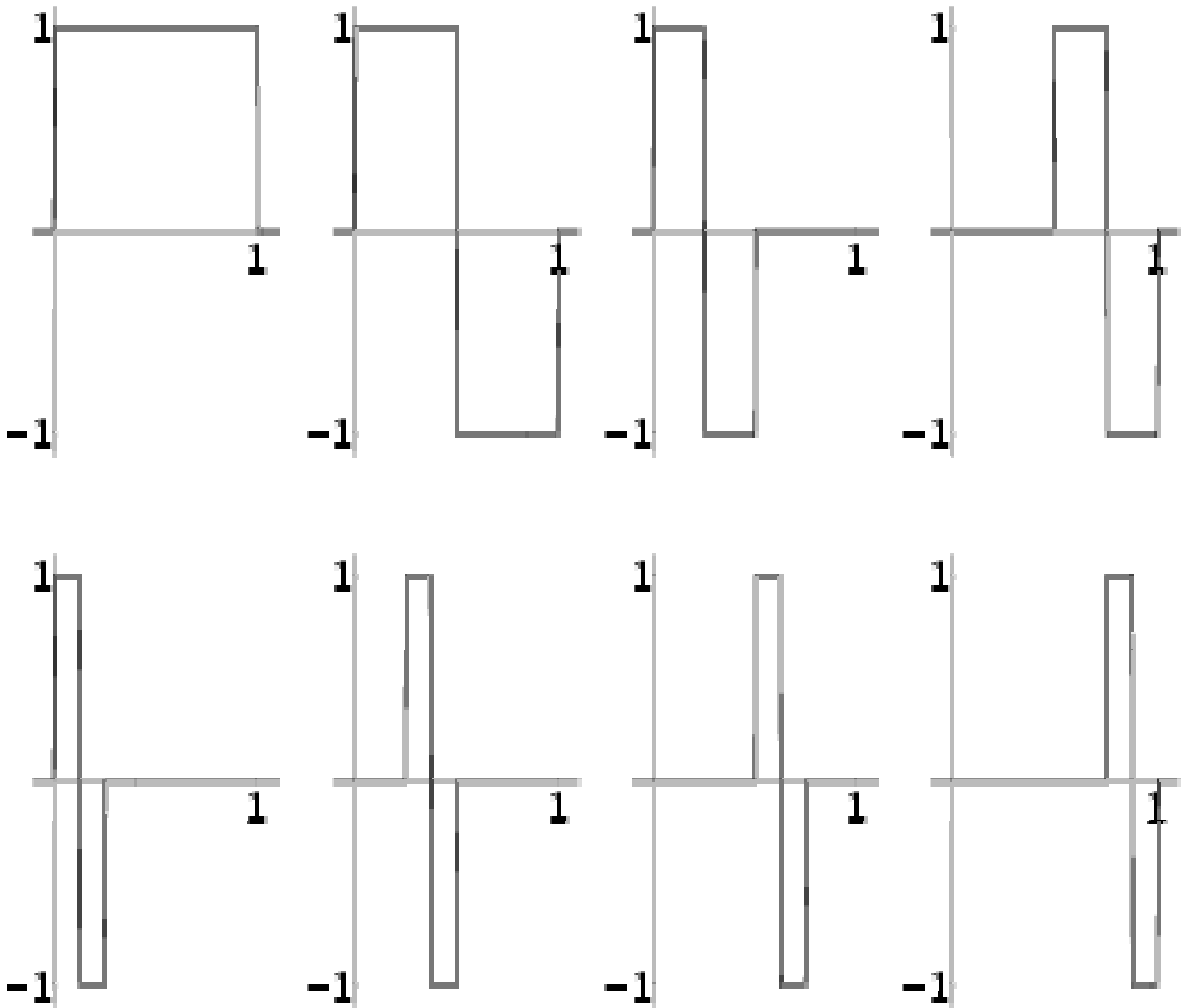
Advanced Transform Methods

Haar Functions

Andy Watson

Haar Functions



$$\psi(x) \equiv \begin{cases} 1 & 0 \le x \le \frac{1}{2} \\ -1 & \frac{1}{2} \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\psi_{jk}(x) \equiv \psi(2^j x - k),$$

