SECTION B (Page 01)

(05) (a) (i) Medical Imaging (MRI, CT, Ultrasound)

- DSP is used to process and enhance medical images obtained from ~~Maget~~ Magnetic Resonance Imaging (MRI), Computed Tomography (CT), and Ultrasound
- It helps in noise reduction, edge detection, and image reconstruction, improving diagnosis accuracy

(ii) Biomedical Signal processing (ECG, EEG, EMG)

- DSP is used to analyze Electrocardiograms (ECG) for heart monitoring, Electroencephalograms (EEG) for brain activity, and Electromyograms (EMG) for ~~musle~~ muscle analysis.
- It enables filtering, feature extraction and pattern recognition to detect abnormalities like arrhythmias, epilepsy, and neuro muscular disorders

Cont'd → SECTION B (Page 02)

(05)(b) (i)

$$w(0) = w(1) = 0$$

$$\mu = 0.1$$

$$\Delta = 3, \text{ hence } x(n) = d(n-3)$$

$$y(n) = w(0)x(n) + w(1)x(n-1)$$

$$e(n) = d(n) - y(n)$$

$$w(0) = w(0) + 0.2e(n)x(n)$$

$$w(1) = w(1) + 0.2e(n)x(n-1)$$

(ii)

for $n=0$; $x(0)=0$, $y(0)=0$, $e(0)=-1$, $w(0)=0$, $w(1)=0$

for $n=1$; $x(1)=0$, $y(1)=0$, $e(1)=1$, $w(0)=0$, $w(1)=0$

for $n=2$; $x(2)=0$, $y(2)=0$, $e(2)=-1$, $w(0)=0$, $w(1)=0$

for $n=3$; $x(3)=-1$, $y(3)=0$, $e(3)=1$, $w(0)=-0.2$, $w(1)=0$

Cont'd:→ SECTION B (Page 03);

(06) (a) Given: $y(n) = T[x(n)] = 5x(n) + 2x(n^2)$.

Soln:

$y(n,k) = 5x(n-k) + 2x(n^2-k)$ —— (i)

$y(n-k) = 5x(n-k) + 2x((n-k)^2)$

$y(n-k) = 5x(n-k) + 2x(n^2+k^2-2nk)$ —— (ii)

Since $y(n,k) \neq y(n-k)$ then the System is

TIME-VARIANT

Cont'd Section B page 04

Q6(b) Given: $s(n) = \left(\frac{1}{5}\right)^{n-1} u(n+1)$

Soln

$$s(n) = \left(\tfrac{1}{5}\right)^{n-1} u(n+1) \equiv \left(\frac{1}{5}\right)^{(n+1)-2} u(n+1) = 25\left(\tfrac{1}{5}\right)^{n+1} \cdot u(n+1).$$

from: $\begin{bmatrix} x(n) \longleftrightarrow X(z) \\ x(n+1) \longleftrightarrow z^{1} X(z) \end{bmatrix}$ properties of z-transform.

taking $x(n)$ as $\left(\tfrac{1}{5}\right)^{n} u(n) \longleftrightarrow \dfrac{1}{1-\frac{1}{5}z^{-1}}$

$\Rightarrow x(n+1)$ which is $\left(\tfrac{1}{5}\right)^{n+1} u(n+1) \longleftrightarrow \dfrac{z}{1-\frac{1}{5}z^{-1}}$

Thus:

$$s(n) = 25\left(\tfrac{1}{5}\right)^{n+1} u(n+1) \longleftrightarrow S(z) = \frac{25z}{1-\frac{1}{5}z^{-1}}$$

