EBU6018
Advanced Transform Methods

Introduction 2023-24

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Brief Self-Introduction

Andy Watson: CEng, FIET

- Delivering education since 1982, previous work in Medical Physics and Industrial Control Engineering.
- Started at QMUL on the JP/JEI with BUPT 2008.
- Started teaching Advanced Transform Methods in 2009 as a masters level module then from 2017 as we now know and love it.
- I also teach other modules across the JP/JEI, and other activities such as the SSLC.

Course Format

- Lectures/Tutorials/Office Hours
  - All sessions are face-to-face
  - These sessions include theory, tutorials, office hours.

  - There are two groups for EBU6018
  - Group 1 comprises classes 1 to 5
  - Group 2 comprises classes 6 to 10
  - Both groups taught by Andy in weeks 7 and 8, and by Yixuan in weeks 13 and 14.

  - Lecture notes will be on QM+ before the teaching sessions start.

Coursework

- Laboratory sessions
  - 2 labs (weeks 9 and 12)
  - The labs are shared with EBU5303 Multimedia Fundamentals to highlight some of the relationships between the background principles of the transforms and their applications in practice
  - Details of the lab exercises to follow
  - Each lab is a computer-based Matlab exercise
  - Reports to be submitted, total weighting 15%.
- Two Class Tests (weeks 10 and 15). Total weighting 5%.
- Exam. Weighting 80%

Lab Schedule

- EBU6018/5303 lab schedule for this semester:

- Weeks 9 and 12

- –Class 1-5: Wednesday, slots 3 and 4, TB4 - 138
- –Class 6-10: Tuesday, slots 6 and 7, TB4 - 138
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The labs use Matlab, which you should all have access to. See here: https://www.its.qmul.ac.uk/support/self-help/software/free-and-discounted-software/matlab/

Detailed Timetable Weeks 7, 8

Telecoms- MultimediaG1						
Module code	Module Name		lecturer1	lecturer2	classes	weeks
EBU6018	Advanced Transform Methods		Andy	Yixuan	1-5	7-8; 13-14
		Monday	Tuesday	Wednesday	Thursday	Friday
Week 7	08:00-09:35			3-535		
	09:50-11:25	4-340				
	11:30-12:15					
	13:00-14:35					
	14:45-16:25				3-537	
	16:35-18:10		3-535			3-535
	18:30-19:15					
	19:20-20:55					
Week 8	08:00-09:35			3-535		
	09:50-11:25	4-340				
	11:30-12:15					
	13:00-14:35					
	14:45-16:25				3-537	
	16:35-18:10		3-535			3-535
	18:30-19:15					
	19:20-20:55					

Telecoms- MultimediaG2						
Module code	Module Name		lecturer1	lecturer2	classes	weeks
EBU6018	Advanced Transform Methods		Andy	Yixuan	6-10	7-8; 13-14
		Monday	Tuesday	Wednesday	Thursday	Friday
Week 7	08:00-09:35					3-535
	09:50-11:25			3-435		
	11:30-12:15					
	13:00-14:35		3-435			
	14:45-16:25					
	16:35-18:10	3-435			3-435	
	18:30-19:15					
	19:20-20:55					
Week 8	08:00-09:35					3-535
	09:50-11:25			3-435		
	11:30-12:15					
	13:00-14:35		3-435			
	14:45-16:25					
	16:35-18:10	3-435			3-435	
	18:30-19:15					
	19:20-20:55					

Detailed Timetable Weeks 13, 14

Week 13	08:00-09:35			3-535				08:00-09:35					3-535
	09:50-11:25	4-340						09:50-11:25			3-435		
	11:30-12:15							11:30-12:15					
	13:00-14:35							13:00-14:35		3-435			
	14:45-16:25				3-537			14:45-16:25					
	16:35-18:10		3-535			3-535		16:35-18:10	3-435			3-435	
	18:30-19:15							18:30-19:15					
	19:20-20:55							19:20-20:55					
Week 14	08:00-09:35			3-535				08:00-09:35					3-535
	09:50-11:25	4-340						09:50-11:25			3-435		
	11:30-12:15							11:30-12:15					
	13:00-14:35							13:00-14:35		3-435			
	14:45-16:25				3-537			14:45-16:25					
	16:35-18:10		3-535			3-535		16:35-18:10	3-435			3-435	
	18:30-19:15							18:30-19:15					
	19:20-20:55							19:20-20:55					

Tutorials/Office Hours

Group 1:

Tutorial Wednesday 11:30-12:15, 3-435

Office Hour Wednesday 13:00-13:45, 3-408

Group 2:

Tutorial Thursday 18:30-19:15, 3-535

Office Hour Wednesday 13:00-13:45, 3-408

Books

1. Introduction to Time-Frequency and Wavelet Transforms
   Author: Shie Qian
   Publisher: Prentice Hall
   ISBN: 0130303607; 978-0130303608
   (Some material in the lecture notes refers to this book).

2. Discrete Fourier and Wavelet Transforms: An Introduction through Linear Algebra with Applications to Signal Processing
   Author: Roe W Goodman
   Publisher: World Scientific
   ISBN: 9789814725774.