$\phi_{00}=\phi(x)$

$\psi_{00}=\psi(x)$

$\psi_{10}=\psi(2x)$

$\psi_{11}=\psi(2x\text{-}1)$

$\psi_{20}=\psi(4x)$

$\psi_{21}=\psi(4x\text{-}1)$

$\psi_{22}=\psi(4x\text{-}2)$

$\psi_{23}=\psi(4x\text{-}3)$

Orthogonal

$$\psi(x) \equiv \begin{cases} 1 & 0 \le x \le \frac{1}{2} \\ -1 & \frac{1}{2} \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\psi_{jk}(x) \equiv \psi(2^j x - k),$$

$$\int_0^1 \psi_{jk}(x)\psi_{lm}(x)dx = 0$$

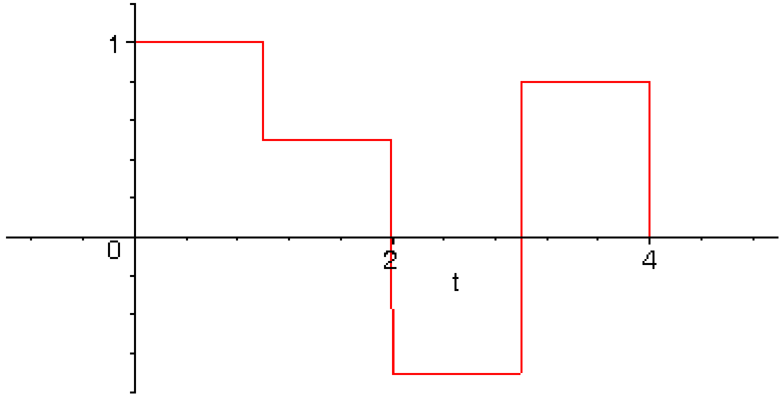
Orthonormal

$$\psi_{jk}(x) = 2^{-j/2}\psi(2^j x - k)$$

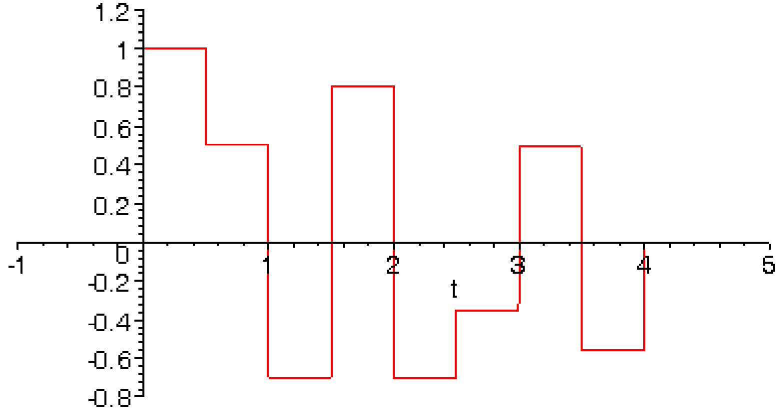
$$\int_0^1 \psi_{jk}(x)\psi_{jk}(x)dx = 1$$

