Chapter 5

Baseband Transmission of Digital Signals

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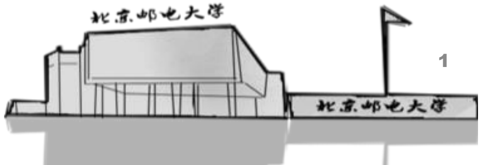
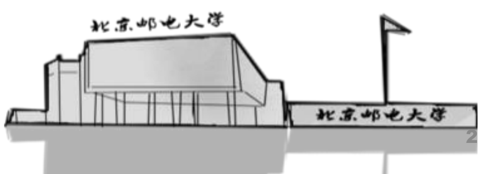


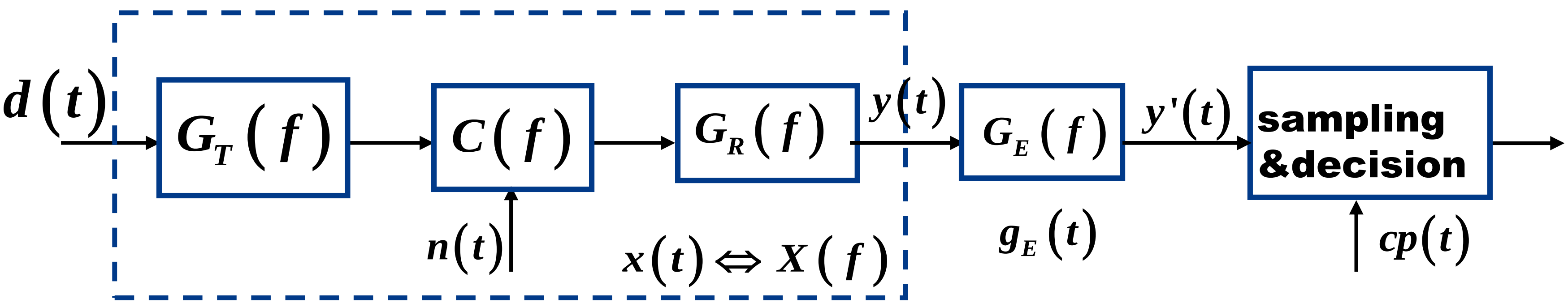
Baseband Transmission of Digital Signals

- Introduction
- Baseband signal and pulse modulation
- Digital PAM signal transmission through AWGN channel
- Digital PAM signal transmission through baseband channel
- Optimal transmission of digital PAM signal through ideal baseband channel and under AWGN condition
- Eye Diagram
- Channel Equalization
- Partial Response System
- Symbol Synchronization
- Summary



Channel Equalization

- Channel Equalization is a scheme to reduce the non-ideality of the channel.
- Time domain equalization
  - Linear equalization
  - Non-linear equalization
- Frequency domain equalization



$$X(f)\cdot G_E(f)=\left|X_{\text{升余}}(f)\right|e^{-j2\pi f t_0}$$

Channel Equalization

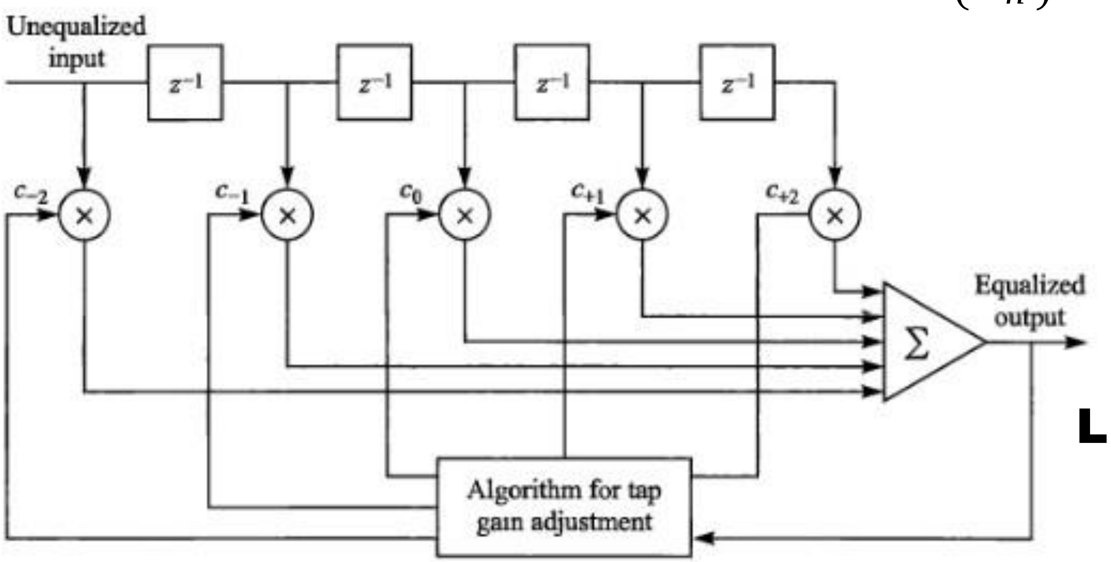
•Linear Equalization

The impulse response:

$$h_k = \sum_{n=-N}^{N} c_n x_{k-n}$$

$\{x_k\}$: **input**

$\{c_n\}$: **tap coefficients**



Linear transversal filter

Channel Equalization

•Peak Distortion Criterion

Peak Distortion

$$D = \frac{1}{h_0} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} |h_k| = \frac{1}{h_0} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \left| \sum_{n=-\infty}^{\infty} c_n x_{k-n} \right|$$

The peak distortion *D* is minimized by adjusting the equalizer coefficients $\{c_n\}$ to force :

$$h_k = \sum_{n=-N}^{N} c_n x_{k-n} = \begin{cases} 0, & 1 \leq |k| \leq N \\ 1, & k = 0 \end{cases}$$

N $\to \infty$, residual ISI$\to$0 $\Longrightarrow$ **zero-forcing filter**

Channel Equalization

•Peak Distortion Criterion

Example: **Determine the tap coefficients of a three-tap zero-forcing equalizer if the ISI spans three symbols and is characterized by the values x(0) = 1, x(-1) = 1/4, x(1) = 1/2.**

$$\begin{bmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} c_{-1} = -\dfrac{1}{3} \\ c_0 = \dfrac{4}{3} \\ c_1 = -\dfrac{2}{3} \end{cases}$$

$$h_k = \sum_{n=-N}^{N} c_n x_{k-n} \Rightarrow \begin{cases} h_{-2} = -\dfrac{1}{12} \\ h_2 = -\dfrac{1}{3} \end{cases}$$

Before: $D_0 = \dfrac{1}{x_0} \sum_{k \neq 0} |x_k| = \dfrac{3}{4}$ **After:** $D = \dfrac{1}{h_0} \sum_{k \neq 0} |h_k| = \dfrac{5}{12}$

Channel Equalization

- **Mean-Square-Error (MSE) Criterion**

Error: $e_m = a_m - \hat{a}_m = a_m - \sum_{n=-N}^{N} w_n x_{k-n}$

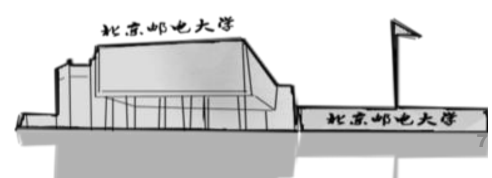
The mean square value: $J = E\left(e_m^2\right) = E\left(a_m - \sum_{n=-N}^{N} w_n x_{k-n}\right)^2$

$\Longrightarrow$ **Minimize the MSE:**

$$\frac{\partial J}{\partial w_k} = 0 \Longrightarrow R_{ax}(k) = \sum_{n=-N}^{N} w_n R_x(n-k), \quad k = 0, \pm 1, \ldots, \pm N$$