Topics to be covered

- Linear Algebra and Basis Functions
- Fourier Transform (FT)
- Short-Time Fourier Transform (STFT), Spectrogram
- Discrete Cosine Transform (DCT)
- Principal Component Analysis (PCA) and the Karhunen-Loeve Transform (KLT)
- Wavelet Transform (WT), Scalogram
- Multiresolution Analysis (MRA)
- Perfect Reconstruction (PR)
- Wigner-Ville Distribution (WVD)
- Uncertainty Principle (UP)

Schedule of Topics

The syllabus splits into 4 blocks.

Block 1 (Basics) : (Week 7)

Mainly revision from DSP

- Introduction
- Linear Algebra
- Fourier Series/Fourier Transform
- Limitations of FT/Stationary and Non-stationary signals
- Sampling and the DFT
- FFT

Schedule of Topics

Block 2 (Time/Frequency Analysis) : (Week 8)

- Discrete Cosine Transform
- Time/Frequency Analysis
- Short-time Fourier Transform
- Gabor Transform

Schedule of Topics

Block 3 (Wavelets) : (Week 13)

- Haar Functions
- Wavelet Transform
- Multi-resolution Analysis
- WT from Filterbanks
- Transform Matrices

Schedule of Topics

Block 4 (Other Advanced Topics and Revision) : (Week 14)

- Karhunen-Loeve Transform
- Wigner-Ville Distribution
- Uncertainty Principle
- Perfect Reconstruction (incl Daubuchie Functions)
- Summary of Applications of Transforms
- Revision/Exam preparation

Some background...

It can be advantageous to process signals and images in the frequency domain.

E.g. convolution in the time domain becomes multiplication in the frequency domain (convolution is used to determine the output from a process given its impulse response and the input applied).

….some background...

The basic method of transforming from the time domain to the frequency domain is the Fourier Transform. There are four versions:

- Fourier Series FS: continuous time (periodic) /discrete frequency
- Fourier Transform FT: continuous time (aperiodic or periodic) /continuous frequency
- Discrete Time Fourier Transform DTFT: discrete time (aperiodic) /continuous frequency
- Discrete Fourier Transform DFT: discrete time (periodic) /discrete frequency **Only practical version!**

Example of time-frequency information

By en:User:Prof.rick - author's "own edition and arrangement",
Public Domain, https://commons.wikimedia.org/w/index.php?curid=5222815

...some background...

A major problem in digital signal and image processing is the trade-off between the time taken to process and the resolution/quality of the output.

The time taken can be reduced by, e.g.,

- Using fast processing techniques e.g. FFT
- Identifying redundancy in the information i.e. removing duplicate information before processing
- Omitting components in the processing that do not noticeably contribute to quality e.g. JPEG

...some background...

- A signal is a function of time, f(t)
- An image is a 2D function, f(x,y), and the "amplitude" at any point (x,y) is the intensity or grey level (and possibly colour information) at that point
- If x, y and intensity are all finite and discrete we have a digital image
- Each (finite) element has a unique location and value and is called a pixel.
- The data for each pixel is location + intensity + colour….a lot of data!

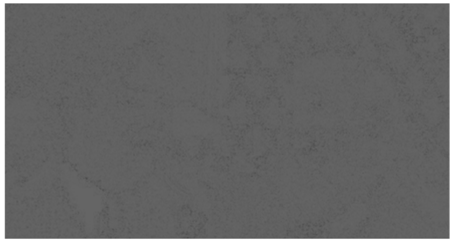
Example of JPEG

Original

Compressed

Error

75%



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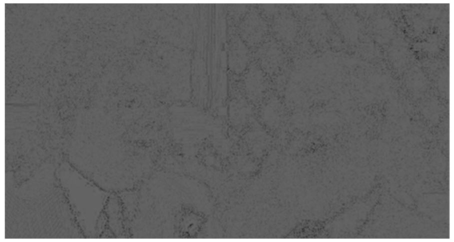

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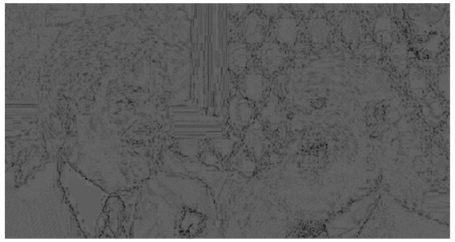

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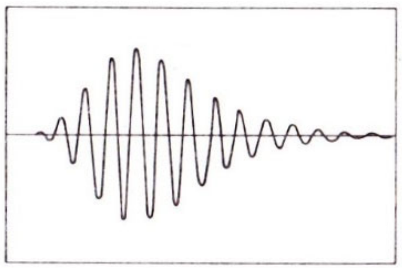
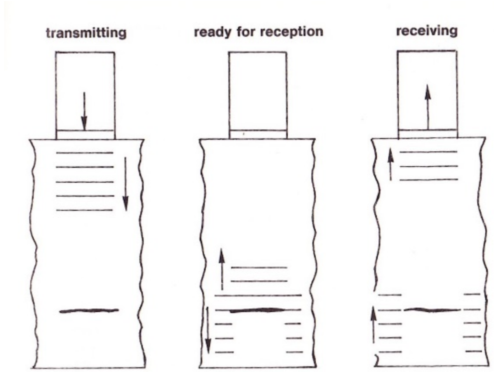
...some background...

