§ 5.3 Magnetic Dipole

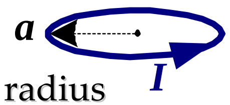
Only a brief introduction and comparison

Electric Dipole

A pair of opposite charges very close to each other.

- Distance: l
- Point charges: $q_1=q$、$q_2=-q$

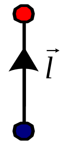
Magnetic Dipole



A circular current with a very small radius

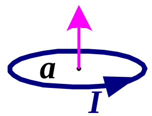
Electric Dipole Moment

$$\vec{p} = q\vec{l}$$

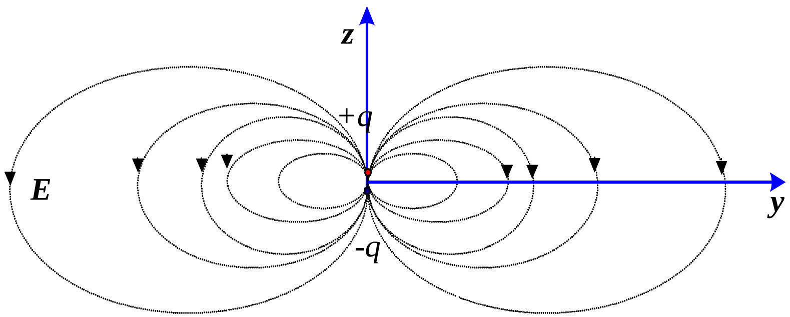


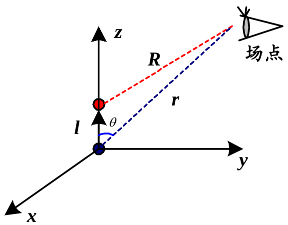
Magnetic Dipole Moment

$$\vec{p}_m = I\vec{S}$$



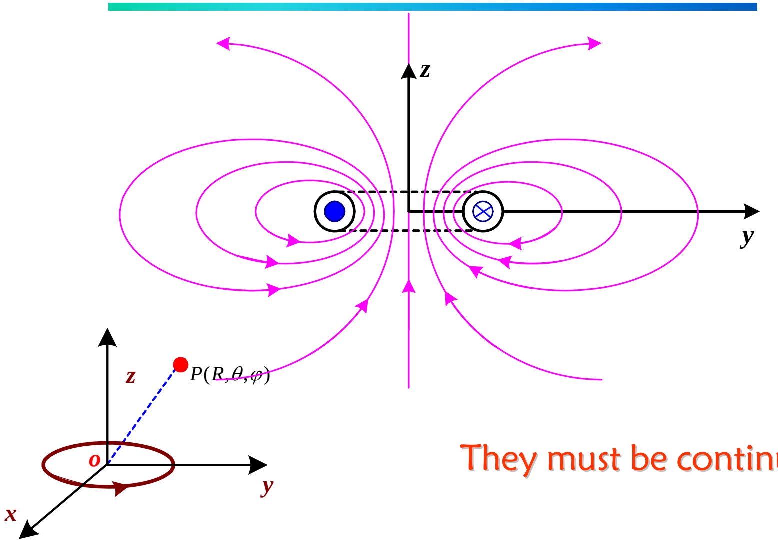
Lines of E-Flux for E-Dipole





Unnecessary to be continuous.

Lines of M-Flux for M-Dipole



They must be continuous.

Key Parameters for M-Dipole

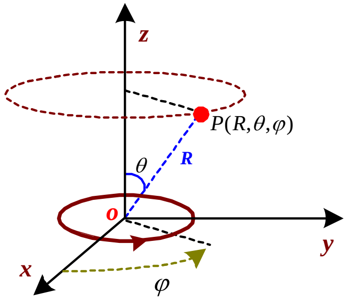
$$\vec{A}=\vec{a}_{\varphi}\left(\frac{\mu_0 I a^2\cdot\sin\theta}{4R^2}\right)$$

$$\vec{A}=\frac{\mu_0\vec{p}_m\times\vec{a}_R}{4\pi\cdot R^2}=-\frac{\mu_0}{4\pi}\vec{p}_m\times\nabla\left(\tfrac{1}{R}\right)$$

$$\vec{B}=\nabla\times\vec{A}$$

$$\vec{B}=\vec{a}_R\frac{\mu_0 P_m}{2\pi r^3}\cos\theta+\vec{a}_\theta\frac{\mu_0 P_m}{4\pi r^3}\sin\theta$$

$$\vec{E}=\vec{a}_R\frac{P_e}{2\pi\varepsilon_0 r^3}\cos\theta+\vec{a}_\theta\frac{P_e}{4\pi\varepsilon_0 r^3}\sin\theta$$

