EBU6018
Tutorial – Sampling and DFT Solutions

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Question 1

With the aid of a diagram, demonstrate the effect of the folding of an aliased signal.

For a sampling frequency of 4000 samples/second, what is the apparent frequency resulting from sampling signals of respectively 1000 Hz, 5000 Hz, 7500 Hz and 11,500 Hz?

Question 1 Solution

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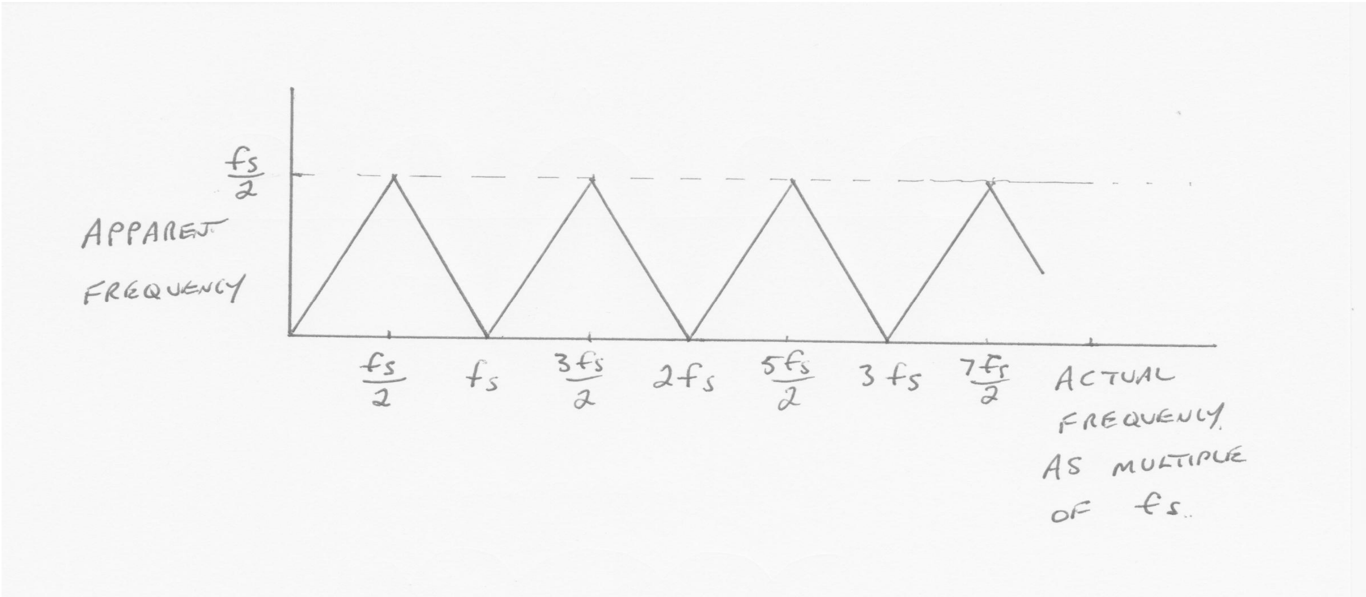
1000 Hz, 5000 Hz, 7500 Hz and 11,500 Hz?



Question 1 Solution

If we sample at 4000 samples/second:

- 1000Hz appears to be 1000Hz (as it should)
- 5000Hz appears to be 1000Hz
- 7500Hz appears to be 500Hz
- 11500Hz appears to be 500Hz

