EBU6018
Advanced Transform Methods

Tutorial Solutions – DCT

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1-Dimensional DCT

$$DCT[k] = c(k)\sum_{n=0}^{N-1} s[n]\cos\frac{\pi(2n+1)k}{2N} \qquad k = 0,1,2 \ldots N-1$$

$$DCT[k] = \langle s, \psi_k \rangle$$

$$c(k) = \begin{cases} \sqrt{1/N} & k = 0 \\ \sqrt{2/N} & k \neq 0 \end{cases}$$

c(k) is the normalisation factor.

- Orthonormal

$$\langle \psi_m, \psi_n \rangle = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

The Basis Functions Ψ_k are the cosine terms in the definition.
They are calculated for each value of k, with n = 0….N-1

DCT Question 1

If N = 4 (input is a 4-point sequence)

For each value of k = 0….N-1, insert n = 0….N-1:

$$\psi_0 = (1,1,1,1)/2$$

$$\psi_1 = \sqrt{1/2}(\cos(\pi/8), \cos(3\pi/8), \cos(5\pi/8), \cos(7\pi/8))$$

$$\psi_2 = \sqrt{1/2}(\cos(\pi/4), \cos(3\pi/4), \cos(5\pi/4), \cos(7\pi/4))$$

$$\psi_3 = \sqrt{1/2}(\cos(3\pi/8), \cos(9\pi/8), \cos(15\pi/8), \cos(5\pi/8))$$

$$DCT[0] = \frac{1}{\sqrt{N}}\sum_{n=0}^{3} s[n] \qquad DCT[1] = \sqrt{\frac{2}{N}}\sum_{n=0}^{3} s[n]\cos\frac{\pi(2n+1)}{8}$$

$$DCT[2] = \sqrt{\frac{2}{N}}\sum_{n=0}^{3} s[n]\cos\frac{\pi(2n+1)}{4} \qquad DCT[3] = \sqrt{\frac{2}{N}}\sum_{n=0}^{3} s[n]\cos\frac{\pi(2n+1)3}{8}$$

Question

These 4 Basis Functions can be written in Matrix format.

- Calculate the elements of the 4x4 Basis Function Matrix.

- Then determine the output sequence if the input sequence is $s[n] = [7, 4, -5, 10]$

4x4 DCT Basis Matrix

$$\Psi = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

4x4 DCT Basis Matrix

$$\Psi = \begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.50 & -0.50 & -0.50 & 0.50 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix}$$

4x4 DCT Transform

$$\text{DCT}=\begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.50 & -0.50 & -0.50 & 0.50 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix}\begin{bmatrix} 7 \\ 4 \\ -5 \\ 10 \end{bmatrix}$$

4x4 DCT Transform

$$\text{DCT} = \begin{bmatrix} 8.00 \\ 0.48 \\ 9.00 \\ -6.66 \end{bmatrix}$$

DCT Question 2

The1D Discrete Cosine Transform (DCT) is defined:

$$DCT[k] = c(k)\sum_{n=0}^{N-1} s[n]\cos\frac{\pi(2n+1)k}{2N}$$

$$c(k) = \begin{cases} \sqrt{1/N} & k = 0 \\ \sqrt{2/N} & k \neq 0 \end{cases}$$

(i) Derive the basis functions for a 3-point DCT, for k=n=0, 1, 2.

(ii) Calculate, to 3 decimal places, the DCT of the input sequence:

s(n)=[7, -2, 5].

Solution

k = 0, $\Psi_0 = \sqrt{\frac{1}{3}}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$

k = 1, $\Psi_1 = \sqrt{\frac{2}{3}}\begin{bmatrix}\cos\frac{\pi}{6} & \cos\frac{3\pi}{6} & \cos\frac{5\pi}{6}\end{bmatrix}$

k = 2, $\Psi_2 = \sqrt{\frac{2}{3}}\begin{bmatrix}\cos\frac{2\pi}{6} & \cos\frac{6\pi}{6} & \cos\frac{10\pi}{6}\end{bmatrix}$

