## Problem 2

a)

The boundary values are and . For the given values the boundary conditions are and . This is known since the internal and external pressures are given and it is said that these pressures correspond to the radial stresses at the inner and outer radii (the bounds).

b)

The given ODE can be put in the general form but normalizing the highest order term and moving any terms without the dependent variable to the right side. Therefore:

Where:

With this the finite difference method can be used to determine the values in the matrix A and in the vector b and solve for the vector x. The following is the implementation in matlab.

### Matlab Code

The code has the function that takes a step size, and a script to run through the different step sizes and plot the results.

Function Code

function [ri, r] = fdODE (n)

%for our case

%returns the ri values used from ro to rn and the solved r values

%example call: [x y] = fdODE (100)

a = 1;

b = 6;

ya = 100;

yb = 0;

v = 0.3;

w = 104.72;

p = 724.638e-6;

ri = linspace(a,b,n+1);

dr = (b-a)/n;

px = @(x)3./x;

qx = @(x)0.\*x;

rx = @(x)-(3+v).\*p.\*w.^2+0.\*x;

diag0 = (-2+dr.^2.\*qx(ri(2:n)));

diagn1 = (1-dr./2.\*px(ri(2:n)));

diag1 = (1+dr./2.\*px(ri(2:n)));

A = diag(diag0) + diag(diag1(1:n-2),1)+diag(diagn1(2:n-1),-1)

b = rx(ri).\*dr.^2;

b = b(2:n);

b(1) = b(1) - diagn1(1)\*ya;

b(n-1) = b(n-1) - diag1(n-1)\*yb

r = ya;

r(2:n) = b/A;

r(n+1)=yb;

#### Script

[x1 y1] = fdODE (4);

plot(x1,y1, '-r')

xy1 (:,1) = x1';

xy1 (:,2) = y1'

hold on

[x2 y2] = fdODE (8);

plot(x2,y2, 'b')

xy2 (:,1) = x2';

xy2 (:,2) = y2'

[x3 y3] = fdODE (16);

plot(x3,y3, 'g')

xy3 (:,1) = x3';

xy3 (:,2) = y3'

hold off

#### Output

Note for xy1, xy2, xy3 the first column is ri and the second column is σri.