- Images do not need to be visible to humans, just as signals do not need to be audible to humans, e.g.
  - Ultrasound (e.g. Non-Destructive Testing, Clinical Diagnostics, etc)
  - Electron microscopy
  - Computer Axial Tomography (CAT) scans (a combination of many X-Rays)
- A variety of transform techniques have been devised and developed to cater for different applications.

Ultrasound Scan



Short pulse
frequency a few MHz



Piezo-electric probe transmits pulses and receives echoes

The magic is the specific transform used for different applications

Then magic happens

...some background.

Digital Signal Processing....many applications.......

Digital Image Processing

- improvement of pictorial information for human interpretation
- processing for storage, transmission and representation for autonomous machine perception

Two types of signal

Stationary:

The frequency content is the same for all time.

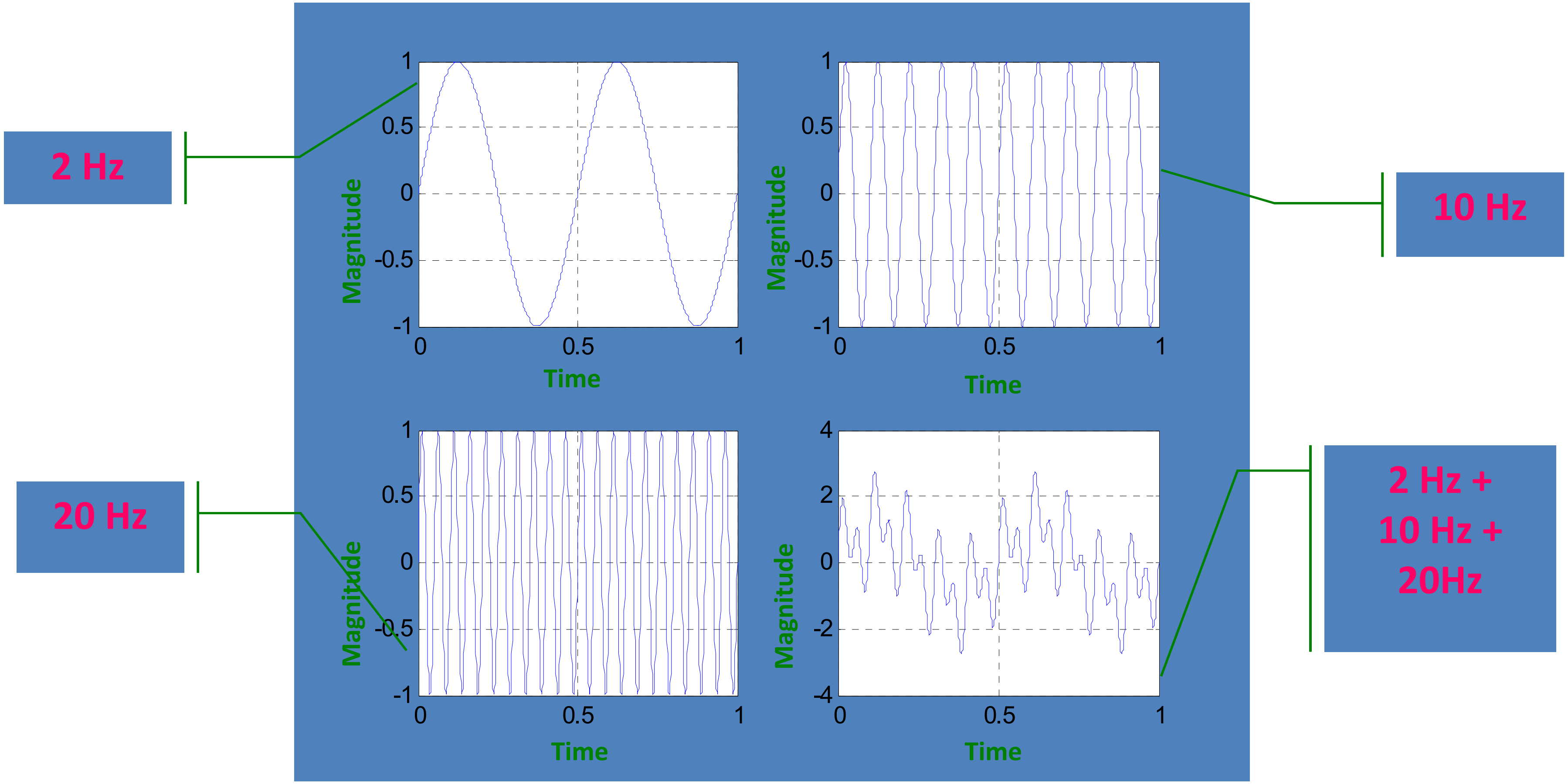
Non-stationary:

The frequency content changes with time.

