EBU6018
Advanced Transform Methods

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Introduction 2018-19

Course Format

- **Lectures/Tutorials**
  - **5 x 2 hours/week over 4 weeks**
  - **(weeks 6, 8, 11, 15)**
  - **Tutorial: Wednesday 18:30-19:30 TB4 Room 238**
- **Laboratory sessions**
  - **2 labs (weeks 9, 16. 2016215101-10，Wed 17:30-20:30，Research building 116)**
  - **(3-hour computer-based exercises). (Reports plus follow-up questions: weighting 15%).**
- **Class Tests. (Weighting 5%)**
- **Exam. (Weighting 80%)**

Some background...

It can be advantageous to process signals and images in the frequency domain.

E.g. convolution in the time domain becomes multiplication in the frequency domain (convolution is used to determine the output from a process given its impulse response and the input applied).

….some background...

The basic method of transforming from the time domain to the frequency domain is the Fourier Transform. There are four versions:

Fourier Series FS: continuous time (periodic) /discrete frequency

Fourier Transform FT: continuous time (aperiodic or periodic) /continuous frequency

Discrete Time Fourier Transform DTFT: discrete time (aperiodic) /continuous frequency

Discrete Fourier Transform DFT: discrete time (periodic) /discrete frequency

...some background...

A major problem in digital signal and image processing is the trade-off between the time taken to process and the resolution/quality of the output.

The time taken can be reduced by, e.g.,

- **Using fast processing techniques**
- **Identifying redundancy in the information**
- **Omitting components in the processing that do not noticeably contribute to quality**

...some background...

- A signal is a function of time, f(t)
- An image is a 2D function, f(x,y), and the "amplitude" at any point (x,y) is the intensity or grey level (and possibly colour information) at that point
- If x, y and intensity are all finite and discrete we have a digital image
- Each (finite) element has a unique location and value and is called a pixel

...some background...

- **Images do not need to be visible to humans, just as signals do not need to be audible to humans,**
- **e.g., ultrasound, electron microscopy, CAT scans, etc.**
- **A variety of transform techniques have been devised and developed to cater for different applications.**

...some background.

Digital Signal Processing....many applications.......

Digital Image Processing

- **improvement of pictorial information for human interpretation**
- **processing for storage, transmission and representation for autonomous machine perception**

