MEASUREMENT ASSIGNMENT 2

1. (a) Define the term error and mistake

   (b) Distinguish random error from systematic error, hence give a practical example of each term and briefly explain how they can be reduced or eliminated

2. The following measurements were taken by a student for the length of piece of rod; 21.02, 20.99, 20.92, 21.11and 20.69. Basing on error analysis, find the true value of the length of a piece of rod and its associated error.

3. A physical quantity P is related to four observable a, b, c and d as follows $P = \frac{a^3 b^2}{c^{\frac{1}{2}} d}$

   The percentage error of measurements in a is1%, b is 3%, c is 4% and d is 2%. What is the percentage error in P?

4. Compute the numerical value of J and error in it for the relation $J = \left(\frac{I^2 R}{W + m}\right)\frac{t}{\theta}$, given that $I =$ 2.5 ± 0.05, $R = 11.36 \pm 0.01$, $W = 21 \pm 1$, $m = 155 \pm 1$, $\theta = 28 \pm 0.5$, and $t = 298 \pm 0.5$

5. (i) Explain what $\pm a$ units, following the value of a parameter, signify in experimental Physics.

   (ii) The specific resistance ρ of a thin circular wire of radius r cm, resistance R in ohms and length l cm is given by $\rho = \frac{\pi r^2 R}{l}$, if $r = (0.26 \pm 0.02)cm$, $l = (78 \pm 0.01)cm$ and $\rho = (0.087 \pm 0.016)\Omega cm$, calculate the percentage error in resistance R

6. If the clock losses 3 seconds in 5 minutes, determine the error in measuring the value of g in equation $T = 2\pi\sqrt{\frac{l}{g}}$, given that T=2.22sec, l = 121.6cm, $\Delta T = \pm 0.1s$ and $\Delta l = \pm 0.05cm$

7. The length l of the simple pendulum is 1m and is known to 1mm accuracy. If it takes about 200.60 seconds for 100 oscillations with a watch of 0.01s resolution, what is the accuracy in the determination of g?

8. In the experiment to determine the equivalent resistance of two resistance connected in parallel, the following measurements obtained; $R_1 = 5 \pm 0.1\Omega$ and $R_2 = 6 \pm 0.2\Omega$

   (i) What is the value of equivalent resistance?

   (ii) Calculate the error in obtaining the value of equivalent resistance

9. Given two resistors with resistances $R_1 = (2 \pm 0.5)\Omega$ and $R_1 = (4 \pm 0.5)\Omega$ calculate percentage error and numerical value of effective resistance when the resistors are arranged in:-

   (i) Series

   (ii) Parallel

(Submission: on Saturday17th June 16, 2023 at 0900AM)

$T = 1.5 \pm 0.002$ sec

$A = 0.3 \pm 0.005$ m

$K = 0.28 \pm 0.005$ m.

T. couple

a = 43.58 μv²

b = 0.04 μV

MEASUREMENTS
ASSIGNMENT 02: MARKING GUIDE.

Q1 (a) i) Error: The difference between actual value and measured value

OR

→ Small deviation of measured value from the true value

→ Uncertainities occuring during experiment

ii) Mistake

→ The unintential wrong way of conducting experiment

→ Large deviation of ~~true~~ measured value from actual value

(b) Random error	Systematic Error
→ Error occuring due to unknown causes	→ Occur due to known causes
→ Involve mixture of both positive and negative values	→ Involves only positive values Conferly manner

Q2. from the data given

Actual value $(\bar{l}) = \dfrac{l_1 + l_2 + l_3 + l_4 + l_5}{5}$

$$\bar{l} = \frac{21.02 + 20.99 + 20.92 + 21.11 + 20.69}{5}$$

Actual value = 20.946 units

consider The table below

$\bar{X} \Rightarrow 20.946$

$\bar{X} - X = \Delta X$	$\Delta X_1 = -0.074$	-0.044	0.026	-0.164	0.256.

$\overline{\Delta X} = D$

Mean absolute error $|\overline{\Delta X}|$.

$$|\overline{\Delta X}| = \frac{(0.074 + 0.044 + 0.026 + 0.164 + 0.256)}{5}$$

$$|\overline{\Delta X}| = 0.1128 \text{ units}$$

Numerical value of length

$$= (20.946 \pm 0.1128) \text{ Units}$$

Q3 Given:-

$$P = \frac{a^3 b^2}{c^{1/2} d}$$

Required Percentage error in P.

Apply ln both Sides:-

$$\ln P = \ln\left(\frac{a^3 b^2}{c^{1/2} d}\right).$$

$$\ln P = \ln a^3 + \ln b^2 - \ln c^{1/2} - \ln d$$

$$\ln P = 3\ln a + 2\ln b - \tfrac{1}{2}\ln c - \ln d.$$

differentiate both sides → (Error is maximized)

$$\Delta P/P = 3\frac{\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}.$$

Since % Error in P = $\Delta P/P \times 100\%$.

$$\% P = 3(1\%) + 2(3\%) + \frac{1}{2}(4\%) + (2\%).$$

$$\% P = 13\%$$

$$\frac{\Delta R}{R} = \frac{0.1}{5} + \frac{0.2}{6} + \frac{0.2+0.1}{5+6}.$$

$$\frac{\Delta R}{R} = 0.08061$$

$$\Delta R = 0.08061 R$$

$$= 0.08061 \times 2.73\Omega$$

$$\Delta R = 0.22\Omega$$

Error in obtaining value of equivalent resistance
$\Rightarrow \pm 0.22\Omega$.

