

# Python Course 1: Introduction to Python

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# Introduction

- **Objectives:**
  - ▶ Learn Python basics
  - ▶ Get an introduction to popular optimization and data science libraries
- **Prerequisites:** Familiarity with basic programming concepts (variables, data types, control structures, and functions) in any programming language
- **Pedagogical approach:** Learn fundamental Python concepts by solving an optimization problem

# Plan

- Introduction
- Python fundamentals
- Data science and optimization libraries:
  - ▶ Manipulating data with `pandas`
  - ▶ Visualizing data with `Matplotlib`
  - ▶ Formulating and solving optimization models with `Pyomo`
- Additional topics (if time permits)

# Creating a virtual environment

- Use a virtual environment to keep your project's dependencies isolated.
- To **create** the environment, in a terminal, navigate to your project folder and run:

```
python -m venv venv
```

- To **activate** the environment:
  - ▶ `venv\Scripts\activate` (Windows)
  - ▶ `source venv/bin/activate` (macOS/Linux)
- When activated, `(venv)` appears at the prompt.
- To **exit** the environment, type `deactivate`.

# Getting started with Python

- Strings:

```
"Hello , world!"      # Double quotes
'Hello , world!'      # Single quotes
"That's fine!"        # Double quotes with a single quote inside
```

- Printing:

```
print("Hello , world!")
```

- Arithmetic:

```
2 + 3      # Addition: 5
3 ** 2     # Exponentiation: 9
```

- Comments:

```
# This is a comment
```

# Transportation problem

A motivating example for learning Python

**Goal:** Distribute goods from suppliers to customers at minimal cost.

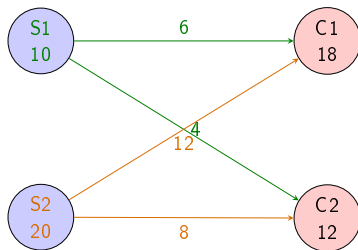
**Given:**

- Supplier capacities
- Customer demands
- Shipping costs

**Constraints:**

- Supply limits per supplier
- Demand requirements per customer

**Objective:** Minimize total transportation cost



# Transportation problem

## Data representation

### Typical data for the transportation problem:

- **Suppliers:** List with supply amounts
- **Customers:** List with demand amounts
- **Cost Matrix:** Table of shipping costs per unit from each supplier to each customer

### Example:

#### Suppliers

- S1: 10 units
- S2: 20 units

#### Customers

- C1: 18 units
- C2: 12 units

#### Cost Matrix

	C1	C2
S1	5	7
S2	4	6

# Variables and data types

## Concepts

- A **variable** is a name (identifier) that refers to an **object** (the value) stored in memory.
- Each object has a **data type**, which determines what kind of data it represents (e.g. integer, string).
- The **name** (the variable) and the **content** (the object/value) are separate:
  - ▶ **Variable:** The label or identifier (e.g. x)
  - ▶ **Object/Value:** The data assigned to the variable (e.g. 42 or "hello")
  - ▶ **Data type:** The kind of the object/value (e.g. int, str)
- *In Python, every value is represented as an object.*



# Variables and data types

## Dynamic typing in Python

- **No type declaration** is needed when defining a variable.
- For example, you simply assign a value without: `x = 10` (no need to specify that it is an integer)
- **Variables can be reassigned** to values of different types:
  - ▶ `x = 10` # x is an integer (`int`)
  - ▶ `x = "hello"` # now x is a string (`str`)
- This is different from languages like Java or C++, where the type must be specified.

# Variables and data types

## Example

The input data from our transportation problem can be stored in variables:

```
# Suppliers
s1 = 10
s2 = 20

# Customers
c1 = 18
c2 = 12

# Cost matrix
s1_c1 = 5
s1_c2 = 7
s2_c1 = 4
s2_c2 = 6
```

# Printing strings with variables

## Examples

- You can print the value of a variable by passing it as an argument to the print function:

```
print(s1)                                # Output: 10
```

- You can print multiple variables by passing them as separate arguments:

```
print(c1, c2)                            # Output: 18 12  
print(s1, c2, s1_c2)                     # Output: 10 12 7
```

- You can include variable values inside a string using an f-string (formatted string literal):

```
print(f"s1: {s1}, s2: {s2}")             # Output: s1: 10, s2: 20
```

# The type function

- Use the type function to determine the type of any object.
- It can be called directly on a value:

```
print(type(23.54))      # Output: <class 'float'>
print(type("Customer")) # Output: <class 'str'>
```

- Or on a variable:

```
s1 = 10
print(type(s1))          # Output: <class 'int'>
```

- The output is always in the form <class 'type'>.

# Python basic data types

## Examples:

- **int**: Integer numbers  
`x = 42`
- **float**: Decimal numbers  
`y = 3.14`
- **str**: Text strings  
`name = "test"`
- **bool**: Boolean values  
`flag = True`
- **list**: Ordered collection  
`nums = [1, 2, 3]`
- **tuple**: Immutable collection  
`coords = (10, 20)`
- **dict**: Key-value pairs  
`info = {"S1": 35, "S2": 25}`
- **set**: Unordered unique items  
`unique = {2, 4, 6}`

## Remarks

- There are many other built-in data types.
- You can also define your own data types (using object-oriented programming).

# Lists

## Concept and syntax

- A **list** is a built-in Python data structure that stores an ordered collection of items.
- Lists can contain elements of any type (integers, strings, other lists, etc.).
- You can access elements in a list by their position, starting from 0.

