SOLUTIONS

Module:	Advanced Transform Methods		
Module Code	EBU6018	Paper	B
Time allowed	2hrs	Filename	Solutions_1819_EBU6018_B
Rubric	ANSWER ALL FOUR QUESTIONS		
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Question 1.

a) (a) Summarise the Wavelet Transform (WT) in terms of:

(i) Kind of decomposition
(ii) Analysing Function
(iii) Variable
(iv) Information obtained
(v) Suitability

[10 marks]

Answer:

[10 marks: 2 marks for each criterion]

Kind of Decom-position	Time-Scale
Analyzing Function	Wave limited in time with fixed number of oscillations. Wavelet is contracted or dilated to change the window size and change the scale at which one looks at the signal. Since number of oscillations is fixed, frequency of the wavelet changes as scale changes.
Variable	Scale, position of the wavelet
Information	Small wavelets provide good time information but poor frequency information. Vice versa for large wavelets.
Suited for	Nonstationary signals, such as brief signals and signals with components at different time scales.

b) Two sets of functions are given by:

$\{\Psi_1\}$=[(2,0),(a,2)]

$\{\Psi_2\}$=[(-16,4),(0,b)]

State the condition required for these two sets to be a Dual Basis and determine the corresponding values of a and b.

[8 marks]

Answer:

For a dual basis, the Kronecker delta must be satisfied to show that the bases are mutually orthogonal. [1 mark]

$$\left\langle \Psi_i, \hat{\Psi}_j \right\rangle = \sum_k \Psi_i(k) \hat{\Psi}_j(k) = \delta_{ij}$$ [1 mark]

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$$\psi_1 = [(2,0),(a,2)]$$
$$\psi_2 = [(-16,4),(0,b)]$$

$$<\psi_{11}\psi_{12}> = \begin{bmatrix} 2 & 0 \end{bmatrix}\begin{bmatrix} a \\ 2 \end{bmatrix} = 2a = 1 \text{ IF } a = \frac{1}{2}$$

[1 MARK]

$$<\psi_{11}\psi_{22}> = \begin{bmatrix} 2 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ b \end{bmatrix} = 0 \text{ [1 MARK]}$$

$$<\psi_{12}\psi_{21}> = \begin{bmatrix} a & 2 \end{bmatrix}\begin{bmatrix} -16 \\ 4 \end{bmatrix} = -16a + 8 = 0$$
$$\text{IF } a = \frac{1}{2}$$

[1 MARK]

$$<\psi_{21}, \psi_{22}> = \begin{bmatrix} -16 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ b \end{bmatrix} = 4b = 1 \text{ IF } b = \frac{1}{4}$$

[1 MARK]

SO, $a = \frac{1}{2}$, $b = \frac{1}{4}$ FOR DUAL BASIS

[2 MARKS]

(c) Explain why it is desirable for a transform to have orthogonal basis functions, and explain what needs to be done if they are not orthogonal.

[7 marks]

Answer:

Any vector in a vector space can be represented by a combination of other vectors in that space [1 mark]. If this set of vectors for the vector space is orthogonal, then any other vector in that space can be represented by a unique set of coefficients [1 mark]. This set of vectors then forms a basis function [1 mark]. This is then used to perform the transform [1 mark]. The same set of orthogonal vectors can then be used to invert the transform [1 mark]. If they are not orthogonal, then another set of vectors that are mutually orthogonal to the first must be found to allow the inverse transform [1 mark]. These two sets of vectors are referred to as a biorthogonal set [1 mark]

Question 2

(a) The Discrete Cosine Transform (DCT) is related to the Fourier Transform, and has several advantages. State and briefly explain THREE advantages of the DCT.

[6 marks]

Answer:

The DCT is reversible (invertible) because the basis functions are orthogonal. [2 marks]

Only cosine functions are used so all calculations are real, not complex. [2 marks]

Implementation is fast because only real calculations. [2 marks]

(b)

(i) Explain what is meant by a transform being *separable*.

(ii) The DCT is separable. Explain how this is implemented when a DCT is applied to a 2D image.

