MOMENTS, SKEWNESS AND KURTOSIS

Beyond the measures of central tendency and dispersion explained earlier, there are measures that further describe the characteristics of a distribution. Some of them are discussed here.

Moments

Moments are a set of statistical parameters to measure a distribution. Four moments are commonly used:

- 1st moment - Mean (describes central value)
- 2nd moment - Variance (describes dispersion)
- 3rd moment - Skewness (describes asymmetry)
- 4th moment - Kurtosis (describes peakedness)

The formula for calculating moments is as follows:

1st moment = $\mu_1 = \frac{\sum f(x - \bar{x})}{n}$

2nd moment = $\mu_2 = \frac{\sum f(x - \bar{x})^2}{n}$

3rd moment = $\mu_3 = \frac{\sum f(x - \bar{x})^3}{n}$

4th moment = $\mu_4 = \frac{\sum f(x - \bar{x})^4}{n}$

Skewness

The term 'skewness' refers to lack of symmetry or departure from symmetry, e.g., when a distribution is not symmetrical (or is asymmetrical) it is called a skewed distribution. The measures of skewness indicate the difference between the manner in which the observations are distributed in a particular distribution compared with a symmetrical (or normal) distribution. The concept of skewness gains importance from the fact that statistical theory is often based upon the assumption of the normal distribution. A measure of skewness is, therefore, necessary in order to guard against the consequence of this assumption.

In a symmetrical distribution, the values of mean, median and mode are alike. If the value of mean is greater than the mode, skewness is said to be positive. In a positively skewed distribution, mean is greater than the mode and the median lies somewhere in between mean and mode. A positively skewed distribution contains some values that are much larger than most other observations. A distribution is positively skewed when the long tail is on the positive side of the peak.

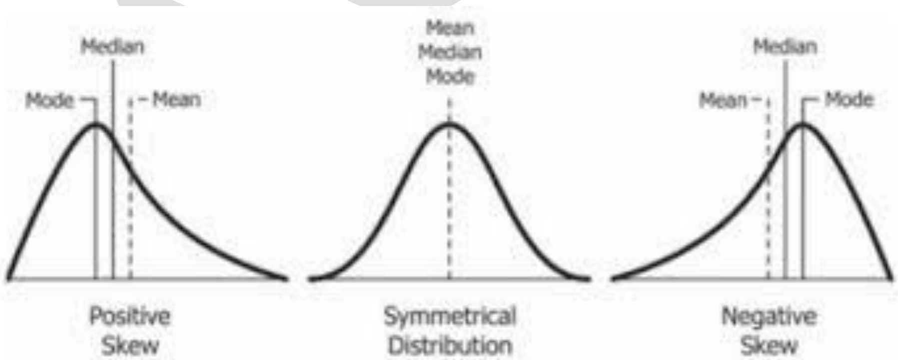
On the other hand, if the value of mode is greater than mean, skewness is said to be negative. The following diagrams could clarify the meaning of skewness. In a negatively skewed distribution, mode is greater than the mean and the median lies in between mean and mode. The mean is pulled towards the low-valued item (that is, to the left). A negatively skewed distribution contains some values that are much smaller than most observations. A distribution is negatively skewed when the long tail is on the negative side of the peak.

Generally,

If Mean > Mode, the skewness is positive.

If Mean < Mode, the skewness is negative.

If Mean = Mode, the skewness is zero.



Skewness is measured in the following ways:

Karl Pearson’s Coefficient of Skewness = $\frac{Mean - Mode}{Standard\ Deviation}$ or $\frac{3(Mean - Median)}{Standard\ Deviation}$

Moment based measure of skewness = $\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3}$

Pearson’s coefficient of skewness = $\gamma_1 = \sqrt{\beta_1}$

Kurtosis

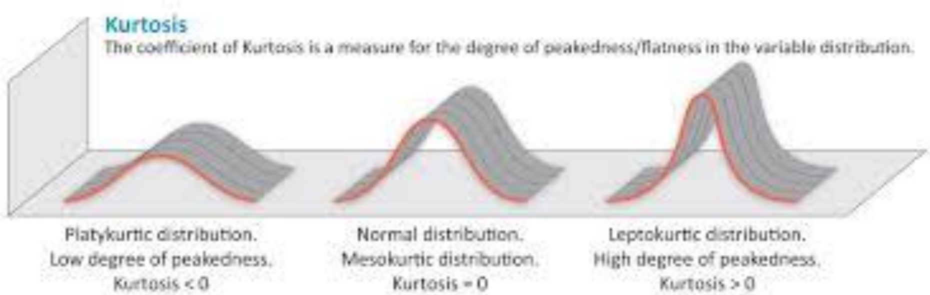
Kurtosis refers to the degree of peakedness of a frequency curve. It tells how tall and sharp the central peak is, relative to a standard bell curve of a distribution.

Kurtosis can be described in the following ways:

- *Platykurtic*– When the kurtosis < 0, the frequencies throughout the curve are closer to be equal (i.e., the curve is more flat and wide)
- *Leptokurtic*– When the kurtosis > 0, there are high frequencies in only a small part of the curve (i.e, the curve is more peaked)
- *Mesokurtic*- When the kurtosis = 0

To show the peakedness of a distribution,

- Platykurtic: flat and spread out
- Leptokurtic: high and thin
- Mesokurtic: normal in shape



Kurtosis is measured in the following ways:

Moment based Measure of kurtosis = $\beta_2 = \dfrac{\mu_4}{\mu_2^{\,2}}$

Coefficient of kurtosis = $\gamma_2 = \beta_2 - 3$

Illustration

Find the **first, second, third and fourth orders of moments, skewness** and **kurtosis** of the following:

i. 11, 11, 10, 8, 13, 15, 9, 10, 14, 12, 11, 8
ii. 52, 55, 45, 50, 55, 42, 48, 57, 55, 52, 53, 45, 50, 48, 50
iii.

x	f
10	3
11	2
12	4
15	3
18	5
20	1