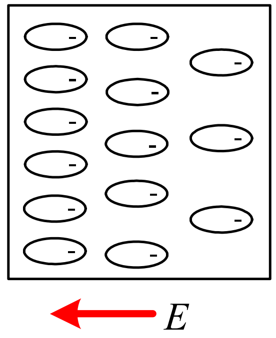


§ 5.4 Material in M-Field

Magnetization

Recall that

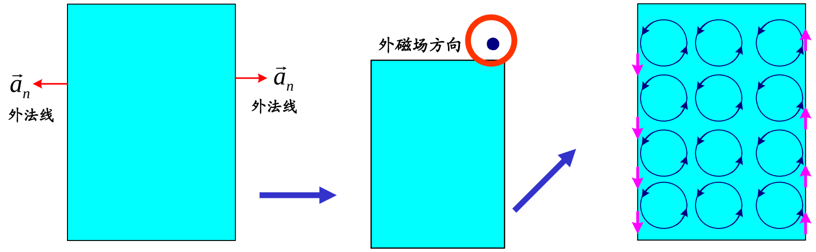
- Materials in E-field will be polarized. Subjected into an E-field, E-dipoles begin to queue orderly which induce bound charges on the surface.



Materials in M-field will be magnetized

- Molecule currents, or atom currents are actually M-dipoles.
- These M-dipoles oriented at random without external M-field.
- With external M-field, all M-dipoles point to the same direction, which is called magnetization.
  - Diamagnetic（反磁性体）: substance inside which the M-intensity is weaker than external M-intensity.
  - Paramagnetic（顺磁性的）: substance inside which the M-intensity is stronger than external M-intensity

- Due to magnetization, all M-dipoles queue orderly and thus yield a kind of surface current, called bound current, or magnetization current.



Magnetization Intensity 磁化强度(Optional)

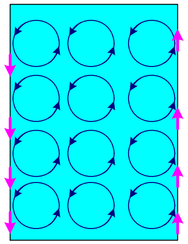
$$\vec{M} = \lim_{\Delta\tau \to 0} \frac{\sum \vec{p}_m}{\Delta\tau} \qquad (A/m)$$

The magnetic dipole moment per unit volume

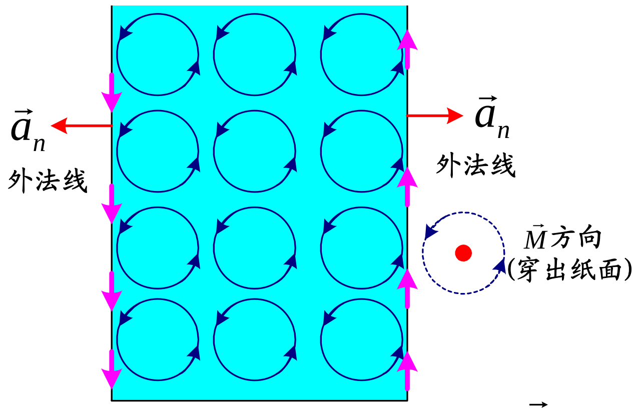
Recall that the polarization intensity is E-moment per unit volume

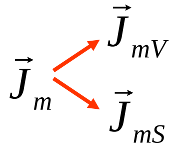
Density of Magnetization current

$$\begin{cases} \vec{J}_m = \nabla \times \vec{M} & (A/m^2) \\ \vec{J}_{ms} = \vec{M} \times \vec{a}_n & (A/m) \end{cases}$$



If a homogeneous substance is magnetized uniformly, the net current inside must be 0.



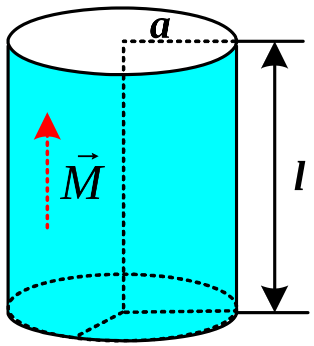


Adjacent M-dipoles will counteract each other

$$\begin{cases} \vec{J}_{mV} = \nabla \times \vec{M} = 0 \\ \vec{J}_{mS} = \vec{M} \times \vec{a}_n = ? \end{cases}$$

例题：选学内容

已知：圆柱形磁性材料，半径为a，长度为l，
被均匀磁化，轴向磁化强度为 $\vec{M}$
求：轴线上磁化磁通密度(磁感应强度**B**)~~~~~?



