Queen Mary
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EBU6018
Advanced Transform Methods

Tutorial: Filterbanks

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Question 1

Multiresolution Analysis (MRA) is used to separate data into course and fine details

Apply the transform defined by

$$x_{n-1},i = (x_{n,2i}+x_{n,2i+1})/2$$

$$d_{n-1},i = (x_{n,2i}-x_{n,2i+1})/2$$

to the sequence $[x_{n,i}] = [6, 8, 3, 11, 9, 5, 7, 2]$

Where i = 0…..7, is the index position in the sequence, and n is the level. The next level is n-1.

At each level, calculate the sequencies for x_{n-1},i and d_{n-1},i. Continue till no further levels are possible.

Question 1

n=3 [6, 8, 3, 11, 9, 5, 7, 2]

n=2 [7, 7, 7, 4.5, -1, -4, 2, 2.5]

n=1 [7, 5.75, 0, 1.25, -1, -4, 2, 2.5]

n=0 [6.375, 0.625, 0, 1.25, -1, -4, 2, 2.5]

Question 2

Multiresolution Analysis (MRA) is used to separate data into course and fine details

Apply the transform defined by

$$x_{n-1},i = (x_{n,2i}+x_{n,2i+1})/2$$

$$d_{n-1},i = (x_{n,2i}-x_{n,2i+1})/2$$

to the sequence $[x_{n,i}] = [12, 8, 13, 10, 6, 7, 11, 14]$

Where i = 0…..7, is the index position in the sequence, and n is the level. The next level is n-1.

At each level, calculate the sequencies for x_{n-1},i and d_{n-1},i. Continue till no further levels are possible.

i) State the significance of the first element in the final level.

ii) Has any information been lost in the process?

iii) Comment on how this process could be used to compress the data.

Question 2

Applying the transform:

n=3 [12, 8, 13, 10, 6, 7, 11, 14]

n=2 [10, 11.5, 6.5, 12.5, 2, 1.5, -0.5, -1.5]

n=1 [10.75, 9.5, -0.75, -3, 2, 1.5, -0.5, -1.5]

n=0 [10.125, 0.625, -0.75, -3, 2, 1.5, -0.5, -1.5]

i) The first element is the average of all the elements in the original sequence.

ii) No information has been lost

iii) Because most of the values in the final level are small, potentially fewer bits would be required to store it. Where there are zeroes, they do not need to be stored, although the positions of the other values would need to be stored. Small values could be replaced by zeroes without significant loss of detail, this can be done by applying a threshold value, the bigger the threshold the greater the loss of detail.

Question 3

Given a Haar wavelet transform analysis filterbank which uses a low-pass filter

$$h_0[0] = h_0[1] = \tfrac{1}{2}$$

and a high-pass filter $h_1[0] = \frac{1}{2} \qquad h_1[1] = -\frac{1}{2}$

. Use the recursive equations:

$$c_{m-1,n} = \sqrt{2} \cdot \frac{1}{2}(c_{m,2n} + c_{m,2n+1})$$

$$= \frac{1}{\sqrt{2}}(c_{m,2n} + c_{m,2n+1})$$

$$d_{m-1,n} = \frac{1}{\sqrt{2}}(c_{m,2n} - c_{m,2n+1})$$

to calculate the Haar wavelet transform for a sampled signal s[n] = [3 2 5 -2] after 1 and 2 stages of the transform filterbank.

Question 3

Start with the signal is the finest resolution coefficient,

$S[n] = [3\ 2\ 5\ -2]$

First level:

$C_{1,0} = 1/\sqrt{2}(3+2) = 5/\sqrt{2}$

$C_{1,1} = 1/\sqrt{2}(5+(-2)) = 3/\sqrt{2}$

$D_{1,0} = 1/\sqrt{2}(3-2) = 1/\sqrt{2}$

$D_{1,1} = 1/\sqrt{2}(5-(-2)) = 7/\sqrt{2}$

Hence the first level of the wavelet transform is [5 3 1 7]

Second level:

$C_{0,0} = ½(5+3) = 4$

$D_{0,0} = ½(5-3) = 1$

Hence the second level of the wavelet transform is $[4\ 1\ 1/\sqrt{2}\ 7/\sqrt{2}]$

Question 4

Given a Haar wavelet transform analysis filterbank which uses a low-pass filter

$$h_0[0] = h_0[1] = \frac{1}{2}$$

and a high-pass filter $h_1[0] = \frac{1}{2} \quad h_1[1] = -\frac{1}{2}$

Using the recursive equations, calculate the Haar wavelet transform for a sampled signal

$$s[n] = [1, 2, 0, -2]$$

after 1 and 2 stages of the transform filterbank.