This demonstrates the need for "Advanced" Transforms

Example 1: Stationary signal

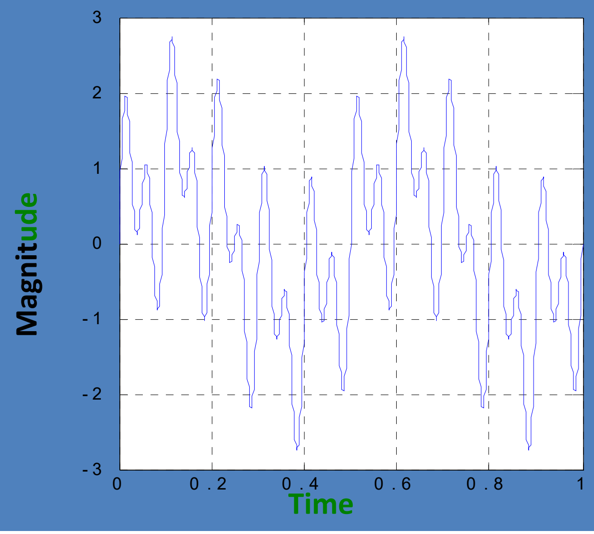
This example is the sum of 3 continuous sinewaves



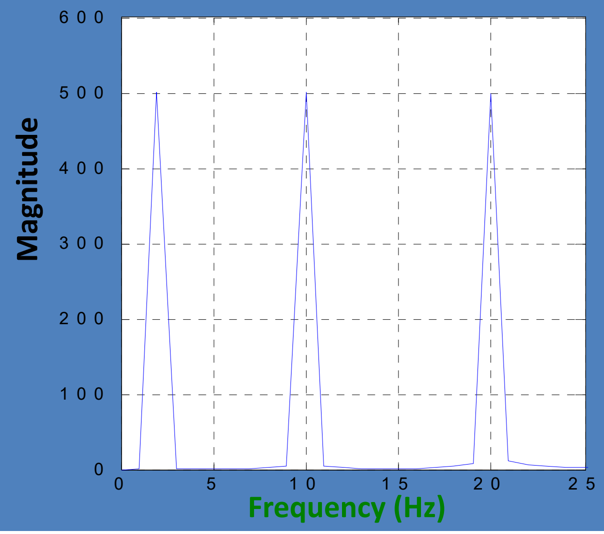
Example 1: Stationary signal

2 Hz + 10 Hz + 20Hz

Stationary



Signal in time domain

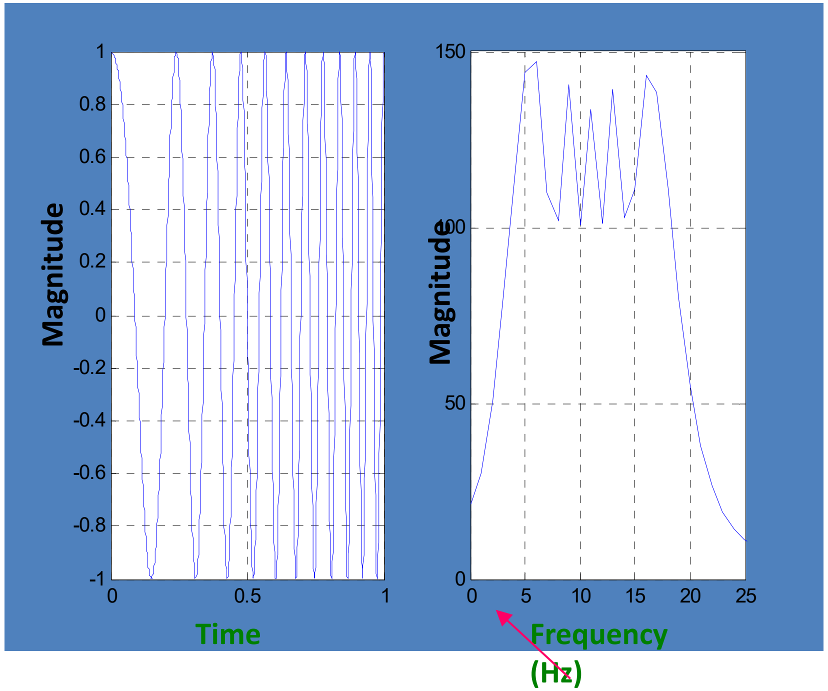


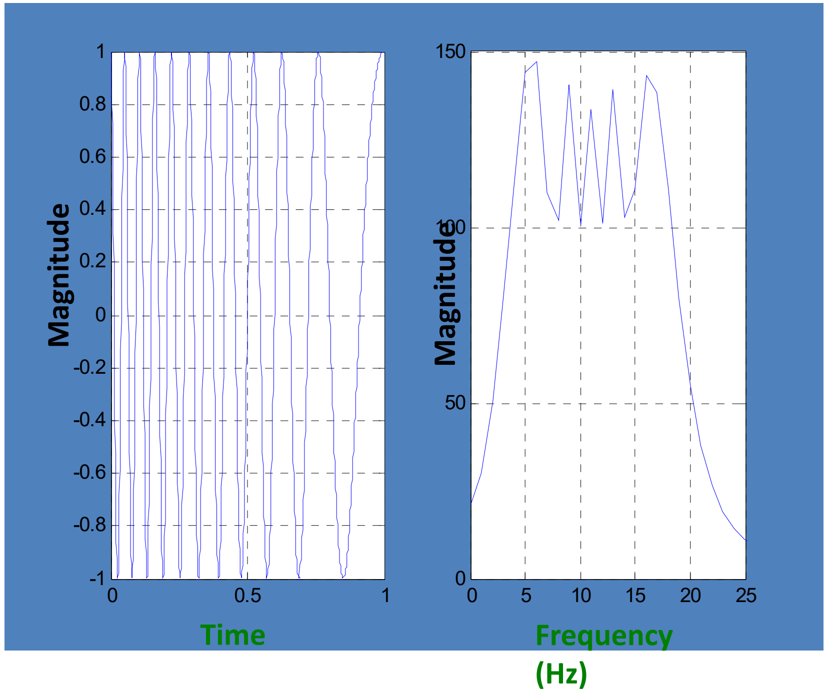
Power spectrum $\left|X(f)\right|^2$

By looking at the Power spectrum (Fourier Transform) of the signal we can recognize the three frequency components (at 2,10,20Hz respectively).

Example 2: Non-stationary signals

Consider two linearly modulated sinusoids (chirps).
The first with increasing frequency and the second with decreasing frequency.





In this case we have two nonstationary signals in time with identical FTs.
Confusion arises and power spectrum (Fourier Transform) is not useful.

Basis Functions

- A function f(x) can sometimes be better analysed as a linear expansion of “expansion functions”. If the expansion is unique, i.e., there is only one set of expansion function coefficients for a given f(x), then they are called “basis functions”.

Basis Functions