Two types of signal

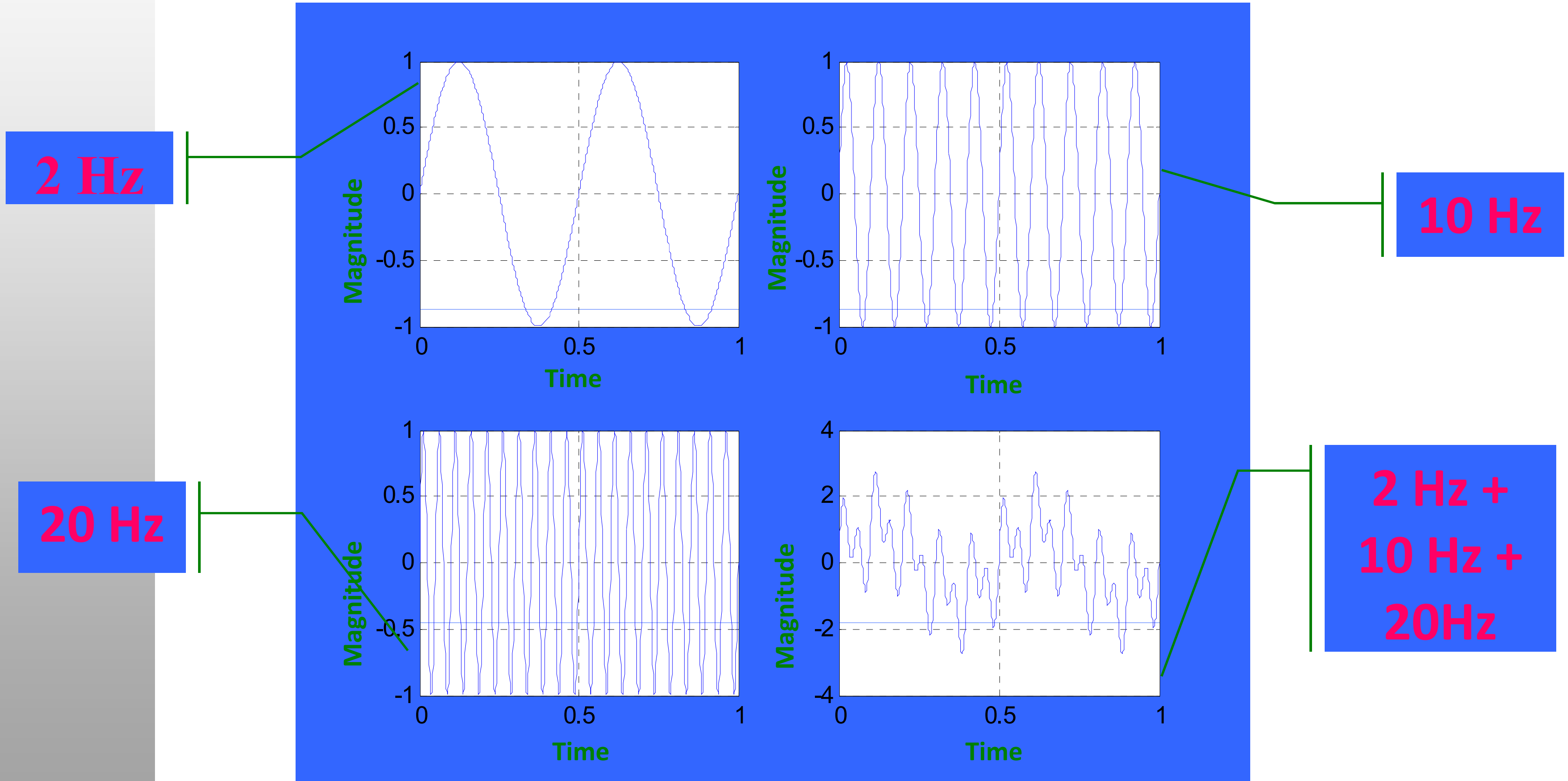
Stationary:

The frequency content is the same for all time.

Non-stationary:

The frequency content changes with time.