### General syntax:

```
my_list = [element1, element2, element3, ...]
```

### Remark

Python lists can mix types, e.g. `mixed = [1, "hello", 3.14, [2, 3]]`

# Lists

## Access elements

- Access elements by index: `my_list[0]` (first element)
- Lists are 0-indexed (the first item is at position 0)
- Access the last element with `my_list[-1]`
- Access a range of elements (a **slice**) with `my_list[start:stop]` (from start up to but not including stop)

### Example:

```
mixed = [1, "hello", 3.14, [2, 3]]

print(mixed[0])      # Output: 1          (first element)
print(mixed[-1])     # Output: [2, 3]    (last element)
print(mixed[1:3])    # Output: ['hello', 3.14]
                    # (slice from index 1 up to 3)
```

# Lists

## Useful functions on lists

- `len(x)`: Returns the number of elements in the list `x`.
- `sorted(x)`: Returns a new sorted list with the same elements as `x` (the original list is unchanged).
- `sum(x)`: Returns the sum of all elements in the list (works with numbers).
- `min(x)`: Returns the smallest element in the list.
- `max(x)`: Returns the largest element in the list.

```
supply = [10, 20]
demand = [18, 12]

print(len(supply))           # Output: 2
print(sorted(demand))        # Output: [12, 18]
print(demand)                # Output: [18, 12]
print(sum(supply))           # Output: 30
print(min(demand))           # Output: 12
print(max(demand))           # Output: 18
```



# Lists

## Sequence of numbers with `range`

- The `range` function creates a sequence of numbers:
- `range(n)` generates a sequence of numbers from 0 to  $n-1$ .
- The sequence generated by `range` is not a list, but can be converted to one using `list()`:

```
print(range(10)) # Output: range(0, 10) NOT A LIST  
  
print(list(range(10))) # [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

- In general, `range(start, stop, step)` generates a sequence:
  - ▶ starting from `start`;
  - ▶ incrementing by `step` (or decrementing if `step < 0`);
  - ▶ stopping before `stop`

```
print(list(range(4, 10, 2))) # [4, 6, 8]  
print(list(range(11, 0, -3))) # [11, 8, 5, 2]
```

# Methods

## What is a method?

- A **method** is a function that is associated with a specific object (like a list or a string).
- You call a method using the “dot notation”:

`object.method(arguments)`

- The method can use or change the contents of the object it belongs to.

### Examples with strings:

```
my_string = "hello world"
print(my_string.upper())           # Output: "HELLO WORLD"
print(my_string.replace("o", "O")) # Output: "hellO wOrld"
print(my_string.split())           # Output: ['hello', 'world']
```

# Methods

## Common list methods

- Lists in Python have several useful methods for modifying their contents:
  - ▶ `append(x)`: Adds element `x` to the end of the list.
  - ▶ `insert(i, x)`: Inserts element `x` at position `i`.
  - ▶ `pop([i])`: Removes and returns the element at position `i` (last element if `i` is omitted).
  - ▶ `remove(x)`: Removes the first occurrence of element `x`.
  - ▶ `extend(iterable)`: Adds all elements from another iterable to the end of the list.
  - ▶ `sort()`: Sorts the list in place.
  - ▶ `reverse()`: Reverses the elements of the list in place.
- Elements of a list can also be changed or accessed using the `[]` (index) operator.

## Note

These methods modify the list directly (in place).

# Methods

## Examples of common list methods

```
lst = [10, 20, 30]

lst.append(40)           # [10, 20, 30, 40]
lst.insert(1, 15)        # [10, 15, 20, 30, 40]
lst.pop()                # [10, 15, 20, 30]
lst.remove(15)           # [10, 20, 30]
lst.extend([50, 60])     # [10, 20, 30, 50, 60]
lst.sort()               # [10, 20, 30, 50, 60]
lst.reverse()            # [60, 50, 30, 20, 10]
lst[0] = 99              # [99, 50, 30, 20, 10]
```

# Mutability

## Concept

- Objects in Python can be *mutable* (can be changed after creation) or *immutable* (cannot be changed after creation).
  - ▶ **Mutable objects:** `list`, `dict`, `set`
  - ▶ **Immutable objects:** `int`, `float`, `str`, `tuple`
- Mutable objects can have their contents changed after they are created, while immutable objects cannot be changed—any operation that seems to modify them actually creates a new object.

# Mutability

## Examples

```
# Mutable example (list)
mixed = [1, "hello", 3.14, [2, 3]]
mixed[0] = 42
mixed.pop()
print(mixed)      # Output: [42, 'hello', 3.14]

# Immutable example (str)
s = "hello"
# s[0] = "H" # This would raise an error
s = "Hello" # A new string object is assigned to s

# Immutable example (int)
x = 10
x = x + 5 # A new int object is assigned to x
```

# Dictionaries

## Concept and syntax

- A **dictionary** is a built-in Python data structure that stores data as key-value pairs.
- Each **key** is unique and is used to access its corresponding **value**.
- Dictionaries are useful for representing mappings, such as supplier to supply amounts, or customer to demand amount.

### General syntax:

```
my_dict = {key1: value1, key2: value2, ...}
```

- Access a value by its key: `my_dict[key1]`
- Keys can be of any **immutable** type (e.g., strings, numbers, tuples).

# Dictionaries

Example: suppliers and customers

```
supply_by_sup = {"S1": 10, "S2": 20}
demand_by_cust = {"C1": 18, "C2": 12}

print(supply_by_sup["S2"]) # Output: 20
print(demand_by_cust["C1"]) # Output: 18
print(len(supply_by_sup)) # Output: 2
```

- In {"S1": 10, "S2": 20}, "S1" and "S2" are the keys, whereas 10 and 20 are the values.
- The len of a dictionary in Python corresponds to the number of key-value pairs it contains.



# Dictionaries

Example: cost matrix