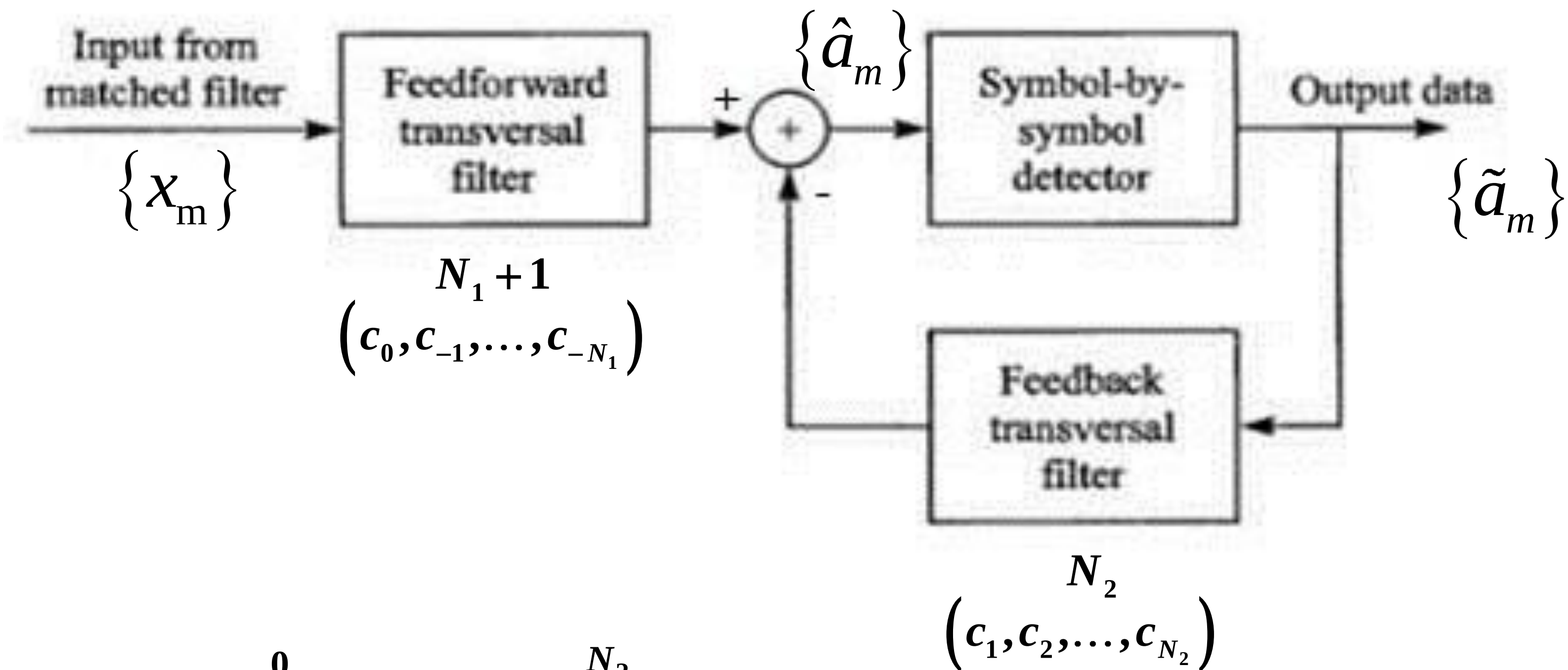
Estimation:

$$\hat{R}_x(k) = \frac{1}{K}\sum_{m=1}^{K} x(m-k)x(m), \quad \hat{R}_{ax}(k) = \frac{1}{K}\sum_{k=1}^{K} x(m-k)a(m)$$



Channel Equalization

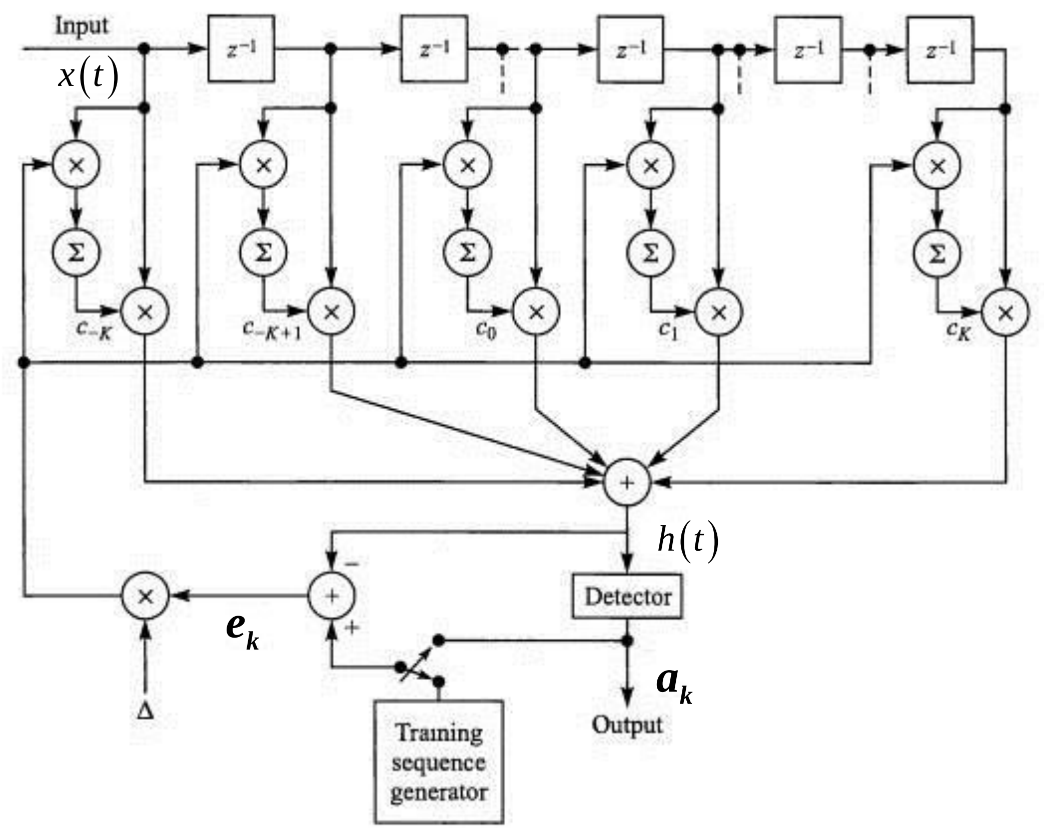
- **Decision-feedback Equalization**



$$\hat{a}_m = \sum_{n=-N_1}^{0} c_n x_{m-n} - \sum_{n=1}^{N_2} c_n \tilde{a}_{m-n}$$