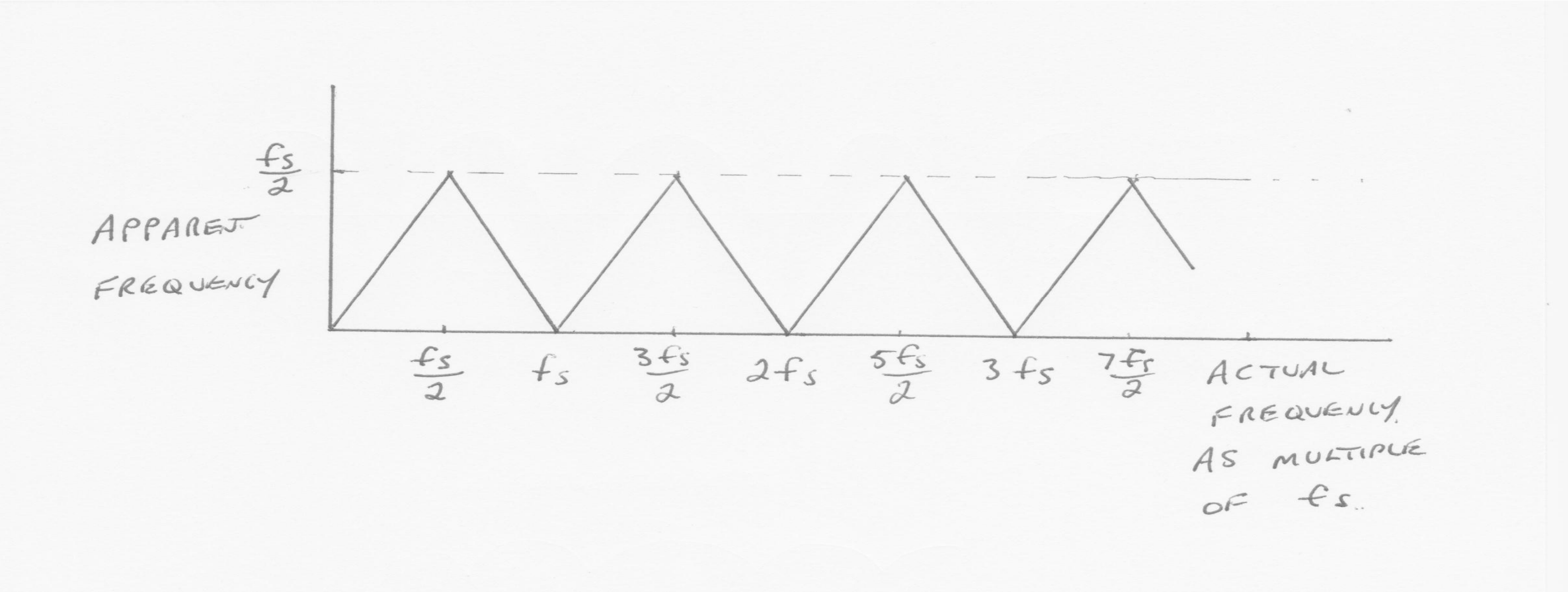
Question 2

With the aid of a diagram, demonstrate the effect of the folding of an aliased signal.

For a sampling frequency of 5000 samples/second, what is the apparent frequency resulting from sampling signals of respectively 2500 Hz, 6250 Hz, 10000 Hz and 13750 Hz?

Question 2 Solution

With the aid of a diagram, demonstrate the effect of the folding of an aliased signal.
For a sampling frequency of 5000 samples/second,
what is the apparent frequency resulting from sampling signals of respectively 2500 Hz, 6250 Hz, 10000 Hz and 13750 Hz?



Question 2 Solution

If we sample at 5000 samples/second:

- 2500Hz appears to be 2500Hz (as it should)
- 6520Hz appears to be 1250Hz
- 10000Hz appears to be 0Hz
- 13750Hz appears to be 1250Hz

Question 3

An FFT is a fast algorithm for implementing a DFT.

Estimate the approximate number of computations that are required to perform the FFT of an 8-point sequence.

One FFT structure is radix-2 decimation-in-time. Illustrate this FFT structure using the following 8-point sequence:

$$S[n] = [7, 3, -5, 2, 6, 4, -1, 8]$$

Question 3 Solution

Number of computations is $N\log_2 N$

8-point sequence, so no of computations = 8x3 = 24.

S[n] = [7, 3, -5, 2, 6, 4, -1, 8]

STEP 1: [7, -5, 6, -1][3, 2, 4, 8]

STEP 2: [7, 6][-5, -1][3, 4][2, 8]

STEP 3: [7][6][-5][-1][3][4][2][8]

Note: the positions in the output are the BIT-REVERSED positions of the input

Question 4

Suppose we have a signal of bandwidth 15kHz that is sampled at a rate of 45,000 samples/second.