```
cost_by_sup_cust = {  
    ("S1", "C1"): 5,  
    ("S1", "C2"): 7,  
    ("S2", "C1"): 4,  
    ("S2", "C2"): 6  
}  
  
print(cost_by_sup_cust[("S1", "C2")]) # 7  
print(cost_by_sup_cust[("S2", "C1")]) # 4  
print(len(cost_by_sup_cust))          # 4
```

- A **tuple** (such as a pair ("S1", "C2")) can be used as a dictionary key because it is **immutable**.
- A **list** cannot be used as a key, because lists are **mutable**.

# Dictionaries

## Common dictionary methods

- Dictionaries in Python have several useful methods for accessing and working with their contents:
  - ▶ `keys()`: Returns a view of all the keys in the dictionary.
  - ▶ `values()`: Returns a view of all the values in the dictionary.
  - ▶ `items()`: Returns a view of all key-value pairs as tuples.
  - ▶ `get(key, default)`: Returns the value for `key` if it exists; otherwise returns `default` (or `None` if `default` is not provided).
  - ▶ `pop(key)`: Removes the item with the specified `key` and returns its value.

# Dictionaries

## Examples of common dictionary methods

```
supply_by_sup.keys()      # dict_keys(['S1', 'S2'])
demand_by_cust.values()   # dict_values([18, 12])
cost_by_sup_cust.items()
# dict_items([(('S1', 'C1'), 5), (('S1', 'C2'), 7),
#            (('S2', 'C1'), 4), (('S2', 'C2'), 6)])

supply_by_sup.get("S3", 0)
# 0 (since "S3" is not in suppliers_dict)
demand_by_cust.pop("C2")
# 12 (and removes "C2" from the dictionary)
```

## Dictionary views vs. lists

- Methods like `mydict.keys()`, `mydict.values()`, and `mydict.items()` return **dictionary view** objects.
- A dictionary view is **not exactly a list**, but you can loop over it like a list.
- If you need a real list (for example, to index or modify it), use `list(dict_view)`.

```
list(supply_by_sup.keys())      # ['S1', 'S2', 'S3']
list(demand_by_cust.values())   # [18, 12]
list(cost_by_sup_cust.items())
# [ (('S1', 'C1'), 5), (('S1', 'C2'), 7) ]
```

# Greedy heuristic for the transportation problem

**Goal:** Assign flow from suppliers to customers in order of increasing unit cost, without violating supply or demand.

## Pseudo code:

- ① Initialize empty flow dictionary.
- ② Sort all (*supplier*, *customer*) pairs by their unit cost (ascending).
- ③ **For each** (*supplier*, *customer*) pair in sorted order:
  - Ⓐ *supply*  $\leftarrow$  *supplier*'s current available units.
  - Ⓑ *demand*  $\leftarrow$  *customer*'s current required units.
  - Ⓒ Set *quantity* to the smaller of *supply* and *demand*.
  - Ⓓ Assign *quantity* from *supplier* to *customer* in the flow dictionary.
  - Ⓔ Subtract *quantity* from *supplier*'s supply and *customer*'s demand.

# Greedy heuristic for the transportation problem

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  - Ⓔ Subtract *quantity* from *supplier*'s supply and *customer*'s demand.

# Greedy heuristic for the transportation problem

Initialize empty flow dictionary

- Create a dictionary to store the final solution: the quantity shipped from each supplier to each customer.
- The keys will be (*supplier*, *customer*) pairs, and the values are quantities shipped.

```
flow_by_sup_cust = {}
```

# Greedy heuristic for the transportation problem

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**Pseudo code:**

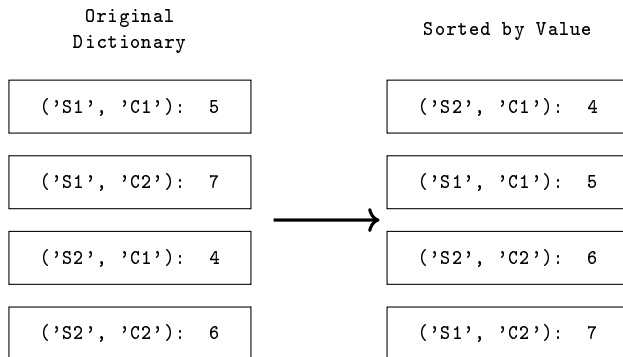
- ➊ Initialize empty flow dictionary.
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- ➌ For each (*supplier*, *customer*) pair in sorted order:
  - ➐ *supply*  $\leftarrow$  *supplier*'s current available units.
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  - ➒ Set *quantity* to the smaller of *supply* and *demand*.
  - ➓ Assign *quantity* from *supplier* to *customer* in the flow dictionary.
  - ➔ Subtract *quantity* from *supplier*'s supply and *customer*'s demand.



# Greedy heuristic for the transportation problem

Sort all (*supplier*, *customer*) pairs by their unit cost

- We want to sort the supplier-customer pairs by cost:



- This can be done with the help of the `sorted` function.

# The sorted() function

## General syntax

- `sorted()` is a built-in function that returns a new sorted list from an iterable.