Channel Equalization

•Adaptive Equalization



$$e_k = h_k - a_k = \sum_{i=-N}^{N} w_i x_{k-i} - a_k$$

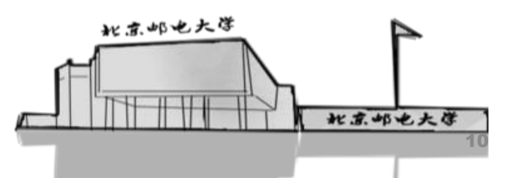
$$J = E\left(e_k^2\right) = E\left(\sum_{i=-N}^{N} w_i x_{k-i} - a_k\right)^2$$

$$\frac{\partial J}{\partial w_i} = 2E\left[e_k x_{k-i}\right] = 0$$

$$E\left[e_k x_{k-i}\right] \approx \frac{1}{m}\sum_{k=1}^{m} e_k x_{k-i}$$

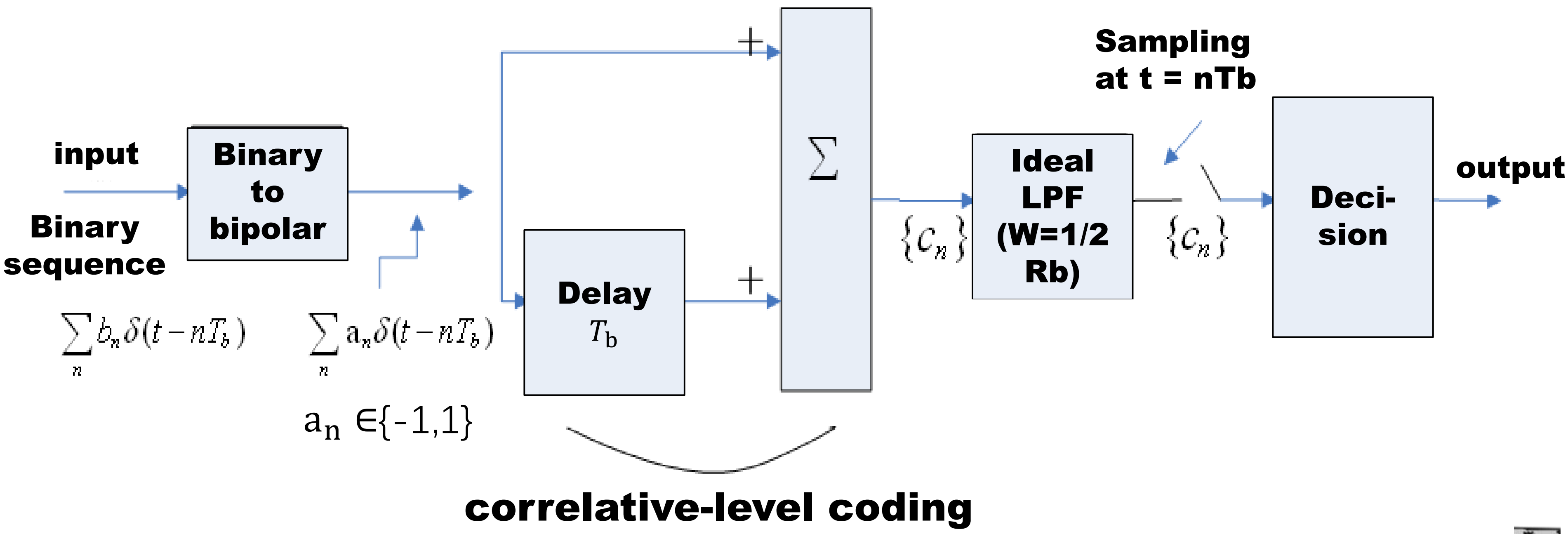
Linear adaptive equalizer based on MSE criterion.

