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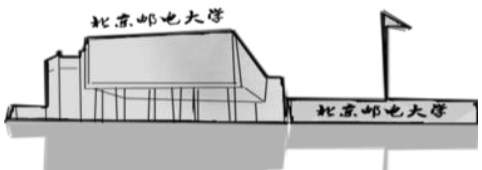
Chapter 6

Bandpass Transmission of Digital Signals

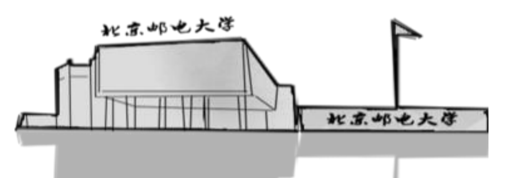
School of Information and Communication Engineering

Beijing University of Posts and Telecommunications



Bandpass Transmission of Digital Signals

- **Introduction**
- **Sinusoidal carrier modulation of the binary digital signal**
- **Quadrature phase shift keying**
- **M-ary digital modulation**



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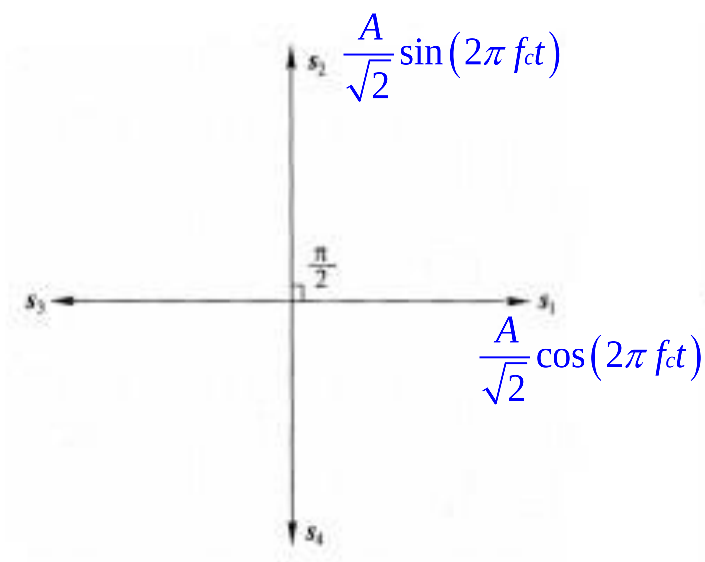
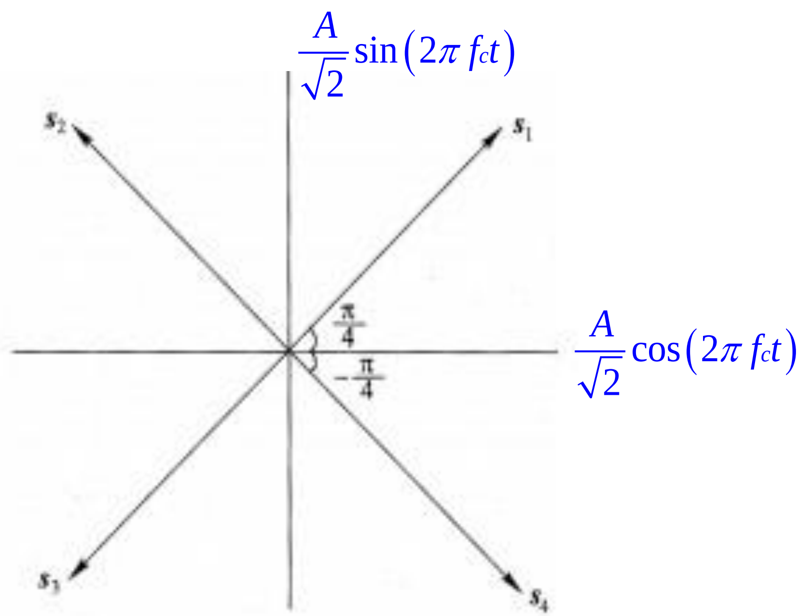
QPSK Signal

$$s_i(t) = A\cos(\omega_c t + \theta_i), \quad i = 1,2,3,4 \quad 0 \le t \le T_s$$



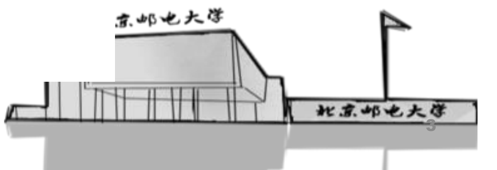


- **Mode A**

  $$\theta_i = (i-1)\frac{\pi}{2}$$

- **Mode B** ~a more general mode

  $$\theta_i = (2i-1)\frac{\pi}{4}$$



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QPSK Signal

QPSK modulation

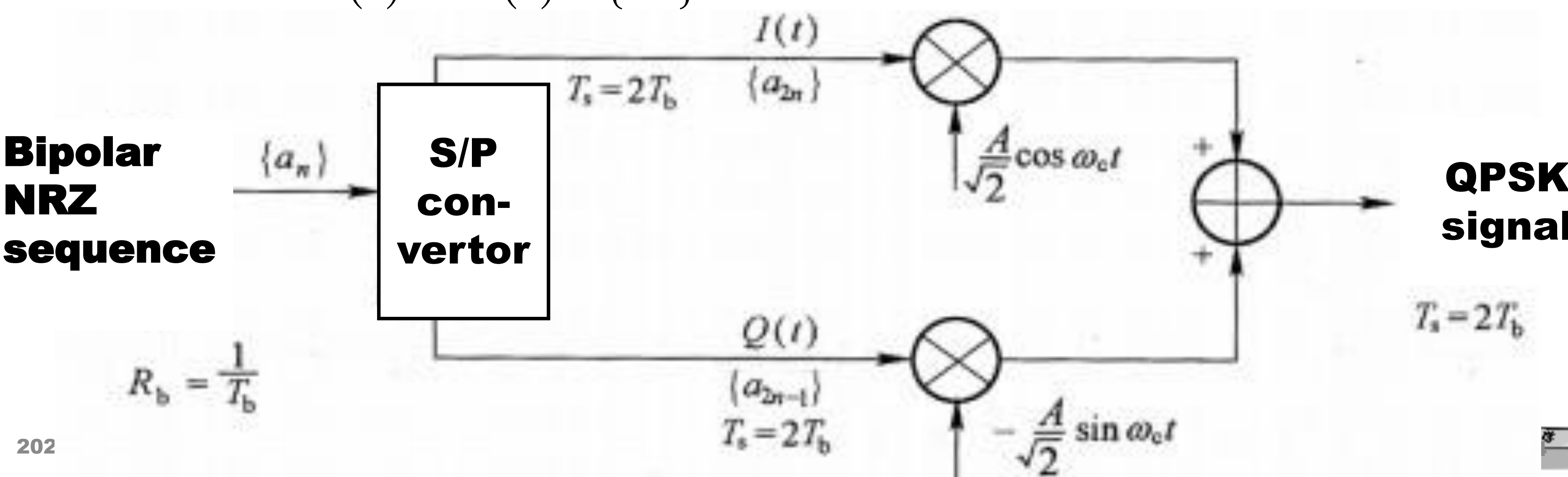
$$s_i(t) = A\cos(\omega_c t + \theta_i) = A(\cos\theta_i \cos\omega_c t - \sin\theta_i \sin\omega_c t)$$

