Probability function/Probability mass function:

i) $p(x) > 0$
ii) $\sum p(x) = 1$

Problem: If $p(x) = x + 1/14$; x=1,2,3,4; then show that p(x) is a probability function and find the value of $p(2 \leq x < 4)$.

Solution: Given that,

$$p(x) = (x + 1)/14$$

We know that p(x) is a probability function if

$$\sum p(x) = 1$$

Now,

$$\sum_{x=a}^{b} p(x) = \sum_{x=1}^{4} \frac{x+1}{14}$$

$$= \frac{1+1}{14} + \frac{2+1}{14} + \frac{3+1}{14} + \frac{4+1}{14}$$

$$= \frac{2+3+4+5}{14} = \frac{14}{14} = 1$$

$$So\ p(x) is\ a\ probability\ function.$$

$$p(2 \leq x < 4) = \frac{2+1}{14} + \frac{3+1}{14} = \frac{7}{14} = 1/2$$

Continuous probability function/probability density function:

i) $f(x) \geq 0$
ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

Problem: Show that $f(x) = \frac{1}{30}(2 + 5x); 2 < x < 5$ is a probability density function and also find $p(x \geq 4)$

Solution: Given that,

$$f(x) = \frac{1}{30}(2 + 5x); 2 < x < 5$$

We know that f(x) is a probability density function if

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Now,

$$\int_{-\infty}^{\infty} f(x)dx = \int_{2}^{5} \frac{1}{30}(2 + 5x)dx$$

$$= \frac{1}{30}\left[2x + 5.\frac{x^2}{2}\right]$$

$$= \frac{1}{30}\left[\left(2.5 + 5.\frac{5^2}{2}\right) - \left(2.2 + 5.\frac{2^2}{2}\right)\right]$$

$$= \frac{1}{30}(58.5) = 1.95$$

So f(x) is not a probability density function.

$$p(x \geq 4) = \int_{4}^{\infty} f(x)dx$$

$$= \int_{4}^{5} \frac{1}{30}(2 + 5x)dx$$

$$=$$

Joint probability function/joint probability mass function:

$$P(x, y); x\ and\ y\ discrete\ random\ variable$$

- $p(x, y) > 0$
- $\sum_x \sum_y p(x, y) = 1$

Marginal probability function of x:

$$g(x) = \sum_{y} p(x, y)$$

Marginal probability function of y:

$$h(y) = \sum_{x} p(x, y)$$

Problem: The joint probability function of two discrete random variable x and y is given below:

$$P(x, y) = \frac{x + 2y}{16}; x = 0,1; y = 0,1,2,3$$

i) Find the marginal probability function of x and y
ii) Find the conditional probability function of x given y.

Solution:

$$P(x,y) = \frac{x+2y}{16}; x = 0,1; y = 0,1,2,3$$

Marginal probability function of x:

$$g(x) = \sum_{y} p(x,y) = \sum_{y=0}^{3} \frac{x+2y}{16}$$

$$= \frac{x+2.0}{16} + \frac{x+2.1}{16} + \frac{x+2.2}{16} + \frac{x+2.3}{16}$$

$$= \frac{x+x+2+x+4+x+6}{16} = \frac{4x+12}{16}$$

Marginal probability function of y:

$$h(y) = \sum_{x} p(x,y) = \sum_{x=0}^{1} \frac{x+2y}{16}$$

$$= \frac{0+2y}{16} + \frac{1+2y}{16}$$

$$= \frac{1+4y}{16}$$

the conditional probability function of x given y,

$$P(x|y) = \frac{p(x,y)}{p(y)} = \frac{\frac{x+2y}{16}}{\frac{1+4y}{16}} = \frac{x+2y}{1+4y}$$

the conditional probability function of y given x,

$$P(y|x) = \frac{p(x,y)}{p(x)} = \frac{\frac{x+2y}{16}}{\frac{4x+12}{16}} = \frac{x+2y}{4x+12}$$

Joint probability density function/joint continuous probability function:

$$f(x,y); x \text{ and } y \text{ continuous random variable}$$

- $f(x,y) \geq 0$
- $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$

Marginal probability density function of x:

$$g(x) = \int_y f(x,y)dy$$

Marginal probability density function of y:

$$h(y) = \int_x f(x,y)dx$$