```
sorted(iterable, key=None, reverse=False)
```

- **Parameters:** (Parameters marked as optional have a default value and can be omitted.)
  - ▶ `iterable`: The sequence (e.g., list, tuple, dict, etc.) to sort.
  - ▶ `key` (optional): A function that tells Python what to sort by.
  - ▶ `reverse` (optional): If `True`, sorts in descending order.
- `sorted()` always returns a new list and does not modify the original data.

# The `sorted()` function

## Example: Sorting shipments

- Applying `sorted` directly to `cost_by_sup_cust` returns a list of the dictionary's keys, sorted in ascending (lexicographic) order:

```
sorted(cost_by_sup_cust)
# [('s1', 'c1'), ('s1', 'c2'), ('s2', 'c1'), ('s2', 'c2')]
```

- To include both the keys and values in the result, sort the dictionary's items:

```
sorted(cost_by_sup_cust.items())
# (('s1', 'c1'), 5), (('s1', 'c2'), 7),
# (('s2', 'c1'), 4), (('s2', 'c2'), 6)]
```

# Defining a function

- General syntax:

```
def function_name(arg1, arg2, ...):  
    # Code block  
    return value
```

- Example:

```
def get_cost(cost_tuple):  
    cost = cost_tuple[1]  
    return cost
```

- The function `get_cost` takes one argument: `cost_tuple`.
- The variable `cost` is assigned the 2nd element of the tuple (the cost).
- The function returns the `cost`.
- For example: `get_cost (('S1', 'C1'), 4)` returns 4.

# The sorted() function

Example: Sorting shipments by cost

- To sort by cost, we pass our function `get_cost` to the `key` parameter:

```
def get_cost(cost_tuple):  
    cost = cost_tuple[1]  
    return cost  
  
...  
  
sorted_costs = sorted(cost_by_sup_cust.items(), key=get_cost)  
# [ (('s2', 'c1'), 4), (('s1', 'c1'), 5),  
#   (('s2', 'c2'), 6), (('s1', 'c2'), 7)]
```

# Lambda Expressions

- A **lambda expression** is an **anonymous function**—a function without a name.
- **General syntax:** `lambda x, y, ...: f(x, y, ...)` (for example: `lambda x, y: x + y`)
- When a function is simple and only used once, a lambda expression makes the code shorter and clearer.

```
# Instead of defining a separate function:  
sorted_costs = sorted(cost_by_sup_cust.items(),  
                      key=lambda x: x[1])
```

# Greedy heuristic for the transportation problem

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## Pseudo code:

- ① Initialize empty flow dictionary.
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  - Ⓓ Assign *quantity* from *supplier* to *customer* in the flow dictionary.
  - Ⓔ Subtract *quantity* from *supplier*'s supply and *customer*'s demand.

# Greedy heuristic for the transportation problem

Iterating over the sorted shipments

- We want to iterate over each pair of `sorted_costs`.

Iteration 1	Iteration 2	Iteration 3	Iteration 4
('S2', 'C1'): 4	('S2', 'C1'): 4	('S2', 'C1'): 4	('S2', 'C1'): 4
('S1', 'C1'): 5	('S1', 'C1'): 5	('S1', 'C1'): 5	('S1', 'C1'): 5
('S2', 'C2'): 6	('S2', 'C2'): 6	('S2', 'C2'): 6	('S2', 'C2'): 6
('S1', 'C2'): 7	('S1', 'C2'): 7	('S1', 'C2'): 7	('S1', 'C2'): 7

- This can be done with the help of a for loop.



# The for loop

## General syntax

- A for loop lets you repeat actions for each item in a sequence (like a list, tuple, or dictionary).

```
for variable in sequence:  
    # code to execute for each item
```

- The loop ends automatically after all items are processed.

### Remark

The code inside the for loop must be indented. Python uses indentation to determine which lines belong inside the loop.

# The for loop

## Unpacking values

```
for s_c_cost in sorted_costs:
    print(s_c_cost)  # 1st output: (("S1", "C1"), 4)

for s_c, cost in sorted_costs:
    print(s_c)  # 1st output: ("S1", "C1")
    print(cost)  # 1st output: 4

for (s, c), cost in sorted_costs:
    print(s)  # 1st output: S2
    print(c)  # 1st output: C3
    print(cost)  # 1st output: 3
```

- The loop visits each item in `sorted_costs` one at a time.
- You can unpack multiple values directly in the loop header.
- You can use the unpacked values (e.g, `s`, `c` and `cost`) inside the loop body.

## More about loops

- The `while` loop:

```
while condition:
    # code block to repeat
```

- Control flow statements for managing loop execution (in both `for` and `while` loops):
  - ▶ `break`: Exits the loop immediately.
  - ▶ `continue`: Skips to the next iteration of the loop.
  - ▶ `else`: Executes after the loop only if it was not exited by `break`.

```
for s in supply_by_sup.values():
    if s == 10:
        print("A supply of 10 was found!")
        break
else:
    print("A supply of 10 was not found.")
```

# Greedy heuristic for the transportation problem

**Goal:** Assign flow from suppliers to customers in order of increasing unit cost, without violating supply or demand.

## Pseudo code:

- 1 Initialize empty flow dictionary.
- 2 Sort all (*supplier*, *customer*) pairs by their unit cost (ascending).
- 3 **For each** (*supplier*, *customer*) pair in sorted order:
  - a *supply*  $\leftarrow$  *supplier's* current available units.
  - b *demand*  $\leftarrow$  *customer's* current required units.
  - c Set *quantity* to the smaller of *supply* and *demand*.
  - d Assign *quantity* from *supplier* to *customer* in the flow dictionary.
  - e Subtract *quantity* from *supplier's* supply and *customer's* demand.

# Greedy heuristic for the transportation problem

Assigning *supply* and *demand*

- The current supply of supplier  $s$  is given by the value for key  $s$  in the dictionary `supply_by_sup`:

```
supply = supply_by_sup[s]
```

- The current demand of customer  $c$  is given by the value for key  $c$  in the dictionary `demand_by_cust`:

```
demand = demand_by_cust[c]
```

# Greedy heuristic for the transportation problem

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# Greedy heuristic for the transportation problem

Deciding how much to ship at each step

- At each step, we want to ship as many units from supplier  $s$  to customer  $c$  as possible, without exceeding either the supplier's supply or the customer's demand.
- Setting *quantity* to the smaller value ensures that neither limit is violated.
- In other words,  $quantity = \min(supply, demand)$ .
- This is equivalent to the following decision process:
  - ▶ If  $supply > demand$ :
    - ★  $quantity \leftarrow demand$
  - ▶ Else if  $supply < demand$ :
    - ★  $quantity \leftarrow supply$
  - ▶ Else:
    - ★  $quantity \leftarrow supply$  (or demand, they are equal)

# Conditional statements

The `if`, `elif`, and `else` statements

```
if condition1:
    # if condition1 is True
elif condition2:
    # if condition2 is True (and condition1 is False)
elif condition3:
    # if condition3 is True (and previous are False)
    # ...
else:
    # if none of the above conditions are True
```

- `if` checks the first condition.
- You can have zero, one, or many `elif` blocks to check additional conditions in order.
- `else` is optional, and runs if none of the above conditions are `True`.



# Conditional statements

Example: deciding how much to ship at each step

- **Decision process:**

- ▶ If *supply* > *demand*:

- ★ *quantity*  $\leftarrow$  *demand*

- ▶ Else if *supply* < *demand*:

- ★ *quantity*  $\leftarrow$  *supply*

- ▶ Else:

- ★ *quantity*  $\leftarrow$  *supply* (or *demand*, they are equal)

- This decision process can be translated in Python as follows:

```
if supply > demand:
    quantity = demand
elif supply < demand:
    quantity = supply
else:
    quantity = supply # or demand
```

# Greedy heuristic for the transportation problem

**Goal:** Assign flow from suppliers to customers in order of increasing unit cost, without violating supply or demand.

## Pseudo code:

- ① Initialize empty flow dictionary.
- ② Sort all (*supplier*, *customer*) pairs by their unit cost (ascending).
- ③ **For each** (*supplier*, *customer*) pair in sorted order:
  - Ⓐ *supply*  $\leftarrow$  *supplier*'s current available units.
  - Ⓑ *demand*  $\leftarrow$  *customer*'s current required units.
  - Ⓒ Set *quantity* to the smaller of *supply* and *demand*.
  - Ⓓ Assign *quantity* from *supplier* to *customer* in the flow dictionary.
  - Ⓔ Subtract *quantity* from *supplier*'s supply and *customer*'s demand.

## Storing the shipment plan

- Record the assigned quantity of shipment in `flow_by_sup_cust` to keep track of how much is shipped from each supplier to each customer:

```
flow_by_sup_cust = {} # Defined previously

# ... some logic ...

for (s, c), cost in sorted_costs:

    # ... determine quantity to ship ...

    flow_by_sup_cust[(s, c)] = quantity
```

## Greedy heuristic for the transportation problem

**Goal:** Assign flow from suppliers to customers in order of increasing unit cost, without violating supply or demand.

### Pseudo code:

- 1 Initialize empty flow dictionary.
- 2 Sort all (*supplier*, *customer*) pairs by their unit cost (ascending).
- 3 **For each** (*supplier*, *customer*) pair in sorted order:
  - a *supply*  $\leftarrow$  *supplier*'s current available units.
  - b *demand*  $\leftarrow$  *customer*'s current required units.
  - c Set *quantity* to the smaller of *supply* and *demand*.
  - d Assign *quantity* from *supplier* to *customer* in the flow dictionary.
  - e Subtract *quantity* from *supplier*'s supply and *customer*'s demand.

# Greedy heuristic for the transportation problem

## Updating supply and demand

- Subtract `quantity` from the supply of supplier `s`:

```
supply_by_sup[s] -= quantity
```

- Subtract `quantity` from the demand of customer `c`:

```
demand_by_cust[c] -= quantity
```

- After each shipment, these updates keep track of what each supplier still has and what each customer still needs.

# Greedy heuristic for the transportation problem

## Printing the solution

- Print the shipment plan (how much is shipped from each supplier to each customer):

```
print(f"Shipment plan: {flow_by_sup_cust}")  
# Shipment plan: {('S2', 'C1'): 18, ('S1', 'C1'): 0,  
#                ('S2', 'C2'): 2, ('S1', 'C2'): 10}
```

- Print the remaining supply for each supplier:

```
print(f"supply_by_sup: {supply_by_sup}")  
# Remaining supply: {'S1': 0, 'S2': 0}
```

- Print the remaining demand for each customer:

```
print(f"demand_by_cust: {demand_by_cust}")  
# Remaining demand: {'C1': 0, 'C2': 0}
```

# Greedy heuristic for the transportation problem

## Creating a function

- To reuse our greedy heuristic, we can put it inside a function:

```
def greedy_solve(supply_by_sup, demand_by_cust,
                 cost_by_sup_cust):
    flow_by_sup_cust = {}

    # ... greedy heuristic ...

    return flow_by_sup_cust
```

- We can then call the function with different input data:

```
cost_by_sup_cust2 = {("S1", "C1"): 8, ("S1", "C2"): 2,
                     ("S2", "C1"): 5, ("S2", "C2"): 9}
supply_by_sup2 = {"S1": 13, "S2": 17}
demand_by_cust2 = {"C1": 9, "C2": 21}

solution2 = greedy_solve(supply_by_sup2, demand_by_cust2,
                         cost_by_sup_cust2)
```

# Greedy heuristic for the transportation problem

## Returning remaining supply and demand

- In addition to the shipment plan, we can return the remaining supply and the remaining demand.
- Python lets you return more than one value from a function (as a tuple).