Baseband Transmission of Digital Signals



Partial Response System

- By introducing deterministic or controlled ISI, we can achieve the Nyquist rate of 2W Baud.
- Class I partial response system or Duobinary System: correlative-level coding



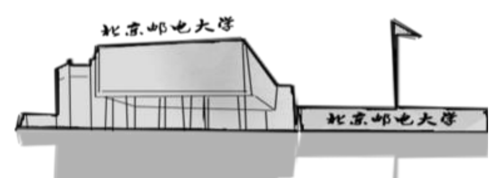
Partial Response System

- **correlative-level coding**: $c_n = a_n + a_{n-1}$

b_n	1	0	1	1	0	0	0	1	0	1	1
a_n	+1	-1	+1	+1	-1	-1	-1	+1	-1	+1	+1
c_n		+0	0	+2	0	-2	-2	0	0	0	+2

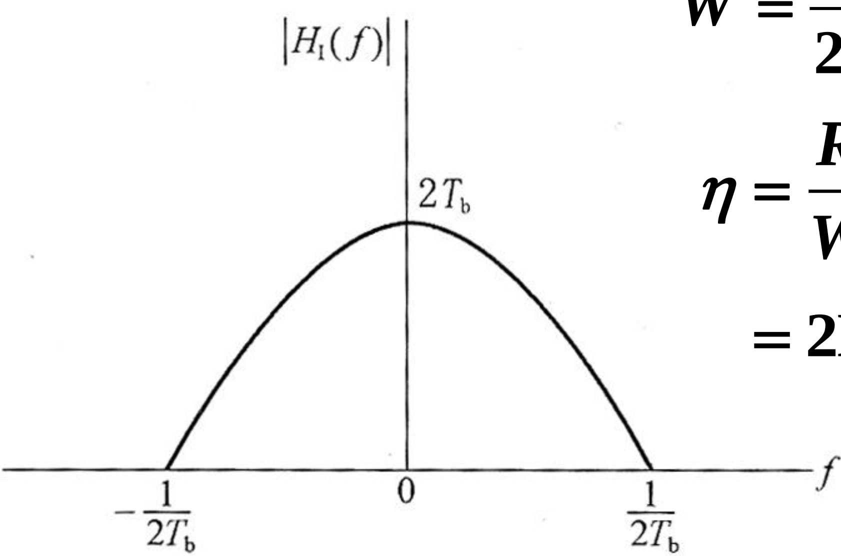
3-level sequence

$$h_1(t) = \delta(t) + \delta(t - T_b)$$



Partial Response System

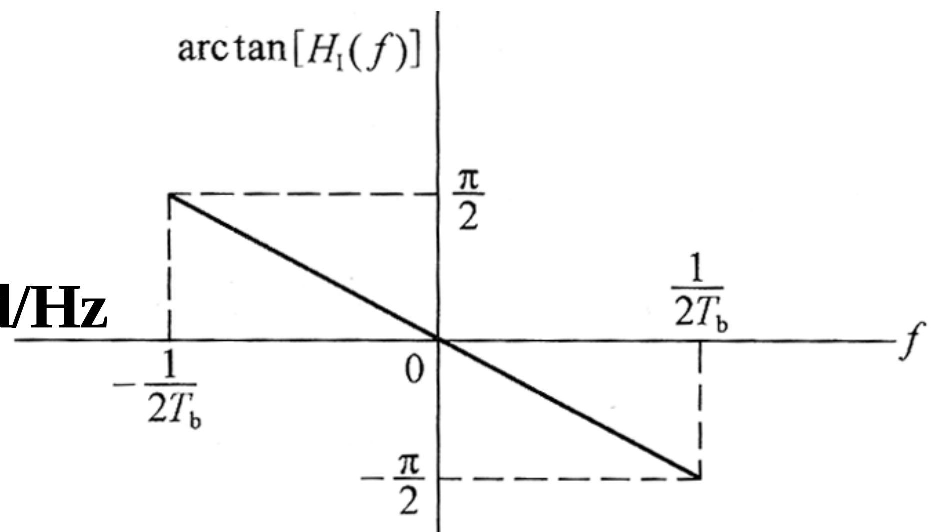
$$
\begin{aligned}
H_I(f) &= H_{Nyquist}(f)\left[1+\exp(-j2\pi fT_b)\right] \\
&= H_{Nyquist}(f)\left[\exp(j\pi fT_b)+\exp(-j\pi fT_b)\right]\exp(-j\pi fT_b) \\
&= 2H_{Nyquist}(f)\cos(\pi fT_b)\exp(-j\pi fT_b) \\
&= \begin{cases} 2\cos(\pi fT_b)\cdot\exp(-j\pi fT_b)\cdot T_b, & |f|\le 1/2T_b \\ 0, & |f|>1/2T_b \end{cases}
\end{aligned}
$$



