Chapter 5

Baseband Transmission of Digital Signals

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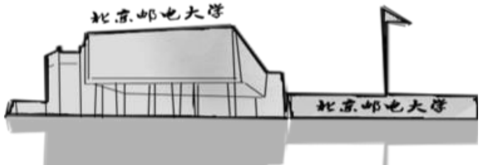
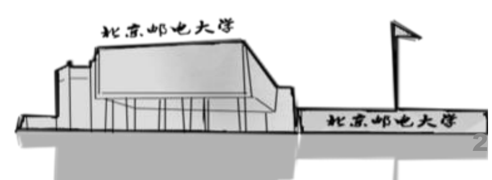


Baseband Transmission of Digital Signals

- Introduction
- Baseband signal and pulse modulation
- Digital PAM signal transmission through AWGN channel
- Digital PAM signal transmission through baseband channel
- Channel Equalization
- Eye Pattern
- Partial Response System
- Symbol Synchronization
- Summary

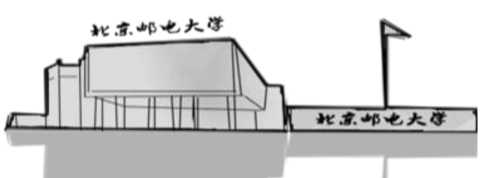


Digital PAM Signal Transmission through Baseband channel

- Baseband Channel
- Inter Symbol Interference (ISI)
- Nyquist Criterion

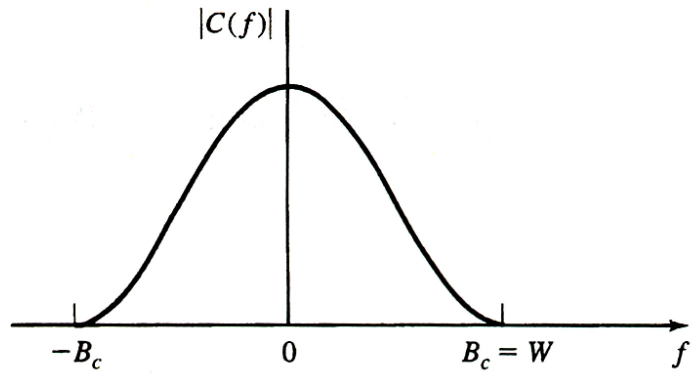
The reasons for modeling PAM signal transmission through baseband channel

- Band limited channel will lead to dispersion in time domain, which might cause Inter Symbol Interference (ISI).
- Baseband transmission is adopted in practical applications, e.g., PCM system.
- Bandpass systems are usually discussed through their equivalent baseband systems.



Baseband Channel Model

$$c(t) \Leftrightarrow C(f)$$



- **For baseband channel**

$$C(f) = |C(f)| e^{j\theta(f)}, \quad |f| \le W$$

$|C(f)|$ **~Amplitude-frequency characteristic**

$\theta(f)$ **~Phase-frequency characteristic**

$$\tau_G(f) = -\frac{1}{2\pi}\frac{\mathrm{d}\theta(f)}{\mathrm{d}f}$$ **~ Group delay characteristic**

