**5.10** Repeat Example 5.7 when both the conductors carry currents in the $z$ direction.

Exercise 5.10 $\vec{B} = \left[ -\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(b-y)} \right] \vec{a}_x$ , $\vec{ds} = -dy\,dz\,\vec{a}_x$

$$\Phi = \int_s \vec{B}.\vec{ds} = \frac{\mu_0 I}{2\pi} \int_a^{b-a} \frac{1}{y} dy \int_0^L dz + \frac{\mu_0 I}{2\pi} \int_a^{b-a} \frac{1}{y-b} dy \int_0^z dz$$

$$= \frac{\mu_0 I L}{2\pi} \ln\left(\frac{b-a}{a}\right) + \frac{\mu_0 I L}{2\pi} \ln\left(\frac{a}{b-a}\right) = 0$$

**5.12** If $\vec{\mathbf{B}} = 12x\vec{\mathbf{a}}_x + 25y\vec{\mathbf{a}}_y + cz\vec{\mathbf{a}}_z$, find $c$.

Exercise 5.12 $\vec{B} = 12x\,\vec{a}_x + 25y\,\vec{a}_y + cz\,\vec{a}_z$

$\nabla.\vec{B} = 12 + 25 + c$. Since $\nabla.\vec{B}$ must be zero, $c = -37$

**5.14** Determine the total flux enclosed in Example 5.10 using the magnetic flux density in the region within the conductors.

Exercise 5.14 $\vec{B} = \frac{\mu_0 I}{2\pi\rho} \vec{a}_\phi$ $\vec{ds} = d\rho\,dz\,\vec{a}_\phi$ $I = 80A$

$$\Phi = \int_s \vec{B}.\vec{ds} = \frac{\mu_0 I}{2\pi} \int_{0.01}^{0.1} \frac{1}{\rho} d\rho \int_0^{100} dz = \frac{100\,\mu_0 I}{2\pi} \ln(10) = 3.68 \text{ mWb}$$

**5.15** A short, straight conductor of length $L$ carries a current $I$ in the $z$ direction. Show that the magnetic vector potential at a point far away from the conductor is

$$\vec{\mathbf{A}} = \frac{\mu_0 IL}{4\pi R} \vec{\mathbf{a}}_z$$

where $R$ is the distance of the point of observation from the origin. What is the magnetic flux density at that point?

Exercise 5.15

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int_{-L/2}^{L/2} \frac{dz}{R}\vec{a}_z = \frac{\mu_0 I}{4\pi R} L \vec{a}_z \text{ when } R \gg L. \quad (\text{Note: } R \cong r)$$

$$= \frac{\mu_0 I}{4\pi R} L \cos\theta \, \vec{a}_r - \frac{\mu_0 I L}{4\pi R}\sin\theta \, \vec{a}_\theta$$

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & r\sin\theta\,\vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\mu_0 I}{4\pi r} L\cos\theta & -\frac{\mu_0 I L}{4\pi}\sin\theta & 0 \end{vmatrix} \frac{1}{r^2 \sin\theta} = \frac{\mu_0 I L}{4\pi r^2}\sin\theta \, \vec{a}_\phi$$