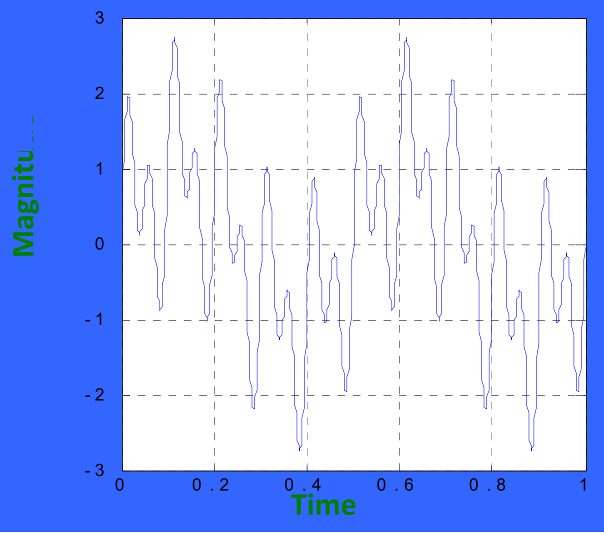
Example 1: Stationary signal

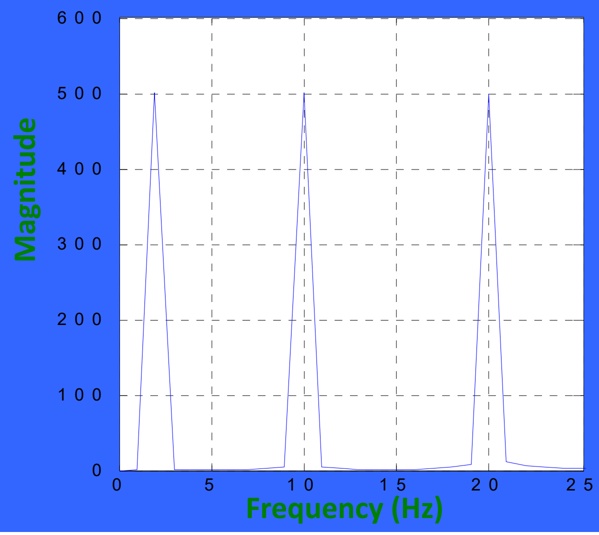


Example 1: Stationary signal

2 Hz + 10 Hz + 20Hz

Stationary



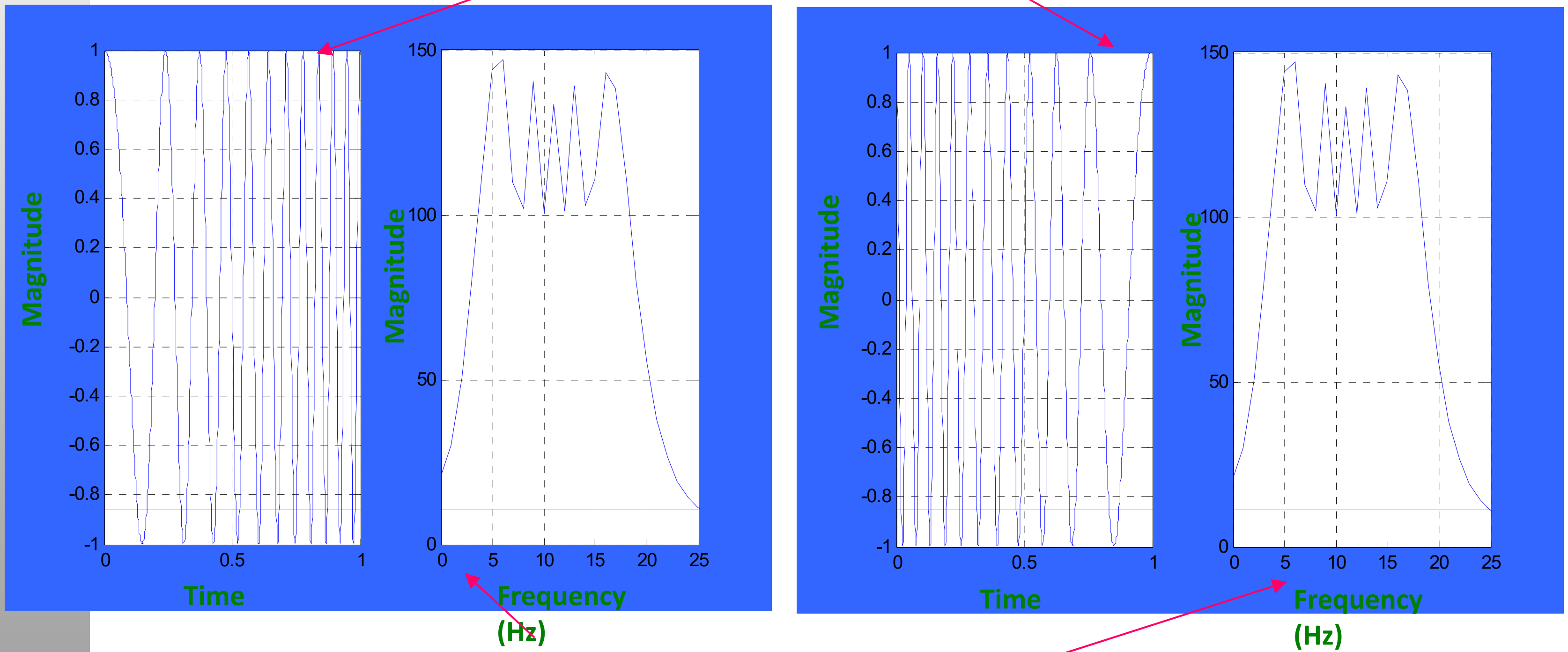


Signal in time domain   Power spectrum $|X(f)|^2$

By looking at the Power spectrum of the signal we can recognize three frequency components (at 2,10,20Hz respectively).