An example of “Basis Functions” are the sine and cosine functions used for the Fourier Transform.

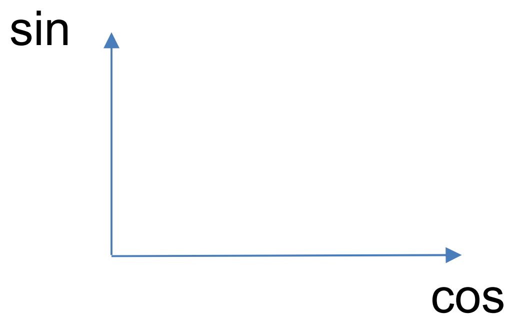
These two basis functions are “Orthogonal”.

This allows the FT to be “Inverted”, that is, the original function can be uniquely reproduced

i.e. we can add the transform coefficients together to obtain the original function.

Orthogonality

- We know that the Fourier Transform is based on sines and cosines.
- We know that sine and cosine are orthogonal, that is, they are separated by a phase angle of 90 degrees.



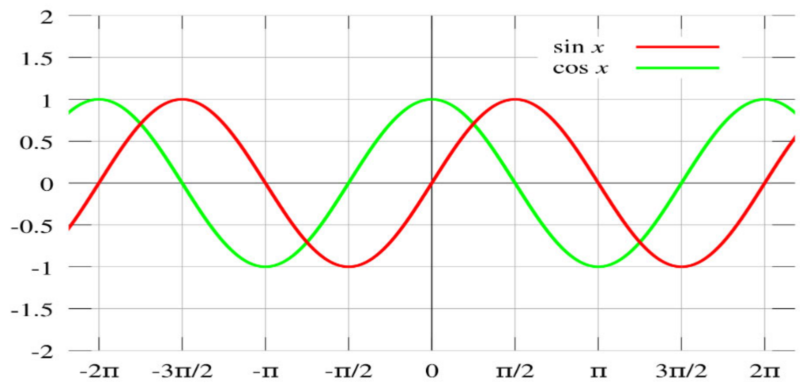


Image from Wikipedia

- **The Fourier Series is one form of the Fourier Transform…..**

Forms of the Fourier Series

- Three forms
  - Original (sine and cosine components)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n.\cos(n.\omega.t) + \sum_{n=1}^{\infty} b_n.\sin(n.\omega.t)$$

  - Cosine-with-phase form

$$x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_k) \qquad -\infty < t < \infty$$

  - Exponential form

$$x(t) = \sum_{n=-\infty}^{\infty} X_n.\, e^{jn\omega t}$$

- Each comprise a set of sine waves plus a set of cosine waves. If these are added together then x(t) is obtained i.e. Invertible!

