Measures of central Tendency

A measure of central tendency is a value that represents a typical or central entry of a data set. The three most commonly used measures of central tendency are the mean, the median and the mode.

Arithmetic mean:

The arithmetic mean is an average or central value of a set of observation obtained by summing these observations and then dividing this sum by the number of such observations.

Symbolically,

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i}{n}$$

where, $\sum_{i=1}^{n} x_i$ = the sum of all values

n = total number of observations

Geometric mean: Geometric mean is the nth root of the product of n non-zero positive values of a series.

Symbolically,

$$G = (x_1 \times x_2 \times \cdots \times x_n)^{\frac{1}{n}}$$

$$\Rightarrow \log G = \log (x_1 \cdot x_2 \cdots x_n)^{\frac{1}{n}}$$

$$\Rightarrow \log G = \frac{1}{n} \log (x_1 \cdot x_2 \cdots x_n)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \log x_i$$

Harmonic mean:

Harmonic mean is the reciprocal of the arithmetic mean of the reciprocal of the values in a series. Harmonic mea H.M

Symbolically $= \dfrac{1}{\frac{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}{n}}$

$$= \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

$$= \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

problem: calculate arithmetic mean, Geometric mean and Harmonic mean from the following frequency distribution

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	4	5	10	12	8	6	4	3

Class Interval	frequency (f_i)	Mid of class interval (x_i)	$f_i x_i$	$\log x_i$	$f_i \log x_i$	$\frac{f_i}{x_i}$
10-20	3	15	45	1.176	3.528	0.2
20-30	4	25	100	1.398	5.592	0.16
30-40	5	35	175	1.544	7.72	0.14
40-50	10	45	450	1.653	16.53	0.22
50-60	12	55	660	1.740	20.88	0.22
60-70	8	65	520	1.813	14.504	0.12
70-80	6	75	450	1.875	11.25	0.08
80-90	4	85	340	1.929	7.716	0.05
90-100	3	95	285	1.978	5.934	0.03
Total:	55		3025		93.654	1.22

Arithmetic mean:

By direct method, Arithmetic mean,

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3025}{55}$$

$$= 55$$

Geometric mean

We know

$$\text{Log } G_r = \frac{\Sigma f_i \text{ Log } x_i}{\Sigma f_i}$$

$$= \frac{93.654}{55}$$

$$= 1.7028$$

$$G_r = \text{Anti-log } (1.7028)$$

$$= 50.44$$

Harmonic mean:

$$H = \frac{\Sigma f_i}{\Sigma \frac{f_i}{x_i}}$$

$$= \frac{55}{1.22}$$

$$= 45.08$$

(*) A.M → weight / kgs / cms

Gr.M → dealing with % 1st - 10%.
2nd → 30%.

H.M → mean of certain ratios - Speed / See

From the following data compute the Arithmetic mean by direct method and Shortcut method.

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
No. of Students	9	42	61	140	250	102	71	23	2

Soln: (a) Direct method:

Marks	No. of Students f_i	Mid value (x_i)	$f_i x_i$
0-9	9	4.5	40.50
10-19	42	14.5	609.00
20-29	61	24.5	1494.50
30-39	140	34.5	4830.00
40-49	250	44.5	11,125.00
50-59	102	54.5	5559.00
60-69	71	64.5	4579.50
70-79	23	74.5	1713.50
80-89	2	84.5	169.00
	$N = 700$ Σf_i		$\Sigma f_i x_i = 30120$

We know,

Arithmetic mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{30120}{700}$

$= 43.02$

(b) Short-cut method :

Marks	No. of students f_i	Mid-value x_i	$d_i = \frac{x_i - A}{C}$	$f_i d_i$
0-9	9	4.5	-4	-36
10-19	42	14.5	-3	-126
20-29	61	24.5	-2	-122
30-39	140	34.5	-1	-140
40-49	250	(44.5) A	0	0
50-59	102	54.5	+1	+102
60-69	71	64.5	+2	+142
70-79	23	74.5	+3	+69
80-89	2	84.5	+4	+8
	$N = \Sigma f_i = 700$			$\Sigma f_i d_i = -103$

We know,

$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} \times c$, where,

$\bar{x}$ = Arithmetic mean

A = Assume mean

~~$\Sigma f_i d_i$ = frequency~~

Σf_i = total frequency

$d_i = \frac{x_i - A}{c}$, c = class interval

$\therefore \bar{x} = 44.5 + \frac{-103}{700} \times 10$

$= 44.5 - 1.47$

$= 43.03$

≈ 43 . (Ans)

H.W Calculate arithmetic mean, geometric mean and harmonic mean from the following frequency distribution

Class interval	50-100	100-150	150-200	200-250	250-300	300-350
Frequency	15	30	50	40	10	5

Ans: Arithmetic mean,

$\bar{x} = 180$

Geometric mean, 171.39

Harmonic mean, 157.23