8.6 The electric field intensity of a uniform plane wave in free space is given by $\vec{\mathbf{E}} = 120\cos(2\pi \times 10^9 t - \beta y)\,\vec{\mathbf{a}}_z$ V/m. Determine (a) the phase constant, (b) the magnetic field intensity, (c) the wavelength, (d) the average power density in the medium, (e) the average energy density in the electric field, and (f) the average energy density in the magnetic field.

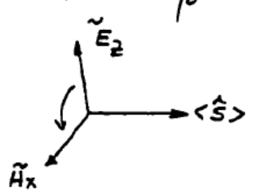
<u>Exercise 8.6</u> $\quad \omega = 2\pi \times 10^9$ rad/s $\quad \beta_o = \frac{\omega}{c} = 20.94$ rad/m $\quad \eta_o \cong 377\Omega$

$\tilde{\vec{E}} = 120\, e^{-j20.94y}\, \vec{a}_z$ V/m

For y- directed propagation,

$\tilde{\vec{H}} = 0.318\, e^{-j20.94y}\, \vec{a}_x$ A/m

$\langle \hat{S} \rangle = \frac{1}{2} Re\left[\tilde{E} \times \tilde{H}^*\right] = 19.1\, \vec{a}_y$ W/m^2



8.7 The magnetic field intensity of a plane wave in free space is given as $\vec{\mathbf{H}} = 0.1\cos(200\pi \times 10^6 t + \beta z)\,\vec{\mathbf{a}}_x$ A/m. Determine (a) the phase constant, (b) the velocity of propagation, (c) the wavelength, (d) the electric field intensity, (e) the displacement current density, and (f) the average power flow per unit area.

<u>Exercise 8.7</u> $\quad \omega = 200\pi \times 10^6$ rad/s $\quad \beta_o = \frac{\omega}{c} = \frac{2}{3}\pi$ rad/m, $\eta_o = 377\Omega$

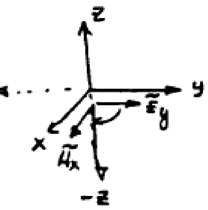
$\vec{u}_p = -3\times 10^8\, \vec{a}_z$ m/s $\quad \lambda_o = \frac{2\pi}{\beta_o} = 3$ m

$\tilde{\vec{H}} = 0.1\, e^{j\beta_o z}\, \vec{a}_x$ A/m

$\tilde{\vec{E}} = 37.7\, e^{j\beta_o z}\, \vec{a}_y$ V/m

$\tilde{J}_d = j\omega\epsilon_0 \tilde{E} = j\, 0.209\, e^{j\beta_o z}\, \vec{a}_y$ A/m^2

$\langle \hat{S} \rangle = \frac{1}{2} Re\left[\tilde{E} \times \tilde{H}^*\right] = -1.885\, \vec{a}_z$ W/m^2



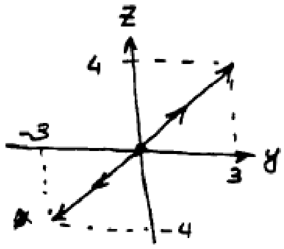
8.15 The electric field intensity of a wave in a region is given by $\vec{\mathbf{E}} = 3\cos(\omega t - \beta x - 45°)\,\vec{\mathbf{a}}_y + 4\sin(\omega t - \beta x + 45°)\,\vec{\mathbf{a}}_z$ V/m. Determine the polarization of the wave.

Exercise 8.15 at $\beta x = 45^\circ$

$E_z = 4 \sin \omega t$, $E_y = 3 \sin \omega t$

$\frac{E_y}{E_z} = 0.75 \Rightarrow E_y = 0.75\, E_z$

St. line relationship $\Rightarrow$ Linear Polarization



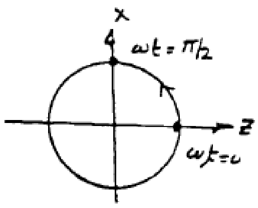
8.16 Find the polarization of a wave if the electric field intensity is given by $\vec{\mathbf{E}} = (-j25\,\vec{\mathbf{a}}_x + 25\,\vec{\mathbf{a}}_z)e^{-(0.01+j120y)}$ V/m.

