§ 2.5 Electric Potential

- **Why do we introduce such a parameter?**
- Static E-field is a conservative field.
- This field has divergence but not curl. $\nabla\times\vec{E}=0$
- Recall related *Identical Equation for a conservative field*
  - The curl of a scalar's gradient is always zero.

$$\nabla\times\nabla U\equiv 0$$

  - It's reversible. If the curl of a vector field is 0, the vector must be a scalar's gradient.

$$\nabla\times\vec{F}\equiv 0 \quad\Rightarrow\quad \vec{F}=\nabla U$$

$$\because \nabla\times\vec{E}=0 \longrightarrow \vec{E}=-\nabla\psi$$

1. Static E-field is a conservative field

$$\because \nabla\times\vec{E}=0 \qquad \therefore \exists U, \quad \vec{E}=\nabla U$$

2. Thus we define a scalar function: ψ

$$\vec{E}=\underline{-}\nabla\psi$$

Negative gradient of this scalar is E-intensity.

3. In Cartesian Coordinates

$$\vec{E}=-\nabla\psi \qquad \nabla\psi=\left\{\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y},\frac{\partial\psi}{\partial z}\right\}$$

$$\vec{E}=-\left(\vec{a}_x\frac{\partial\psi}{\partial x}+\vec{a}_y\frac{\partial\psi}{\partial y}+\vec{a}_z\frac{\partial\psi}{\partial z}\right)$$

How about gradient in other coordinate?

Gradient in different coordinates

Cartesian Coordinates

$$\nabla=\vec{a}_x\frac{\partial}{\partial x}+\vec{a}_y\frac{\partial}{\partial y}+\vec{a}_z\frac{\partial}{\partial z}$$

Cylindrical Coordinates

$$\nabla=\vec{a}_r\frac{\partial}{\partial r}+\vec{a}_\varphi\frac{1}{r}\frac{\partial}{\partial\varphi}+\vec{a}_z\frac{\partial}{\partial z}$$

Spherical Coordinates

$$\nabla=\vec{a}_R\frac{\partial}{\partial R}+\vec{a}_\theta\frac{1}{R}\frac{\partial}{\partial\theta}+\vec{a}_\varphi\frac{1}{R\cdot\sin\theta}\frac{\partial}{\partial\varphi}$$

4. Potential Difference Between Spot A & B

$$\psi_B - \psi_A = \int_B^A \vec{E} \bullet d\vec{l} \qquad \vec{E} = -\nabla \psi$$

Physical Meaning:

$$\psi_B - \psi_A = \int_B^A (1\vec{E}) \bullet d\vec{l}$$

Work by electrostatic force when moving unit charge from B to A.

This work is related only to the starting- & ending spots, regardless of the path.

Similar to that by gravity.

5. Reference Potential

$$\psi_B - \psi_A = \int_B^A \vec{E} \bullet d\vec{l} \qquad \text{Let} \quad \psi_A = 0$$

$$\boxed{\therefore \psi_B = \int_B^{\text{Ref. Spot}} \vec{E} \bullet d\vec{l}}$$

$$\psi_\infty = 0$$

Usually, we assume potential at infinity as 0.

6. Potential in a Field of a Point Charge

$$\psi_P = \int_P^\infty \vec{E} \bullet d\vec{l} = \int_P^\infty \frac{q}{4\pi\varepsilon_0 R^2} dR = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{R}$$

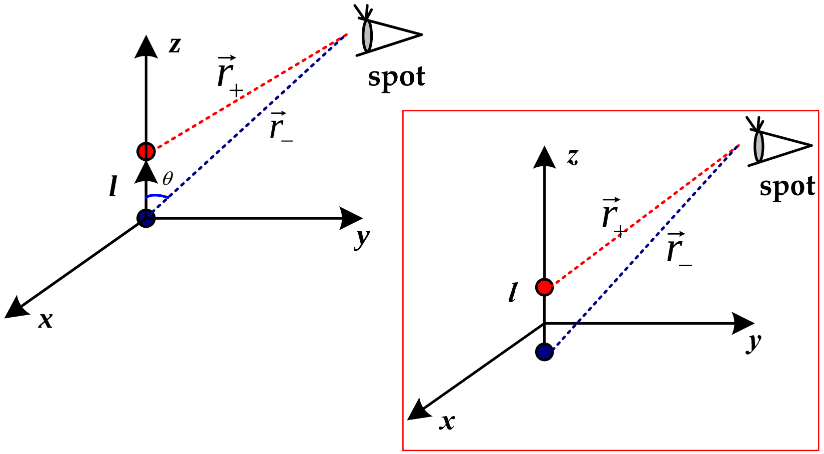
$$\boxed{\psi_P = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{R}}$$

Potential in Complicate Distribution of Charges

$$\boxed{\psi = \int \frac{dq}{4\pi\varepsilon R}}$$

- *dq* should be obtained according to the distribution of charges.
  - ***Bulk*** charges, ***surface*** charges, ***line*** charges——integral
  - Scattering charges——sum
- If in space, use ε_0 in above equation.
- If E-Intensity is known, just apply $\psi_B = \int_B^\infty \vec{E} \bullet d\vec{l}$

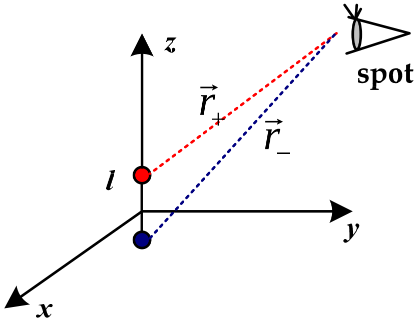
Example 1. Potential in Field of Electric Dipole



Scattering charges——sum

$$\psi = \frac{q}{4\pi\varepsilon_0}\cdot\left(\frac{1}{r_+}+\frac{-1}{r_-}\right)$$

In spherical coordinates



Summary of Potential Calculation

$$\boxed{\therefore \psi_B = \int_B^{\text{Ref. Spot}} \vec{E}\bullet d\vec{l}}$$

$$\boxed{\psi_B = \int_B^{\infty} \vec{E}\bullet d\vec{l}}$$

$$\boxed{\psi_P = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{R}}$$

$$\boxed{\psi = \int \frac{dq}{4\pi\varepsilon R}}$$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——Now, let’s go on.