7.3 Let the amplitude of the magnetic field intensity $\boldsymbol{H}$ of a uniform plane wave in the air be $\frac{1}{3\pi}$A/m. Suppose the wave propagates in the direction of $-\boldsymbol{e}_z$ with phase constant $k = 30$rad/m. The direction of $\boldsymbol{H}$ is $(-\boldsymbol{e}_y)$ when $t = 0$ and $z = 0$.

(1) Write the expressions of $\boldsymbol{H}$ and $\boldsymbol{E}$;

(2) Find the frequency and wavelength.

7.3 (1) $\boldsymbol{H} = -\boldsymbol{e}_y \dfrac{1}{3\pi} \cos(9 \times 10^9 t + 30z)\text{A/m}, \boldsymbol{E} = \boldsymbol{e}_x 40 \cos(9 \times 10^9 t + 30z)\text{V/m}$

(2) $f = \dfrac{9}{2\pi} \times 10^9\text{Hz}, \lambda = \dfrac{\pi}{15}\text{m}$

7.7 Point out the polarization of the following plane wave.

(1) $\boldsymbol{E} = 3\left(\boldsymbol{e}_x + \mathrm{j}\boldsymbol{e}_y\right)\mathrm{e}^{-\mathrm{j}kz}$;

(2) $\boldsymbol{E} = \left(3\boldsymbol{e}_x + 2\boldsymbol{e}_y\right)\mathrm{e}^{-\mathrm{j}kz}$;

(3) $\boldsymbol{E} = (3\boldsymbol{e}_x + \boldsymbol{e}_y 4\mathrm{e}^{\mathrm{j}\frac{\pi}{3}})\mathrm{e}^{-\mathrm{j}kz}$;

(4) $\boldsymbol{E} = (-\boldsymbol{e}_x - 2\sqrt{3}\boldsymbol{e}_y + \sqrt{3}\boldsymbol{e}_z)\mathrm{e}^{-\mathrm{j}0.04\pi(\sqrt{3}x-2y+3z)}$.

7.7 (1) Levorotatory circular polarization; (2) Line polarization wave; (3) Levorotatory elliptic polarization; (4) Line polarization wave

7.20 The conductivity of sea water is $\sigma = 4\text{S/m}$ and $\varepsilon_\text{r} = 8$. Find the attenuation constant, wavelength and wave impedance of the electromagnetic wave in the sea with frequencies of 10kHz, 1MHz, 10MHz and 1GHz.

7.20 When $f = 10\text{kHz}$, $\alpha = 0.126\pi$, $\lambda = 15.87\text{m}$, $\eta = 0.0316\pi(1+j)(\Omega)$

When $f = 1\text{MHz}$, $\alpha = 1.26\pi, \lambda = 1.587\text{m}$, $\eta = 0.316\pi(1+j)(\Omega)$

When $f = 10\text{MHz}$, $\alpha = 4\pi, \lambda = 0.5\text{m}$, $\eta = \pi(1+j)(\Omega)$

When $f = 1\text{GHz}$, $\alpha = 24.65\pi, \beta = 2\pi \times 32.4\text{rad/m}$, $\lambda = 0.03\text{m}$, $\eta = 42/\sqrt{1 - j \times 0.89}(\Omega)$

8.1 A uniform plane wave is incident from air normally upon the surface of perfect conductor at $z = 0$ (xOy plane), given the electric field of incident wave $E_x^+ = E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - kz)}$, try to find:

(1) magnetic field of incident wave H_y^+;

(2) magnetic field of reflected wave H_y^-;

(3) magnetic field of the total wave $H_y = H_y^+ + H_y^- = ?$

8.1 (1) $H_y^+ = \dfrac{k}{\omega\mu_0} E_x^+ = \left(\dfrac{1}{\eta_0}\right) E_x^+ = \dfrac{1}{\eta_0} E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - kz)}$

(2) $H_y^- = \dfrac{1}{\eta_0} E_0^+ \mathrm{e}^{\mathrm{j}(\omega t + kz)}$

(3) $H = 2\left(\dfrac{1}{\eta_0}\right) E_0^+ \cos(kz)\, \mathrm{e}^{\mathrm{j}\omega t}$

8.5 A harmonically varying uniform plane wave is incident from air upon the surface of perfect conductor at $z=0$ (xOy plane), given that the incident electric field is: $\boldsymbol{E}^{+}=\boldsymbol{e}_y 10 \mathrm{e}^{\mathrm{j}(\omega t-6x-8z)}\mathrm{V/m}$. Find:

(1) the incident angle θ_{i} of the wave;

(2) the frequency f and the wavelength λ;

(3) write the complex form of electric field for reflected wave;

(4) write the expression of the electric field of the total wave.