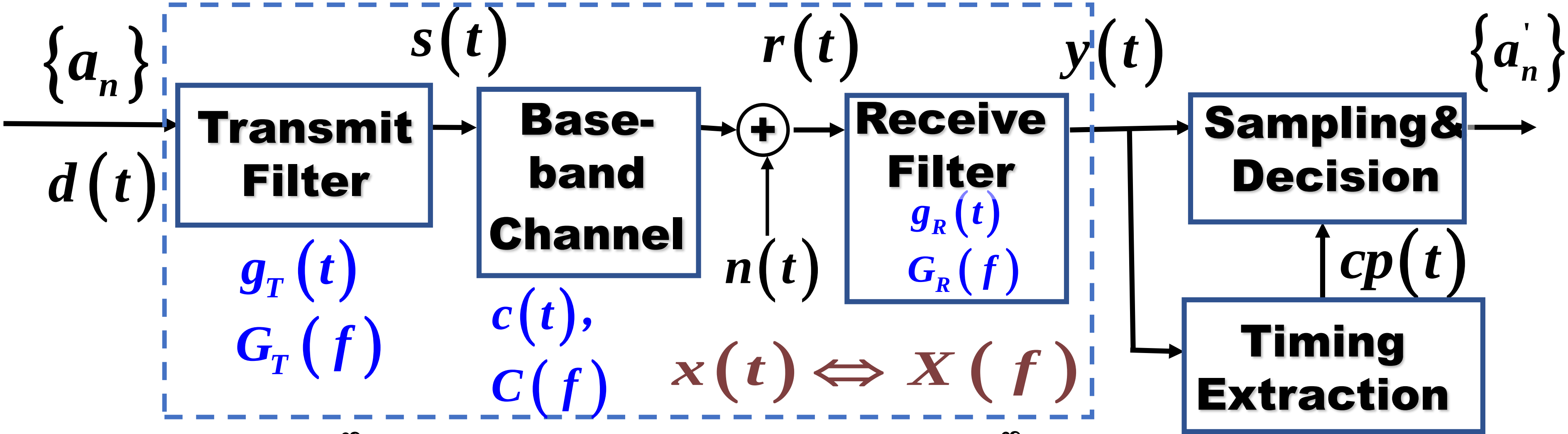
Ideal Baseband Channel Model

- **For ideal baseband channel**

  $\left|C(f)\right|$ **and** $\tau_G(f)$ **are both constant.**

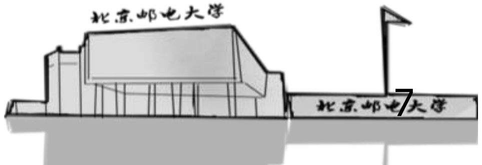
- **Since the frequency spectrum of the transmitted signal is filtered by the channel, its time domain pulse might be dispersed.**
- **Thus, ISI might be introduced.**

Inter-symbol Interference



- $d(t)=\sum\limits_{n=-\infty}^{\infty} a_n \delta(t-nT_s)$
- $s(t)=\sum\limits_{n=-\infty}^{\infty} a_n g_T(t-nT_s)$,
- $r(t)=\sum\limits_{n=-\infty}^{\infty} a_n h(t-nT_s)+n(t)$, $\quad h(t)=g_T(t)*c(t)$
- $y(t)=\sum\limits_{n=-\infty}^{\infty} a_n x(t-nT_s)+\gamma(t)$, $\quad \gamma(t)=n(t)*g_R(t)$

$$x(t)=g_T(t)*c(t)*g_R(t) \Longleftrightarrow X(f)=G_T(f)\cdot C(f)\cdot G_R(f)$$



Inter- symbol Interference

Output signal of receive filter

$$y(t)=\sum_{n=-\infty}^{\infty} a_n x(t-nT_s)+\gamma(t)$$

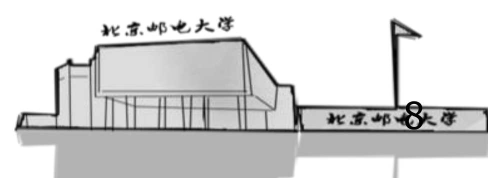
At the sampling moment, $t=mT_s+t_0 (t_0=0)$,

$$y(mT_s)=\sum_{n=-\infty}^{\infty} a_n x(mT_s-nT_s)+\gamma(mT_s)$$

Or $$y_m=\sum_{n=-\infty}^{\infty} a_n x_{m-n}+\gamma_m = x_0 a_m + \underline{\sum_{n\neq m} a_n x_{m-n}} + \underline{\gamma_m}$$

Inter-symbol Interference(ISI)

Random noise



Nyquist Criterion

- It is possible to eliminate ISI at the sampling moments by ensuring that the system satisfies the Nyquist criterion.
- At the moment t=mTs,

$$y_m = x_0 a_m + \sum_{\substack{n=-\infty \\ m\neq n}}^{\infty} a_n x_{m-n} + \gamma_m$$

- If there is no ISI

$$x(nT_s) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

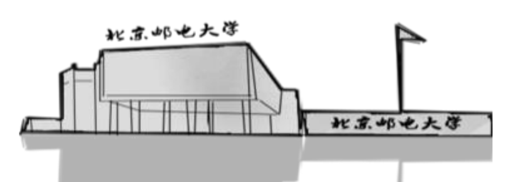
Nyquist Criterion

- **Then**

$$y_m = a_m + \gamma_m$$

- **In frequency domain, the Nyquist criterion becomes**

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T_s}\right) = T_s, \quad |f| \le \frac{1}{2T_s}$$