[6 marks]

Answer:

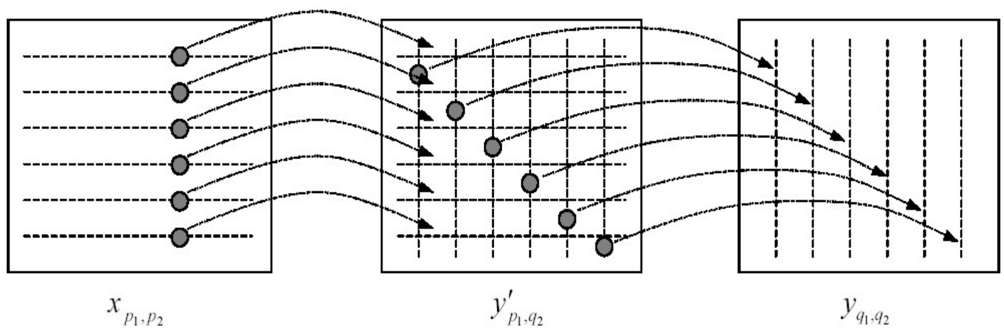
(i) An nD transform is separable if it can be carried out as a series of n 1D transforms. [2 marks]

(ii) Each row of the 2D image is transformed individually as a1D transform and then the columns of this 1D transform are transformed to produce the 2D transformed data. [2 marks]

[2 marks for the following diagram}

Separable Transforms

May be implemented by applying the one dimensional transform first to the rows of the image and then to its columns (note that changing the application order does not change the result).

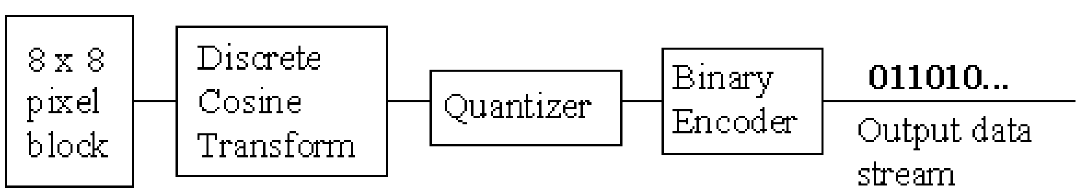


(c) The DCT is used for JPEG compression. Use a suitable diagram to explain to explain this process.

[7 marks]

Answer:

The image is divided into 8x8 (64 pixel) blocks and each block transformed separately.[1 mark]
The transformed image is quantised to reduce the precision of the DCT coefficients.[1 mark]
Coding is carried out to convert colour information from RGB to YUV.[1 mark]



[4 marks for the diagram:1 for each block]

(d) An FFT is a fast algorithm for implementing a DFT. One FFT structure is radix-2 decimation-in-time. Illustrate this FFT structure using the following 8-point sequence:

S[n] = [2, 6, 3, 9, 7, 4, 1, 11]

[6 marks]

Answer:

S[n] = [, 6, 3, 9, 7, 4, 1, 11]

STEP 1: [2 3 7 1][6 9 4 11]

STEP 2: [2 7][3 1][6 4][9 11]

STEP 3: [2][7][3][1][6][4][9][11]

[6 marks: 2 for each step]

Question 3

(a) The Karhunen-Loeve Transform (KLT) is used for data compression, and maximises the quality of compression for any given level of compression. Briefly explain how it achieves this.

[4 marks]

Answer:

The KLT uses principal component analysis (PCA) [1 mark] to statistically decouple multidimensional data by computing the covariance matrix [1 mark]. The eigenvectors of this covariance matrix are then the basis vectors for transforming the data [1 mark]. The transformed data on the least principal component can then be discarded [1 mark],

(b) Evaluate the normalised eigenvectors of the covariance matrix of the following 2D dataset.