分析：
(1) 已知什么？
(2) 求什么？
(3) 怎么建立坐标系？
(4) 怎么入手？

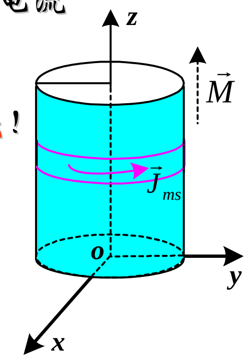
解题：

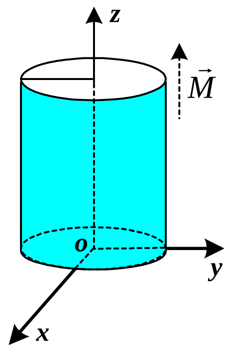
(1)建立坐标系
(2)磁棒内磁化强度是一个常矢量，…
(3)只有侧表面有"磁化面电流"

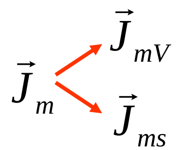
磁体等价于
一个载有面电流的圆柱壳！

$$\vec{J}_{mV} = \nabla \times \vec{M} = 0$$

$$\vec{J}_{mS} = \vec{M} \times \vec{a}_n = M\vec{a}_z \times \vec{a}_r = M\vec{a}_\varphi$$







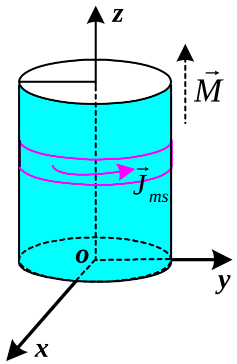
磁化圆柱等价于一个载有面电流的圆柱壳!

为什么是“圆柱壳”，不是“有盖桶”?

磁化圆柱两个“底面”

$$\vec J_{mS} = \vec M \times \vec a_n = (\vec a_z M)\times \vec a_z \equiv 0$$

$$\vec J_{mS} = \vec M \times \vec a_n = (\vec a_z M)\times (-\vec a_z) = 0$$



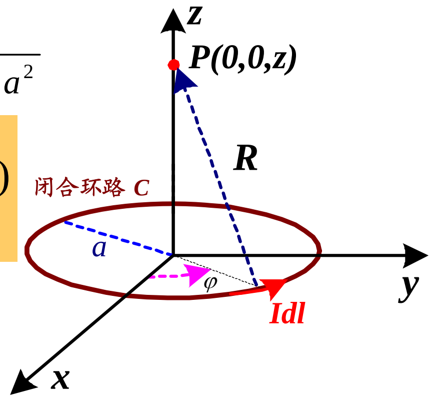
由直接求解法可得电流环在轴线上的磁场

$$\vec B = \oint_S d\vec B = \frac{\mu_0}{4\pi}\oint_C \frac{I_{源} d\vec l_{源}}{R^2_{源-场}} \times \vec a_R$$

$$I d\vec l = \vec a_\varphi (I \cdot a \cdot d\varphi)$$

$$R = |\vec R| = |\overrightarrow{源点到场点}| = \sqrt{z^2 + a^2}$$

$$\boxed{\vec B = \vec a_z \frac{\mu_0 I a^2}{2(z^2+a^2)^{\frac{3}{2}}} \quad (T)}$$



求磁通密度

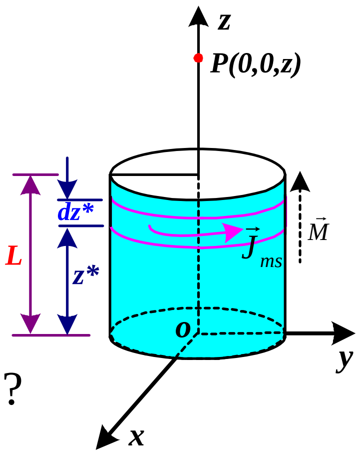
$$\vec J_{mS} = M \vec a_\varphi$$

“电流带”在P点处的磁通密度:

$$dI = J_{mS} \cdot dz^*$$

$$d\vec B = \vec a_z \frac{\mu_0 a^2 dI}{2\left((z-z^*)^2 + a^2\right)^{\frac{3}{2}}}$$

$$\vec B = \vec a_z \int_0^L \frac{\mu_0 a^2 M}{2\left((z-z^*)^2+a^2\right)^{\frac{3}{2}}} dz^* = ?$$

