

第三章

- T3. (1) 真 (2) 假 (3) 真 (4) 真
 (5) 真 (6) 假 (7) 真 (8) 假

T4. (1) 假 . 反例: $A = \{a\}$ $B = \{b\}$ $C = \{\{a\}\}$

(2) 假 . 反例: $A = \{a\}$ $B = \{\{a\}\}$ $C = \{\{a\}\}$

(3) 假 . 反例: $A = \{a\}$ $B = \{a, b\}$ $C = \{\{a\}\}$

T5. (1) 真

(2) 假 反例: $A = \{a\}$ $B = \{\{a\}\}$ $C = \{\{a\}, \{b\}\}$

(3) 假 反例: $A = \{a\}$ $B = \{a, b\}$ $C = \{\{a, b\}\}$

T6. (1) $2^{\{a, b, c\}} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

(2) $2^{\{a, \{b, c\}\}} = \{\emptyset, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

(3) $2^{\{\emptyset\}} = \{\emptyset, \{\emptyset\}\}$

(4) $2^{\{\emptyset, \{\emptyset\}\}} = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

T8. 证明: (1) $(A \setminus B) \setminus C = (A \cap B') \cap C'$

$$= A \cap (B' \cap C')$$

$$= A \cap (B \cup C)'$$

$$= A \setminus (B \cup C)$$

(2) 由(1)已知 $(A \setminus B) \setminus C = A \setminus (B \cup C)$

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$$\begin{aligned} A \setminus (B \cup C) &= A \setminus [(B \cup C) \cap X] \\ &= A \setminus [(B \cup C) \cap (C \cup C')] \\ &= A \setminus [(C \cup B) \cap (C \cup C')] \\ &= A \setminus [(C \cap C') \cup (B \cap C')] \\ &= A \setminus [C \cup (B \cap C')] \quad (\text{由 (1) 律}) \\ &= (A \setminus C) \setminus (B \cap C') \\ &= (A \setminus C) \setminus (B \setminus C) \end{aligned}$$

(3) $(A \setminus B) \setminus C = A \setminus (B \cup C)$ (1) 结论

$$\begin{aligned} &= A \setminus (C \cup B) \\ &= (A \setminus C) \setminus B \quad (1) \text{ 结论} \end{aligned}$$

T9. 证明：循环论证。

① 证明 $A \subseteq B \Rightarrow A' \cup B = X$:

$$\begin{aligned} A' \cup B &= A' \cup (A \cup B) \quad (A \subseteq B) \\ &= (A' \cup A) \cup B \quad (\text{交换律}) \\ &= X \cup B = X \end{aligned}$$

② 证明 $A' \cup B = X \Rightarrow A \cap B' = \emptyset$:

$$\begin{aligned} A \cap B' &= (A')' \cap B' = (A' \cup B)' \\ &= X' = \emptyset \end{aligned}$$

③ 证明 $A \cap B' = \emptyset \Rightarrow A \subseteq B$:

$$A = A \cap X = A \cap (B \cup B') = (A \cap B) \cup (A \cap B')$$

$$= (A \cap B) \cup \emptyset = A \cap B$$

$$\Rightarrow A \subseteq B$$

故命题 $A \subseteq B$, $A' \cup B = X$, $A \cap B' = \emptyset$ 相互等价.

T10. (1) 不一定成立. 反例: $A = \{a\}$, $B = \{b\}$, $C = \{a, b\}$

$$A \cup B = A \cup C = \{a, b\}, \text{ 但 } B \neq C.$$

(2) 不一定成立. 反例: $A = \{a\}$, $B = \{a\}$, $C = \{a, b\}$,

$$A \cap B = A \cap C = \{a\}, \text{ 但 } B \neq C.$$