∴ Also: recall that $s(n)$ = step response of an LTI system, hence the Input was a unit step discrete signal ie

$$x(n) = u(n)$$

$$\Rightarrow X(z) = \frac{1}{1-z^{-1}}$$

Now:

$$H(z) = \frac{S(z)}{X(z)} = S(z) \cdot \left[1 - z^{-1}\right]$$

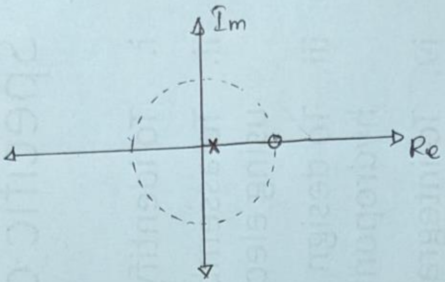
$$\Rightarrow H(z) = \frac{25z\left(1-z^{-1}\right)}{1-\frac{1}{5}z^{-1}} = \frac{25z - 25}{1-\frac{1}{5}z^{-1}} = \frac{25(z-1)}{1-\frac{1}{5}z^{-1}}$$

(i) ∴ $H(z) = \dfrac{25(z-1)}{1-\frac{1}{5}z^{-1}}$; ROC $|z| > \frac{1}{5}$

$H(z)$ has a zero at $\underline{z=1}$ and a pole at $\underline{z=\frac{1}{5}}$

Contd Section B page 05

Q6(b) (i)



(ii) Determining $h(n)$:

From: $H(z) = \dfrac{25(z-1)}{1-\frac{1}{5}z^{-1}} = 25\left[\dfrac{1}{1-\frac{1}{5}z^{-1}}\right]z - 25\left[\dfrac{1}{1-\frac{1}{5}z^{-1}}\right]$

Upon inverse Z-Transform, we get

$$h(n) = 25\left(\frac{1}{5}\right)^{n+1} u(n+1) - 25\left(\frac{1}{5}\right)^{n} u(n)$$

(iii) ⇒ The System is <u>STABLE</u> since the pole ($z = \frac{1}{5}$) is inside the Unit Circle.

⇒ The System is <u>NOT CASUAL</u>

Cont'd SECTION 'B' | page 06

Qn07 (a) Given: $K_1 = 1/2$, $K_2 = -1/3$ and $K_3 = 1$.

Soln:

for FIR filter coefficients for the Direct form structure.

- In our case $m = 3$ (three stage lattice filter). and

Always $\alpha_3(0) = 1$

So required to find $\alpha_3(1)$, $\alpha_3(2)$ and $\alpha_3(3)$ but.

$K_3 = \alpha_3(3) = 1$ Thus we are required to find the remaining i.e $\alpha_3(1)$ and $\alpha_3(2)$

Formulas to be used:

Always: $\alpha_m(0) = 1$ —— (i)

$\alpha_m(m) = K_m$ —— (ii) and finally the most important one is

$$\alpha_m(k) = \alpha_{m-1}(k) + K_m \alpha_{m-1}(m-k) \; ; \; 1 \le k \le m-1 \text{ —— (iii)}$$

Thus: $(m=3)$ and $\alpha_3(0) = 1$

$\alpha_1(1) = K_1 = 1/2$, $\alpha_2(2) = K_2 = -1/3$ and $\alpha_3(3) = K_3 = 1$

Now:

let $m = 2$ and $k = 1$ so that, upon using eqn (iii)

$$\Rightarrow \alpha_2(1) = \alpha_1(1) + K_2 \alpha_1(1)$$

$$\therefore \alpha_2(1) = \frac{1}{2} + \left[-\frac{1}{3} \times \frac{1}{2}\right] = \frac{1}{3}$$

$$\therefore \alpha_2(1) = \frac{1}{3} \text{ —— eqn (iv)}$$

Let $m = 3$ and $k = 1$ so that, upon using eqn (iii)

$$\Rightarrow \alpha_3(1) = \alpha_2(1) + K_3 \alpha_2(2)$$

$$\Rightarrow \alpha_3(1) = \frac{1}{3} + \left[1 \times -\frac{1}{3}\right] = 0$$

$$\therefore \alpha_3(1) = 0 \text{ and } \Rightarrow$$

Letting $m = 3$ and $k = 2$, using eqn (iii)