A =

-2.0000 1.8333 0

0.4643 -2.0000 1.5357

0 0.6053 -2.0000

b =

-57.6412 -40.9746 -40.9746

xy1 =

1.0000 100.0000

2.2500 51.0407

3.5000 95.7172

4.7500 93.9845

6.0000 0

A =

Columns 1 through 6

-2.0000 1.5769 0 0 0 0

0.5833 -2.0000 1.4167 0 0 0

0 0.6739 -2.0000 1.3261 0 0

0 0 0.7321 -2.0000 1.2679 0

0 0 0 0.7727 -2.0000 1.2273

0 0 0 0 0.8026 -2.0000

0 0 0 0 0 0.8256

Column 7

0

0

0

0

0

1.1974

-2.0000

b =

Columns 1 through 6

-52.5513 -10.2436 -10.2436 -10.2436 -10.2436 -10.2436

Column 7

-10.2436

xy2 =

1.0000 100.0000

1.6250 50.4665

2.2500 82.9399

2.8750 112.8550

3.5000 133.8102

4.1250 139.4038

4.7500 123.2344

5.3750 78.9003

6.0000 0

A =

Columns 1 through 6

-2.0000 1.3571 0 0 0 0

0.7115 -2.0000 1.2885 0 0 0

0 0.7581 -2.0000 1.2419 0 0

0 0 0.7917 -2.0000 1.2083 0

0 0 0 0.8171 -2.0000 1.1829

0 0 0 0 0.8370 -2.0000

0 0 0 0 0 0.8529

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

Columns 7 through 12

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

1.1630 0 0 0 0 0

-2.0000 1.1471 0 0 0 0

0.8661 -2.0000 1.1339 0 0 0

0 0.8770 -2.0000 1.1230 0 0

0 0 0.8864 -2.0000 1.1136 0

0 0 0 0.8944 -2.0000 1.1056

0 0 0 0 0.9013 -2.0000

0 0 0 0 0 0.9074

0 0 0 0 0 0

0 0 0 0 0 0

Columns 13 through 15

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

0 0 0

1.0987 0 0

-2.0000 1.0926 0

0.9128 -2.0000 1.0872

0 0.9176 -2.0000

b =

Columns 1 through 6

-66.8466 -2.5609 -2.5609 -2.5609 -2.5609 -2.5609

Columns 7 through 12

-2.5609 -2.5609 -2.5609 -2.5609 -2.5609 -2.5609

Columns 13 through 15

-2.5609 -2.5609 -2.5609

xy3 =

1.0000 100.0000

1.3125 63.4632

1.6250 84.4366

1.9375 105.7740

2.2500 126.5607

2.5625 145.8816

2.8750 162.8218

3.1875 176.4664

3.5000 185.9005

3.8125 190.2090

4.1250 188.4772

4.4375 179.7899

4.7500 163.2323

5.0625 137.8895

5.3750 102.8464

5.6875 57.1882

6.0000 0

#### Plot

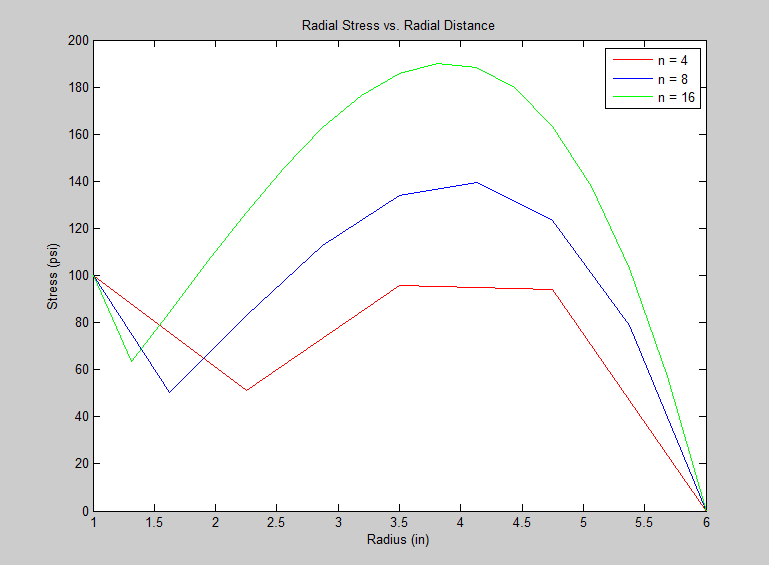


Figure - Finite Difference Method n Value Comparison

### Discussion

As can be seen from the plot, the number of iterations can greatly change the accuracy of the solution to the differential equation. This method requires a substantially small step size or large number of steps to be meaningful.

## Problem 3

For this problem the code for problem 2 has been modified for PDEs. The code outputs the A matrix and b vector, and returns the temperature matrix T reshaped so that each entry (i,j) in the matrix corresponds to Ti,j for i = 1,2,...nx-1, j = 1,2,...ny-1. The function call is T = fdPDE (dx, dy).

### Matlab Code

function T = fdPDE (dx, dy)

%define rectangular bounds

a = 15;

b = 20;

%define values at rectangular bounds

Ta = 50;

Tb = 200;

Tyo = 150;

Txo = 100;

nx = a/dx;

ny = b/dy;

num\_nodes = (nx-1)\*(ny-1);

A = zeros(num\_nodes);

b = zeros(1,num\_nodes);

nodef = @(i,j)(j-1)\*(nx-1)+i;

for j = 1:ny-1

for i = 1:nx-1,

node = nodef(i,j);

A (node,node) = -4;

T(node,1) = i\*dx;

T(node,2) = j\*dy;

%setting up rest of A matrix and b vector

if (i==1),

b(node) = b(node) - Txo;

else

A(node,nodef(i-1,j)) = 1;

end

if (i==nx-1),

b(node) = b(node) - Ta;

else

A(node,nodef(i+1,j)) = 1;

end

if (j==1),

b(node) = b(node) - Tyo;

else

A(node,nodef(i,j-1)) = 1;

end

if (j==ny-1),

b(node) = b(node) - Tb;

else

A(node,nodef(i,j+1)) = 1;

end

end

end

A

b

T = reshape(b/A,nx-1,ny-1)';

### Output

For this function, the two function calls used were:

>> T = fdPDE(5,5)

>> T = fdPDE(2.5,2.5)

The output was:

>> T = fdPDE(5,5)

A =

-4 1 1 0 0 0

1 -4 0 1 0 0

1 0 -4 1 1 0

0 1 1 -4 0 1

0 0 1 0 -4 1

0 0 0 1 1 -4

b =

-250 -200 -100 -50 -300 -250

T =

116.0455 103.0021

111.1801 95.9627

132.7122 119.6687

>> T = fdPDE(2.5,2.5)