[21 marks]

x	y
-3.01	-2.67
0.18	-2.12
-6.56	-4.24
1.60	1.92
2.78	-1.20
2.03	2.14

Answer:

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$\bar{x} = -0.50$ [1 MARK]

$\bar{y} = -1.03$ [1 MARK]

$$\text{VAR IN } x = \frac{\sum_{1}^{N} (x-\bar{x})^2}{N-1} = 13.00 \quad \text{[2 MARKS]}$$

$$\text{VAR IN } y = \frac{\sum_{1}^{N} (y-\bar{y})^2}{N-1} = 6.60 \quad \text{[2 MARKS]}$$

$$\text{COV}(x,y) = \frac{\sum_{1}^{N} (x-\bar{x})(y-\bar{y})}{N-1} = 7.30 \quad \text{[2 MARKS]}$$

$$\text{COV. MATRIX} = \begin{bmatrix} 13.00 & 7.30 \\ 7.30 & 6.60 \end{bmatrix} \quad \text{[1 MARK]}$$

FOR EIGENVALUES: $\left| \begin{bmatrix} 13.00 & 7.30 \\ 7.30 & 6.60 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$$\therefore \begin{vmatrix} (13.00-\lambda) & 7.30 \\ 7.30 & (6.60-\lambda) \end{vmatrix} = 0 \quad \text{[2 MARKS]}$$

$$\lambda^2 - 19.6\lambda + 32.51 = 0$$

$$\therefore \lambda = 17.77,\ 1.83 \quad \text{[2 MARKS]}$$

FOR EIGENVECTORS:

$$\lambda_1 = 17.77 \quad \begin{bmatrix} -4.77 & 7.30 \\ 7.30 & -11.17 \end{bmatrix} \begin{bmatrix} \phi_{11} \\ \phi_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore [\phi_1] = \begin{bmatrix} 1.53 \\ 1.00 \end{bmatrix} \quad \text{[3 MARKS]}$$

$$\lambda_2 = 1.83 \quad \begin{bmatrix} 11.17 & 7.30 \\ 7.30 & 4.77 \end{bmatrix} \begin{bmatrix} \phi_{21} \\ \phi_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore [\phi_2] = \begin{bmatrix} -0.65 \\ 1.00 \end{bmatrix} \quad \text{[3 MARKS]}$$

$$\text{NORMALISE } \phi_1 = \frac{1}{\sqrt{3.34}} \begin{bmatrix} 1.53 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 0.84 \\ 0.55 \end{bmatrix} \quad \text{[1 MARK]}$$

$$\text{NORMALISE } \phi_2 = \frac{1}{\sqrt{1.42}} \begin{bmatrix} -0.65 \\ 1.00 \end{bmatrix} = \begin{bmatrix} -0.55 \\ 0.84 \end{bmatrix} \quad \text{[1 MARK]}$$

Question 4

(a) Explain what is meant by *multiresolution analysis*. Illustrate this concept with a diagram showing *piecewise approximation* of a signal.

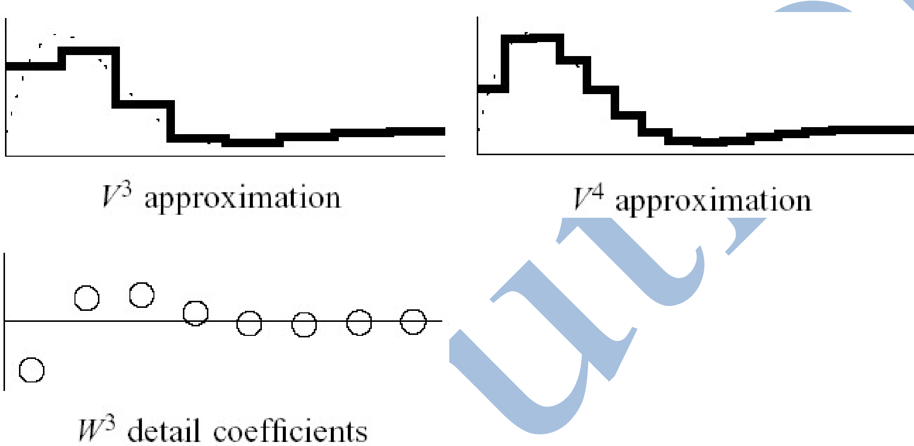
[4 marks]

Answer

The basic concept is to decompose a fine-resolution signal into

A coarse-resolution version of the signal, [1 mark]

And the differences left over. [1 mark]



The fine resolution signal on the right is decomposed into the coarse resolution signal and fine detail shown on the left. [2 marks: 1 for each decomposition]

(b) Multiresolution Analysis (MRA) is used to separate data into course and fine detail.