Exercise 8.16 $E_x = 25\, e^{-0.01y} \sin(\omega t - 120y)$ $E_z = 25\, e^{-0.01y} \cos(\omega t - 120y)$

at $y=0$, $E_x = 25 \sin \omega t$ $E_z = 25 \cos \omega t$

$E_x^2 + E_z^2 = 25^2$ [Circular]

$$\left.\begin{array}{lll} \omega t = 0 & E_x = 0, & E_z = 25 \\ \omega t = \pi/2 & E_x = 25 & E_z = 0 \end{array}\right\} \text{Right handed}$$



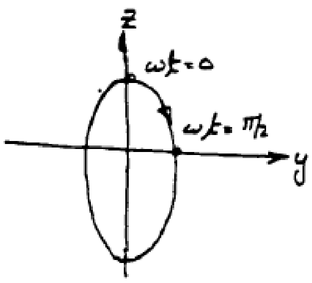
8.17 The electric field intensity of a wave in a region is given by $\vec{\mathbf{E}} = 30\cos(\omega t - \beta x - 30^\circ)\,\vec{\mathbf{a}}_y + 40\cos(\omega t - \beta x + 60^\circ)\,\vec{\mathbf{a}}_z$ V/m. Determine the polarization of the wave.

Exercise 8.17 Let $\beta x = 60^\circ$, then

$E_y = 30 \sin \omega t$ $E_z = 40 \cos \omega t$

$\left(\frac{E_y}{30}\right)^2 + \left(\frac{E_z}{40}\right)^2 = 1$ [Elliptical]

$$\left.\begin{array}{lll} \omega t = 0 & E_y = 0 & E_z = 40 \\ \omega t = \pi/2 & E_y = 30 & E_z = 0 \end{array}\right\} \text{Left-handed}$$



Problem:

8.7 If $\vec{\mathbf{H}} = 100\cos(30{,}000t + \beta z)\,\vec{\mathbf{a}}_x$ A/m is the magnetic field intensity in free space of a uniform plane wave, determine (a) the phase constant, (b) the wavelength, (c) the velocity of propagation, (d) the $\vec{\mathbf{E}}$ field, and (e) the time average power flow per unit area.

Problem 8.7 $\tilde{\vec{H}} = 100\, e^{j\beta_0 z}\, \vec{a}_x$ A/m, $\eta_0 = 120\pi \cong 377\,\Omega$

$\tilde{\vec{E}} = 37{,}700\, e^{j\beta_0 z}\, \vec{a}_y$ V/m $\qquad \vec{u} = -3\times 10^8\, \vec{a}_z$ m/s $\qquad \omega = 30{,}000$ rad/s

$\beta_0 = \frac{\omega}{c} = 0.0001$ (rad/m) $\Rightarrow \lambda_0 = \frac{2\pi}{\beta_0} = 62.83$ km

$\langle \hat{\vec{S}} \rangle = -\frac{1}{2} \times 37{,}700 \times 100\, \vec{a}_z$

$= 1.885\, \vec{a}_z$ MW/m^2

8.8 A 100-MHz uniform plane wave is traveling in the y direction in a lossless unbounded medium ($\epsilon_r = 4$ and $\mu_r = 1.0$). If the $\vec{\mathbf{E}}$ field has only an x component and its amplitude is 500 V/m when $t = 0$ and $y = 0$, determine (a) the phase velocity, (b) the phase constant, (c) the $\vec{\mathbf{H}}$ field, (d) the wavelength, and (e) the average power flow through a cross-sectional area of 16 cm^2.

Problem 8.8 $f = 100$ MHz $\qquad \omega = 2\pi f = 6.28\times 10^8$ rad/s $\qquad \mu_r = 1 \quad \epsilon_r = 4$

$\tilde{E}_x = 500\, e^{-j\beta y}$ V/m $\qquad u_p = \frac{c}{n} = 1.5\times 10^8$ m/s $\qquad n = \sqrt{\mu_r \epsilon_r} = 2$

$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} = 60\pi\,\Omega$ $\qquad \beta = \frac{\omega}{u_p} = 4.19$ rad/m

$\tilde{H}_z = -\frac{500}{60\pi}\, e^{-j\beta y}$ A/m

$\langle \hat{\vec{S}} \rangle = \frac{1}{2} \mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^* \right] = 663.15\, \vec{a}_y$ W/m.