$$= \frac{A}{\sqrt{2}}\left[I(t)\cos\omega_c t - Q(t)\sin\omega_c t\right]$$

$$I(t) = \sqrt{2}\cos\theta_i \sim \text{inphase component}$$

$$Q(t) = \sqrt{2}\sin\theta_i \sim \text{quadrature component}$$

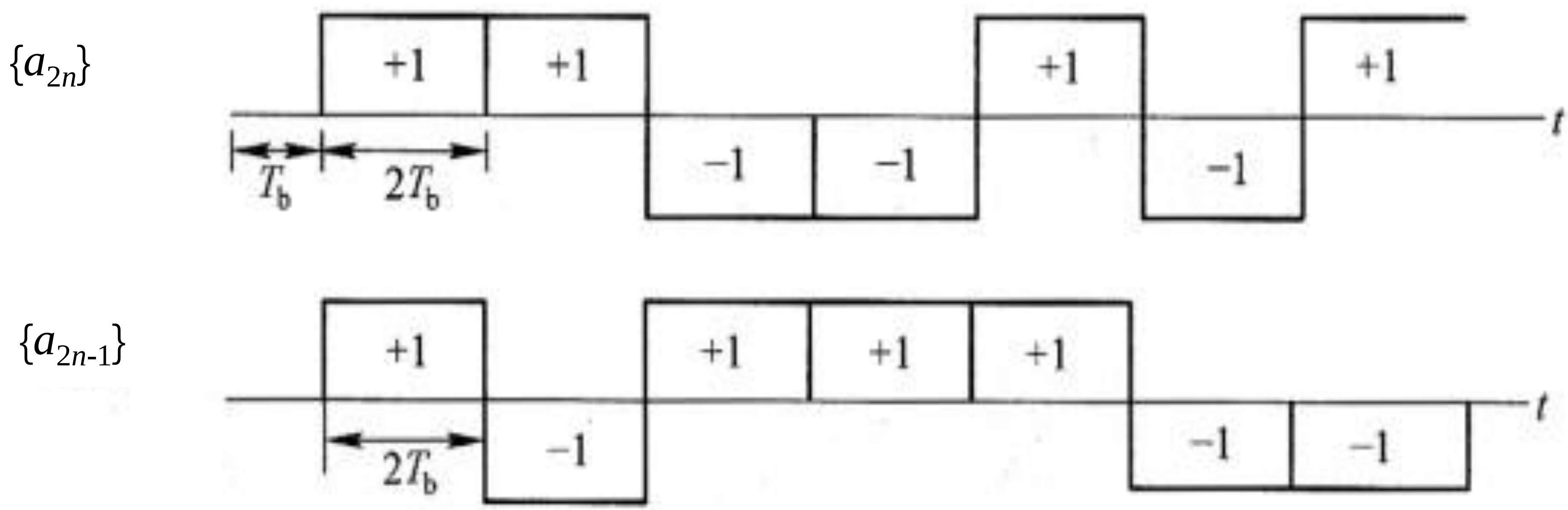
thus, $I(t)$、$Q(t) \in \{\pm 1\}$



QPSK Signal

- **Example**

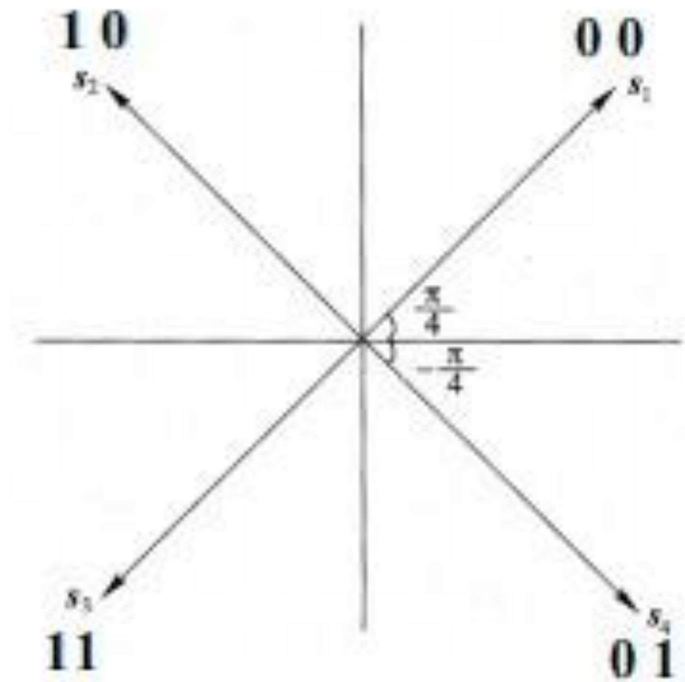
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\{a_n\}$	+1	+1	-1	+1	+1	-1	+1	-1	+1	+1	-1	-1	-1	+1
$I(t)$, $\{a_{2n}\}$		+1		+1		-1		-1		+1		-1		+1
$Q(t)$, $\{a_{2n-1}\}$		+1		-1		+1		+1		+1		-1		-1