A =

Columns 1 through 11

-4 1 0 0 0 1 0 0 0 0 0

1 -4 1 0 0 0 1 0 0 0 0

0 1 -4 1 0 0 0 1 0 0 0

0 0 1 -4 1 0 0 0 1 0 0

0 0 0 1 -4 0 0 0 0 1 0

1 0 0 0 0 -4 1 0 0 0 1

0 1 0 0 0 1 -4 1 0 0 0

0 0 1 0 0 0 1 -4 1 0 0

0 0 0 1 0 0 0 1 -4 1 0

0 0 0 0 1 0 0 0 1 -4 0

0 0 0 0 0 1 0 0 0 0 -4

0 0 0 0 0 0 1 0 0 0 1

0 0 0 0 0 0 0 1 0 0 0

0 0 0 0 0 0 0 0 1 0 0

0 0 0 0 0 0 0 0 0 1 0

0 0 0 0 0 0 0 0 0 0 1

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

Columns 12 through 22

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0 0

0 1 0 0 0 0 0 0 0 0 0

0 0 1 0 0 0 0 0 0 0 0

0 0 0 1 0 0 0 0 0 0 0

1 0 0 0 1 0 0 0 0 0 0

-4 1 0 0 0 1 0 0 0 0 0

1 -4 1 0 0 0 1 0 0 0 0

0 1 -4 1 0 0 0 1 0 0 0

0 0 1 -4 0 0 0 0 1 0 0

0 0 0 0 -4 1 0 0 0 1 0

1 0 0 0 1 -4 1 0 0 0 1

0 1 0 0 0 1 -4 1 0 0 0

0 0 1 0 0 0 1 -4 1 0 0

0 0 0 1 0 0 0 1 -4 0 0

0 0 0 0 1 0 0 0 0 -4 1

0 0 0 0 0 1 0 0 0 1 -4

0 0 0 0 0 0 1 0 0 0 1

0 0 0 0 0 0 0 1 0 0 0

0 0 0 0 0 0 0 0 1 0 0

0 0 0 0 0 0 0 0 0 1 0

0 0 0 0 0 0 0 0 0 0 1

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

Columns 23 through 33

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0 0

0 1 0 0 0 0 0 0 0 0 0

0 0 1 0 0 0 0 0 0 0 0

0 0 0 1 0 0 0 0 0 0 0

1 0 0 0 1 0 0 0 0 0 0

-4 1 0 0 0 1 0 0 0 0 0

1 -4 1 0 0 0 1 0 0 0 0

0 1 -4 0 0 0 0 1 0 0 0

0 0 0 -4 1 0 0 0 1 0 0

0 0 0 1 -4 1 0 0 0 1 0

1 0 0 0 1 -4 1 0 0 0 1

0 1 0 0 0 1 -4 1 0 0 0

0 0 1 0 0 0 1 -4 0 0 0

0 0 0 1 0 0 0 0 -4 1 0

0 0 0 0 1 0 0 0 1 -4 1

0 0 0 0 0 1 0 0 0 1 -4

0 0 0 0 0 0 1 0 0 0 1

0 0 0 0 0 0 0 1 0 0 0

Columns 34 through 35

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

0 0

1 0

0 1

0 0

0 0

1 0

-4 1

1 -4

b =

Columns 1 through 11

-250 -150 -150 -150 -200 -100 0 0 0 -50 -100

Columns 12 through 22

0 0 0 -50 -100 0 0 0 -50 -100 0

Columns 23 through 33

0 0 -50 -100 0 0 0 -50 -300 -200 -200

Columns 34 through 35

-200 -250

T =

122.6422 129.1659 128.1572 120.3343 100.6227

111.4027 115.8643 113.1284 102.5574 82.1565

107.1044 109.7602 105.9348 94.6102 75.4459

107.2546 110.1373 106.2405 94.5028 75.0168

111.7768 117.2939 114.3869 102.1439 80.1183

122.5588 132.8746 131.8692 119.5677 93.3126

145.5838 159.7765 160.6477 150.9449 123.5644

### Tabulated Results

For dx=dy=5”

|  |  |  |
| --- | --- | --- |
| T(xi,yj)[°F] | i=1 | i=2 |
| j=1 | 116.0455 | 103.0021 |
| j=2 | 111.1801 | 95.9627 |
| j=3 | 132.7122 | 119.6687 |

For dx=dy=2.5”

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| T(xi,yj)[°F] | i=1 | i=2 | i=3 | i=4 | i=5 |
| j=1 | 122.6422 | 129.1659 | 128.1572 | 120.3343 | 100.6227 |
| j=2 | 111.4027 | 115.8643 | 113.1284 | 102.5574 | 82.1565 |
| j=3 | 107.1044 | 109.7602 | 105.9348 | 94.6102 | 75.4459 |
| j=4 | 107.2546 | 110.1373 | 106.2405 | 94.5028 | 75.0168 |
| j=5 | 111.7768 | 117.2939 | 114.3869 | 102.1439 | 80.1183 |
| j=6 | 122.5588 | 132.8746 | 131.8692 | 119.5677 | 93.3126 |
| j=7 | 145.5838 | 159.7765 | 160.6477 | 150.9449 | 123.5644 |