Sketch a portion of the frequency spectrum of the sampled signal showing the baseband spectrum and the first upper sideband.

Estimate the order of low-pass filter required to reduce the amplitude of alias frequencies to less than 5% of the baseband amplitudes.

Question 4: Notes

- From the data given in the question, the upper limit of the baseband spectrum is 15kHz and the lower limit of the upper sideband spectrum is 30 kHz.
- The difference between 15 and 30 is 1 Octave (an octave is a range of a doubling of frequency).

- Using a low-pass filter to separate the sidebands from the baseband spectrum is called BANDLIMITING, and is the only way to prevent aliasing.

Question 4 Solution

$f_B = 15\ kHz \qquad f^* = 45{,}000$ SAMPLES/SEC

LPF — ROLL-OFF RATE = $-6n$ dB/OCTAVE

n = FILTER ORDER

BASEBAND AND ALIAS FREQUENCIES ARE SEPARATED BY 1 OCTAVE

EACH ORDER (n) REDUCES THE AMPLITUDE BY $\times 0.5$

$n=1$ 30 kHz REDUCED BY 50%

$n=2$ " " TO 25%

$n=3$ " " " TO 12.5%

$n=4$ " " " TO 6.25%

$n=5$ " " " TO 3.125%

Question 5

Suppose we have a recording of a piece of music of duration 5 minutes.

We wish to obtain a DFT of the music.

Assume the quality of the recording is high with a bandwidth of 15kHz.

What is the minimum sampling rate?

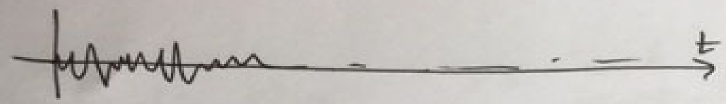
What length of sequence is obtained from this sampling rate?

How could the implementation time of the DFT be reduced, and what would be the effect on the accuracy of the output?

Question 5: Notes

- The only practical way of performing a Fourier Transform is to use a DFT. Any other method would involve infinity, which is obviously not practical.
- But the DFT assumes that the finite input sequence is periodic, ie, repeats itself.
- So the only way to get an accurate FT for the piece of music is to perform the DFT on the complete sampled length.
- If we find the DFT of segments of the music, these will not be identical to each other, so the calculation would be much faster because each segment has fewer samples, but the overall FT will not be accurate.
- If we use an FFT, then the number of calculations would be in the order of NLog_2N, ie 9000000x23.1 = a big number!

Question 5 Solution



5 MINUTES = 300 SECONDS

15 kHz BANDWIDTH $\therefore$ MIN $f^* = 30\text{kHz}$

$\therefore$ NUMBER OF SAMPLES = 30,000 × 300

= 9,000,000

[$\log_2(9000000) = 23.101.$]

A 9×10^6 POINT SEQUENCE WOULD BE TIME-CONSUMING.

SEGMENTING THE MUSIC WOULD REDUCE THE OVERALL PROCESSING TIME.

BUT BECAUSE THE SEGMENTS WOULD NOT BE PERIODIC, THE ACCURACY IS REDUCED.

Question 6

An FFT is a fast algorithm for implementing a DFT.

Estimate the approximate number of computations that are required to perform the FFT of an 8-point sequence.

One FFT structure is radix-2 decimation-in-time. Illustrate this FFT structure using the following 8-point sequence:

$$S[n] = [2, 6, 3, 9, 7, 4, 1, 11]$$

Question 6 Solution

Number of computations is $N\log_2 N$

8-point sequence, so no of computations = 8x3 = 24.

S[n] = [2, 6, 3, 9, 7, 4, 1, 11]

STEP 1: [2, 3, 7, 1][6, 9, 4, 11]

STEP 2: [2, 7][3, 1][6, 4][9, 11]

STEP 3: [2][7][3][1][6][4][9][11]