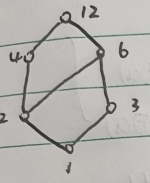
No: W11

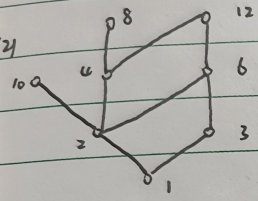
Date:

(P249)

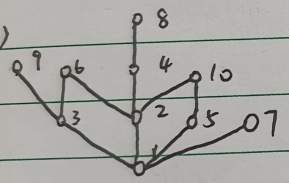
11. 已知 $(L, \leq)$ 是半序集，若 $(L, \leq)$ 为格，则 L 中任意两个元素都有上、下确界存在。对 (1) (2) (3) 作 Hasse 图判断：

(1) 

，任意两元素均有上、下确界，是格.

(2) 

不是格，如 对 (8, 12) 上确界不存在

(3) 

不是格，如 (8, 9) 上确界不存在.

证明：

(1) $\left.\begin{array}{l} a*b \leq a \leq a \oplus c \\ a*b \leq b \leq b \oplus d \end{array}\right\} \Rightarrow a*b \leq (a \oplus c)*(b \oplus d)$

$\left.\begin{array}{l} c*d \leq c \leq a \oplus c \\ c*d \leq d \leq b \oplus d \end{array}\right\} \Rightarrow c*d \leq (a \oplus c)*(b \oplus d)$

$\Rightarrow (a*b) \oplus (c*d) \leq (a \oplus c)*(b \oplus d)$

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(2) $a*b \le a \le a \oplus b$

$a*b \le b \le b \oplus c$

$a*b \le a \le c \oplus a$

$\Rightarrow a*b \le (a \oplus b)*(b \oplus c)*(c \oplus a)$

同理可得 $b*c \le (a \oplus b)*(b \oplus c)*(c \oplus a)$

$c*a \le (a \oplus b)*(b \oplus c)*(c \oplus a)$

$\Rightarrow (a*b) \oplus (b*c) \oplus (c*a) \le (a \oplus b)*(b \oplus c)*(c \oplus a)$