SOLUTIONS

Module:	Telecoms Systems		
Module Code	EBU5302	Paper	A
Time allowed	2hrs	Filename	Solutions_1516-1_EBU5302_A
Rubric	ANSWER ALL FOUR QUESTIONS		
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Question 1

Let $x(t)$ be a band-limited signal to W = 2 kHz, amplitude $0 \leq x(t) \leq 2$ and power $P = 1$. Signal $x(t)$ is sampled at a rate 20% higher than the Nyquist rate to provide a guard band. The maximum acceptable error in the sample amplitude (the maximum quantization error) is 0.5% of the peak amplitude. The quantized samples are binary coded.

Assume "Sr" is an M=8 symbol source. Symbol A.... H represent each of the symbol amplitude values generated by the quantiser. The probability p_m of each symbol is shown in the following table:

m	A	B	C	D	E	F	G	H
P(m)	0.3	0.1	0.06	0.25	0.04	0.05	0.18	0.02

a) Using diagrams to explain why in general sampling has to meet the Nyquist sampling theorem.

[4 marks]

b) Illustrate what is the sample rate for $x(t)$.

[1 mark]

c) Find the minimum bandwidth of a channel required to transmit the encoded binary signal. **[6 marks]**

d) If 24 such signals are time-division-multiplexed, determine the minimum transmission bandwidth required to transmit the multiplexed signal.

[1 marks]

e) What is the information content for each symbol of Sr?

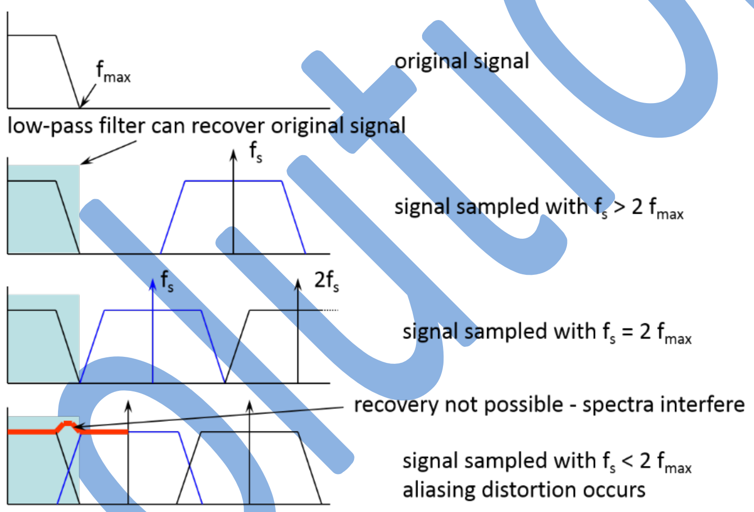
[9 marks]

f) What are the source entropy and source efficiency for Sr?

[4 marks]

Answer

a) Nyquist rate is defined as twice the signal bandwidth W for low-pass band signal. [4 marks]



b) The Nyquist sampling rate for $x(t)$ is R_N = 2* 2000 = 4000 Hz (samples per second). The actual sampling rate is Rs = 4000*1.2=4800 Hz. [1 mark]

c) The quantization step is q, and the maximum quantization error is ±q/2.
Therefore

q/2 =0.5% *2, [1 mark]

So quantization level M = 100, [1 mark]

For binary coding, L must be a power of 2. Hence, the next higher value of L that is a power of 2 is L=128. [1 mark]

So we need $n=\log_2 128 = 7$ bits per sample. [1 mark]

As we required to transmit a total of C= 7*4800=33,600 bit/s. [1 mark]

Because for binary, we can transmit up to 2 bits per hertz of bandwidth, we require a minimum transmission bandwidth B_T=C/2=16.8kHz. [1 mark]

d) Multiplexed signal has a total of C_M= 24*33,600 = 0.806 Mbit/s, which requires a minimum of 0.806/2= 0.403 MHz of transmission bandwidth. [1 mark]

e) The information content I of symbol is defined as

$$I = \log_2(1/p)$$

[1 mark]

so

m	A	B	C	D	E	F	G	H
I_m	1.74	3.32	4.06	2	4.64	4.32	2.47	5.64

[8 marks]

f) The entropy is defined as

- $H = \sum_i p_i \log_2(1/p_i)$ [1 mark]

The resulting entropy will then be H = 2.55 bits/symbol. [1 mark]

For an information source that produces 8 symbols with the same probability (uniform information source), the entropy is H_{max} = log 8 = 3. [1 mark]

So the source efficiency is 2.55/3= 85%. [1 mark]

Question 2

A digital information source produces binary sequences at a rate of 5 kbps. The probability of producing the value 0 is $p_0 = 0.2$. A Hamming code with the following parity check matrix **H** is employed to protect information against errors:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The resulting binary sequences are transmitted through a wireless channel where power falloff with distance follows the formula $P_r(d) = P_t(d_0/d)^3$ for d_0=10m. Assume the channel has bandwidth B=30 kHz and AWGN with noise PSD (power spectral density) $N_0/2$, where $N_0 = 10^{-9}$ W/Hz.

a) For a transmit power of 1 W, find the capacity of this cannel for a transmit-receive distance of 100m and 1km.

