# EBU6018
# Advanced Transform Methods

## Fourier Transform_2
## Fourier Transform

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# Fourier Transform

- The Fourier Series can only be applied to periodic signals, giving an infinite number of discrete frequencies. However, periodic signals are non-informational.
- Non-periodic signals (signals containing information) cannot be analysed using the Fourier Series, the Fourier Transform (FT) is required.
- This gives us the bandwidth of a signal as the sum of an continuous infinity of sinusoids.

Sinusoids: sine, cosine, arbitrary phase angle

# Fourier Transforms

The Fourier Transform (FT) is defined as:

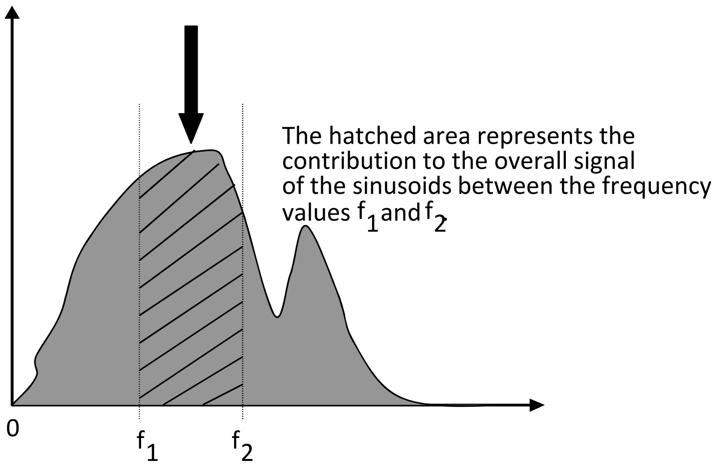
$$X(f) = \int_{t=-\infty}^{t=\infty} e^{-j\omega t} \cdot x(t)dt$$

X(f) in LHS is values in the amplitude spectral density (ASD).

x(t) is the signal

A frequency domain diagram showing spectral density

Y axis represents the contribution of each of the sinusoids to the overall amplitude of of the original signal




Frequency of the sinewave components

The Fourier transform will always be denoted by an uppercase letter or symbol, whereas signals will usually be denoted by lowercase letters or symbols.

**Example:**
**R&S spectrum analyzer (R&S FSP40) F**

Note: the spectrum analyser will sample the signal before analysing it.

# Fourier Transforms

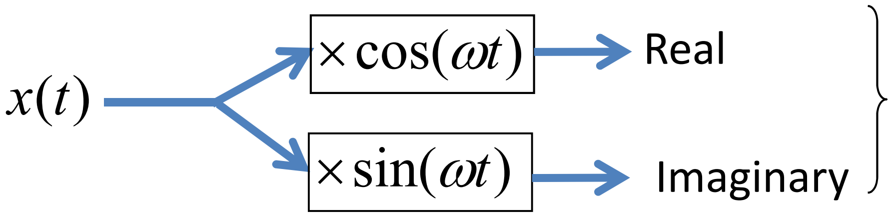
The Fourier Transform (FT) is defined as:

$$X(f) = \int_{t=-\infty}^{t=\infty} e^{-j\omega t} \cdot x(t)\,dt$$

**By Euler's formula**

$$= \int_{t=-\infty}^{t=\infty} \left(\cos(\omega t) - j\sin(\omega t)\right) \cdot x(t)\,dt$$

Implement the FT:




$$X(f) = \text{Re} + j\,\text{Im}$$

# The Conditions for an FT

- A signal is said to have a Fourier transform in the ordinary sense if the integral in the following equation converges (i.e. exists) .

$$X(f) = \int_{t=-\infty}^{t=\infty} e^{-j\omega t} \cdot x(t)dt$$

x(t) is "well behaved" if:

1. the signal x(t) has a finite number of discontinuities, maxima, and minima within any finite interval of time.
2. if x(t) is absolutely integrable $\int_{t=-\infty}^{t=\infty} |x(t)|dt \ < \ \infty$

These are the Dirichlet Conditions.

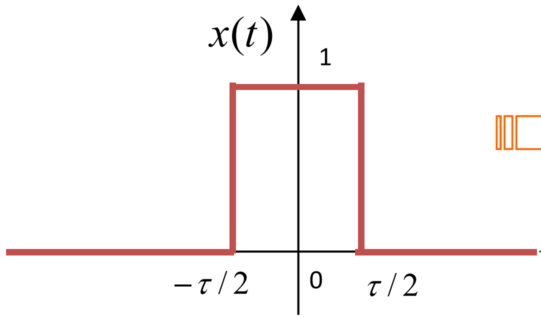
However some common signals are not absolutely integrable, such as a constant DC signal.

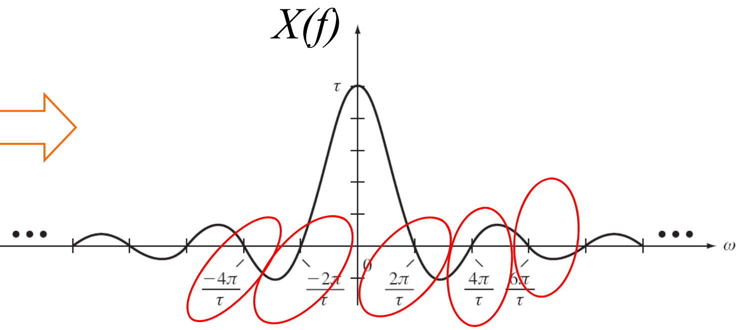
But in practice this could not exist for an infinite time.