Nyquist Criterion

☐ In time domain

$$x(t)\sum_{n=-\infty}^{\infty}\delta\left(t-nT_s\right)$$

$$=\sum_{n=-\infty}^{\infty}x\left(nT_s\right)\delta\left(t-nT_s\right)$$

$$=\delta\left(t\right)$$

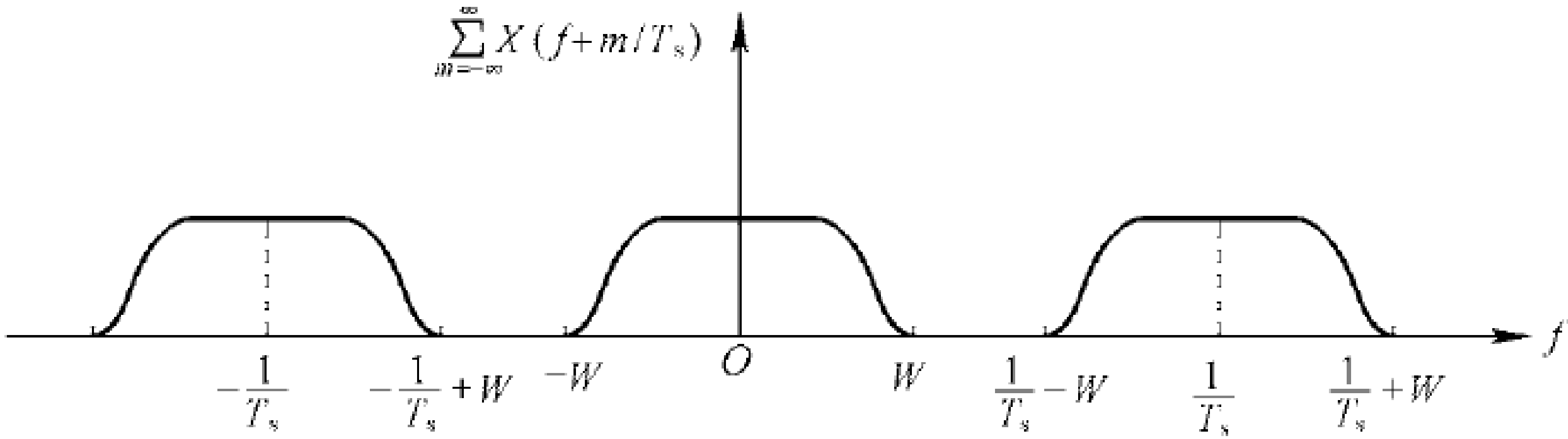
☐ In frequency domain

$$X(f)*\frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_s}\right)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X\left(f-\frac{n}{T_s}\right)=1 \Rightarrow \sum_{n=-\infty}^{\infty}X\left(f-\frac{n}{T_s}\right)=T_s$$

Nyquist Criterion

Observation 1:

- **When W< 1/2Ts, the Nyquist Criterion can not be satisfied.**

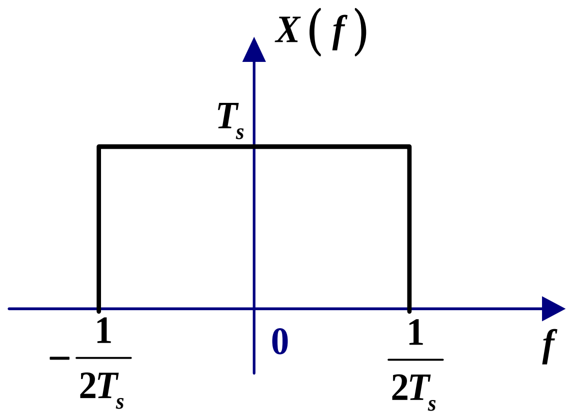


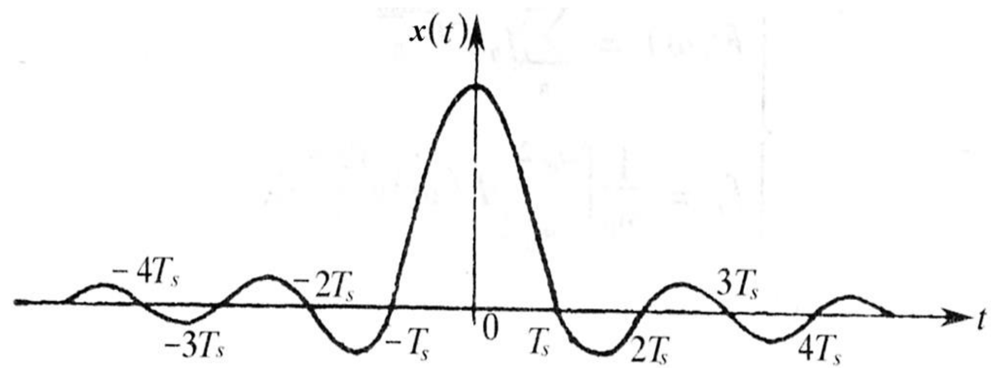
- **In the example illustrated above, it is very clear that gaps between spectra make a constant overlapping impossible.**

Nyquist Criterion

Observation 2:

- The minimum bandwidth of X(f) for no ISI transmission: W=1/2Ts



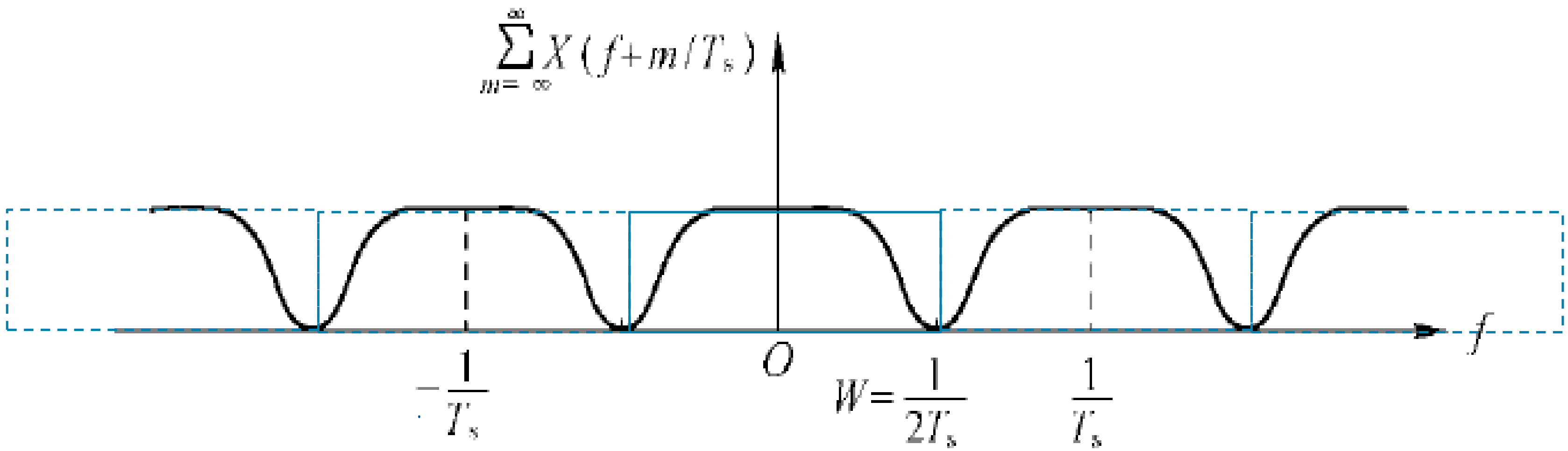


$$X(f)=\begin{cases} T_s, & |f| \le \dfrac{1}{2T_s} \\ 0, & \text{otherwise} \end{cases} \qquad x(t)=\mathrm{sinc}\left(\frac{t}{T_s}\right)$$

- But, this ideal filter is very difficult to be produced.