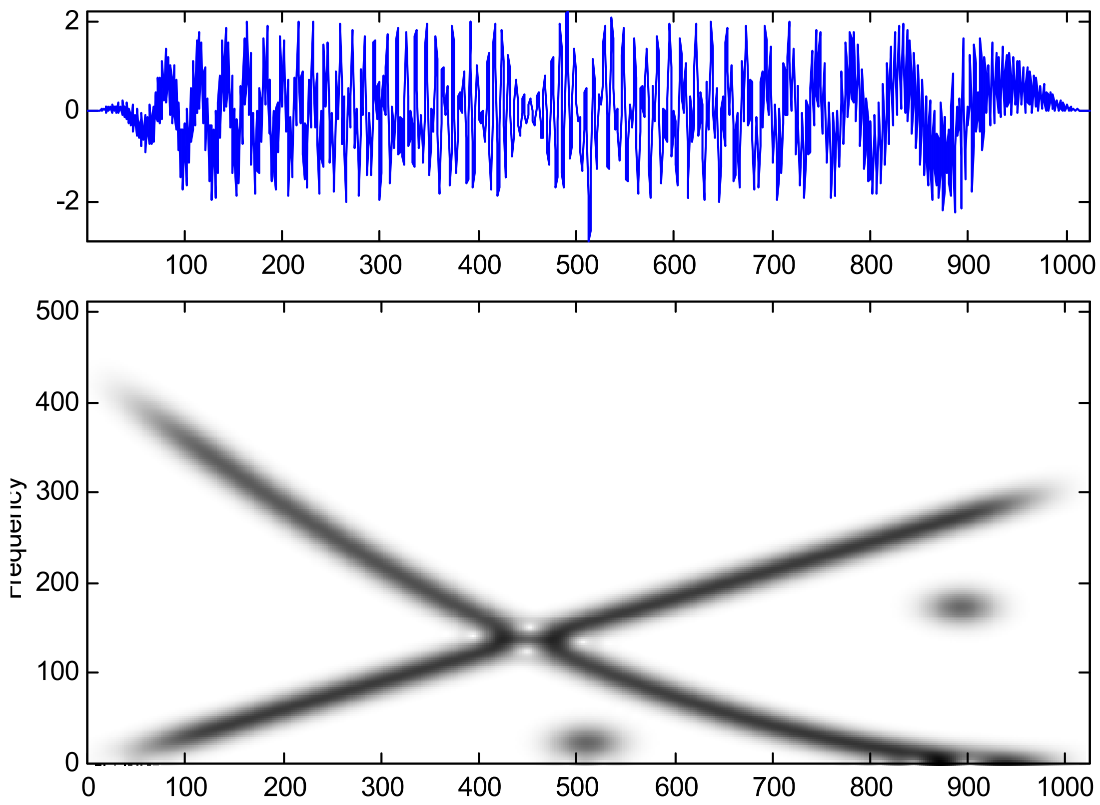
Example 2:non stationary signals

Consider two linearly modulated sinusoids (chirps). The first with increasing frequency and the second with decreasing.



In this case we have two nonstationary signals in time with identical FTs. Confusion arises and power spectrum is not very useful.

So, what is in a signal?



Topics to be covered

- Linear Algebra and Basis functions
- Fourier Transform, STFT, Spectrogram
- Discrete Cosine Transform
- Karhunen-Loeve Transform and Principal Component Analysis (PCA)
- Wavelet Transform, Scalogram
- Multiresolution Analysis
- Wigner-Ville distribution

Basis Functions

- **A function f(x) can sometimes be better analysed as a linear expansion of "expansion functions". If the expansion is unique, i.e., there is only one set of expansion function coefficients for a given f(x), then they are called "basis functions".**

Basis Functions

An example of “Basis Functions” are the sine and cosine functions used for the Fourier Transform.

These two basis functions are “Orthogonal”.