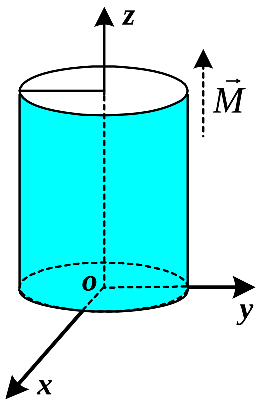


结 果

已知: 圆柱形磁性材料，半径为a，长度为l，
被均匀磁化，轴向磁化强度为 $\vec M$

求: 轴线上磁通密度

$$\vec B = \vec a_z \frac{\mu_0 M}{2}\left[\frac{z}{\sqrt{z^2+a^2}} - \frac{z-L}{\sqrt{(z-L)^2+a^2}}\right]$$



M-Intensity & Relative Permeability(磁导率)

Question:

External M-field + Magnetized Substance → New M-field
How to describe new M-field inside the magnetized substance?

Solution:

Recall that for magnetostatics in free space we have

$$\begin{cases} \nabla \bullet \vec{B} = 0 \\ \dfrac{1}{\mu_0} \nabla \times \vec{B} = \vec{J} \end{cases}$$

For new M-field inside the magnetized substance

$$\frac{1}{\mu_0} \nabla \times \vec{B} = \underbrace{\vec{J}}_{\text{Corresponding to free current}} + \underbrace{\vec{J}_M}_{\text{Corresponding to magnetization current}}$$

$$\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{J} + \vec{J}_M = \vec{J} + \nabla \times \vec{M} \quad \therefore \nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}$$

Magnetic Field Intensity

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \qquad (A/m)$$

$$\therefore \nabla \times \vec{H} = \vec{J} \quad \text{(volume density of free current)}$$

In comparison with electrostatics:

$$\oint_S \vec{E} \bullet d\vec{S} = \frac{\sum q_{fc} + \sum q_{pc}}{\varepsilon_0} \qquad \nabla \bullet (\varepsilon_0 \vec{E} + \vec{P}) = \rho_{fc}$$

$$\nabla \bullet \vec{D} = \rho_{fc}$$

$\nabla \times \vec{H} = \vec{J}$ (volume density of free current)

so

$$\int_S \left(\nabla \times \vec{H} \right) \bullet d\vec{S} = \int_S \left(\vec{J} \right) \bullet d\vec{S}$$

Applying Stokes's Law, we have

$$\oint_C \vec{H} \bullet d\vec{l} = I$$

What is the Unit of "Magnetic Field Intensity" ?

$$\vec{H}: \quad (A/m)$$

How about "Electric Field Intensity" ?

$$\boxed{\vec{E}: \quad (V/m)}$$

In Linear & Isotropic Materials

$$\vec{M} = \chi_m \vec{H}$$

χ_m : susceptibility（磁化率 无量纲）

$$\because \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{B} = \ldots = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$\vec{B}: \quad (Wb/m^2)$ $\qquad$ $\vec{H}: \quad (A/m)$

μ_r : relative permeability（相对磁导率）

$$\boxed{\mu_r = 1 + \chi_m = \mu / \mu_0}$$

μ : absolute permeability（绝对磁导率）

In comparison with electrostatics:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon \vec{E}$$

A Discussion on Relative Permeability μ_r

1. *diamagnetic*抗磁性材料 $\mu_r \le 1$ $\chi_m \approx -0$

   Copper, lead, gold, silver, etc..

2. *paramagnetic*顺磁性材料 $\mu_r \ge 1$ $\chi_m \approx +0$

   Aluminum, tungsten (钨), etc..

3. *ferromagnetic*铁磁性材料 $\mu_r >> 1$ $\chi_m >> 0$

   Cobalt (钴), iron, etc..

Summary on Material Parameters

真空中磁导率(*Permeability*)：

$$\mu_0 = 4\pi \cdot 10^{-7} (H / m)$$

真空中介电常数(*Dielectric Constant*)：

$$\varepsilon_0 = \frac{1}{4\pi \cdot 9 \times 10^9} = 8.85 \times 10^{-12} (F / m)$$

$$\frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = c$$

简单媒质——线性、均匀、各向同性

磁化率χ_m：无单位、常数

相对磁导率μ_r：无单位、常数