Nyquist Criterion

- The minimum bandwidth of X(f) for no ISI transmission: W=1/2Ts

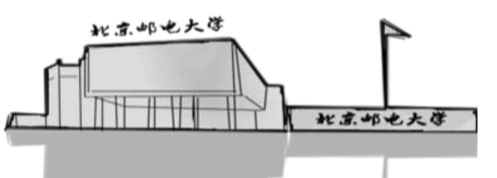


- **Maximum frequency efficiency**:

$$\eta = \frac{R_s}{W} = 2(\mathrm{Baud/Hz})$$

- also

$$\eta = \frac{R_b}{W} = 2\log_2 M = 2k(\mathrm{bit/s/Hz})$$



Discussion

For a given baseband transmission system with bandwidth W

- **Its maximum transmission rate without ISI: 2W**
- **Its maximum frequency efficiency: 2Baud/Hz**

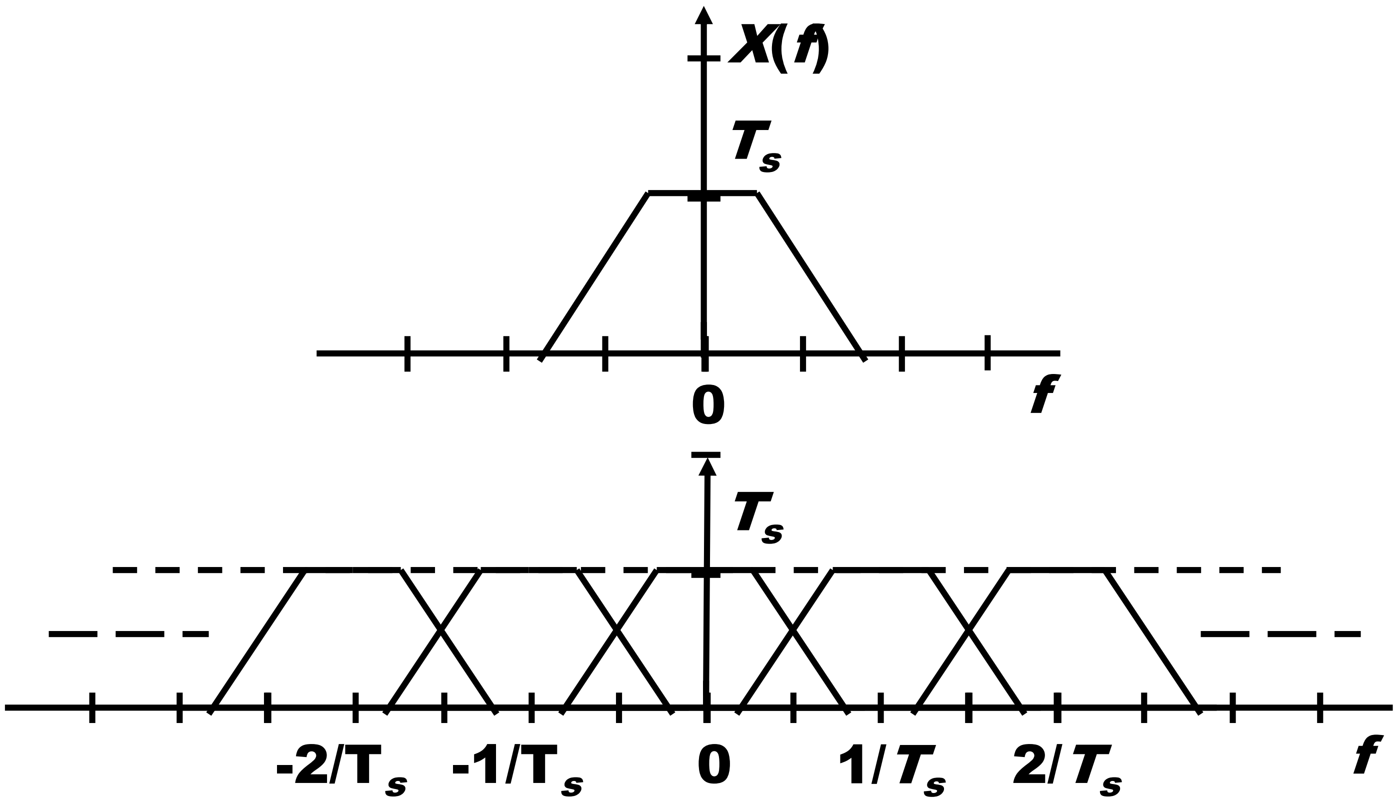
For a given transmitted baseband digital signal with symbol rate Rs