$\Rightarrow \alpha_3(2) = \alpha_2(2) + K_3\alpha_2(1)$

$\Rightarrow \alpha_3(2) = -\frac{1}{3} + \left[1 \times \frac{1}{3}\right] = 0$

$\therefore \alpha_3(2) = 0$

Hence, we can write

$$H_m(z) = 1 + \sum_{k=1}^{m} \alpha_m(k) z^{-k} \quad ; m \geq 1$$

i.e

$$H_3(z) = 1 + \sum_{k=1}^{3} \alpha_3(k) \cdot z^{-k} \quad \text{i.e}$$

$$H_3(z) = 1 + \alpha_3(1)z^{-1} + \alpha_3(2)z^{-2} + \alpha_3(3)z^{-3}$$

But $\alpha_3(1) = \alpha_3(2) = 0$ ~~Hence~~ But $\alpha_3(3) = K_3 = 1$ hence

$$H_3(z) = 1 + z^{-3}$$

Coefficients: $\alpha_3(0) = 1$, $\alpha_3(1) = \alpha_3(2) = 0$ and $\alpha_3(3) = \underline{1}$

Coefficients: $\alpha_3(0) = 1$, $\alpha_3(1)$ ...

(Sect B Cont'd) | Page 08 |

Qn07: (b)(i) $H(z) = B(z) = 1 - 0.8z^{-1} + 0.15z^{-2}$ (FIR)

Soln:

We have: $\alpha_2(0) = 1$, $\alpha_2(1) = -0.8$ and $\alpha_2(2) = K_2 = 0.15$

But we don't have: $\alpha_1(1) = K_1$:

from: $$\alpha_{m-1}(k) = \left[\frac{\alpha_m(k) - \alpha_m(m) \cdot \alpha_m(m-k)}{1 - \alpha_m^2(m)}\right]$$

Set $k=1$ and $m=2$ hence

Contd SECT B

07:(b)(i) $\alpha_1(1) = \left[\dfrac{\alpha_2(1) - \alpha_2(2)\cdot\alpha_2(1)}{1 - \alpha_2^2(2)}\right]$

$$\alpha_1(1) = \frac{(-0.8) - (0.15 \times -0.8)}{1 - (0.15)^2}$$

$$\therefore \alpha_1(1) = K_1 = -16/23 = -0.696$$

Now that we have $K_1 = -0.696$ and $K_2 = 0.15$

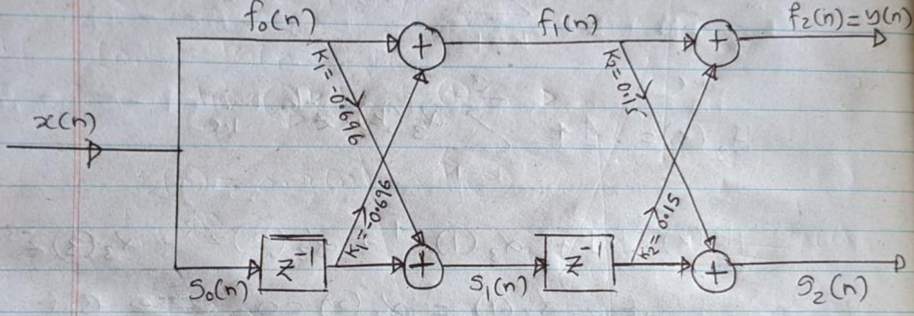


Fig 1: A lattice realization for the system $H(z) = B(z)$

07:(b)(ii) $H(z) = \dfrac{1}{A(z)} = \dfrac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}$

$$= \frac{1}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}} \quad \text{(IIR)}$$

$\Rightarrow$ $\alpha_2(0) = 1$ $\alpha_2(1) = -1/6$ and $\alpha_2(2) = K_2 = -1/6$

from $\alpha_{m-1}(k) = \left[\dfrac{\alpha_m(k) - \alpha_m(m)\cdot\alpha_m(m-k)}{1 - \alpha_m^2(m)}\right]$

Set $m=2$ and $k=1$ for $\alpha_1(1) = K_1$

$$\alpha_1(1) = K_1 = \frac{\alpha_2(1) - \alpha_2(2)\cdot\alpha_2(1)}{1 - \alpha_2^2(2)} = \frac{-1/6 - (-1/6 \cdot -1/6)}{1 - \left(-1/6\right)^2}$$

$$\therefore \left(\alpha_1(1) = K_1 = -1/5 = -0.2\right)$$

(07)(b)(ii)