This allows the FT to be “Inverted”, that is, the original function can be uniquely reproduced.

STFT, Spectrogram

- The Short-Time Fourier Transform is used to find a frequency spectrum snapshot by calculating FT of a short time interval of the signal.
- Useful if the signal is not stationary- can then see the changes in spectrum with time
- Assumes the signal is stationary within a "window"
- A spectrogram is an image that shows how the spectral density varies with time.

Discrete Cosine Transform

- **Most of the energy is contained in just a few of the transform coefficients**
- **Used in JPEG image compression formats and in MPEG video compression formats.**
- **Uses only the real part of a FT, that is, only the cosine terms (computation is therefore much simpler).**

Karhunen-Loeve Transform and PCA

- Gives optimum error resulting from truncating the transform coefficients
- Has disadvantages (one being that it requires a lot of computation)
- But it can be approximated by the DCT for the majority of images

Wavelet Transform,Scalogram

- “Substitute” the window in the STFT by a wave packet (“wavelet”)
- Produces a transform that is a function of 2 real variables a and b (scale and translation) instead of a function of t and f.
- Can be used on non-stationary data (where the FT is not very useful).

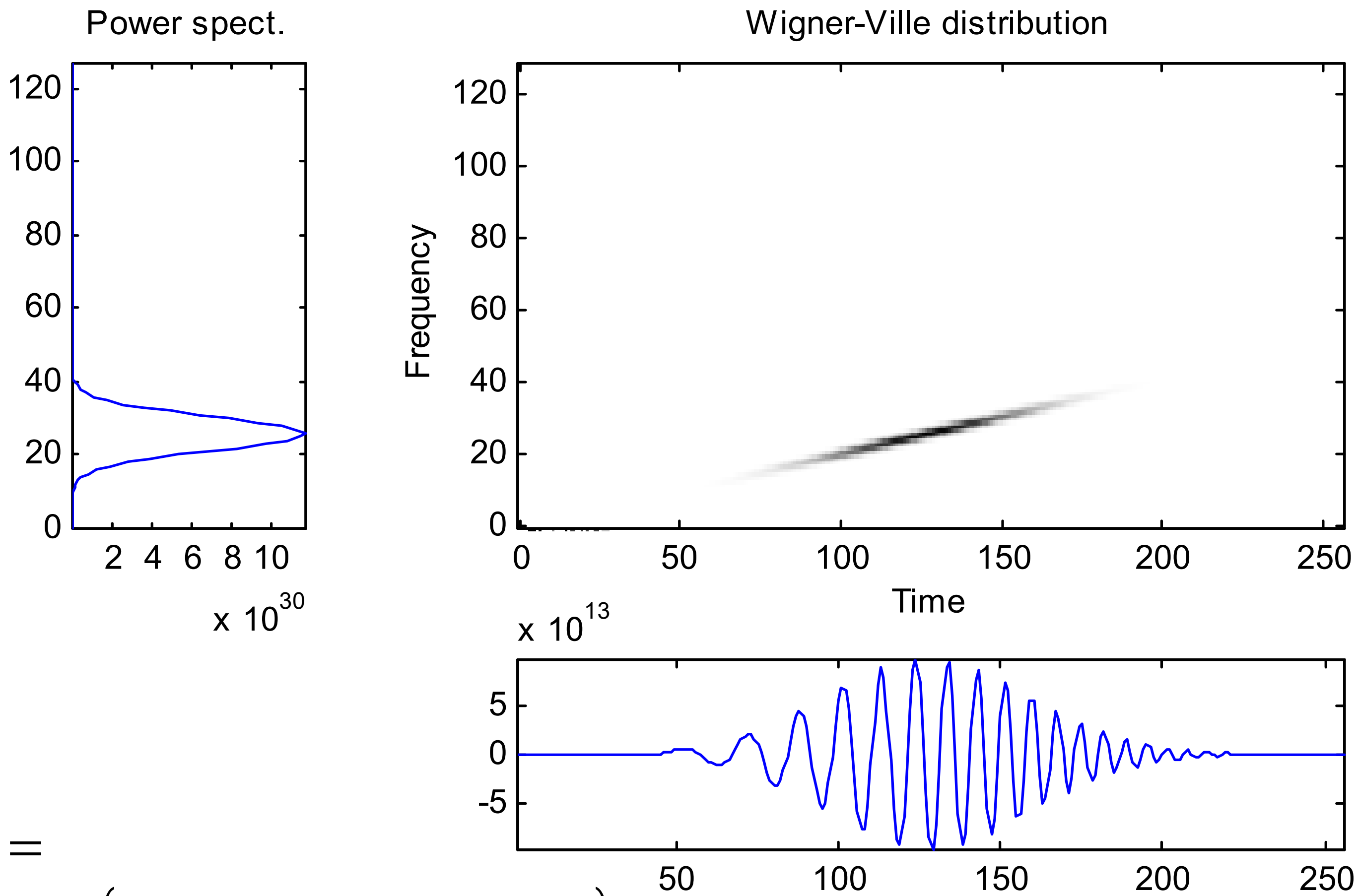
Multiresolution Analysis

- **Decomposing a signal or image into a set of bandlimited components (subbands) that can then later be reassembled to reconstruct the original signal.**
- **One way of implementing discrete wavelet transforms.**

Wigner-Ville distribution

- **High resolution time-frequency distribution**
- **Better trade-off between time and frequency than the STFT**
- **Does not suffer from the disadvantage of STFT and CWT where high resolution cannot be achieved simultaneously in both time and frequency domains (due to the uncertainty principle)**
- **Applications include analysis of physiological signals (e.g. cardiovascular) and geological exploration (e.g. hydrocarbons)**

Illustration: chirplet



$$s(t) = \sqrt[4]{\frac{\alpha}{\pi}} \exp\left\{-\frac{\alpha}{2}(t-t_0)^2 + j\frac{\beta}{2}t^2\right\}$$

Summary: Fourier Transform

Kind of Decomposition	**Frequency**