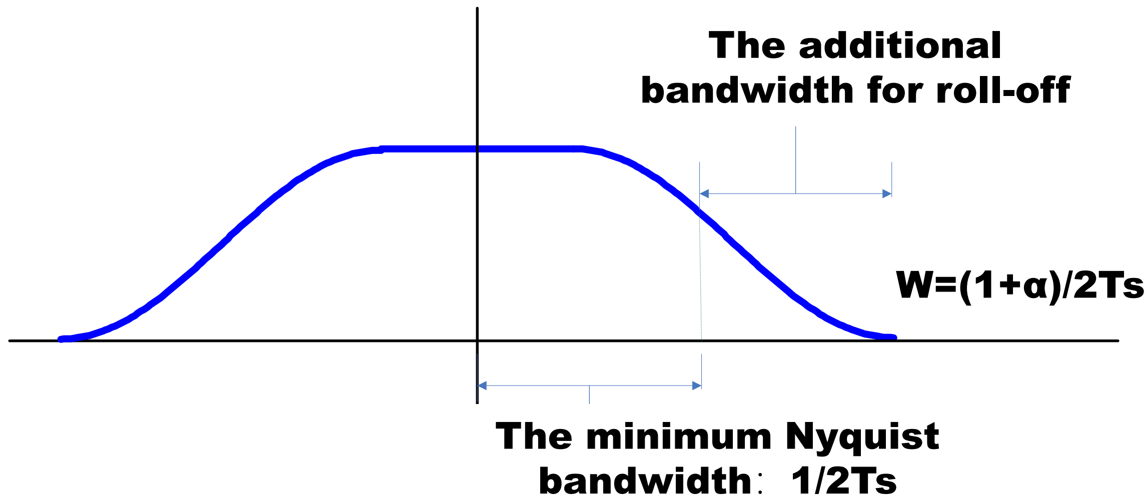
- **For no ISI transmission, the minimum required bandwith of system : Rs/2**

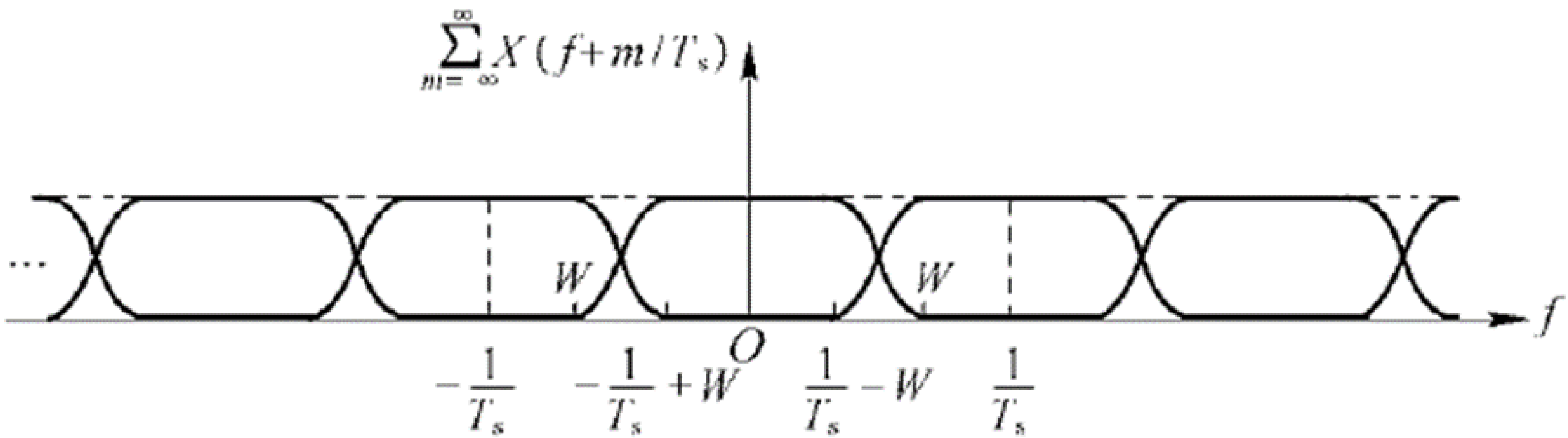
Nyquist Criterion

Observation 3:

- Shape of X(f) when W>1/2Ts




The additional bandwidth for roll-off
W=(1+α)/2Ts
The minimum Nyquist bandwidth: 1/2Ts

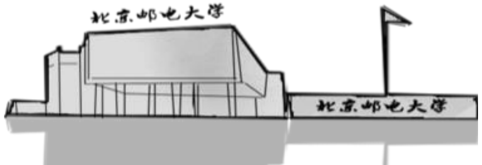

$\sum_{m=-\infty}^{\infty} X(f+m/T_s)$
W
W
O
f
$-\frac{1}{T_s}$
$-\frac{1}{T_s}+W$
$\frac{1}{T_s}-W$
$\frac{1}{T_s}$

RC Filter

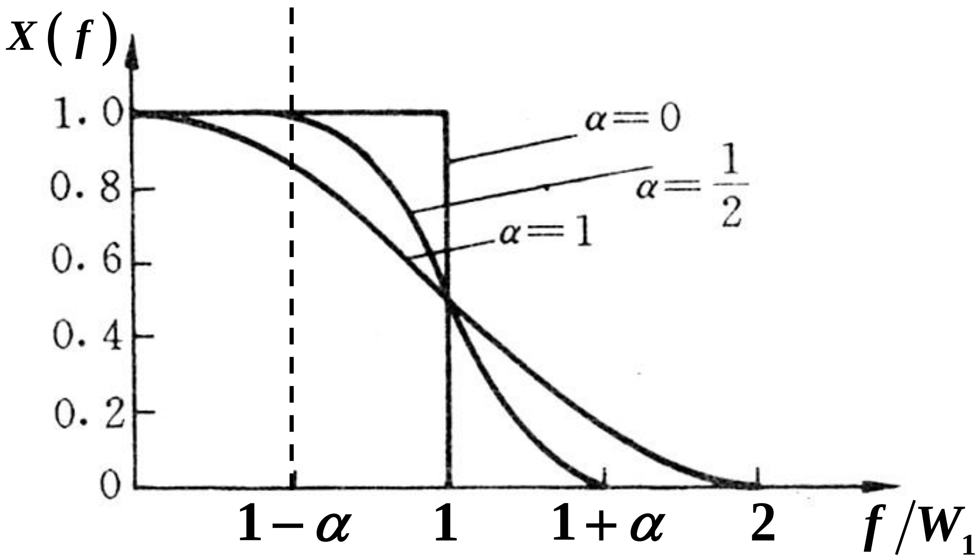
- A reasonable scheme: **Raised Cosine Filter**

$$X(f)=\begin{cases} T_s, & 0\le |f|\le \dfrac{1-\alpha}{2T_s} \\ \dfrac{T_s}{2}\left[1+\cos\dfrac{\pi T_s}{\alpha}\left(|f|-\dfrac{1-\alpha}{2T_s}\right)\right], & \dfrac{1-\alpha}{2T_s}<|f|\le\dfrac{1+\alpha}{2T_s} \\ 0, & |f|>\dfrac{1+\alpha}{2T_s} \end{cases}$$

With Roll-off factor: $0\le\alpha\le 1$



RC Filter



- **Bandwidth and symbol rate:**

$$\boxed{W_1 = R_s/2}$$

$$W = W_1 + W_2 = \frac{R_s}{2}(1+\alpha)$$

- **Spectrum efficiency:**

$$\eta = \frac{2}{1+\alpha} \quad (\text{Baud/Hz})$$

$\alpha = 0$ ⇒ **Ideal LPF**

$\alpha = 1$ ⇒

$$X(f) = \begin{cases} \frac{T_s}{2}\left[1+\cos \pi f T_s\right], & |f| \le \frac{1}{T_s} \\ 0, & |f| > \frac{1}{T_s} \end{cases}$$