$\langle P \rangle = \int_S \langle \hat{\vec{S}} \rangle \cdot \vec{ds} = 1.06$ W

8.22 Find the polarization of the following waves:

a) $\tilde{\vec{\mathbf{E}}} = 100 e^{-j300x}\, \vec{\mathbf{a}}_y + 100 e^{-j300x}\, \vec{\mathbf{a}}_z$ V/m

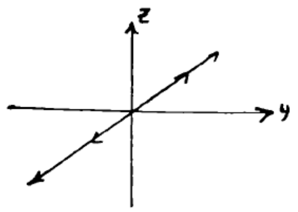
b) $\tilde{\vec{\mathbf{E}}} = 16 e^{j\pi/4} e^{-j100z}\, \vec{\mathbf{a}}_x - 9 e^{-j\pi/4} e^{-j100z}\, \vec{\mathbf{a}}_y$ V/m

c) $\tilde{\vec{\mathbf{E}}} = 3\cos(t - 0.5y)\, \vec{\mathbf{a}}_x - 4\sin(t - 0.5y)\, \vec{\mathbf{a}}_z$ V/m

Problem 8.22 a) At $x=0$

$$\left.\begin{array}{l} E_y = 100\cos\omega t \\ E_z = 100\cos\omega t \end{array}\right\} \Rightarrow E_y = E_z$$

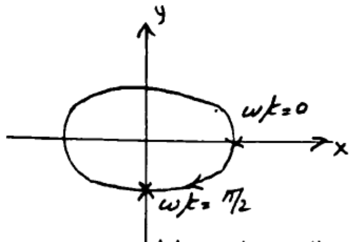
Linear polarization



b. When $100z = \pi/4$

$$\left.\begin{array}{l} E_x = 16\cos\omega t \\ E_y = -9\sin\omega t \end{array}\right\} \Rightarrow \left(\frac{E_x}{16}\right)^2 + \left(\frac{E_y}{9}\right)^2 = 1$$

(Elliptical)

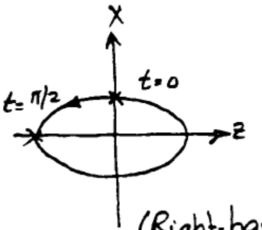


(Left-handed)

c) In a $y=0$ plane:

$$\left.\begin{array}{l} E_x = 3\cos t \\ E_z = -4\sin t \end{array}\right\} \Rightarrow \left(\frac{E_x}{3}\right)^2 + \left(\frac{E_z}{4}\right)^2 = 1$$

Elliptical



(Right-handed)

8.23 A uniform plane wave with an $\vec{\mathbf{E}}$ of $12\cos(\omega t - \beta z)\vec{\mathbf{a}}_x - 5\sin(\omega t - \beta z)\vec{\mathbf{a}}_y$ V/m is propagating in a lossless medium ($\epsilon_r = 2.5$, $\mu_r = 1$) at 200 Mrad/s. Determine the corresponding $\vec{\mathbf{H}}$ field, the phase constant β, the wavelength λ, the intrinsic impedance η, the phase velocity $\vec{\mathbf{u}}_p$, and the polarization of the wave.

Problem 8.23

$\omega = 2\times10^8$ rad/s

$\epsilon_r = 2.5$

$\lambda = \frac{2\pi}{\beta} = 5.96$ m

$\beta = \frac{\omega}{c}\sqrt{\epsilon_r} = 1.054$ rad/m

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = 238.43\,\Omega$

Use Superposition Theorem:

$\tilde{E}_x = 12e^{-j\beta z} \Rightarrow \tilde{H}_y = \frac{12}{\eta}e^{-j\beta z}$

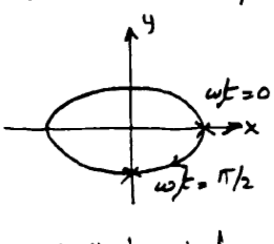
$\tilde{E}_y = j5e^{-j\beta z} \Rightarrow \tilde{H}_x = -j\frac{5}{\eta}e^{-j\beta z}$

$\vec{H} = \frac{1}{\eta}\left[12e^{-j\beta z}\vec{a}_y - j5e^{-j\beta z}\vec{a}_x\right]$

$\vec{H} = 0.05\cos(\omega t - \beta z)\vec{a}_y$
$+ 0.02\sin(\omega t - \beta z)\vec{a}_x$ A/m

at $z=0$ $E_x = 12\cos\omega t$, $E_y = -5\sin\omega t$

$\left(\frac{E_x}{12}\right)^2 + \left(\frac{E_y}{5}\right)^2 = 1$ Elliptical.



Left-handed