$$W=\frac{1}{2T_b}$$

$$\eta=\frac{R_s}{W}$$

$$=2\text{Baud/Hz}$$

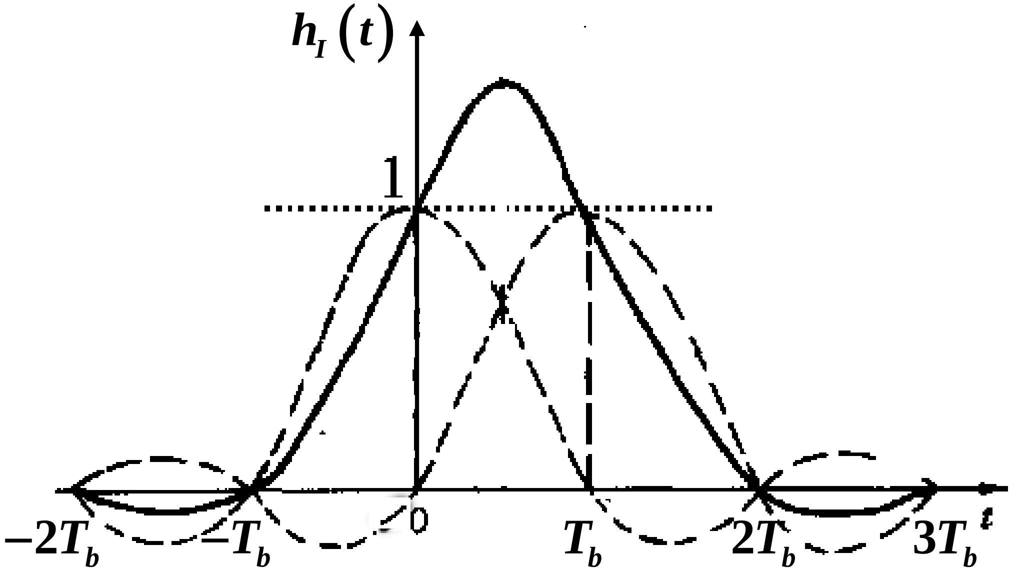


Partial Response System

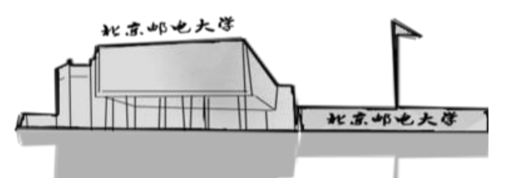
- **Class I partial response system:**

The impulse response:

$$h_I(t) = \text{sinc}\left(\frac{t}{T_b}\right) + \text{sinc}\left[\frac{(t-T_b)}{T_b}\right] = \frac{T_b^2 \sin\frac{\pi t}{T_b}}{\pi t\left(T_b - t\right)}$$



$$h_I(nT_b) = \begin{cases} 1, & n = 0,1 \\ 0, & n \neq 0,1 \end{cases}$$



Partial Response System

- **Class I partial response system**
  - **For the optimal transmission:**

$$\left|G_T(f)\right| = \left|G_R(f)\right| = \left|H_I(f)\right|^{1/2}, \quad |f| \le \frac{1}{2T_b}$$

  - **Data detection:**

Symbol-by-symbol suboptimum detection

$$c_n = a_n + a_{n\text{-}1} \quad \Longrightarrow \quad \hat{a}_n = c_n - \hat{a}_{n-1}$$