Apply the transform defined by

$$x_{n-1,i} = (x_{n,2i}+x_{n,2i+1})/2$$
$$d_{n-1,i} = (x_{n,2i}-x_{n,2i+1})/2$$

to the sequence

$$[x_{n,i}] = [6, 7, 3, 4, 8, 2, 1, 9]$$

Where i = 0…..7, is the index position in the sequence, and
n is the level. The next level is n-1.
At each level, calculate the sequencies for x_{n-1},i and d_{n-1},i
Continue till no further levels are possible.

i) State the significance of the first element in the final level.

ii) Has any information been lost in the process?

iii) Comment on how this process could be used to compress the data.

[12 marks]

Answer:

Applying the transform:

n=3	[6, 7, 3, 4, 8, 2, 1, 9]
n=2	[6.5, 3.5, 5.0, 5.0, -0.5, -0.5, 3.0, -4.0]
n=1	[5.0, 5.0, 1.5, 0.0, -0.5, -0.5, 3.0, -4.0]
n=0	[5.0, 0.0, 1.5, 0.0, -0.5, -0.5, 3.0, -4.0]

[6 marks: 2 for each row]

i) The first element is the average of all the elements in the original sequence [1 mark].
ii) No information has been lost [1 mark]
iii) Because most of the values in the final level are small, potentially fewer bits would be required to store it [1 mark]. Where there are zeroes, they do not need to be stored, although the positions of the other values would need to be stored [1 mark]. Small values could be replaced by zeroes without significant loss of detail [1 mark], this can be done by applying a threshold value, the bigger the threshold the greater the loss of detail [1 mark].

(b) (i) With the aid of a diagram, compare the time-frequency tiling of the Wavelet Transform (WT) with that of the short-time Fourier transform (STFT).

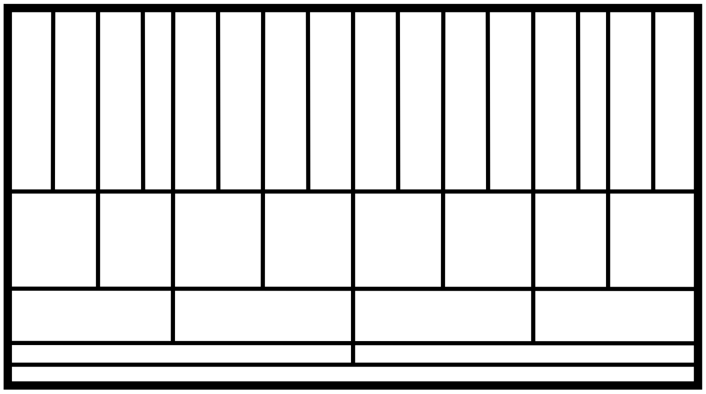
[4 marks]

(ii) Explain the difference between the tiling of the WT and the STFT..

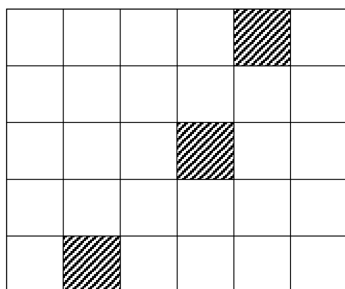
[5 marks]

Answer:

i) Time-frequency tiling of wavelet transform:



Compare t-f tiling of STFT:



[4 marks: 2 marks for each diagram]

(ii) For the STFT, the window is fixed width, so that Δ_t is constant [1 mark]. Because of the uncertainty principle, $\Delta_t.\Delta_f = k$, therefore Δ_f is also fixed [1 mark].

However for the WT, the wavelet function is scalable [1 mark] and therefore can be compressed resulting in Δ_t being variable [1 mark]. This results in Δ_f being inversely proportional to Δ_t [1 mark].