SECT B Cont'd (page no 10)

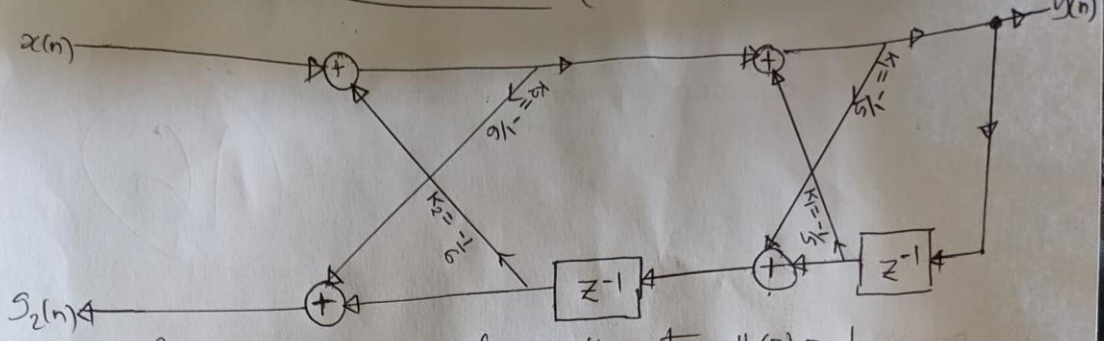


fig 2: A lattice realization for the system $H(z) = \frac{1}{A(z)}$

---

(08)(a) Soln:

To obtain the desired length of 5, a delay of $\alpha = \frac{5-1}{2} = 2$ is incorporated into $H_d(\omega)$. Hence;

$$y(n) = H_d(\omega) = \begin{cases} 1e^{-j2\omega}, & |\omega| \leq \pi/6,\ \pi/3 \leq |\omega| \leq \pi \\ 0, & \pi/6 \leq |\omega| \leq \pi/3 \end{cases}$$

(08)(a) Remember, this is a bandstop, hence:

$$h_d(n) = \begin{cases} 5/6, & \text{for } n=2 \\ \frac{\left[\sin \frac{\pi}{6}(n-2) - \sin \frac{\pi}{3}(n-2)\right]}{\pi[n-2]}, & \text{for } n \neq 2 \end{cases}$$

08 (a) Blackman Window formula

$$\Rightarrow W_{blk}(n) = 0.42 - 0.5 \cos\left[\frac{2\pi n}{M-1}\right] + 0.08 \cos\left[\frac{4\pi n}{M-1}\right] \quad \text{and } [M=5]$$

for $0 \leq n \leq 4$

➢The type of crop pipes used are PVC pipes of 2mm.

n	$h_d(n)$	$\omega_{blk}(n)$	$h(n) = h_d(n) \cdot \omega_{blk}(n)$
0	0.0000	0.0000	0.0000
1	-0.1166	0.3400	-0.0396
2	0.8333	1.0000	0.8333
3	-0.1166	0.3400	-0.0396
4	0.0000	0.0000	0.0000

(Page no 11)

Q8. (b)

$\therefore h(n) = \{ 0 \quad -0.0396 \quad 0.8333 \quad -0.0396 \quad 0 \}$

coefficients

Qn (8) (c) Magnitude Response

Since we have the coefficients i.e

$h(n) = \{0 \quad -0.0396 \quad 0.8333 \quad -0.0396 \quad 0\}$

Hence:

$$H(z) = -0.0396z^{-1} + 0.8333z^{-2} - 0.0396z^{-3}$$

$$\because z = e^{j\omega}$$

$$\Rightarrow H(e^{j\omega}) = -0.0396e^{-j\omega} + 0.8333e^{-j2\omega} - 0.0396e^{-j3\omega}$$

factor out $e^{-j2\omega}$ i.e

$$H(e^{j\omega}) = e^{-j2\omega}\left[-0.0396e^{j\omega} + 0.8333 - 0.0396e^{-j\omega}\right]$$