RC Filter

Impulse response of RC Filter

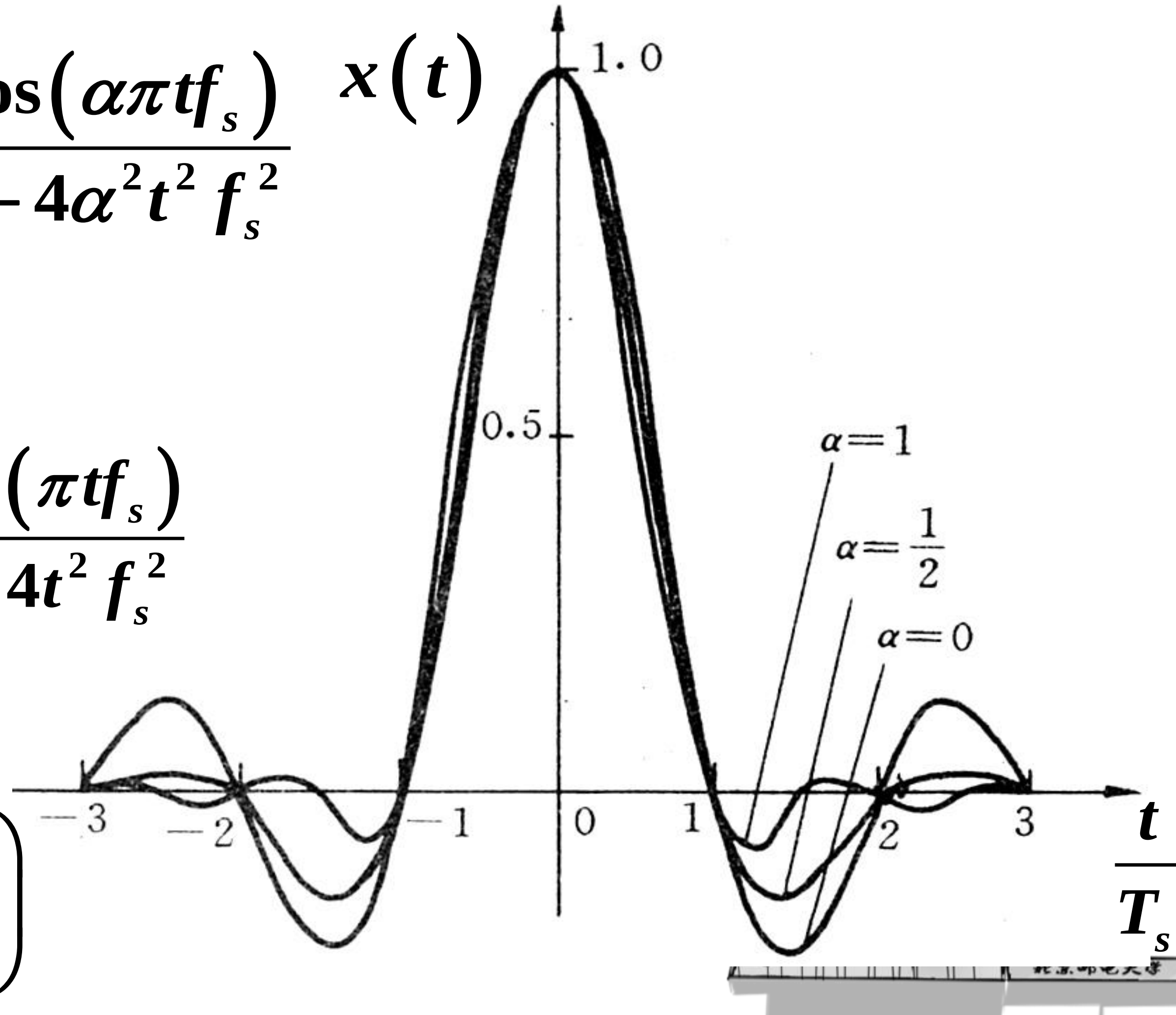
$$x(t) = \text{sinc}\left(\frac{t}{T_s}\right) \cdot \frac{\cos(\alpha \pi t f_s)}{1 - 4\alpha^2 t^2 f_s^2}$$

$\alpha = 1$ →

$$x(t) = \text{sinc}\left(\frac{t}{T_s}\right) \cdot \frac{\cos(\pi t f_s)}{1 - 4 t^2 f_s^2}$$

$\alpha = 0$ →

$$x(t) = \text{sinc}\left(\frac{t}{T_s}\right)$$



Summary

- Digital Baseband signal transmission through Baseband channel might encounter ISI.
- To eliminate ISI, we should properly design the impulse response and transfer function of the system, i.e., $x(t)$ and $X(f)$.
- The Nyquist Criterion must be satisfied for no ISI transmission.