QPSK Signal

- **Gray coding: only one bit is different between adjacent symbols, thus the BER can be decreased.**

b_{2n}	b_{2n-1}	a_{2n}	a_{2n-1}	Carrier phase θ_i
0	0	+1	+1	$\frac{\pi}{4}$
1	0	-1	+1	$\frac{3\pi}{4}$
1	1	-1	-1	$\frac{5\pi}{4}$
0	1	+1	-1	$\frac{7\pi}{4}$



Mode B

QPSK Signal

● PSD of QPSK signal

It’s the sum of two orthogonal BPSK PSDs

$$\because P_{2PSK}(f)=\frac{A^{*2}}{4}\left[P_b(f-f_c)+P_b(f+f_c)\right]$$

$$where \quad P_b(f)=T_s\mathrm{sinc}^2(fT_s)$$

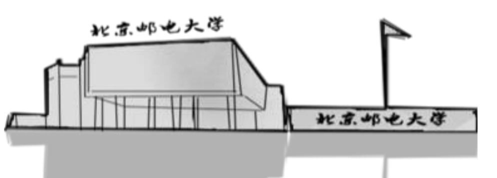
$$\boxed{s(t)=\frac{A}{\sqrt{2}}\left[I(t)\cos\omega_c t-Q(t)\sin\omega_c t\right]}$$

$$for\ QPSK,\ A^*=A/\sqrt{2},\quad T_s=2T_b$$

$I(t)$ and $Q(t)$ are bipolar NRZ sequences with period $T_s=2T_b$

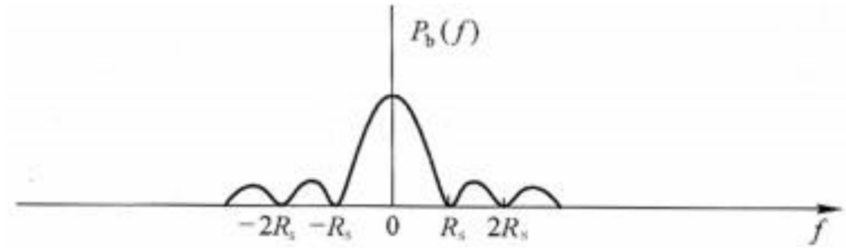
$$\therefore P_{QPSK}(f)=\frac{A^{*2}}{2}\left[P_b(f-f_c)+P_b(f+f_c)\right]$$

$$=\frac{A^2T_b}{2}\left\{\mathrm{sinc}^2\left[2(f-f_c)T_b\right]+\mathrm{sinc}^2\left[2(f+f_c)T_b\right]\right\}$$

