作 业

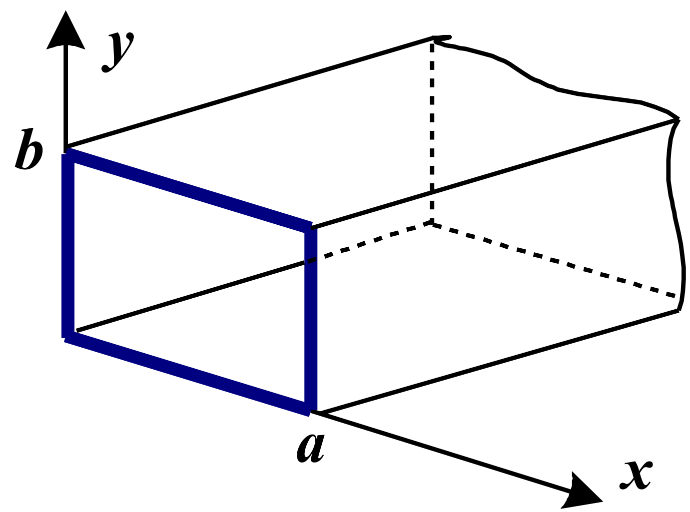
已知矩形理想金属波导(xy方向为矩形, z向无穷)内传输的电场和磁场分别为: $\vec{E}=\vec{a}_y E_y \qquad \vec{H}=\vec{a}_x H_x+\vec{a}_z H_z$

其中: $$E_y=-j\omega\mu\cdot\frac{\pi}{a}\cdot H_0\cdot\sin\left(\frac{\pi}{a}\cdot x\right)$$

$$H_x=j\beta\cdot\frac{\pi}{a}\cdot H_0\cdot\sin\left(\frac{\pi}{a}\cdot x\right)$$

$$H_z=H_0\cdot\cos\left(\frac{\pi}{a}\cdot x\right)$$

H_0、ω、μ、β 是常数

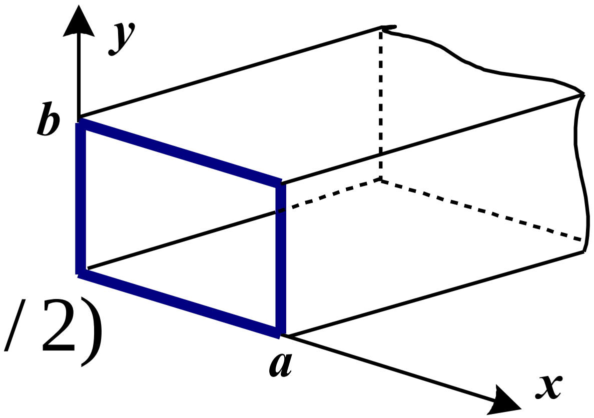


求: 内部的金属四壁的

(1)面电荷密度

(2)面电流密度

(1)面电荷密度



$$E_y = -j\omega\mu \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right)$$

$$E_y = \omega\mu \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) e^{jwt - j\pi/2}$$

$$E_y(t) = \omega\mu \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$$

$$D_y(t) = \varepsilon_0\omega\mu \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$$

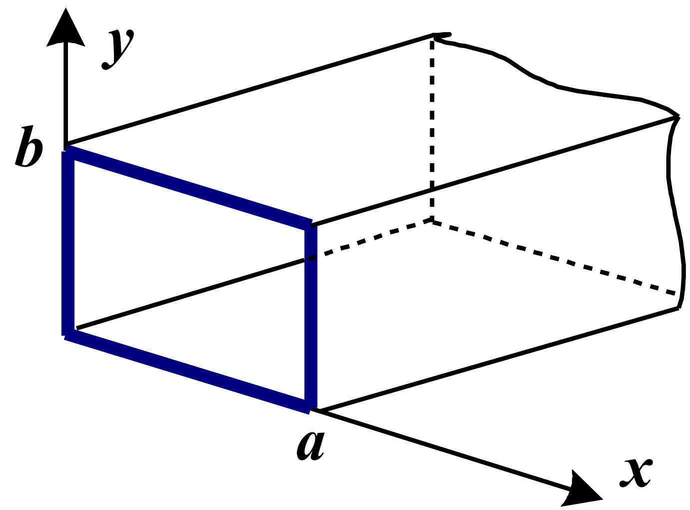
对于理想导体边界 $D_n = \rho_S$

At x=0&a $\rho_S = D_n = 0$

At y=0 $\rho_{S,y=0} = D_y(t) = \varepsilon_0\omega\mu \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$

$$\rho_{S,y=b} = -D_y(t) = -\varepsilon_0\omega\mu \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$$