# Isolated Rectangular Pulse

$$x(t)=\begin{cases}1, & -\dfrac{\tau}{2}\le t\le\dfrac{\tau}{2}\\ 0, & all\ other\ t\end{cases}$$

also denoted as $p_\tau(t)$




FT definition

$$X(f)=\int_{t=-\infty}^{t=\infty} e^{-j\omega t}\cdot x(t)dt=\int_{t=-\infty}^{t=\infty}\Big(\overset{\text{even}}{\cos(\omega t)}-j\,\overset{\text{odd}}{\sin(\omega t)}\Big)\cdot x(t)dt$$

and x(t) is an even signal.

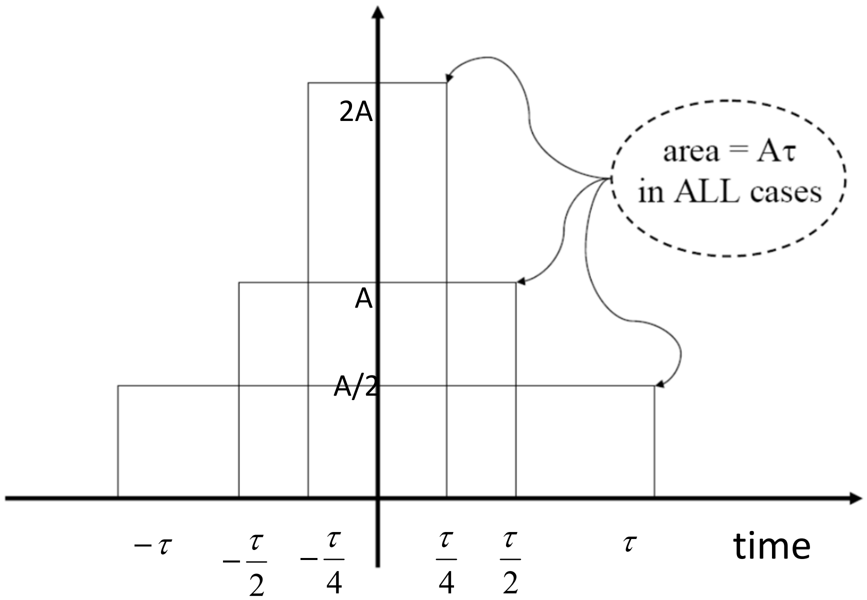
$$X(f)=2\int_0^{\tau/2}(1)\cos(\omega t)dt=\frac{2}{\omega}\left[\sin(\omega t)\Big|_{t=0}^{t=\tau/2}\right]=\frac{2}{\omega}\sin\frac{\omega\tau}{2}$$

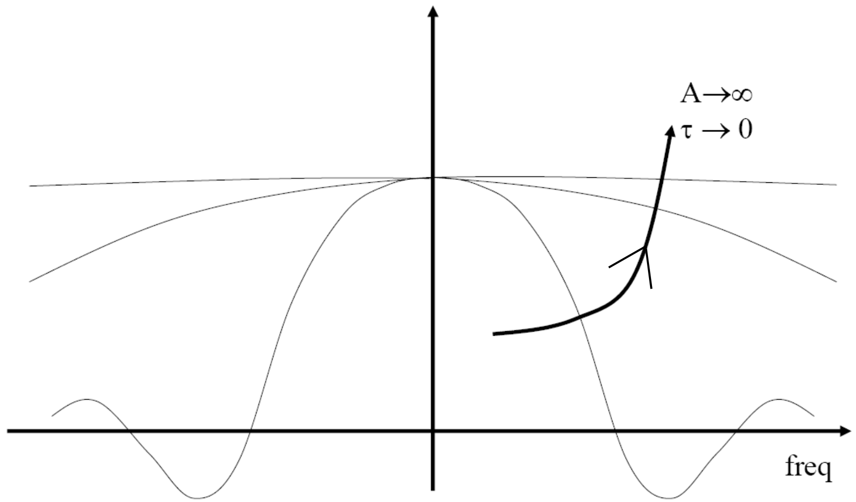
Let's recall the sinc function $\quad \mathrm{sinc}(a\omega)=\dfrac{\sin(a\pi\omega)}{a\pi\omega}$ $\qquad$ Setting $\quad a=\dfrac{\tau}{2\pi}$

$$\mathrm{sinc}\left(\frac{\tau\omega}{2\pi}\right)=\frac{2}{\tau\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

Thus, $\quad X(f)=\tau\,\mathrm{sinc}\left(\dfrac{\tau\omega}{2\pi}\right)$

# Isolated Rectangular Pulse




As the pulse becomes narrower, the lobes of the frequency spectrum become wider (more energy at higher frequencies).
Also, the pulse is “time-limited”, but the frequency spectrum is infinite.

# Inverse Fourier Transform

Given a signal x(t) with Fourier transform X(f), x(t) can be recomputed from X(f) by application of the inverse  Fourier transform give by:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) \cdot e^{j\omega t} df$$

Note: the definitions of the FT and IFT are a pair. The 1/2π and the sign of the exponential can be interchanged.

To denote the fact that X(f) is the Fourier transform of x(t), or that X(f) is the inverse Fourier transform of x(t), the transform pair notation:

$$x(t) \leftrightarrow X(f)$$

will sometimes be used.

One of most fundamental transform pairs in the Fourier Theory is the pair

$$p_\tau(t) \leftrightarrow \tau \,\mathrm{sinc}\, \frac{\tau\omega}{2\pi}$$