[6 marks]

b) Based on the parity check matrix **H**, determine the length of the input information sequences and the length of the code words. Calculate the code rate of this Hamming code and the resulting transmission rate. **[4 marks]**

c) How can the systematic linear block code words of this Hamming code be obtained? Calculate the code words corresponding to the information sequences 0110 and 1010.

[5 marks]

d) Determine the number of errors can be detected and corrected in this Hamming code.

[5 marks]

e) Decode the following received sequence **r** = 1111010.

[5 marks]

Answer

a) The received $SNR = P_r(d)/ N_0 B$ [1 mark] and $C = B \log_2 (1+SNR)$ [1 mark]
For d_1=100m, $SNR_1 = (10/100)^3 / (10^{-9}*30*10^3)$=33= 15 dB [1 mark]
$C_1 = 30000 \log_2 (1+33) = 152.6$ kbps [1 mark]
For d_2=1km, $SNR_1 = (10/1000)^3 / (10^{-9}*30*10^3)$=0.033= -15 dB [1 mark]
$C_2 = 30000 \log_2 (1+0.033) = 1.4$ kbps [1 mark]

b) The dimensions of the parity check matrix are m x n, where n is the length of a code word, $m = n - k$, and k is the length of information sequences. [1 mark]

This is then a (7,4) Hamming code and its code rate is R_C =4/7. [1 mark]

The resulting transmission rate can be obtained as R_B = 5kbps*1/R_c=5000 x 7/4= 8.75 kbps. [2 marks]

c) Based on the parity check matrix **H**, we first obtain the matrix **P**:

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ [1 mark]}$$

The generator matrix **G** of a systematic linear block code will then be

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \text{ [1 mark]}$$

Code words **c** can be obtained by multiplying each 4-bit information sequence **x** by the generator matrix **G**, **c** = **xG**. [1 mark]

By using this expression, the code words corresponding to the sequences 0100 and 1000 are, respectively, 0110100 and 1010001. [2 marks]

d) Hamming codes belong to the family of linear block codes. Hence, the minimum distance can be obtained as the minimum weight (except all-zero codeword), where the weight of a code word is defined as the number of bits of value 1 in each sequence. [2 marks]

The main property of Hamming codes is that their minimum distance is always 3. [1 mark]

d_{min}>e+t+1, Since the minimum distance is 3, up to 2 errors will be detected or 1 error will be corrected. [2 marks]

e) In order to decode the received sequence, we first compute its syndrome **s** =r$\mathbf{H}^T$ =e$\mathbf{H}^T$. [1 mark]

The syndrome sequence corresponding to 1111010 is **s** = 100. [1 mark]

The error sequence corresponding to this syndrome is **e** = 0000100. [1 mark]

Hence, the transmitted code word is **c** = 1111010 + 0000100 = 1111110. [1 mark]

This code word corresponds to the information sequence **x** =1111. [1 mark]

Question 3

a) A multilevel digital communication system sends one of 16 possible levels over the channel every 0.8 ms.

i) What is the number of bits corresponding to each level?

ii) What is the baud (Symbol) rate?

iii) What is the bit rate?

[6 marks]

Solutions:

(a) $L = 2^{\ell} = 16 \Rightarrow$ $\underline{\underline{\ell = 4 \text{ bits/level}}}$

(b) $D = \frac{N}{T} = \frac{1 \text{ symbol}}{0.8 \times 10^{-3} \text{ sec}} = \underline{\underline{1,250 \text{ baud}}}$

(c) $R = \ell D = 4(1,250) = \underline{\underline{5 \text{ kbits/sec}}}$

b) Multilevel data with an equivalent bit rate of 2,400 bits/s is sent over a channel using a four level line code that has a rectangular pulse shape at the output of the transmitter. The overall transmission system (i.e. the transmitter, channel and receiver) has an r=0.5 raised cosine roll-off Nyquist filter characteristic.

i) Find the baud (symbol) rate of the received signal.

ii) Find the 6-dB bandwidth for this transmission system.

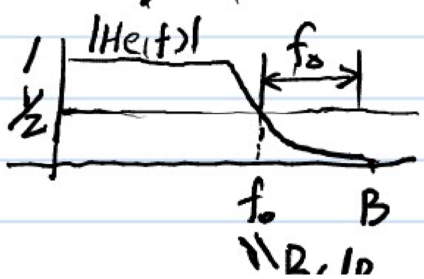
iii) Find the absolute bandwidth for the system.