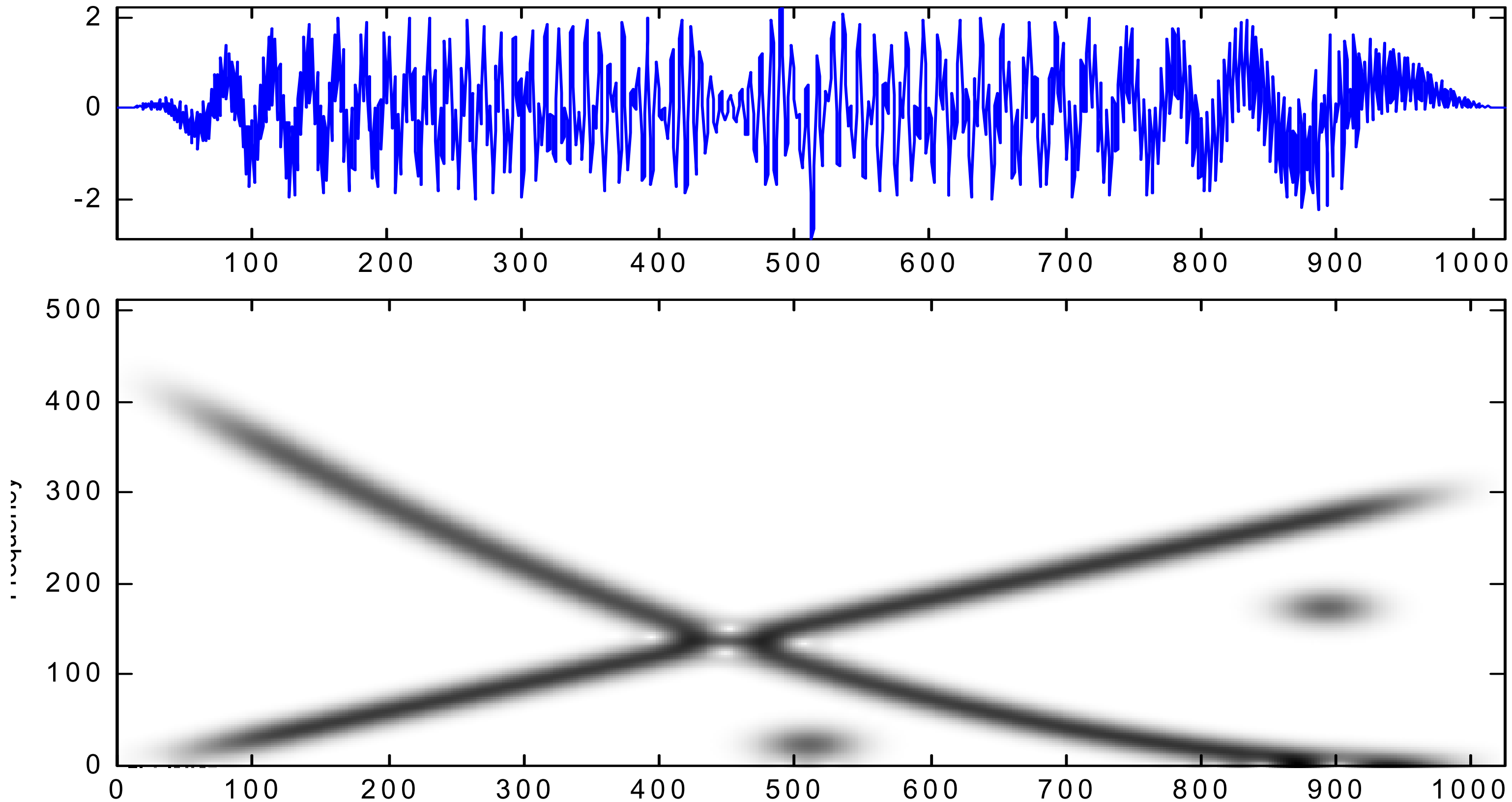
STFT, Spectrogram

- The Short-Time Fourier Transform is used to find a frequency spectrum snapshot by calculating FT of a short time interval of the signal.
- Useful if the signal is not stationary- can then see the changes in spectrum with time
- Assumes the signal is stationary within a “window” (“Quasi Stationary”)
- A spectrogram is an image that shows how the spectral density varies with time.

So, what is in a signal?

Example of a composite signal and its spectrogram

We can take short segments of the signal and find the segment FT



Discrete Cosine Transform

- Used where most of the energy is contained in just a few of the transform coefficients (e.g. visual images).
- Used in JPEG image compression formats and in MPEG video compression formats (you learn about these in EBU5303 Multimedia Fundamentals).
- DCT uses only the real part of a FT, that is, only the cosine terms (computation is therefore much simpler).

Karhunen-Loeve Transform and PCA (Principal Component Analysis)

- Gives optimum error resulting from truncating the transform coefficients (that is, best quality of compressed image)
- Has disadvantages (one being that it requires a lot of computation)
- But it can be approximated by the DCT for the majority of images and applications.

Wavelet Transform,Scalogram

- “Substitute” the window in the STFT by a wave packet (“wavelet”)
- Produces a transform that is a function of 2 real variables a and b (scale and translation) instead of t and f (time and frequency).
- Is used with non-stationary data (for both “feature extraction” and “trend analysis”)

