Moments, skewness and kurtosis:

Moments:

i) **Raw moment:**

$$\mu_r' = \frac{\sum_{i=1}^{n} f_i (x_i - A)^r}{N};$$

$$A = any\ arbitrary\ value; r = 1, 2, 3, \ldots$$

1st raw moment, $\mu_1' = \frac{\sum_{i=1}^{n} f_i (x_i - A)^1}{N}$

2nd raw moment, $\mu_2' = \frac{\sum_{i=1}^{n} f_i (x_i - A)^2}{N}$

ii) **Central moment:**

$$\mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}{N};$$

$$\bar{x} = mean; r = 1, 2, 3, \ldots$$

1st central moment, $\mu_1 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^1}{N}$

2nd central moment, $\mu_2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{N} = \sigma^2 = variance$

Now,

$$\mu_1 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^1}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i \bar{x}}{N}$$

$$= \bar{x} - \bar{x} = 0$$

Relation between raw moment and central moment:

i) $\mu_1 = 0$

ii) $\mu_2 = \mu_2' - (\mu_1')^2$

$$2nd\ central\ moment = 2nd\ raw\ moment - (1st\ raw\ moment)^2$$

Symmetrical distribution:

i) **Mean=median=mode**

ii) **3rd quartile-median=median-1st quartile**

iii) $\mu_1 = \mu_3 = \mu_5 = \cdots . = 0$

Skewness:

Skewness means" lack of symmetry"

i) **Positive skewness:**

$$mean > median > mode$$

ii) **Negative skewness:**

$$mean < median < mode$$

Measure of skewness:

i) **Pearson's 1st measure of skewness:**

$$Skewness = \frac{mean - mode}{SD}$$

ii) **Pearson's 2nd measure of skewness:**

$$Skewness = \frac{3(mean - median)}{SD}$$

iii) **Skewness based on moments:**

$$Skewness, \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{{\mu_2}^3}}$$

$$= \frac{3rd\ central\ moment}{\sqrt{(2nd\ central\ moment)^3}}$$

$$\mu_2 = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{N}$$

$$\mu_3 = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^3}{N}$$