Problems for Chapter One**
  - **By Proakis and Manolakis**

End!!!!!!!