[8 marks]

ANS: $L = 2^{\ell} = 4 \Rightarrow \ell = 2$ [2 marks]

i) $D = R/\ell = 2400/2 = 1200$ baud (

ii) $B = \frac{1}{2}(1+r)D$ where [2 marks] $r = f_\Delta / f_0$: [2 marks]



$\Rightarrow B_{6dB} = \frac{1}{2}(1+$ [2 marks]

c) The following table illustrates the operation of an FHSS system for one complete period of the PN sequence.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	1	0	1	0
Frequency	f_{11}	f_2	f_{11}	f_3	f_3	f_3	f_{22}	f_{10}	f_0	f_0	f_1	f_{22}	f_9	f_1	f_{23}	f_3	f_{22}	f_{11}	f_3	f_{31}
PN Sequence	001	110	011	001	001	001	110	011	001	001	001	110	011	001	001	001	110	011	001	001

To determine:

i) What is the period of the PN sequence?

ii) The system makes use of a form of FSK. What form of FSK is it?

iii) What is the number of bits per symbol?

iv) What is the number of FSK frequencies?

v) What is the length of a PN sequence per hop?

vi) Is this a slow or fast FH system?

vii) What is the total number of possible hops?

viii) Show the variation of the dehopped frequency with time.

[11 marks]

Answer:

i) Period of the PN sequence is $2^4 - 1 = 15$ [1 mark]
ii) MFSK [1 mark]
iii) $L = 2$ [1 mark]
iv) $M = 2^L = 4$ [1 mark]
v) $k = 3$ [1 mark]
vi) fast FHSS [1 mark]
vii) $2^k = 8$ [1 mark]
viii) [4 marks, each 2 for 1 mark]

Time	0	1	2	3	4	5	6	7	8	9	10	11
Input data	0	1	1	1	1	1	1	0	0	0	1	0
Frequency	f_1		f_3		f_3		f_2		f_0		f_2	

Time	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	0	1	0
Frequency	f_1		f_3		f_2		f_2	

Question 4

a) If the received signal level for a particular digital system is -151dBW and the receiver system effective noise temperature is 1500 K, what is E_b/N_0 for a link transmitting 2400bps?

[2 marks]

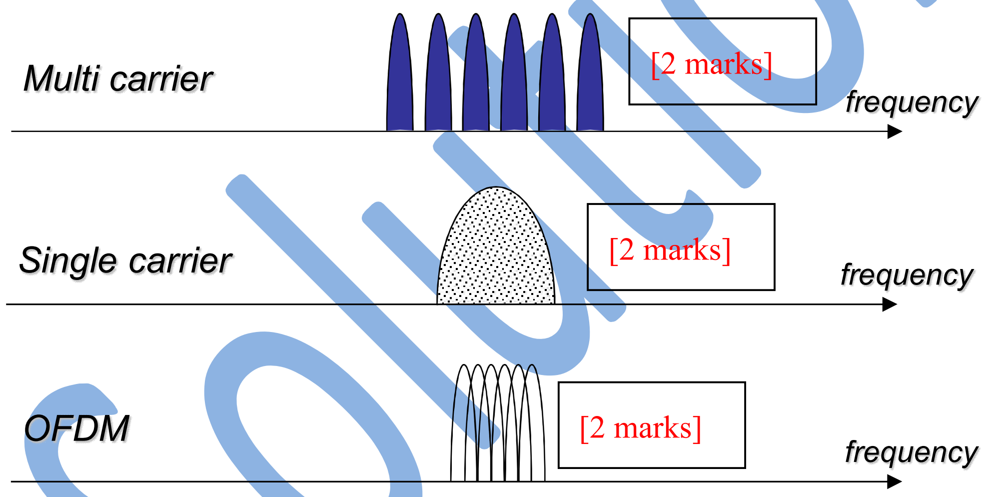
Answer:

$(E_b/N_0) = S_{dBw} - 10\log R - 10\log K - 10\log T$ **[1 mark]** $= -151$ dBW – 10 log 2400 – 10 log 1500 + 228.6 dBW = 12 dBW **[1 mark]**

b) Using diagrams and engineering terms to compare for same data rate transmission by using single carrier, multi-carrier and OFDM modulations, respectively.

[11 marks]

Solutions:



OFDM is multi carrier modulation [1 mark]

OFDM sub-carrier spectrum is overlapping [1 mark]

In FDMA, band-pass filter separates each transmission [1 mark]

In OFDM, each sub-carrier is separated by DFT because carriers are orthogonal [1 mark]

Each sub-carrier is modulated by PSK, QAM [1 mark]

c) Derive the power spectral density (PSD) equation for the polar NRZ signalling.

[12 marks]

ANS: For polar NRZ signaling, the possible le [3 marks]
are +A and −A. For equally likely occurr
−A, and assuming the data are indepe [2 marks]
bit. $R(0)=\sum_{i=1}^{2}(a_n a_n)P_i = A^2\frac{1}{2}+(-A)^2$ [2 marks]

For $k\neq 0$,
$R(k)=\sum_{i=1}^{4}(a_n a_{n+k})P_i = A^2\frac{1}{4}+(-A$ [2 marks]
$=0$

Thus, $R_{polar}(k)=\begin{cases} A^2 & k=0 \\ 0 & k\neq 0 \end{cases}$ [2 marks] [3 marks]

The general expression for the PSD of