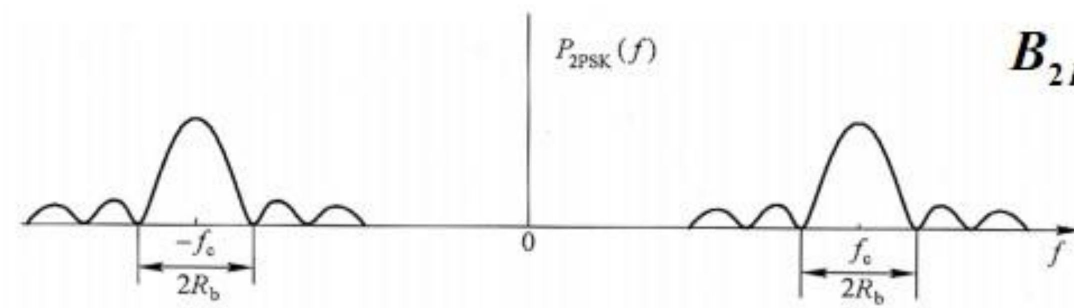


QPSK Signal

- **PSD of QPSK signal**

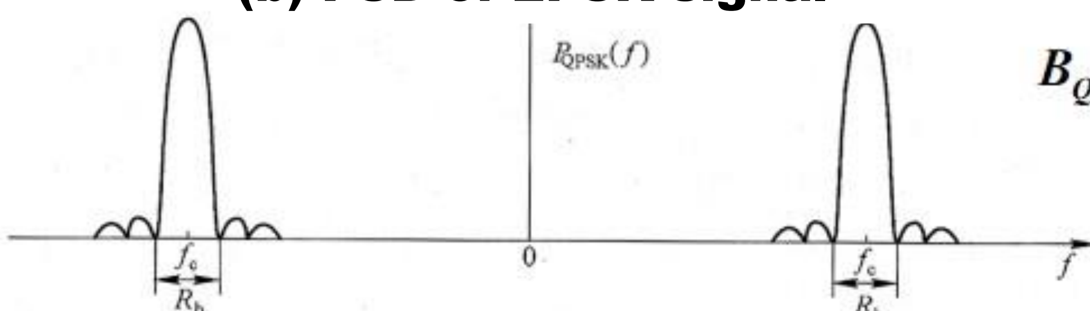


(a) PSD of bipolar rectangle NRZ sequence



$$B_{2PSK} = 2R_s = 2R_b$$

(b) PSD of 2PSK signal



$$B_{QPSK} = 2R_s = R_b$$

(c) PSD of QPSK signal

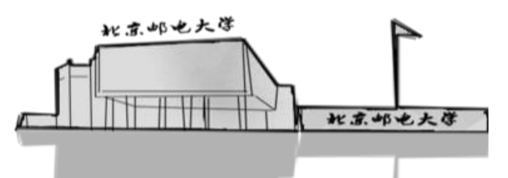
QPSK demodulation

- **The optimal reception of QPSK signal (with AWGN)**

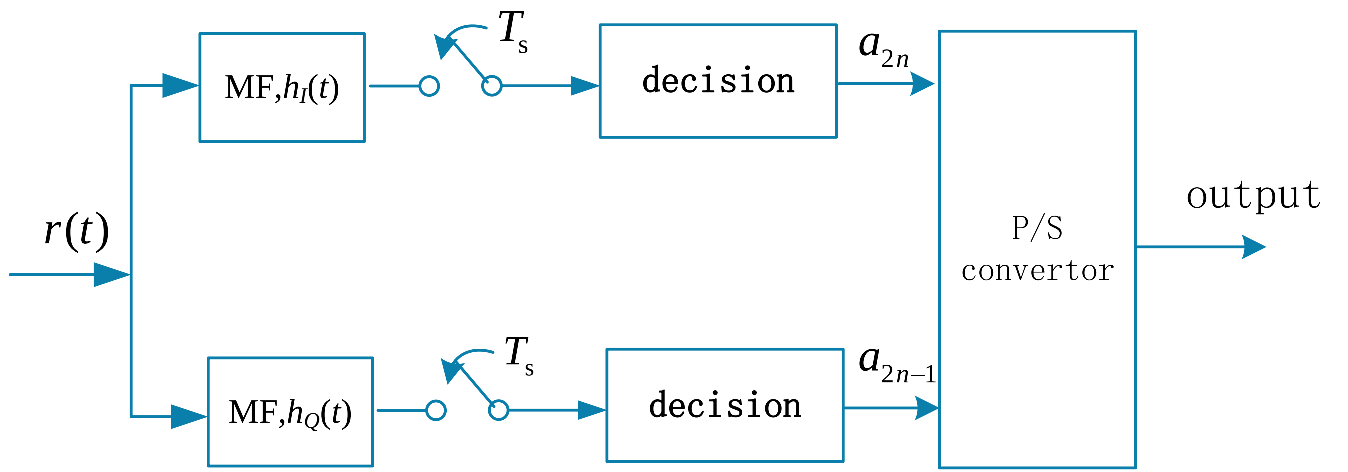
$$s(t) = \frac{A}{\sqrt{2}}\left[I(t)\cos\omega_c t - Q(t)\sin\omega_c t\right]$$

$$= s_I(t) + s_Q(t), \qquad 0 \le t \le T_s$$

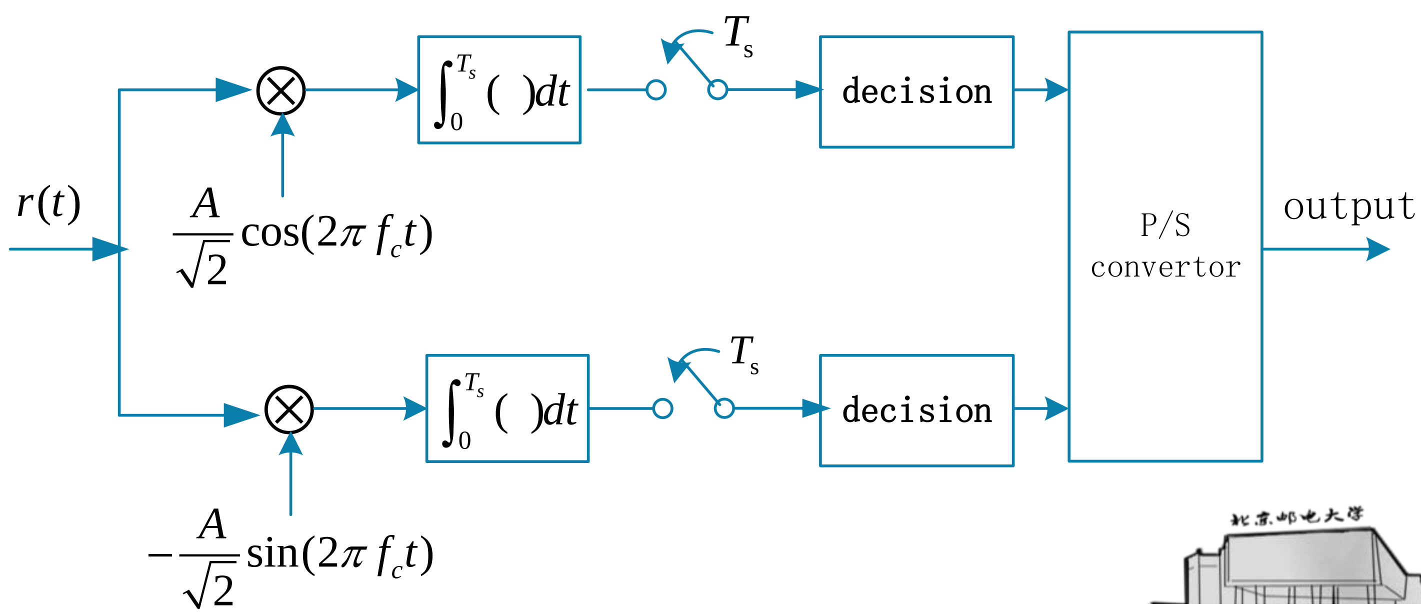
$$\begin{cases} h_I(t) = s_I(T_s - t) \\ h_Q(t) = s_Q(T_s - t) \end{cases}$$



QPSK demodulation



a. Optimal reception of QPSK signal (with MF)



b. Optimal reception of QPSK signal (with coherent demodulator)