(2)面电流密度



$$H_x = j\beta \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right)$$

$$H_z = H_0 \cdot \cos\left(\tfrac{\pi}{a} \cdot x\right)$$

$$\vec{H} = \vec{a}_x H_x + \vec{a}_z H_z$$

$$= \vec{a}_x \beta \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right)\cos(wt + \tfrac{\pi}{2})$$

$$+\vec{a}_z H_0 \cdot \cos\left(\tfrac{\pi}{a} \cdot x\right)\cos(wt)$$

在理想导体边界 $\vec{a}_n \times \vec{H} = \vec{J}_{ST}$

At x=0 $\vec{a}_n = \vec{a}_x$

$$\vec{J}_{ST,x=0} = \vec{a}_n \times \vec{H} = \vec{a}_x \times (\vec{a}_x H_x + \vec{a}_z H_z) = -\vec{a}_y H_z = -\vec{a}_y H_0$$

At x=a $\vec{a}_n = -\vec{a}_x$

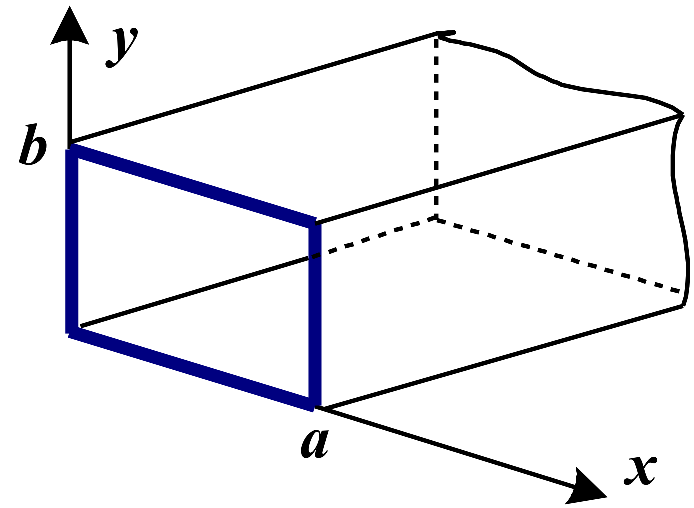
$$\vec{J}_{ST,x=a} = \vec{a}_n \times \vec{H} = -\vec{a}_x \times (\vec{a}_x H_x + \vec{a}_z H_z) = \vec{a}_y H_z = -\vec{a}_y H_0$$

(2)面电流密度

$$\vec{H} = \vec{a}_x H_x + \vec{a}_z H_z$$

$$= \vec{a}_x \beta \cdot \tfrac{\pi}{a} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt + \tfrac{\pi}{2})$$

$$+ \vec{a}_z H_0 \cdot \cos\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt)$$



在理想导体边界 $\vec{a}_n \times \vec{H} = \vec{J}_{ST}$

At y=0 $\vec{a}_n = \vec{a}_y$

$$\vec{J}_{ST,y=0} = \vec{a}_n \times \vec{H} = \vec{a}_y \times (\vec{a}_x H_x + \vec{a}_z H_z) = -\vec{a}_z H_x + \vec{a}_x H_z$$

At y=b $\vec{a}_n = -\vec{a}_y$

$$\vec{J}_{ST,y=b} = \vec{a}_n \times \vec{H} = -\vec{a}_y \times (\vec{a}_x H_x + \vec{a}_z H_z) = \vec{a}_z H_x - \vec{a}_x H_z$$

题目修正：

其中：

$$E_y = -j\omega\mu \cdot \boxed{\frac{\pi}{a}} \cdot H_0 \cdot \sin\left(\frac{\pi}{a} \cdot x\right)$$

$$H_x = j\beta \cdot \boxed{\frac{\pi}{a}} \cdot H_0 \cdot \sin\left(\frac{\pi}{a} \cdot x\right)$$