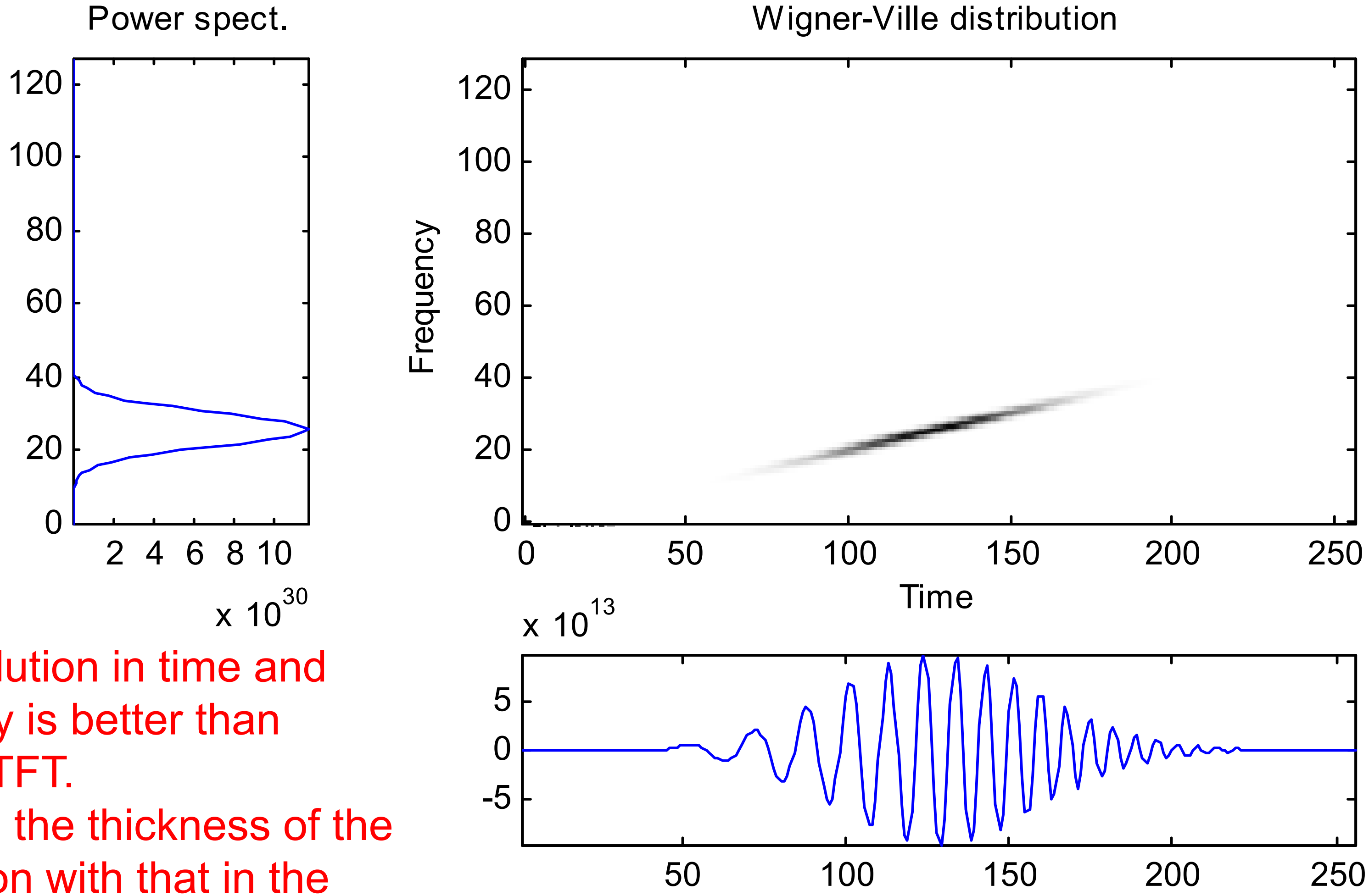
Multiresolution Analysis

- Decomposing a signal or image into a set of bandlimited components (subbands) that can then later be reassembled to reconstruct the original signal.
- One way of implementing discrete wavelet transforms.

Wigner-Ville distribution

- High resolution time-frequency distribution based on the signal's auto-correlation function
- Better trade-off between time and frequency than the STFT because it is a different type of transform
- But suffers from a major disadvantage with composite input signals
- Uncertainty Principle always true
- Applications include analysis of physiological signals (e.g. cardiovascular) and geological exploration (e.g. hydrocarbons)

Illustration: chirplet



The resolution in time and frequency is better than for the STFT.
Compare the thickness of the distribution with that in the following STFT.

