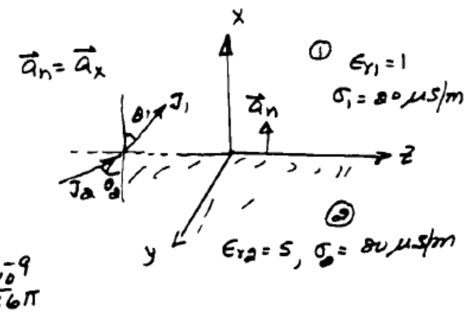
4.20 The volume current density in medium $1(x \geq 0, \epsilon_{r1} = 1$, and $\sigma_1 = 20\,\mu\text{S/m})$ is $\vec{\mathbf{J}}_1 = 100\vec{\mathbf{a}}_x + 20\vec{\mathbf{a}}_y - 50\vec{\mathbf{a}}_z$ A/m^2. Obtain the volume current density in medium $2(x \leq 0, \epsilon_{r2} = 5, \sigma_2 80\,\mu\text{S/m})$. Also compute θ_1, θ_2, and ρ_s at the interface. What are the $\vec{\mathbf{E}}$ and $\vec{\mathbf{D}}$ fields on both sides of the interface?

<u>Exercise 4.20</u> $\vec{J}_1 = 100\,\vec{a}_x + 20\,\vec{a}_y - 50\,\vec{a}_z\ A/m^2$



$J_{n1} = J_{n2} = J_n \Rightarrow J_{x2} = 100$

$J_{t2} = \frac{\sigma_2}{\sigma_1} J_{t1} \Rightarrow J_{y2} = \frac{80}{20} \times 20 = 80$

$J_{z2} = \frac{80}{20}(-50) = -200$

$\vec{J}_2 = 100\,\vec{a}_x + 80\,\vec{a}_y - 200\,\vec{a}_z\ A/m^2$

$\rho_s\Big|_{\text{Interface}} = J_n\left[\frac{\epsilon_1}{\sigma_1} - \frac{\epsilon_2}{\sigma_2}\right] = 100\left[\frac{10^6}{20} - \frac{5\times10^6}{80}\right]\frac{10^{-9}}{36\pi}$

$= 11.05\ \mu C/m^2$

$J_1 = 113.578\ A/m^2,\ J_2 = 237.487\ A/m^2 \quad \theta_1 = \cos^{-1}\left(\frac{100}{113.578}\right) = 28.3^\circ,\ \theta_2 = \cos^{-1}\left(\frac{100}{237.487}\right) = 65.1^\circ$

4.28 The region between two parallel metal plates, each of area 1 m^2, is filled with three conducting media of thicknesses 0.5 mm, 0.2 mm, and 0.3 mm, and of conductivities 10 kS/m, 500 S/m, and 0.2 MS/m, respectively. What is the effective resistance between the two plates? If a potential difference of 10 mV is maintained between the plates, calculate the $\vec{\mathbf{J}}$ and $\vec{\mathbf{E}}$ fields in each region. How much power is dissipated in each medium? What is the total power dissipation?

<u>Problem 4.28</u>

$R_1 = \frac{0.5\times10^{-3}}{10\times10^3\times1} = 50\ n\Omega$

$R_2 = \frac{0.2\times10^{-3}}{500\times1} = 400\ n\Omega$

$R_3 = \frac{0.3\times10^{-3}}{0.2\times10^6\times1} = 1.5\ n\Omega$

$R = R_1 + R_2 + R_3 = 451.5\ n\Omega$

$I = 10\times10^{-3}/451.5\times10^{-9} = 22.148\ kA$

$J_1 = J_2 = J_3 = \frac{I}{A} = 22.148\ kA/m^2$

Since $\vec{J} = \sigma\vec{E} \Rightarrow E_1 = 2.215\ V/m$

$E_2 = 44.296\ V/m$

$E_3 = 0.111\ V/m$

$P = \frac{V^2}{R} = 221.48\ W$

4.31 Using boundary conditions, determine the surface charge densities at the two interfaces between the three conducting media in Problem 4.28 if the dielectric constant of each region is unity.

Problem 4.31

$\vec{D}_1 = -\epsilon_0 (2.215)\, \vec{a}_z \;,\quad \vec{D}_2 = -44.296\, \epsilon_0\, \vec{a}_z$

$\vec{D}_3 = -0.111\, \epsilon_0\, \vec{a}_z$

$\rho_{s_1} = \vec{a}_z \cdot (\vec{D}_1 - \vec{D}_2) = 372.08 \ pC/m^2$

$\rho_{s_2} = \vec{a}_z \cdot (\vec{D}_2 - \vec{D}_3) = -390.68 \ pC/m^2$