Maximum-likelihood sequence detection

Partial Response System

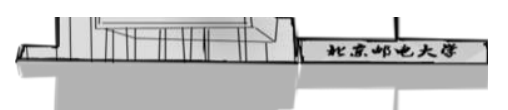
- **Class I partial response system**

Error-propagation

$$\boxed{c_n = a_n + a_{n-1}}$$

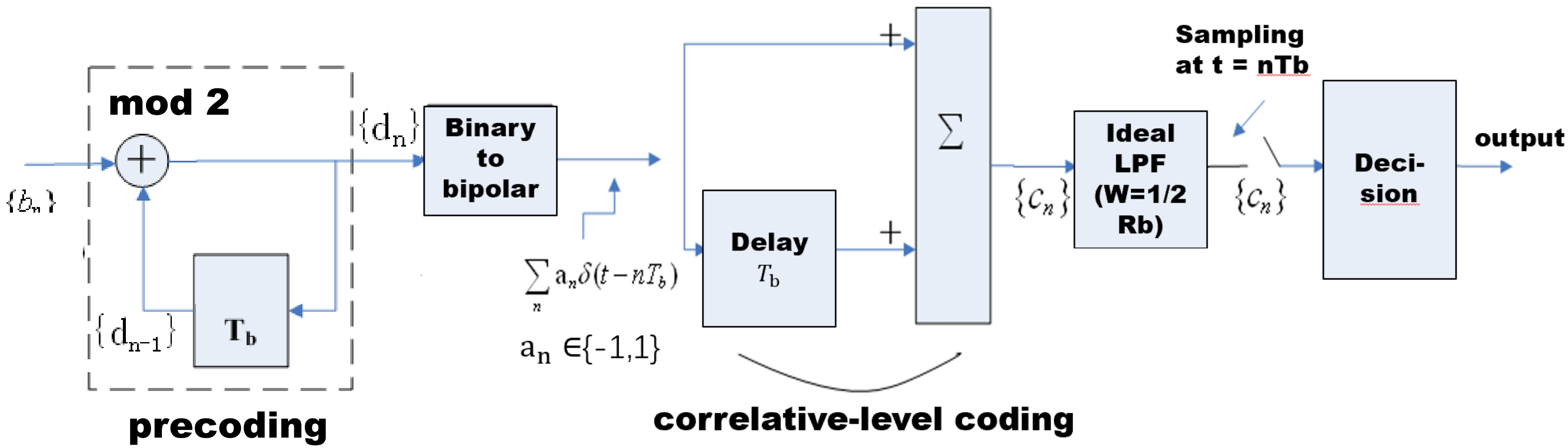
$$\boxed{:a_n = c_n - a_{n-1}}$$

b_n:	1	0	1	1	0	0	0	1	0	1	1
a_n:	+1	-1	+1	+1	-1	-1	-1	+1	-1	+1	+1
c_n:		+0	0	+2	0	-2	-2	0	0	0	+2
$\hat{a}_n$:	+1	-1	+1	+1	-1	-1	-1	+1	-1	+1	+1
c'_n		+0	0	+2	0	-2	-2	-2	0	0	+2
$\hat{a}_n'$:	+1	-1	+1	+1	-1	-1	-1	-1	+1	-1	+3



Partial Response System

- **Class I partial response system with precoding**



Precoding: $\{b_n\} \to \{d_n\}$

$$b_n = d_n \oplus d_{n-1} \Longrightarrow d_n = b_n \oplus d_{n-1}$$ ~ **modulo-2 addition**

Partial Response System

- **Corresponding 2-level sequence**： $a_n = 2d_n - 1$
- **Correlative-level coding** ： $c_n = a_n + a_{n-1} = 2\left(d_n + d_{n-1} - 1\right)$

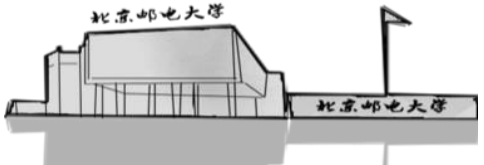
$$\therefore \quad d_n + d_{n-1} = \frac{c_n}{2} + 1$$

- **Modulo-2 addition:**

$$b_n = d_n \oplus d_{n-1} = \left[\frac{c_n}{2} + 1\right]_{\mathrm{mod}\,2}$$

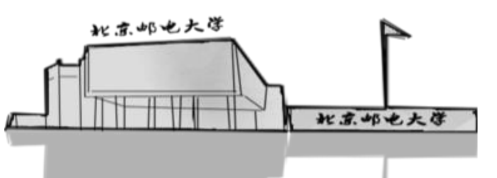
- **Decision rule**：

$$b_n = \begin{cases} 1, & c_n = 0 \\ 0, & c_n = \pm 2 \end{cases} \xrightarrow{y_n = c_n + n_n} b_n = \begin{cases} 1, & |y_n| < 1 \\ 0, & |y_n| \geq 1 \end{cases}$$



Partial Response System

b_n		1	1	1	0	0	1	0	1	1	1	0	0
d_n	0	1	0	1	1	1	0	0	1	0	1	1	1
a_n	-1	+1	-1	+1	+1	+1	-1	-1	+1	-1	+1	+1	+1
c_n		0	0	0	+2	+2	0	-2	0	0	-2	+2	+2
$\hat{b}_n$		1	1	1	0	0	1	0	1	1	0	0	0