QPSK demodulation

- **Average BER**

$$E_{b-2PSK} = \frac{\left(A/\sqrt{2}\right)^2 \cdot 2T_b}{2} = \frac{A^2 T_b}{2} \triangleq E_b$$

average bit energy of BPSK signal

$$P_{eI} = P_{eQ} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_{b-2PSK}}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

average bit energy of QPSK signal

$$P_b = P_I P_{eI} + P_Q P_{eQ}$$

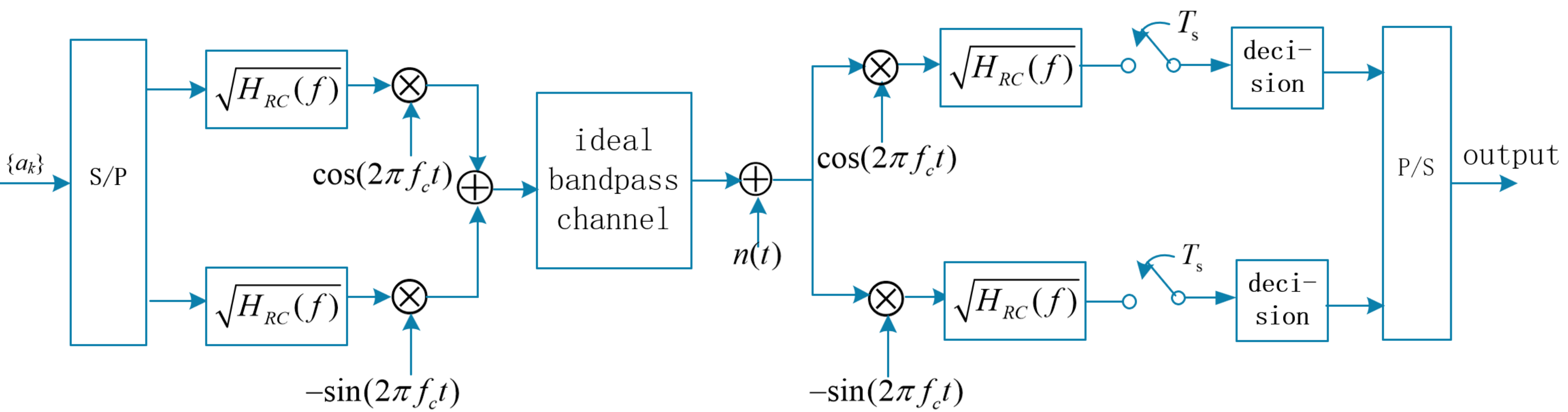
$$P_I = P_Q = 1/2 \Longrightarrow P_b = P_{eI} = P_{eQ}$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

P_I and P_Q are the probabilities of a binary bit appearing at each branch respectively.

QPSK demodulation

- **Optimal reception through ideal bandpass channel and with AWGN**

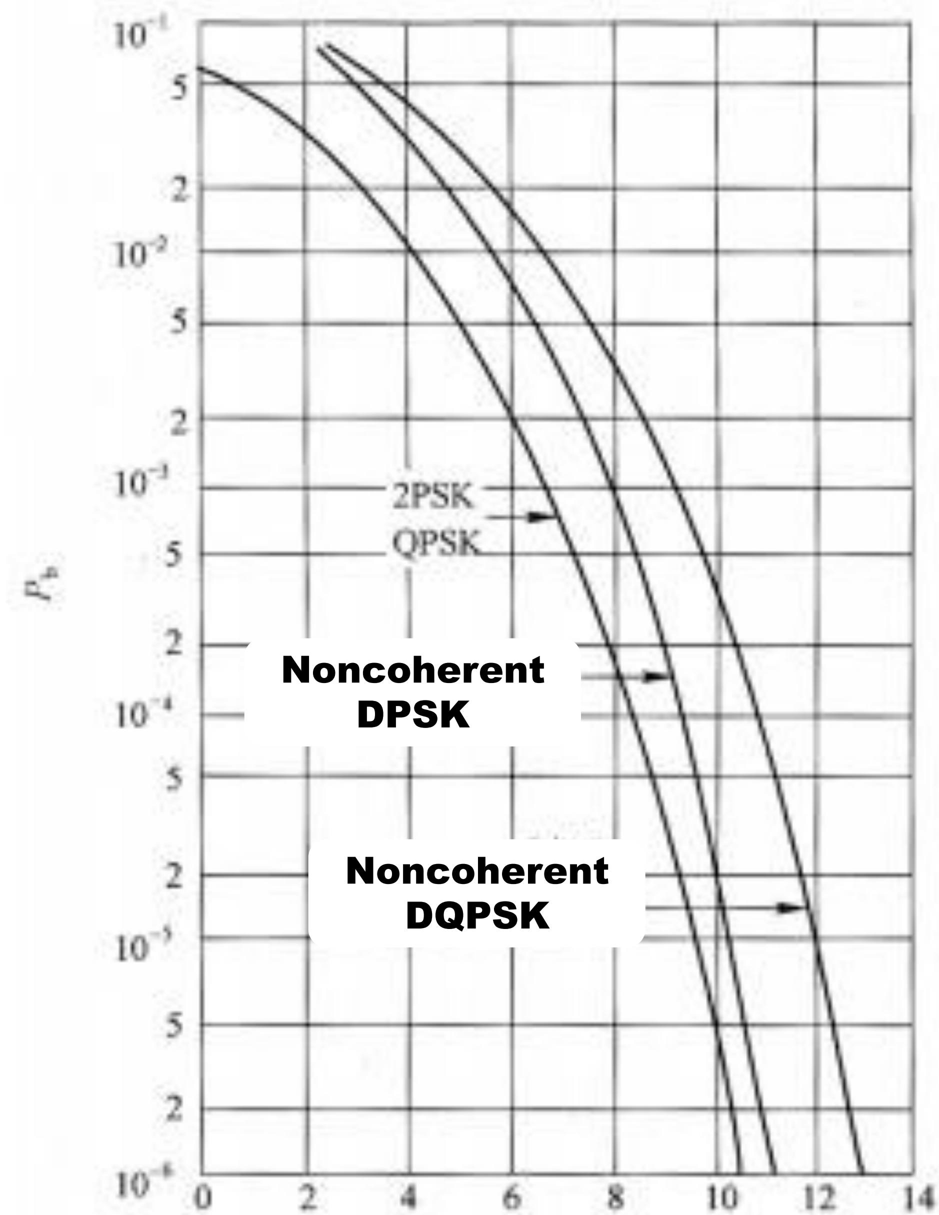


$$P_b = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

With the same R_b, transmitted signal power, and noise PSD, the average BERs of QPSK and 2PSK are the same, whereas the bandwidth of QPSK signal is just half of the bandwidth of BPSK signal.