09. Case I : Series arrangement

$$R_T = R_1 + R_2$$

$$R_T = (2 \pm 0.5)\Omega + (4 \pm 0.5)\Omega$$

$$= (2+4) \pm (0.5+0.5)\Omega$$

$$R_T = (6 \pm 1)\Omega$$

Numerical value of effective resistance in series was $(6 \pm 1)\Omega$

Case II : In parallel arrangement

$$R_T = \frac{R_1 R_2}{R_1+R_2} = \frac{4\times 2}{4+2} = 1.33\Omega$$

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1+R_2}$$

$$= \frac{0.5}{4} + \frac{0.5}{2} + \frac{0.5+0.5}{4+2}$$

$$\frac{\Delta R}{R} = 0.54 \cancel{\Omega}$$

$$\Delta R = 0.54 R$$

$$= 0.54 \times 1.33\Omega$$

$$\Delta R = 0.7182\Omega$$

08 Given:

$R_1 = 5 \pm 0.1\Omega$, $R_2 = 6 \pm 0.2\Omega$

from R_{eff} - Parallel

$$1/R_{eff} = 1/R_1 + 1/R_1$$

$$1/R_t = \frac{R_1 + R_2}{R_1 R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Actual value of R will be

$$R = \frac{5 \times 6}{5 + 6} = 2.73$$

$$R = 2.73\Omega$$

from $R = \frac{R_1 R_2}{R_1 + R_2}$

Apply ln both sides

$$\ln R = \ln\left(\frac{R_1 R_2}{R_1 + R_2}\right).$$

$$\ln R = \ln R_1 + R_2 - \ln(R_1 + R_2).$$

$$\Delta R/R = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta(R_1 + R_2)}{R_1 + R_2}$$

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{(\Delta R_1 + \Delta R_2)}{R_1 + R_2}$$

Since errors are always maximized

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}.$$

Q4) Given;

$$J = \left(\frac{I^2 R}{W+m}\right) t/\theta$$

Required: Numerical value of J

Actual value of J

$$J = \left(\frac{2.5^2(11.36)}{21+155}\right)\frac{298}{28}$$

$$J = \cancel{9.509} \quad 4.293$$

Now,

$$J = \left(\frac{I^2 R}{W+m}\right) t/\theta$$

Apply natural logarithm both sides

$$\ln J = \ln\left(I^2R/W+m\right) t/\theta$$

$$\ln J = \ln I^2 + \ln R - \ln(W+M) + \ln t - \ln\theta$$

Differentiate both sides

$$\Delta J/J = 2\Delta I/I + \Delta R/R - \frac{\Delta(M+W)}{M+W} + \frac{\Delta t}{t} - \frac{\Delta\theta}{\theta}$$

$$\frac{\Delta J}{J} = \frac{2\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta M + \Delta W}{M+W} + \frac{\Delta t}{t} + \frac{\Delta\theta}{\theta}$$

$$\frac{\Delta J}{J} = 2\left(\frac{0.05}{2.5}\right) + \frac{0.01}{11.36} + \frac{1+1}{21+155} + \frac{0.5}{298} + \frac{0.5}{28}$$

$$\Delta J/J = 0.0718$$

$$\Delta J = 0.0718J$$

$$= 0.0718 \times 4.293$$

$$\Delta J = 0.308$$

Numerical value of J = $(4.293 + 0.308)$ units

05 i/ It Means the absolute error (value) of physical quantity

ii/ Given

$f = \frac{\pi r^2 R}{l}$ Required Percentage error in f.

Apply ln both sides

$\ln f = \ln \pi + \ln r^2 + \ln R - \ln l$

Differentiate both sides:-

$\Delta f/f = 0 + 2\Delta r/r + \frac{\Delta R}{R} - \Delta l/l$

Since errors are always maximized

$\frac{\Delta f}{f} = \frac{2\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta l}{l}$

~~$\Delta f/f = 2\left(\frac{0.02}{0.26}\right) + D$~~

for R

$\frac{\Delta R}{R} = \frac{\Delta f}{f} + \frac{\Delta l}{l} + \frac{2\Delta r}{r}$

$\frac{\Delta R}{R} = \left(\frac{0.016}{0.087}\right) + \left(\frac{0.01}{78}\right) + 2\left(\frac{0.02}{0.26}\right)$

$\Delta R/R = 0.3379$

Percentage error in R

$\frac{\Delta R}{R} \times 100\% = 0.3379 \times 100\%$

$= 33.79\%$

$\therefore$ Percentage error in R was 33.79%

Actual value of "g"

$$g = \frac{4\pi^2}{AT^2}\left(A^2+K^2\right).$$

$$g = \frac{4\pi^2}{1.5(3)^2}\left(1.5^2+2^2\right).$$

$$g = 18.27\ m/s^2$$

$$\Delta g = 0.0359\, g$$

$$= 0.0359 \times 18.27\ m/s^2$$

$$= 0.656\ m/s^2$$

Numerical value of $g \Rightarrow (18.27 \pm 0.656)\ m/s^2$