Analyzing Function	**Sines and cosines, oscillating indefinitely**
Variable	**Frequency**
Information	**The frequencies that make up the signal**
Suited for	**Stationary signals**
Notes	**With the FFT, it takes *n*log*n* computations to compute the Fourier transform of *n* points**

Summary: Windowed Fourier Transform (Short time Fourier Analysis)

Kind of Decomposition	Time-Frequency
Analyzing Function	Wave limited in time, multiplied by sinusoidal oscillations. Window size is fixed, but the frequency inside the window varies.
Variable	Frequency, position of the window
Information	The smaller the window, the better the time information. The cost is low frequency information. Large windows give better frequency information but less timing precision.
Suited for	Quasi-stationary signals (stationary at scale of window)
Notes	When a Gaussian is used as the window envelope this is the Gabor transform. Though the Fourier transform is orthogonal, in general the windowed transform is not.

Summary: Wavelet Transform

Kind of Decom-position	Time-Scale
Analyzing Function	Wave limited in time with fixed number of oscillations. Wavelet is contracted or dilated to change the window size and change the scale at which one looks at the signal. Since number of oscillations is fixed, frequency of the wavelet changes as scale changes.
Variable	Scale, position of the wavelet
Information	Small wavelets provide good time information but poor frequency information. Vice versa for large wavelets.
Suited for	Nonstationary signals, such as brief signals and signals with components at different time scales.
Notes	Wavelet transform can be continuous or discrete. Orthogonal and biorthogonal wavelets are special discrete cases. With orthogonal wavelets, the transform can be computed with *cn* computations. *C* depends on the complexity of the wavelet.

Summary: Wigner-Ville Distribution

Kind of Decomposition	Time-Frequency
Analyzing Function	Uses the signal itself. Motivated by time-frequency energy density (c.f. a probability density).
Variable	Time and Frequency. Has high resolution in both time and frequency.
Suited for	Simple signals, e.g. linear chirp, gaussian pulse
Notes	More complex signals lead to undesired “cross-terms”. Can be suppressed with smoothing, but lose high resolution in the process.