Random Variable: When a variable may have the values which can not be known in advance, rather the values depends on chance then the variable might be termed as random variable. We generally use upper-case letter such as x, y, z etc. to denote the random variables.

Let x be a random variable and its probability $p(x)$ then

(i) $p(x) \geq 0$ (ii) $\sum p(x) = 1$

Types of random variable:

A random variable that can take a countable number of possible values (or can be defined over a discrete sample space) is called a discrete random variable.

Example: The number of heads in two successive tosses of a fair coin.

Continuous random variable:

A random variable which takes on an uncountable number of values i.e the values within a range is called a continuous random variable.

Example: The lifetime of Bangladeshi

Probability Distribution:

The set of all possible values of a random variable together with the associated probabilities is called a probability distribution.

If x be a random variable that may have the values $x_1, x_2, \ldots, x_n$ with respective probabilities $p(x_1), p(x_2), \ldots, p(x_n)$ such that $\Sigma P(x) = 1$. Then the probability distribution is written as

x	x_1	x_2	. . . .	x_n
$p(x)$	$p(x_1)$	$p(x_2)$		$p(x_n)$

Probability function or Discrete probability function or probability mass function:

A function $p(x)$ is called a probability mass function or a probability function of the discrete random variable x if for each possible outcome x it satisfies the following properties:

(i) $p(x) > 0$; $p(x)$ is non-negative

(ii) $\sum p(x) = 1$ i.e the sum of all probability $p(x)$ is equal to 1.

(iii) For any real number x, $P(X = x) = f(x)$;
i.e $P(a < x < b) = \sum_{a}^{b} f(x)$

Continuous probability function or probability density function:

The function $f(x)$ is called the probability density function or density function of a continuous random variable x if $f(x)$ defined for all real $x \in (-\infty, \infty)$ satisfies the following properties:

(i) $f(x) \geq 0$ i.e $f(x)$ is non-negative

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) For any interval (a, b)
$P(a \leq x \leq b) = P(a < x < b) = \int_{a}^{b} f(x)\,dx$

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joint probability function:

To be the joint probability function, the function $f(x,y)$ should have the following properties:

(i) $f(x,y) \geq 0$

(ii) $\sum_x \sum_y f(x,y) = 1$, where x, y discrete

or, (iii) $\int_x \int_y f(x,y)\, dx\, dy = 1$, x, y continuous

joint probability density function:

The joint function $f(x,y)$ of the two continuous random variable x and y is called the joint probability density function if it satisfies the following two conditions:

(i) $f(x,y) \geq 0$

(ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

Marginal probability density function:

If the joint probability density function of two continuous random variables x and y be $f(x,y)$, $a<x<b$ and $c<y<d$, then the marginal probability density function can be defined in marginal probability density function of x is

$$g(x) = \int_c^d f(x,y)\,dy, \quad a<x<b$$

$$= \int_y f(x,y)\,dy$$

Marginal probability density function of y is

$$h(y) = \int_a^b f(x,y)\,dx, \quad c<y<d$$

$$= \int_x f(x,y)\,dx$$