```
def greedy_solve(supply_by_sup, demand_by_cust,
                 cost_by_sup_cust):

    # ... greedy heuristic ...

    return flow_by_sup_cust, supply_by_sup, demand_by_cust
```

- You can assign each return value to a different variable:

```
sol, rem_sup, rem_dem = greedy_solve(supply_by_sup,
                                     demand_by_cust,
                                     cost_by_sup_cust)
```



# Greedy heuristic for the transportation problem

## Adding an optional verbose parameter

- To control whether the solution is printed, we can add an **optional** verbose parameter (default: False).
- Optional arguments should always appear after positional arguments in the function definition.

```
def greedy_solve(supply_by_sup, demand_by_cust,
                 cost_by_sup_cust, verbose=False):
    # ... greedy heuristic ...
    if verbose:
        print(f"Shipment plan: {flow_by_sup_cust}")
        print(f"Remaining supply: {supply_by_sup}")
        print(f"Remaining demand: {demand_by_cust}")

    return flow_by_sup_cust
...

greedy_solve(sup, dem, costs) # No output
greedy_solve(sup, dem, costs, verbose=True) # Prints solution
```

# Modifying arguments

## Example

- What are the values of `sup` and `dem` after calling `greedy_solve` in the following example?

```
cost = {("S1", "C1"): 5, ("S1", "C2"): 7,  
        ("S2", "C1"): 4, ("S2", "C2"): 6}  
sup = {"S1": 10, "S2": 20}  
dem = {"C1": 18, "C2": 12}  
  
greedy_solve(sup, dem, costs) # Are the dicts modified?  
print(f"sup={sup}") # Output: ?  
print(f"dem={dem}") # Output: ?
```

# Modifying arguments

## Example

- What are the values of `sup` and `dem` after calling `greedy_solve` in the following example?

```
cost = {("S1", "C1"): 5, ("S1", "C2"): 7,
        ("S2", "C1"): 4, ("S2", "C2"): 6}

sup = {"S1": 10, "S2": 20}
dem = {"C1": 18, "C2": 12}

greedy_solve(sup, dem, costs) # Are the dicts modified?
print(f"sup={sup}") # Output: ?
print(f"dem={dem}") # Output: ?
```

- Output of `print(f"sup={sup}")` and `print(f"dem={dem}")`:

```
sup={'S1': 0, 'S2': 0}
dem={'C1': 0, 'C2': 0}
```

- The dictionaries were modified! **Why?**

# Mutable vs immutable arguments in functions

- The type of object you pass to a function—**mutable** or **immutable**—affects how it can be changed.
- **Immutable objects** (e.g., `int`, `float`, `str`, `tuple`):
  - ▶ The function **cannot change the original object**.
  - ▶ If "modified", a **new object** is created and the original remains unchanged.
- **Mutable objects** (e.g., `list`, `dict`, `set`):
  - ▶ The function **can change the original object**.
  - ▶ Changes made inside the function are **visible outside the function**.

# Mutable vs immutable arguments in functions

Example: Direct modification

```
def greedy_solve(supply_by_sup, demand_by_cust,
                 cost_by_sup_cust):
    # ... some logic ...
    for (s, d), c in sorted(cost_by_sup_cust.items()):
        # ... determine quantity ...
        supply_by_sup[s] -= quantity # MODIFIES sup
        demand_by_cust[d] -= quantity # MODIFIES dem
    # ...

    return flow_by_sup_cust, supply_by_sup, \
           demand_by_cust

greedy_solve(sup, dem, costss)
```

- The lines marked above directly modify the dictionaries `sup` and `dem` passed as arguments.

# Mutable vs immutable arguments in functions

## Example: Making copies

```
def greedy_solve(supply_by_sup, demand_by_cust,
                  cost_by_sup_cust):
    rem_sup = supply_by_sup.copy()
    rem_dem = demand_by_cust.copy()
    # ... some logic ...
    for (s, d), c in sorted(cost_by_sup_cust.items()):
        supply = rem_sup[s]
        demand = rem_dem[d]
        # ... determine quantity ...
        rem_sup[s] -= quantity
        rem_dem[d] -= quantity
    # ...

    return flow_by_sup_cust, rem_sup, rem_dem
```

- To avoid modifying the original arguments, make a copy of the dictionaries inside the function before modifying them.
- Now, the original dictionaries remain unchanged after the function call.

# Modules

- Putting the greedy heuristic in a function lets us use it multiple times in the same script with different data.
- What if we want to reuse `greedy_solve` in other programs, without copying the code each time?
- We can make any function or variable reusable by putting it in a **module** (a separate `.py` file), and then **import** it when needed.

## Creating a module

```
# File: mymodule.py

def my_function(arg1, arg2):
    # ... implementation ...
```

## Importing from a module

```
import mymodule                # import the whole module
from mymodule import my_function # import only my_function
```

# Modules

Example: Moving `greedy_solve` to `greedy.py`

- Place the `greedy_solve` function in its own file, `greedy.py`, to make it reusable.