QPSK demodulation

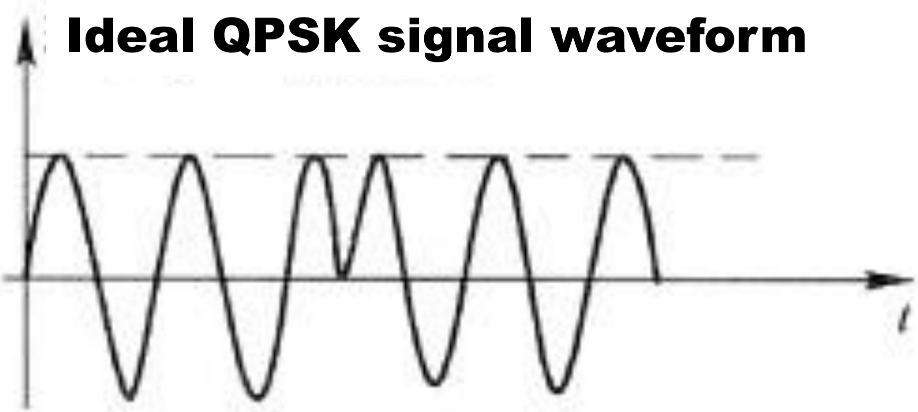
- **Average BER**



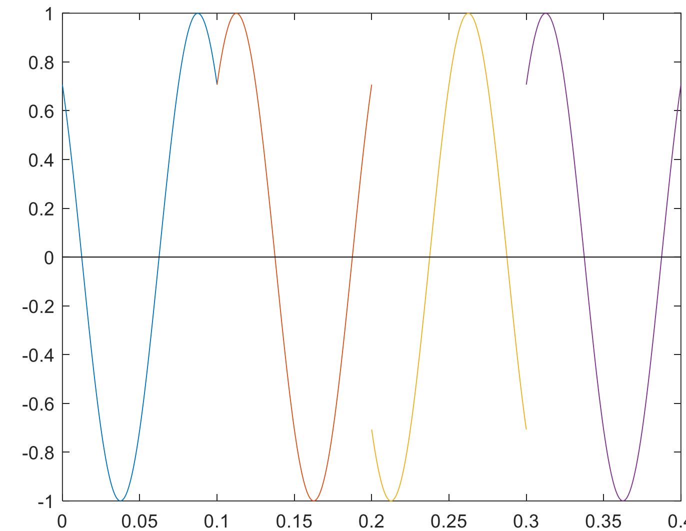
OQPSK

- **Constant envelope**
  - **Ideal QPSK signal has a constant envelope.**
- **Variable envelope problem**
  - **Variable envelope might lead to higher PSD sidelobes.**
  - **Reasons for variable envelope problem**
    - **Large phase shift**
    - **Band-limited channel**
- **Offset QPSK (OQPSK)**
  - **OQPSK is an effective scheme to deal with the phase shift problem of QPSK modulation.**

