Chapter 2

2.1 The electric field strength $E = e_x(yz - 2x) + e_y xz + e_z xy$, find: (1) Can the electric field be the solution of electrostatic field? (2) If it is an electrostatic field, find the potential corresponding to the electric field strength.

Solution:

$$① \ \nabla \times \vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz-2x & xz & xy \end{vmatrix} = \vec{e}_x(x-x) + \vec{e}_y(y-y) + \vec{e}_z(z-z) \equiv 0$$

$$② \ \vec{E} = -\nabla\varphi \Rightarrow \nabla\varphi = (2x - yz)\vec{e}_x - xz\vec{e}_y - xy\vec{e}_z \qquad \varphi = x^2 - xyz + C$$

2.5 A very long semi-cylinder with a radius of a, and surface charge density ρ_s is uniformly distributed on the surface of the cylindrical (Figure of Exercise 2.5). Find the field strength on the cylinder axis.

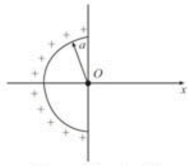

Figure of Exercise 2.5

Solution:

$$d_D \cdot 2\pi a \cdot 1 = \rho_s \cdot dl \cdot 1 = \rho_s \cdot a\, d\varphi$$

$$\Rightarrow d\vec{E} = \frac{\rho_s}{2\pi\varepsilon} d\varphi \cdot \vec{e}_r$$

$$d\vec{E} = \frac{\rho_s}{2\pi\varepsilon} d\varphi\, (\cos\varphi\, \vec{e}_x + \sin\varphi\, \vec{e}_y)$$

$$\Rightarrow \vec{E} = \int_{-\pi/2}^{\pi/2} \frac{\rho_s\, d\varphi}{2\pi\varepsilon} (\cos\varphi\, \vec{e}_x + \sin\varphi\, \vec{e}_y) = \frac{\rho_s}{\pi\varepsilon} \vec{e}_x$$

2.8 There are concentric conductor spherical shells whose inner and outer radii are respectively a and b, the voltage between the two spherical shells is U. Find this electric field strength between the two spherical shells.

Solution:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \cdot \vec{e}_r$$

$$\therefore U = \int_a^b \vec{E} \cdot d\vec{r} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\therefore q = \frac{4\pi\varepsilon_0 ab}{b-a} U \Rightarrow \vec{E} = \frac{ab}{b-a} \cdot \frac{U}{r^2} \cdot \vec{e}_r$$

2.11 There is an electric field inside and outside of a spherical region whose radius is a,

$$\boldsymbol{E} = \begin{cases} \boldsymbol{e}_r A\left(\dfrac{r}{3\varepsilon_0} - \dfrac{r}{3a^2\varepsilon_0}\right), & r < a \\ \boldsymbol{e}_r \dfrac{Ba^2}{\varepsilon_0 r^2}, & r > a \end{cases}$$

Find the charge distribution which generates the electric field.

Solution:

$$\rho = \nabla\cdot\vec{D} = \varepsilon_0\nabla\cdot\vec{E}$$

$$\rho = \begin{cases} A\left(1-\frac{5r^2}{3a^2}\right) & (r<a) \\ B & (r=a). \end{cases}$$

2.12 A conductor ball whose radius is a carries a charge q, the center of the ball locates at the boundary surface of two kinds of media (Figure of Exercise 2.12). Find:

(1) Electric field distribution;

(2) The electrostatic charge distribution on the spherical surface;

Solution:

$$D_1\cdot 2\pi r^2 + D_2\cdot 2\pi r^2 = q. \quad \therefore \varepsilon_1 E\cdot 2\pi r^2 + \varepsilon_2 E\cdot 2\pi r^2 = q.$$

$$\vec{E} = \frac{q}{2\pi r^2(\varepsilon_1+\varepsilon_2)}\cdot\vec{e}_r \quad (r>a). \quad [r<a \text{ 时 } E=0]$$

$$\rho_{上} = D_1 = \frac{q}{2\pi a^2}\cdot\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}$$

$$\rho_{下} = D_2 = \frac{q}{2\pi a^2}\cdot\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}$$

2.14 A concentric sphere capacitor is formed by a conductor ball whose radius is a and a conductor concentric spherical shell, the inner radius of the shell is b, the space between the ball and half of the shell (separated along the radial) is filled with uniform medium whose permittivity is ε_1, The other half is filled with uniform medium whose permittivity is ε_2 (Figure of Exercise 2.14). Find the capacitance of the spherical capacitor.

Solution:

$$\vec{E} = \vec{e}_r\cdot\frac{q}{2\pi r^2(\varepsilon_1+\varepsilon_2)}$$

$$\Rightarrow U = \int_a^b \vec{E}\cdot d\vec{r} = \frac{q}{2\pi(\varepsilon_1+\varepsilon_2)}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\Rightarrow C = \frac{q}{U} = \frac{2\pi(\varepsilon_1+\varepsilon_2)}{\frac{1}{a}-\frac{1}{b}}$$

2.20 There are two infinitely long coaxial cylinders, their radii are respectively a and $r = b$(with $b > a$). These surface charge densities are respectively ρ_{s1} and ρ_{s2}. Find: (1) electric field strength $\boldsymbol{E}$; (2) if $\boldsymbol{E} = 0$ at $r > b$, what relationship should the ρ_{s1} and ρ_{s2} have?

Solution:

$$E = 0 \quad (r<a)$$

$$E = \frac{2\pi\rho_{s1}a}{2\pi r\varepsilon_0} = \frac{\rho_{s1}a}{\varepsilon_0 r} \quad (a<r<b).$$

$$E = \frac{\rho_{s1}a+\rho_{s2}b}{\varepsilon_0 r} \quad (b<r)$$

$$r>b \text{ 处 } E = 0 = \frac{\rho_{s1}a+\rho_{s2}b}{\varepsilon_0 r} \Rightarrow \rho_{s1}/\rho_{s2} = -b/a.$$