```
# File: greedy.py

def greedy_solve(supply_by_sup, demand_by_cust,
                 cost_by_sup_cust):
    # ...implementation...
```

- Now, import and use it in any script:

```
# File: myscript.py

from greedy import greedy_solve

# Use the function as needed
greedy_solve(sup, dem, costs)
```



# Pandas

## What is pandas?

**Pandas** is a popular Python library for working with structured data:

- Efficiently loads, manipulates, and analyzes tabular data (like spreadsheets, CSV files, or SQL tables).
- The main data structure is the **DataFrame**, which looks like a table. Pandas also provides the **Series**, for one-dimensional data.
- Great for data cleaning, exploration, and preparation.

### Installation:

```
pip install pandas
```

### Import pandas:

```
import pandas as pd
```

# Pandas

## Reading CSV files with pandas

### Example: Loading data for the transportation problem

```
costs_df = pd.read_csv("data/costs.csv")
```

- `pd.read_csv` loads a CSV file into a **DataFrame**.

		columns		
		supplier	customer	cost
rows	0	S1	C1	5
	1	S1	C2	7
	3	S2	C1	4
	4	S2	C2	6

- The row labels are called the **index** (accessible with `.index`).
- The column labels are called **columns** (accessible with `.columns`).

# Pandas

## Inspecting DataFrames

```
costs_df.head()      # Show the first 5 rows
costs_df.head(10)    # Show the first 10 rows
costs_df.tail()      # Show the last 5 rows

costs_df['cost']      # Select a single column as a Series
costs_df[['cost']]    # Select a single column as a DataFrame
costs_df[['cost', 'supplier']] # Select multiple columns
                                # as a DataFrame
```

- `head(n)` shows the first `n` rows (`n` is optional, default is 5).
- `tail(n)` shows the last `n` rows (`n` is optional, default is 5).
- `df['supplier']` (single square brackets) returns a Series, not a DataFrame.
- `df[['col']]` (double square brackets) returns a DataFrame, even for one column.
- Double brackets are required to select multiple columns:  
`df[['col1', 'col2']]`.

# Pandas

## Accessing data with loc

- The `loc` method allows you to select rows and columns by label.

```
# Select a single row by index label
costs_df.loc[0]

# Select specific rows and columns by label
costs_df.loc[0, 'cost']

# Select multiple rows and columns by label
costs_df.loc[[0, 3], ['supplier', 'cost']]
```

- `loc[index, columns]` selects data by label, not position.
- Useful for precise data selection, filtering, and subsetting.

# Pandas

## Basic statistics with DataFrames

- `describe()` gives count, mean, std deviation, min, max, and percentiles:

```
costs_df.describe() # Summary statistics for the entire
                    # DataFrame

costs_df['cost'].describe() # Summary statistics for the
                           # cost column
```

- You can also use `mean()`, `std()`, `min()`, `max()`, and `median()` directly.

```
costs_df['cost'].mean() # Mean
costs_df['cost'].std() # Standard deviation
costs_df['cost'].min() # Minimum value
costs_df['cost'].max() # Maximum value
print(costs_df['cost'].median()) # Median
```

# Pandas

## Filtering rows with boolean conditions

```
# Show all rows where cost is less than 12
costs_df[costs_df['cost'] < 12]

# Combine conditions (e.g., cost is less than 12
# and supplier is 'S2')
costs_df[(costs_df['cost'] < 12)
          & (costs_df['supplier'] == 'S2')]

# Use | for "or" (e.g., cost is 7 or 19)
costs_df[(costs_df['cost'] == 7)
          | (costs_df['cost'] == 19)]
```

- Use boolean expressions inside `[]` to select rows.
- Combine conditions with `&` (and), `|` (or), and surround each condition with parentheses.
- The result is a filtered DataFrame.

# Pandas

## Converting DataFrames to dictionaries

```
# Convert supply and demand DataFrames to dict
supply_df.set_index('supplier').to_dict()['supply']
demand_df.set_index('customer').to_dict()['demand']

# Convert costs DataFrame to a dictionary
costs_df.set_index(
    ['supplier', 'customer']).to_dict()['cost']
```

- The `set_index` method sets one or more columns as the dictionary keys.
- The `to_dict()` method converts the DataFrame (or Series) to a Python dictionary.

# Matplotlib

## What is Matplotlib?

**Matplotlib** is a popular Python library for creating visualizations such as graphs and charts.

- Lets you turn data into plots: line charts, bar charts, heatmaps, and more.
- Integrates well with pandas and plain Python lists.
- The main module for plotting is `matplotlib.pyplot`, commonly imported as `plt`.

## Installation:

```
pip install matplotlib
```

## Import Matplotlib:

```
import matplotlib.pyplot as plt
```



# Matplotlib

## The `plot` function

```
plt.plot(x, y, ...)
```

- Creates a **line plot**.
- `x`: Sequence of positions or labels.
- `y`: Sequence of numeric values.
- Common options: `marker`, `linestyle`, `color`.
- `plt.show()` renders and displays the current figure.

### Example:

```
plt.plot(supply.keys(), supply.values(), marker="o",  
         color="orange")  
plt.show()
```

# Matplotlib

## Adding labels, titles, and legends

- `plt.xlabel("...")` sets the label for the x-axis.
- `plt.ylabel("...")` sets the label for the y-axis.
- `plt.title("...")` sets the plot title.
- `plt.legend()` shows a legend for labeled artists (e.g., lines or bars with `label=`).

### Example:

```
plt.plot(supply.keys(), supply.values(), marker="o",  
         color="orange", label="Supply")  
plt.xlabel("Supplier")  
plt.ylabel("Units")  
plt.title("Supply")  
plt.legend()  
plt.show()
```

# Matplotlib

## The `bar` function

```
plt.bar(x, height, ...)
```

- Creates a **bar chart**.
- `x`: Sequence of labels or positions.
- `height`: Sequence of numeric values.
- Additional options: `color`, `label`, etc.

### Example:

```
plt.bar(demand.keys(), demand.values(), color="blue",  
        label="Demand")  
plt.xlabel("Customer")  
plt.ylabel("Units")  
plt.title("Demand")  
plt.legend()  
plt.show()
```

# Matplotlib

## Saving a plot to a file

- Use `plt.savefig("filename.png")` to save the current plot to an image file.
- You can specify the file format by changing the extension (e.g., `.png`, `.jpg`, `.pdf`, `.svg`).
- `plt.savefig` must be called **before** `plt.show()`.

### Example: Saving a line plot

```
plt.plot(supply.keys(), supply.values(), marker="o",  
         color="orange", label="Supply")  
plt.xlabel("Supplier")  
plt.ylabel("Units")  
plt.title("Supply")  
plt.legend()  
plt.savefig("supply_plot.png")  
plt.show()
```

# Pyomo

## What is Pyomo?

- **Pyomo** is a Python-based, open-source optimization modeling language.
- It allows you to define mathematical models for linear, integer, and nonlinear optimization problems.
- Pyomo provides a high-level, readable, and flexible interface for expressing variables, constraints, and objectives.
- Pyomo models can be solved by a variety of external solvers (e.g., GLPK, CBC, Gurobi, CPLEX).

## Installation:

```
pip install pyomo
```

## Import Pyomo:

```
from pyomo.environ import *
```

# Pyomo

## Transportation problem: sets, parameters and variables

- Sets:
  - ▶  $I$ : set of suppliers
  - ▶  $J$ : set of customers
- Parameters:
  - ▶  $s_i$ : supply available at supplier  $i \in I$
  - ▶  $d_j$ : demand required at customer  $j \in J$
  - ▶  $c_{ij}$ : cost per unit shipped from supplier  $i$  to customer  $j$
- Decision variables:
  - ▶  $x_{ij}$ : quantity shipped from supplier  $i$  to customer  $j$

# Pyomo

## Transportation problem: objective function and constraints

- Objective function:

$$\text{Minimize } Z = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

- Constraints:

- ▶ Supply capacity:

$$\sum_{j \in J} x_{ij} \leq s_i \quad \forall i \in I$$

- ▶ Demand satisfaction:

$$\sum_{i \in I} x_{ij} \geq d_j \quad \forall j \in J$$

- ▶ Non-negativity:

$$x_{ij} \geq 0 \quad \forall i \in I, \forall j \in J$$

# Pyomo

## Basic building blocks in Pyomo

- **ConcreteModel**: The main object that holds sets, parameters, variables, constraints, and objectives.
- **Set**: Represents a finite collection of elements (e.g., suppliers, customers).
- **Var**: Decision variables whose values are determined by the optimization solver.
- **Constraint**: Mathematical conditions that must be satisfied.
- **Objective**: The function to be minimized or maximized.



# Pyomo

## Defining a Pyomo model

- A `ConcreteModel` in Pyomo is a container for sets, parameters, variables, constraints, and objectives.
- `ConcreteModel()` creates a model where all components are fully specified when the script runs.

```
model = ConcreteModel()
```

# Pyomo

## Defining sets

- Sets represent index collections for variables and constraints.
- The `initialize` argument fills the set from Python objects.

```
model.S = Set(initialize=supply.keys()) # Suppliers  
model.C = Set(initialize=demand.keys()) # Customers
```

- `S` and `C` are user-defined names for sets.

# Pyomo

## Defining variables

- Variables represent the decisions to be made (e.g., how much to ship from each supplier to each customer).
- In the code:

```
model.x = Var(model.S, model.C, domain=NonNegativeReals)
```

- `model.x[i,j]` is the amount shipped from supplier  $i$  to customer  $j$ .
- `NonNegativeReals` ensures variables are  $\geq 0$ .

# Pyomo

## Defining the objective function

- Minimize the total transportation cost:

```
model.obj = Objective(  
    expr=sum(costs[(i, j)] * model.x[i, j]  
             for i in model.S for j in model.C),  
    sense=minimize  
)
```

- `costs[(i, j)] * model.x[i, j]` is computed for every supplier  $i$  and customer  $j$ .
- `for i in model.S for j in model.C` loops over all supplier-customer pairs.
- The expression inside `sum(...)` is called a Python **generator expression**—it produces terms one at a time, making this both concise and memory efficient.

# Pyomo

## Defining the constraints

- Each supplier's total shipment does not exceed its available supply.

```
model.sup_const = Constraint(  
    model.S,  
    rule=lambda m, i: sum(m.x[i, j]  
                           for j in m.C) <= supply[i]  
)
```

- Each customer's demand is satisfied.

```
model.dem_const = Constraint(  
    model.C,  
    rule=lambda m, j: sum(m.x[i, j]  
                           for i in m.S) >= demand[j]  
)
```

# Pyomo

## Choosing and configuring a solver

- Pyomo uses external solvers (like GLPK, CBC, Gurobi, CPLEX) to find optimal solutions.
- You can choose a solver by name:

```
solver = SolverFactory("cbc")
```

- Replace "cbc" with the name of your solver (e.g., "gurobi", "cplex").

# Pyomo

## Solving the model

- Once the solver is configured, you can solve the model:

```
results = solver.solve(model, tee=True)
```

- `solve()` runs the optimization and returns the results.
- The `tee` argument controls whether to display solver output in the console.

# Pyomo

## Extracting the results

- After solving, you can access the optimal variable values and objective value:

```
flows_opt = {}  
for i in model.S:  
    for j in model.C:  
        flows_opt[(i, j)] = value(model.x[i, j])  
  
opt_cost = value(model.obj)
```

- `value()` extracts the numerical value from a Pyomo variable or expression.