Solution

$$\Psi = \begin{bmatrix} 0.577x1 & 0.577x1 & 0.577x1 \\ 0.816x0.866 & 0.816x0 & 0.816x(-0.866) \\ 0.816x0.5 & 0.816x(-1) & 0.816x0.5 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ 0.707 & 0 & -0.707 \\ 0.408 & -0.816 & 0.408 \end{bmatrix}$$

Solution

$$\text{DCT} = \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ 0.707 & 0 & -0.707 \\ 0.408 & -0.816 & 0.408 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \\ 5 \end{bmatrix}$$

Solution

$$\text{DCT} = \begin{bmatrix} 5.77 \\ 1.414 \\ 6.528 \end{bmatrix}$$

DCT Question 3

Determine the DCT of the 8-point input sequence:

$S[n] = [7, -6, -5, 4, 5, -3, -2, 6]$

Comment on the output sequence obtained.

8x8 DCT Matrix

$$\Psi = \frac{1}{2}\begin{bmatrix} .71 & .71 & .71 & .71 & .71 & .71 & .71 & .71 \\ .98 & .83 & .56 & .19 & -.19 & -.56 & -.83 & -.98 \\ .92 & .38 & -.38 & -.92 & -.92 & -.38 & .38 & .92 \\ .83 & -.19 & -.98 & -.56 & .56 & .98 & .19 & -.83 \\ .71 & -.71 & -.71 & .71 & .71 & -.71 & -.71 & .71 \\ .56 & -.98 & .19 & .83 & -.83 & -.19 & .98 & -.56 \\ .38 & -.92 & .92 & -.38 & -.38 & .92 & -.92 & .38 \\ .19 & -.56 & .83 & -.98 & .98 & -.83 & .56 & -.19 \end{bmatrix}$$

To calculate output sequence

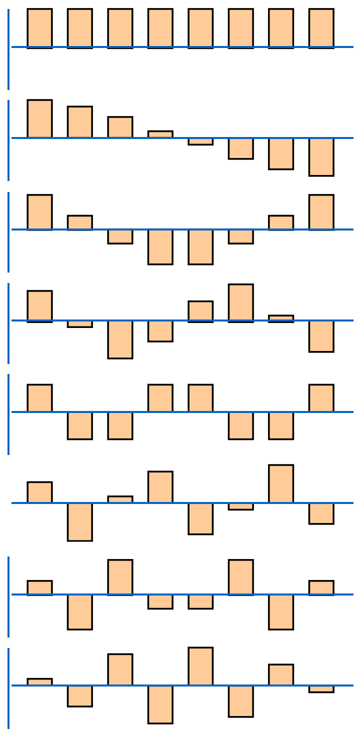
$$\Psi = \frac{1}{2}\begin{bmatrix} .71 & .71 & .71 & .71 & .71 & .71 & .71 & .71 \\ .98 & .83 & .56 & .19 & -.19 & -.56 & -.83 & -.98 \\ .92 & .38 & -.38 & -.92 & -.92 & -.38 & .38 & .92 \\ .83 & -.19 & -.98 & -.56 & .56 & .98 & .19 & -.83 \\ .71 & -.71 & -.71 & .71 & .71 & -.71 & -.71 & .71 \\ .56 & -.98 & .19 & .83 & -.83 & -.19 & .98 & -.56 \\ .38 & -.92 & .92 & -.38 & -.38 & .92 & -.92 & .38 \\ .19 & -.56 & .83 & -.98 & .98 & -.83 & .56 & -.19 \end{bmatrix}\begin{bmatrix} 7 \\ -6 \\ -5 \\ 4 \\ 5 \\ -3 \\ -2 \\ 6 \end{bmatrix}$$

Output sequence

$$Output = \begin{bmatrix} 2.13 \\ -1.825 \\ 1.84 \\ 1.775 \\ 13.49 \\ 0.175 \\ 0.76 \\ 0.875 \end{bmatrix}$$

Comment

$$Output = \begin{bmatrix} 2.13 \\ -1.825 \\ 1.84 \\ 1.775 \\ 13.49 \\ 0.175 \\ 0.76 \\ 0.875 \end{bmatrix}$$



Input sequence data varies like this frequency