Synthesis using the Haar function

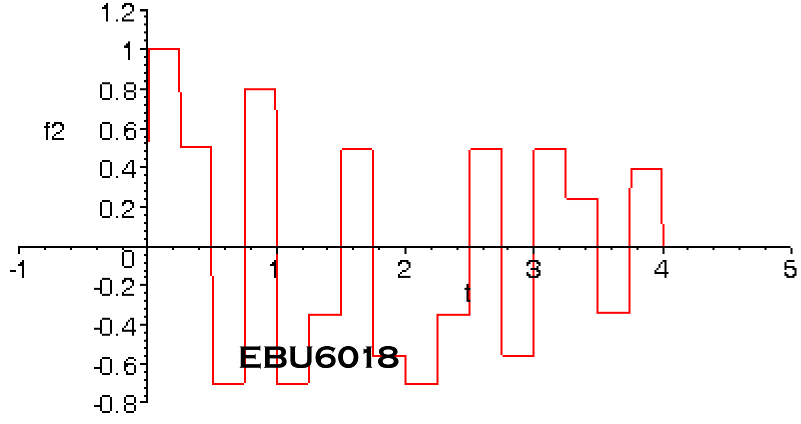
Using just $\phi(x)$



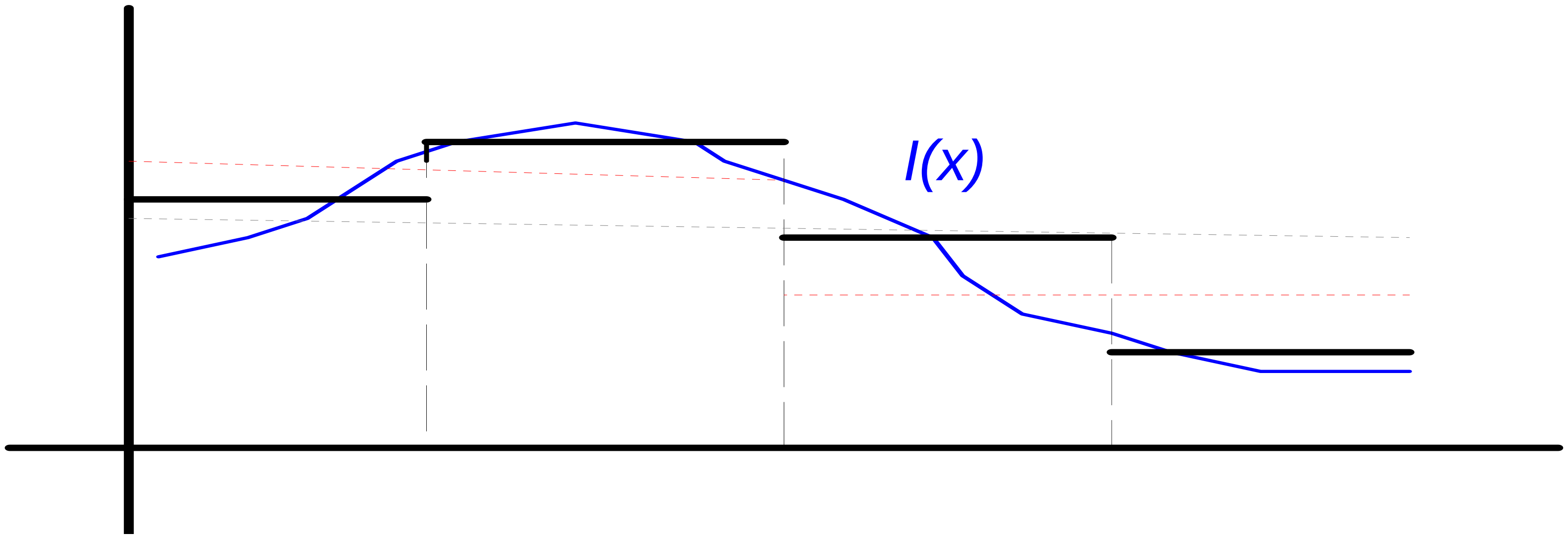
Using ψ_{j0} and ϕ



Using ψ_{j1} , ψ_{j0} and ϕ



Haar's Theorem



- Haar's theorem (1905):

All Haar functions $\psi_{j,k}$, together with the constant function 1, consist into an orthonormal basis for the Hilbert space of all square integrable functions on [0, 1].

- This basis will be applied later in the course in the introduction of wavelets