8.5 (1) $\theta_{\mathrm{i}}=36.87°$

(2) f=477.465 MHz, λ =0.6283m

(3) The complex representation of the reflective electric field is $\boldsymbol{E}^{-}=-\boldsymbol{e}_y 10\cdot \mathrm{e}^{\mathrm{j}(\omega t-6x+8z)}$

(4) The electric field of composite wave is $\boldsymbol{E}=-\mathrm{j}\boldsymbol{e}_y 20\sin(8z)\cdot \mathrm{e}^{\mathrm{j}(\omega t-6x)}$

8.14 A uniform plane wave with electric field intensity $\boldsymbol{E}^{+} = E_0\,(\boldsymbol{e}_x + \mathrm{j}\boldsymbol{e}_y)\,\mathrm{e}^{-\mathrm{j}\beta z}$, is normally incident from air to a lossless dielectric board (it is dielectric for $z \geqslant 0$, its $\mu_\mathrm{r} = 1$, $\varepsilon_\mathrm{r} = 4$). Find:

(1) the electric field intensity of reflected wave;

(2) the magnetic field intensity of transmission wave;

(3) the respective polarization situation of incident wave, reflected wave and transmission wave.

8.14 (1) $\boldsymbol{E}^{-} = -\dfrac{1}{3}E_0(\boldsymbol{e}_x + \mathrm{j}\boldsymbol{e}_y)\mathrm{e}^{\mathrm{j}kz}$

(2) $\boldsymbol{H}^{T} = \dfrac{1}{90\pi}E_0(\boldsymbol{e}_y - \mathrm{j}\boldsymbol{e}_x)\mathrm{e}^{-\mathrm{j}2kz}$

(3) The incident wave is levorotatory circular polarization, the reflected wave is dextrorotatory circular polarization and the transmitted wave is levorotatory circular polarization.

10.5 Rectangular waveguide with $a = 7\text{cm}$, $b = 3\text{cm}$, which is filled with air or lossless medium with $\varepsilon_r = 4$, $\mu_r = 1$.

(1) Find the cutoff frequency and guided wavelength for TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11} modes.

(2) Which modes can propagate at $f = 3 \times 10^9\text{Hz}$.

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$f_c = \frac{v}{\lambda_c} = \frac{1}{\lambda_c}\frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$f_c = \frac{1}{2}\frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

10.5 (1) When filled with air, λ_c are 14, 7, 6, 5.51, 5.51cm respectively

f_c are 2.14, 4.29, 5.00, 5.44×10^9Hz respectively

When filled with medium, λ_c are 14, 7, 6, 5.51, 5.51cm respectively

f_c are 1.07, 2.14, 2.50, 2.72×10^9Hz respectively

(2) When filled with air, there is only TE_{10} mode; When filled with medium, there are TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11} totally 5 modes.

If the working frequency is 3GHz, please determine the mode of propagation in the waveguide.

EXAMPLE 10.7 The phase constant of the TE_{10} mode of an air-filled waveguide with $b = 1$ cm is 102.65 rad/m. If the operating frequency of the waveguide is 12 GHz, and the only mode of propagation is TE_{10}, calculate the length a of the waveguide.

Solution The cutoff frequency of TE_{10} is obtained from

$$\beta_{10} = \beta\sqrt{1 - \left(\frac{f_{c10}}{f}\right)^2}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} = 2\pi \times 12 \times 10^9\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}$$

$$= 251.44 \text{ rad/m}$$

$$102.65 = 251.44\sqrt{1 - \left(\frac{f_{c10}}{12 \times 10^9}\right)^2}$$

Thus, $f_{c10} = 10.95$ GHz

$$f_{c10} = \frac{u_p}{2a}$$

Therefore,

$$a = \frac{u_p}{2f_{c10}} = \frac{3 \times 10^8}{2 \times 10.95 \times 10^9} = 0.0136 \text{ m}$$