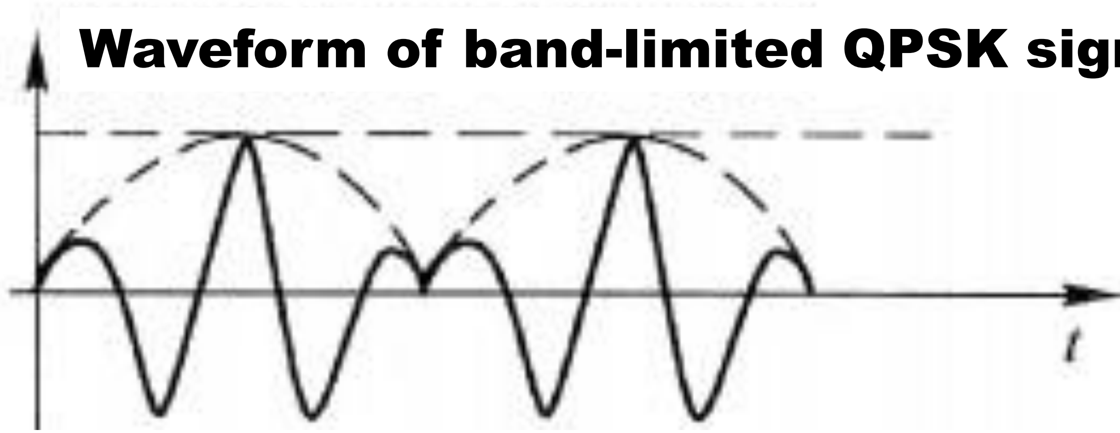
OQPSK

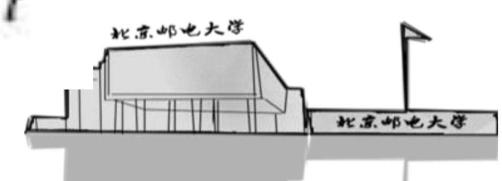


Mode A

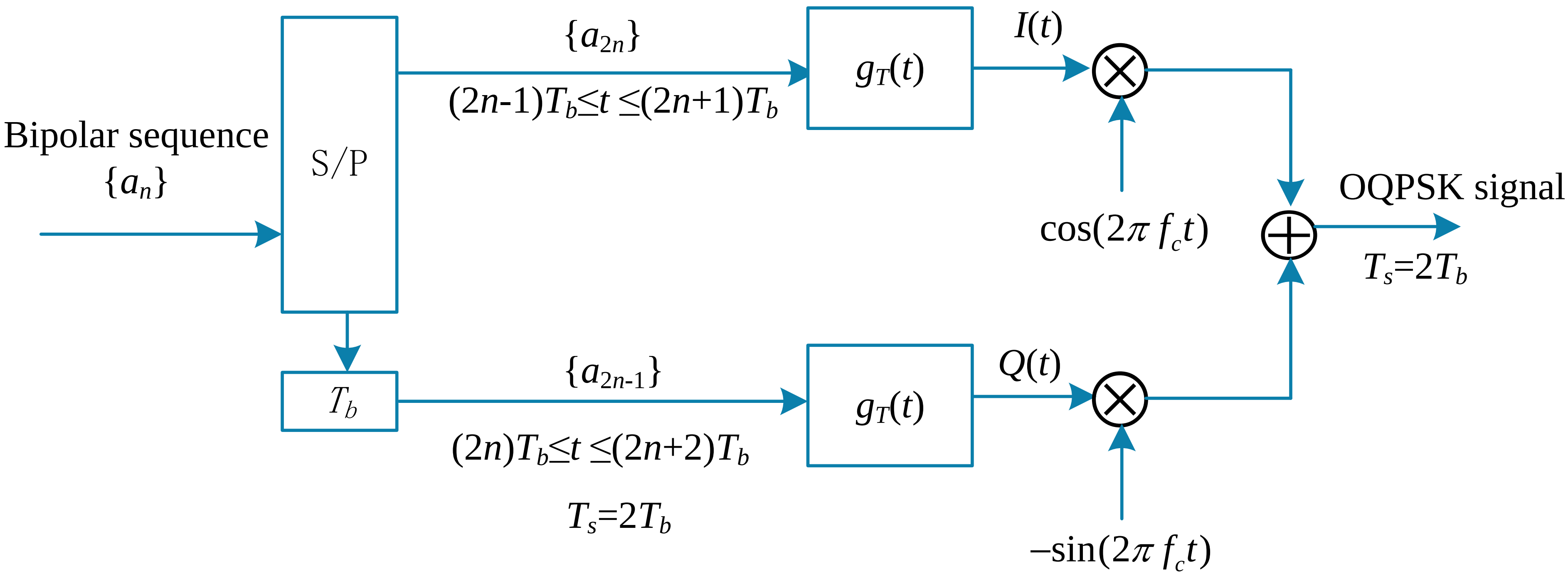


Mode B





OQPSK Modulation



$$s_{OQPSK}(t) = A\left[I(t)\cos\omega_c t - Q(t)\sin\omega_c t \right]$$

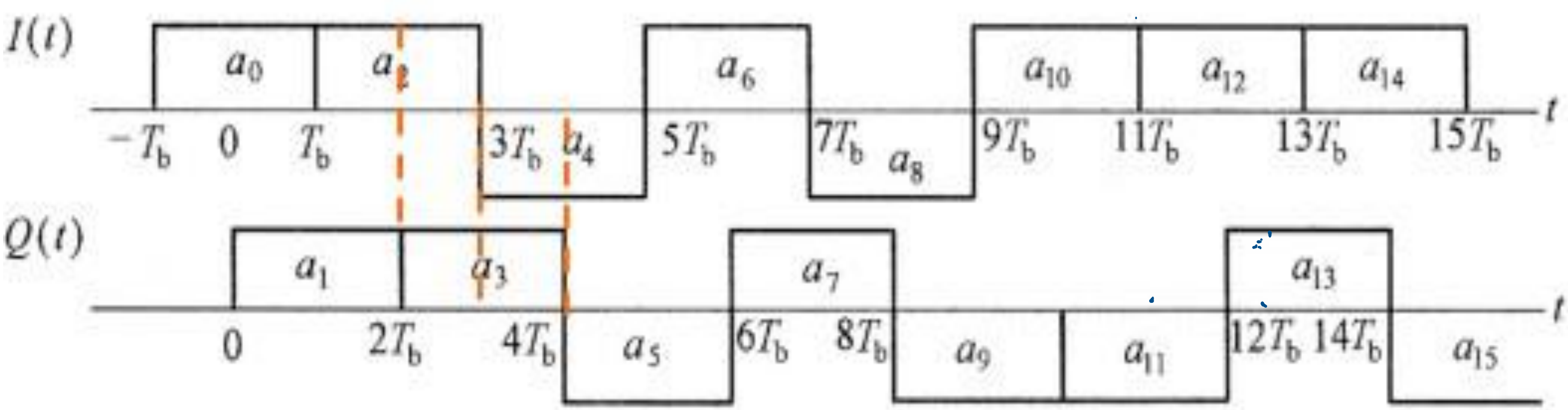
where $$I(t) = \sum_{n=-\infty}^{\infty} a_{2n} g_T(t - 2nT_b)$$