修改为：

$$E_y = -j\omega\mu \cdot \boxed{\frac{a}{\pi}} \cdot H_0 \cdot \sin\left(\frac{\pi}{a} \cdot x\right)$$

$$H_x = j\beta \cdot \boxed{\frac{a}{\pi}} \cdot H_0 \cdot \sin\left(\frac{\pi}{a} \cdot x\right)$$

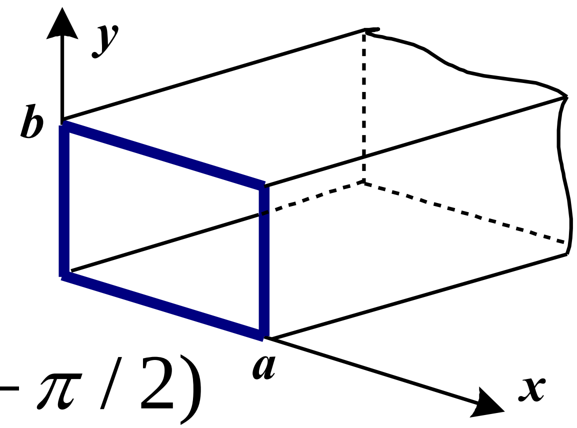
答案修正：

$$E_y = -j\omega\mu \cdot \tfrac{a}{\pi} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right)$$

$$E_y = \omega\mu \cdot \tfrac{a}{\pi} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) e^{jwt - j\pi/2}$$

$$E_y(t) = \omega\mu \cdot \tfrac{a}{\pi} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$$

$$D_y(t) = \varepsilon_0 \omega\mu \cdot \tfrac{a}{\pi} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$$



对于理想导体边界：

At x=0&a $\rho_S = D_n = 0$

At y=0 $\rho_{S,y=0} = D_y(t) = \varepsilon_0 \omega\mu \cdot \boxed{\tfrac{a}{\pi}} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$

$\rho_{S,y=b} = -D_y(t) = -\varepsilon_0 \omega\mu \cdot \boxed{\tfrac{a}{\pi}} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt - \pi/2)$

$$H_x = j\beta \cdot \tfrac{a}{\pi} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right)$$

$$H_z = H_0 \cdot \cos\left(\tfrac{\pi}{a} \cdot x\right)$$

$$\vec{H} = \vec{a}_x H_x + \vec{a}_z H_z$$

$$= \vec{a}_x \beta \cdot \boxed{\tfrac{a}{\pi}} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right)\cos(wt + \tfrac{\pi}{2})$$

$$+\vec{a}_z H_0 \cdot \cos\left(\tfrac{\pi}{a} \cdot x\right)\cos(wt)$$

对于理想导体边界： $\vec{a}_n \times \vec{H} = \vec{J}_{ST}$

At x=0 $\vec{a}_n = \vec{a}_x$

$$\vec{J}_{ST,x=0} = \vec{a}_n \times \vec{H} = \vec{a}_x \times (\vec{a}_x H_x + \vec{a}_z H_z) = -\vec{a}_y H_z = -\vec{a}_y H_0$$

At x=a $\vec{a}_n = -\vec{a}_x$

$$\vec{J}_{ST,x=a} = \vec{a}_n \times \vec{H} = -\vec{a}_x \times (\vec{a}_x H_x + \vec{a}_z H_z) = \vec{a}_y H_z = -\vec{a}_y H_0$$

$$\vec{H} = \vec{a}_x H_x + \vec{a}_z H_z$$

$$= \vec{a}_x \beta \cdot \boxed{\tfrac{a}{\pi}} \cdot H_0 \cdot \sin\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt + \tfrac{\pi}{2})$$

$$+ \vec{a}_z H_0 \cdot \cos\left(\tfrac{\pi}{a} \cdot x\right) \cos(wt)$$

在理想导体边界 $\vec{a}_n \times \vec{H} = \vec{J}_{ST}$

At y=0 $\quad \vec{a}_n = \vec{a}_y$

$$\vec{J}_{ST,y=0} = \vec{a}_n \times \vec{H} = \vec{a}_y \times (\vec{a}_x H_x + \vec{a}_z H_z) = -\vec{a}_z H_x + \vec{a}_x H_z$$

At y=b $\quad \vec{a}_n = -\vec{a}_y$

$$\vec{J}_{ST,y=b} = \vec{a}_n \times \vec{H} = -\vec{a}_y \times (\vec{a}_x H_x + \vec{a}_z H_z) = \vec{a}_z H_x - \vec{a}_x H_z$$