Calculate the inverse wavelet transform of your result using the resynthesis filterbank, to confirm that the result of the inverse transform is the original sequence.

Question 4

Input sequence: $[1, 2, 0, -2]$

$$c_{1,0} = \tfrac{1}{\sqrt{2}}(1+2) = 3/\sqrt{2}$$

$$c_{1,1} = \tfrac{1}{\sqrt{2}}(0+(-2)) = -2/\sqrt{2}$$

$$d_{1,0} = \tfrac{1}{\sqrt{2}}(1-2) = -1/\sqrt{2}$$

$$d_{1,1} = \tfrac{1}{\sqrt{2}}(0-(-2)) = 2/\sqrt{2}$$

$$c_{0,0} = \tfrac{1}{\sqrt{2}} \cdot \tfrac{1}{\sqrt{2}}(3+(-2)) = 1/2$$

$$d_{0,0} = \tfrac{1}{\sqrt{2}} \cdot \tfrac{1}{\sqrt{2}}(3-(-2)) = 5/2$$

After 1 level: $\frac{1}{\sqrt{2}}(3, -2, -1, 2)$

After 2 levels: $\frac{1}{2}(1, 5, -\sqrt{2}, 2\sqrt{2})$

Question 4

From level 0 to level 1:

$$c_{1,0} = \sqrt{2}\left(\tfrac{1}{2}c_{0,0} + \tfrac{1}{2}d_{0,0}\right) = \sqrt{2}\left(\tfrac{1}{2}1/2 + \tfrac{1}{2}5/2\right) = \sqrt{2}(6/4) = 3/\sqrt{2}$$

$$c_{1,1} = \sqrt{2}\left(\tfrac{1}{2}c_{0,0} - \tfrac{1}{2}d_{0,0}\right) = \sqrt{2}\left(\tfrac{1}{2}1/2 - \tfrac{1}{2}5/2\right) = \sqrt{2}(-4/4) = -2/\sqrt{2}$$

The *d* coefficients in the 2nd half of the WT are unchanged, so we have

$$\frac{1}{\sqrt{2}}(3, -2, -1, 2)$$

which is the same as the first stage transform in the analysis direction.

Question 4

From level 1 to level 2:

$$c_{2,0} = \sqrt{2}\left(\tfrac{1}{2}c_{1,0} + \tfrac{1}{2}d_{1,0}\right) = \sqrt{2}\left(\tfrac{1}{2}3/\sqrt{2} + \tfrac{1}{2}(-1/\sqrt{2})\right) = \frac{1}{2}(3-1) = 1$$

$$c_{2,1} = \sqrt{2}\left(\tfrac{1}{2}c_{1,0} - \tfrac{1}{2}d_{1,0}\right) = \sqrt{2}\left(\tfrac{1}{2}3/\sqrt{2} - \tfrac{1}{2}(-1/\sqrt{2})\right) = \frac{1}{2}(3+1) = 2$$

$$c_{2,2} = \sqrt{2}\left(\tfrac{1}{2}c_{1,1} + \tfrac{1}{2}d_{1,1}\right) = \sqrt{2}\left(\tfrac{1}{2}.-2/\sqrt{2} + \tfrac{1}{2}2/\sqrt{2}\right) = \frac{1}{2}(-2+2) = 0$$

$$c_{2,3} = \sqrt{2}\left(\tfrac{1}{2}c_{1,1} - \tfrac{1}{2}d_{1,1}\right) = \sqrt{2}\left(\tfrac{1}{2}.-2/\sqrt{2} - \tfrac{1}{2}2/\sqrt{2}\right) = \frac{1}{2}(-2-2) = -2$$

$$[1, 2, 0, -2]$$

Thank you

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