$$Q(t) = \sum_{n=-\infty}^{\infty} a_{2n+1} g_T(t - 2(n+1)T_b)$$

OQPSK Modulation

- **S/P conversion and signal waveform of OQPSK signal**

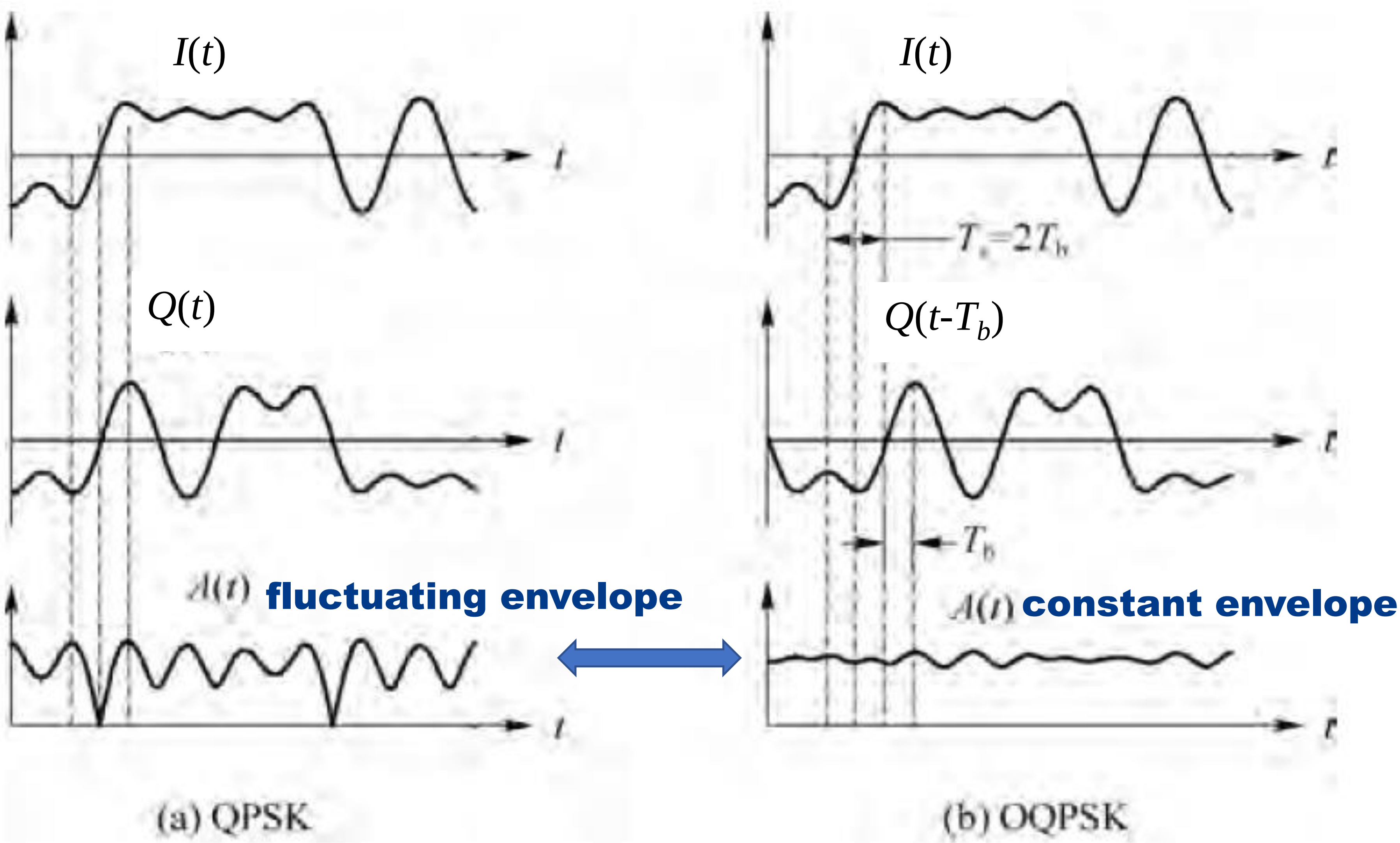
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\{a_n\}$	+1	+1	+1	+1	−1	−1	+1	+1	−1	−1	+1	−1	+1	+1	+1	−1



a_{2n}	a_{2n-1}	**Carrier phase**
+1	+1	$\pi/4$
-1	+1	$3\pi/4$
-1	-1	$5\pi/4$

OQPSK Modulation

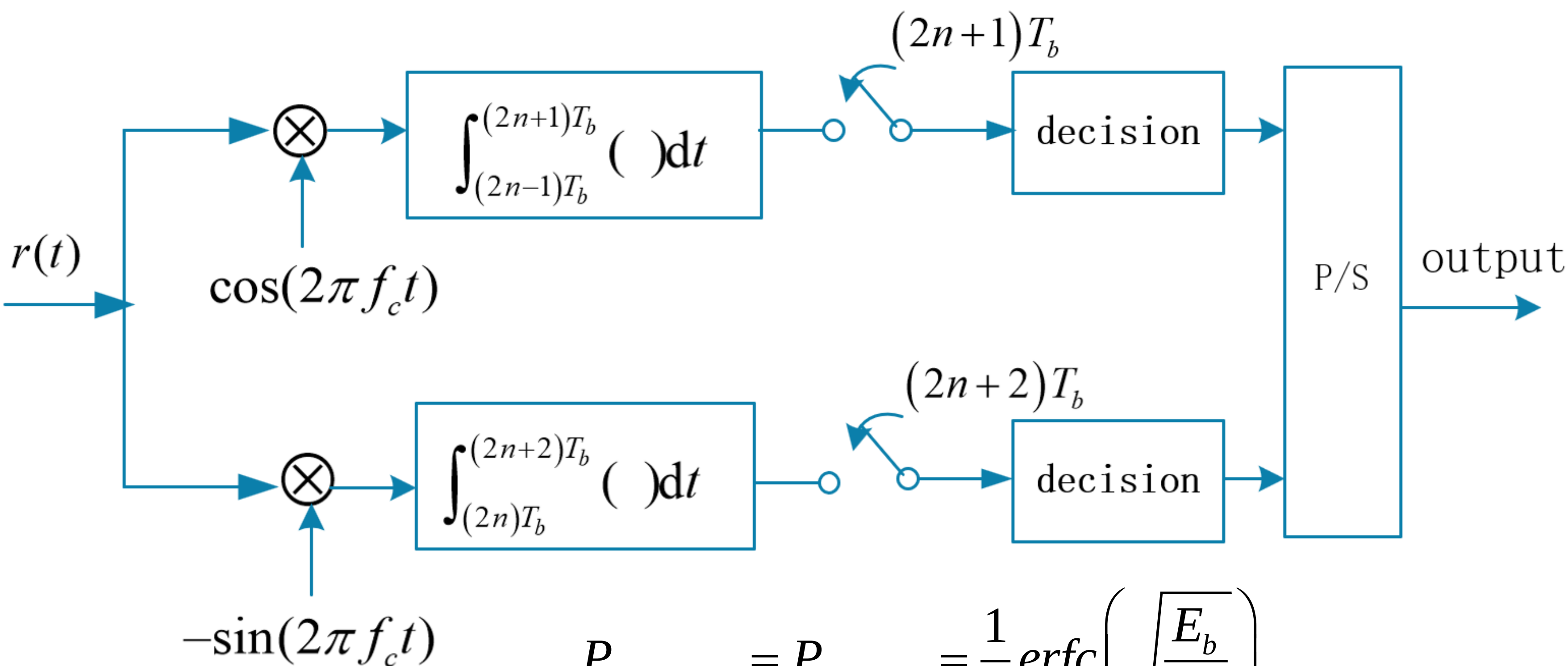
- **Signal waveform of OQPSK and QPSK signal**



OQPSK

- **The PSD of OQPSK signal is the same as the PSD of QPSK signal.**

- **Optimal demodulation of OQPSK signal**



$$P_{b-OQPSK} = P_{b-QPSK} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$