Simplify the bracketed term using Euler's formula i.e

$$e^{j\omega} + e^{-j\omega} = 2\cos(\omega)$$

$$\Rightarrow -0.0396(e^{j\omega} + e^{-j\omega}) + 0.8333 = 0.8333 - 0.0792\cos(\omega)$$

Thus the Magnitude is ;-

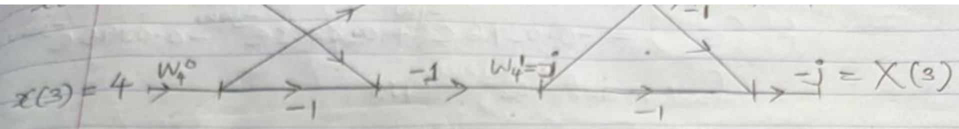
$$|H(e^{j\omega})| = |0.8333 - 0.0792\cos(\omega)|$$

rans	$f = \omega/2\pi$ (Hz)	$0.8333 - 0.0792\cos(\omega)$	$\lvert H(e^{j\omega})\rvert$	$\lvert H(e^{j\omega})\rvert$ dB
π				

Qn(08)(c) Assuming $f_s = 1000\,Hz$.

ω (radians)	$f = \frac{\omega f_s}{2\pi}$ (Hz)	$0.8333 - 0.0792\cos(\omega)$	$\lvert H(e^{j\omega})\rvert$	$\lvert H(e^{j\omega})\rvert_{dB}$
0	0	0.7541	0.7541	-2.45
$\pi/4$	125	0.7773	0.7773	-2.19
$\pi/2$	250	0.8333	0.8333	-1.58
$3\pi/4$	375	0.8893	0.8893	-1.02
π	500	0.9125	0.9125	-0.795

NB: $$\lvert H(e^{j\omega})\rvert_{dB} = 20\log_{10} \lvert H(e^{j\omega})\rvert$$



Qn (08) (d) Since:

$h(n) = h(M-1-n)$ where $M=5$

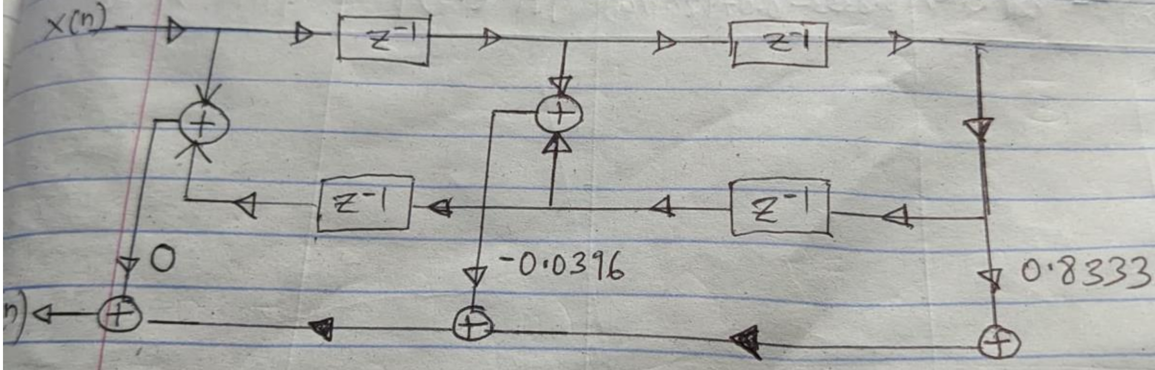
$h(n) = h(4-n)$

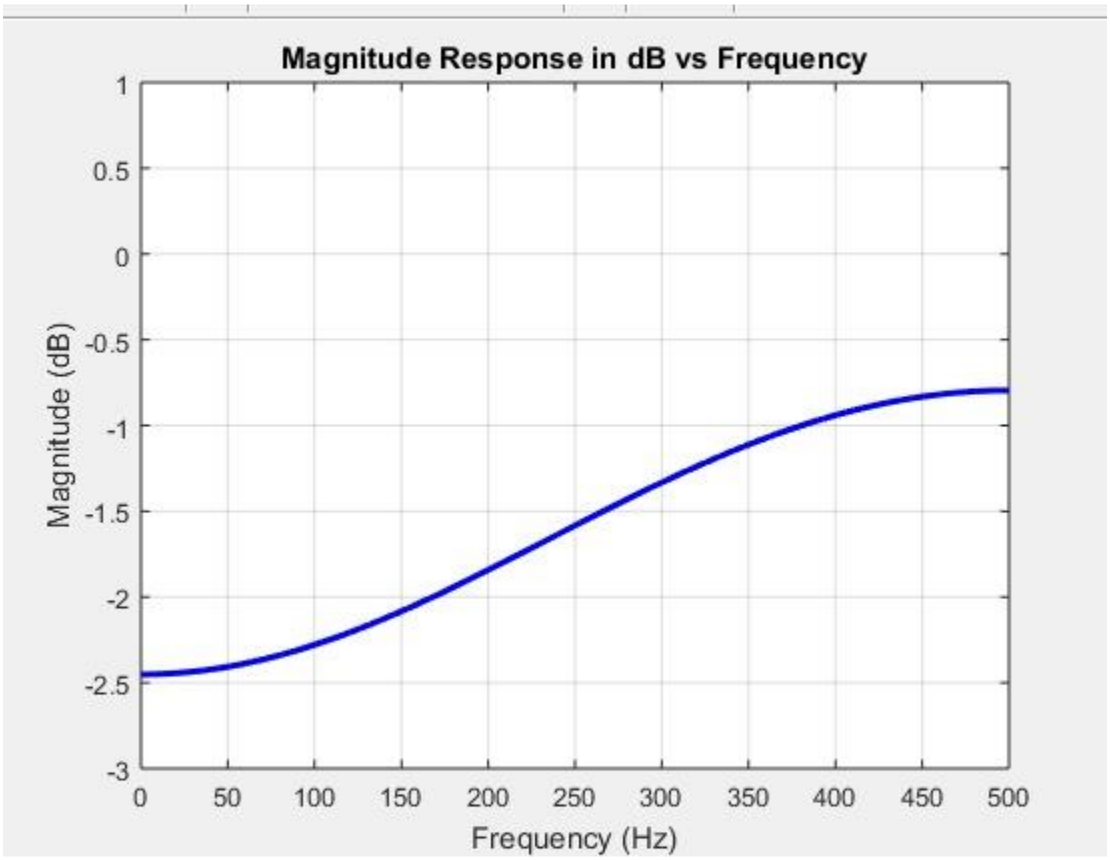
for $n=0$

$h(0) = h(4) = 0$

$h(1) = h(3) = -0.0396$

$h(2) = h(2) = 0.8333$




Magnitude Response in dB vs Frequency
Magnitude (dB)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
50
100
150
200
250
300
350
400
450
500
Frequency (Hz)

UE 2024

Qn 03: (a) 30 coefficients.

(b) for $i = 0, \dots, 29,$

$w(i) = 0$

$$y(n) = \sum_{i=0}^{29} w(i)\,x(n-i-\Delta)$$

$e(n) = d(n) - y(n)$

for $i = 0, \dots, 29$

$w(i) = w(i) + 2\mu e(n) x(n-i)$

Reason ya Qn03(a).

(a) To model each sinusoid, two coeff icients are needed, So again/hence $15 \times 2 = 30$ coefficients.

01.(a) Soln

Causal system = is a system where the output is determined by the current and past inputs but not future inputs

Hence:

$Y(z) = 1 + (-1.5)z^{-1} + 0.5z^{-2}$

and

$X(z) = 1$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 1.5z^{-1} + 0.5z^{-2}}{1}$$

Cross-multiply

$\Rightarrow X(z) - 1.5z^{-1}X(z) + 0.5z^{-2}X(z) = Y(z)$

01.(a) Applying Inverse Z-Transform we get

$y(n) = x(n) - 1.5x(n-1) + 0.5x(n-2)$

$\therefore$ Input-Output eqn =

$y(n) = x(n) - 1.5x(n-1) + 0.5x(n-2)$

01.(b) Impulse response $[h(n)]$

here ~~Impulse response~~ Input $x(n) = \delta(n)$

$X(z) = 1.$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - 1.5z^{-1} + 0.5z^{-2}$$