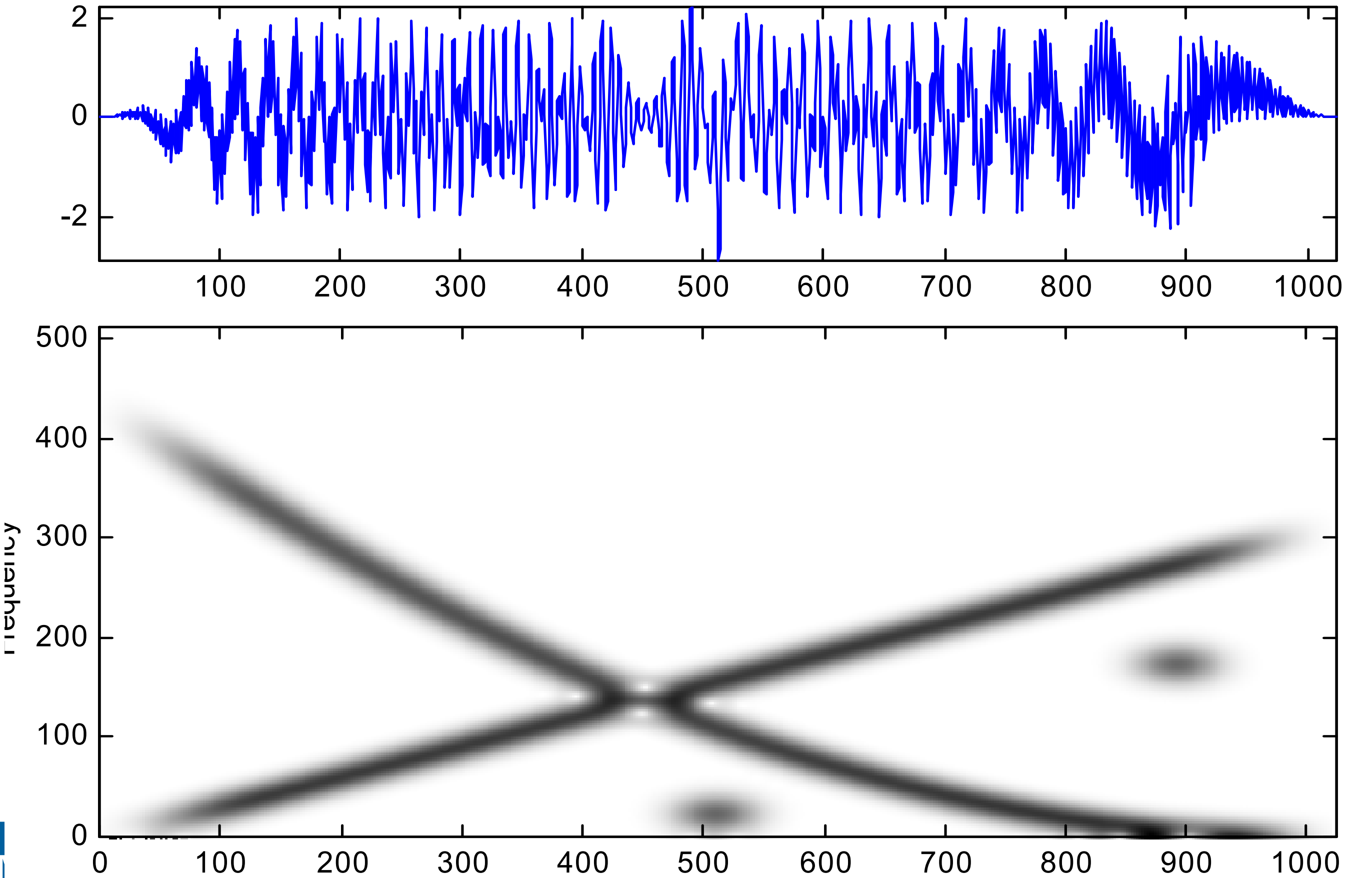
$$s(t) = \sqrt[4]{\frac{\alpha}{\pi}} \exp\left\{-\frac{\alpha}{2}(t-t_0)^2 + j\frac{\beta}{2}t^2\right\}$$

So, what is in a signal?

Example of a composite signal and its spectrogram

We can take short segments of the signal and find the segment FT

This signal contains 2 chirps and 2 short bursts of frequency.



Summary: Fourier Transform

Kind of Decomposition	Frequency
Analyzing Function	Sines and cosines, oscillating indefinitely
Variable	Frequency
Information	The frequencies that make up the signal
Suited for	Stationary signals
Notes	With the FFT, it takes *nlogn* computations to compute the Fourier transform of *n* points

Summary: Windowed Fourier Transform (Short time Fourier Analysis)

Kind of Decomposition	Time-Frequency
Analyzing Function	Wave limited in time, multiplied by sinusoidal oscillations. Window size is fixed, but the frequency inside the window varies.
Variable	Frequency, position of the window
Information	The smaller the window, the better the time information. The cost is low frequency information. Large windows give better frequency information but less timing precision.
Suited for	Quasi-stationary signals (stationary at scale of window)
Notes	When a Gaussian is used as the window envelope this is the Gabor transform. Though the Fourier transform is orthogonal, in general the windowed transform is not.

Summary: Wavelet Transform

Kind of Decom-position	Time-Scale
Analyzing Function	Wave limited in time with fixed number of oscillations. Wavelet is contracted or dilated to change the window size and change the scale at which one looks at the signal. Since number of oscillations is fixed, frequency of the wavelet changes as scale changes.
Variable	Scale, position of the wavelet
Information	Small wavelets provide good time information but poor frequency information. Vice versa for large wavelets.
Suited for	Nonstationary signals, such as brief signals and signals with components at different time scales.
Notes	Wavelet transform can be continuous or discrete. Orthogonal and biorthogonal wavelets are special discrete cases. With orthogonal wavelets, the transform can be computed with *cn* computations. *C* depends on the complexity of the wavelet. n is the number of samples.

Summary: Wigner-Ville Distribution

Kind of Decomposition	Time-Frequency
Analyzing Function	Uses the signal itself. Motivated by time-frequency energy density (c.f. a probability density).
Variable	Time and Frequency. Has high resolution in both time and frequency.
Suited for	Simple signals, e.g. linear chirp, gaussian pulse
Notes	More complex signals lead to undesired “cross-terms”. Can be suppressed with smoothing, but lose high resolution in the process.

Questions?