$\Downarrow$

$h(n) = \delta(n) - 1.5\delta(n-1) + 0.5\delta(n-2)$

~~Other transform~~ i.e.

$$h(n) = \begin{cases} 1, & n=0 \\ -1.5, & n=1 \\ 0.5, & n=2 \\ 0, & \text{otherwise} \end{cases}$$

01.(c) Z-Transform

The Z-transform of the Impulse-response $h(n)$ is

$H(z) = 1 - 1.5z^{-1} + 0.5z^{-2}$

(02)(a) Soln:

$$Y(z) = \frac{5}{6} z^{-1} Y(z) - \frac{1}{6} z^{-2} Y(z) + X(z)$$

$$\Rightarrow \left(1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}\right) Y(z) = X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = H(z) = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}$$

$$\Rightarrow H(z) = \frac{1}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{3} z^{-1}\right)}$$

Also:

$$X(z) = \left(1 - \frac{1}{3} z^{-1}\right)$$

Now since $Y(z) = H(z) X(z)$

hence:

$$Y(z) = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

NB: This is called Pole-Zero Cancellation Effect

$$\Rightarrow y(n) = \left(\frac{1}{2}\right)^n u(n)$$

(02)(b) Soln:

(i) $f_{max} = 125\,Hz$

Minimum Sampling rate required to avoid aliasing $= 2 f_{max}$

$$= 2 \times 125\,Hz$$
$$= 250\,Hz$$
$$= 250 \text{ samples/sec}$$

Q2(b)

(ii) Given:

$$x_a(t) = 3\cos 150\pi t + 2\sin 250\pi t$$

$$x_a(n) = 3\cos \frac{150\pi n}{150} + 2\sin \frac{250\pi n}{150}$$

$$x_a(n) = 3\cos(\pi n) + 2\sin \frac{5\pi n}{3}$$

alias

Hence:

$$2\sin \frac{5\pi}{3}\pi = 2\sin\left(2\pi - \frac{\pi}{3}\right)n$$

$$= 2\sin\left(\frac{\pi n}{3}\right)$$

Hence:

$$x_a(n) = 3\cos(\pi n) + 2\sin\left(\frac{\pi n}{3}\right)$$

(04.) Soln:

$x(n) = \{ \underset{\uparrow}{1}, 3, 1, 4 \}$

$N = 4$

(i) No of Complex multiplications

$= \frac{N}{2} \log_2 N$

$= \frac{4}{2} \log_2 4$

$=$ 4 complex multiplications.

(ii) DFT-method:

$X_4 = W_4 x_4$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 4 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 9 \\ j \\ -5 \\ -j \end{bmatrix} = X[k]$$

(iii) DIT-FFT

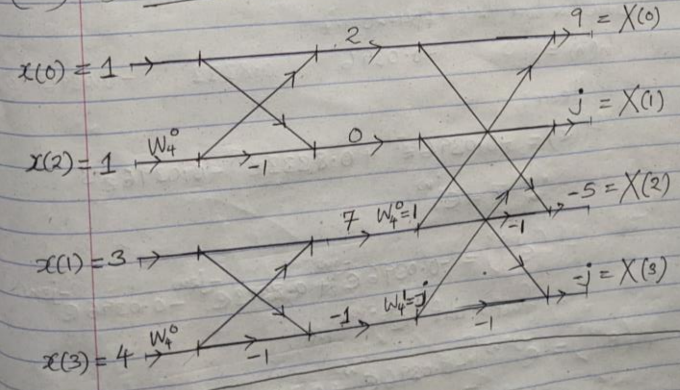
$0 \to 00$ Bit $00 \to 0$ } even

$1 \to 01$ Reversal $10 \to 2$

$2 \to 10$ $\longrightarrow$ $01 \to 1$ } odd

$3 \to 11$ $11 \to 3$

(04) (iii) DIT-FFT.



Qn (08) (d) Since:

where M=5