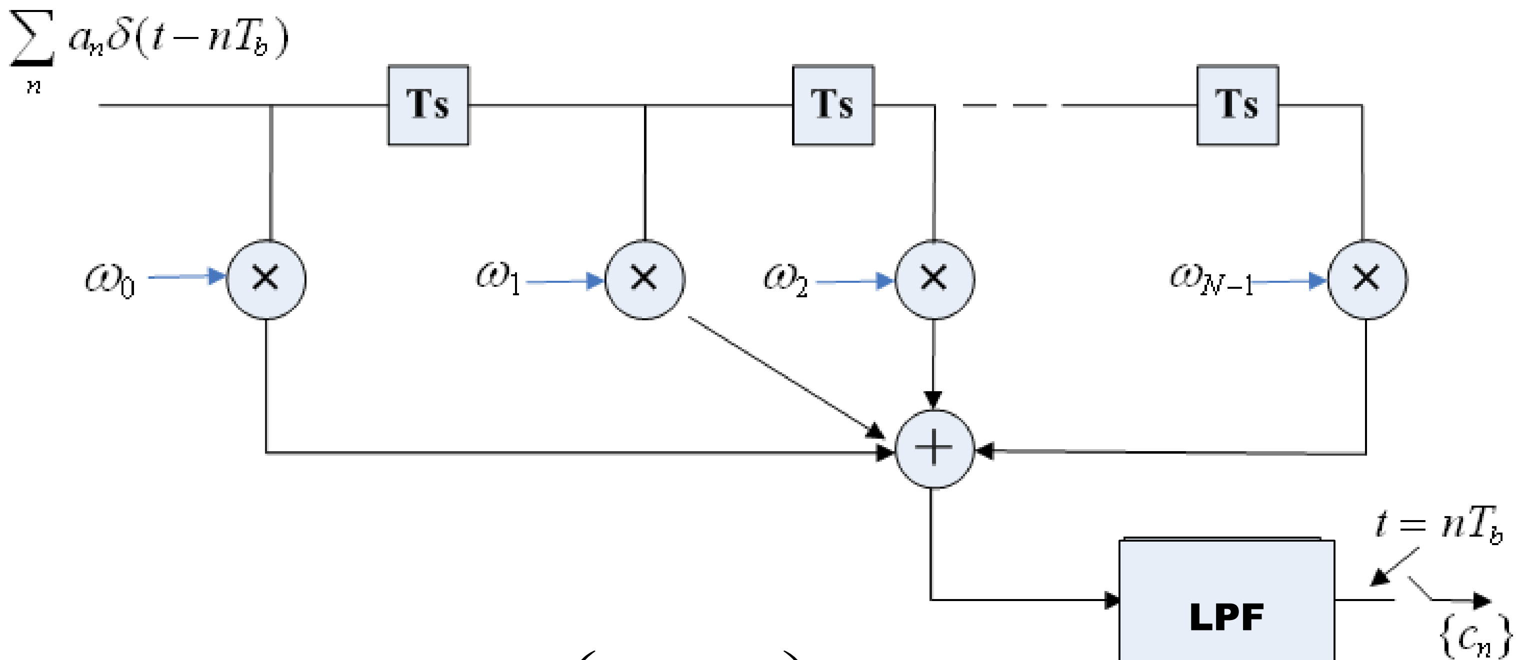


Discussion

Class I partial response system （duobinary system） can achieve 2Baud/Hz frequency efficiency without ISI.

But the BER is a little higher(with the same Eb/N0), since 3-level code is adopted.

General Partial Response System



$$h(t)=\sum_{n=0}^{N-1}\omega_n \operatorname{sinc}\left(\frac{t}{T_b}-n\right)$$

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Symbol Synchronization

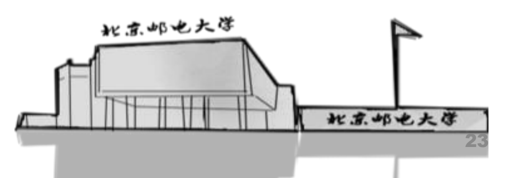
- **Symbol synchronization/timing recovery:**
  - **the precise sampling time instant:**

$$t_m = mT_s + \tau_0$$

- **Timing recovery methods:**
  - **External synchronization**
  - **Transmit the timing signal along with the data, e.g., as a low power pilot. And recover it with a narrowband filter at the receiver.**
  - **Self-synchronization:**

$$y(t) = s(t) + n(t) = \sum_{n=-\infty}^{\infty} a_n x(t - nT_s - \tau_0) + n(t)$$

T_s :the symbol period， τ_0: a nominal time delay



Symbol Synchronization

Line spectrum method

$E[s(t)]=0;$ $$E\left[s^2(t)\right]=E\left[\sum_m\sum_n a_m a_n x(t-mT_s-\tau_0)\cdot x(t-nT_s-\tau_0)\right]$$

$$=\sigma_a^2\sum_n x^2(t-nT_s-\tau_0)$$

$$\overset{T_s}{=\!=}\frac{\sigma_a^2}{T_s}\sum_m c_m e^{j2\pi m(t-\tau_0)/T_s}\leftrightarrow\frac{\sigma_a^2}{T_s}\sum_m c_m\delta\left(f-\frac{m}{T_s}\right)e^{j2\pi m\tau_0/T_s}$$

where, $$c_m=\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}}\sum_n x^2(t-nT_s)e^{-j2\pi mt/T_s}dt=\int_{-\infty}^{\infty}x^2(t')e^{-j2\pi mt'/T_s}dt'$$

$$=\int_{-\infty}^{\infty}X(f)\cdot X\left(\frac{m}{T_s}-f\right)df$$ $\boxed{t'=t-nT_s}$

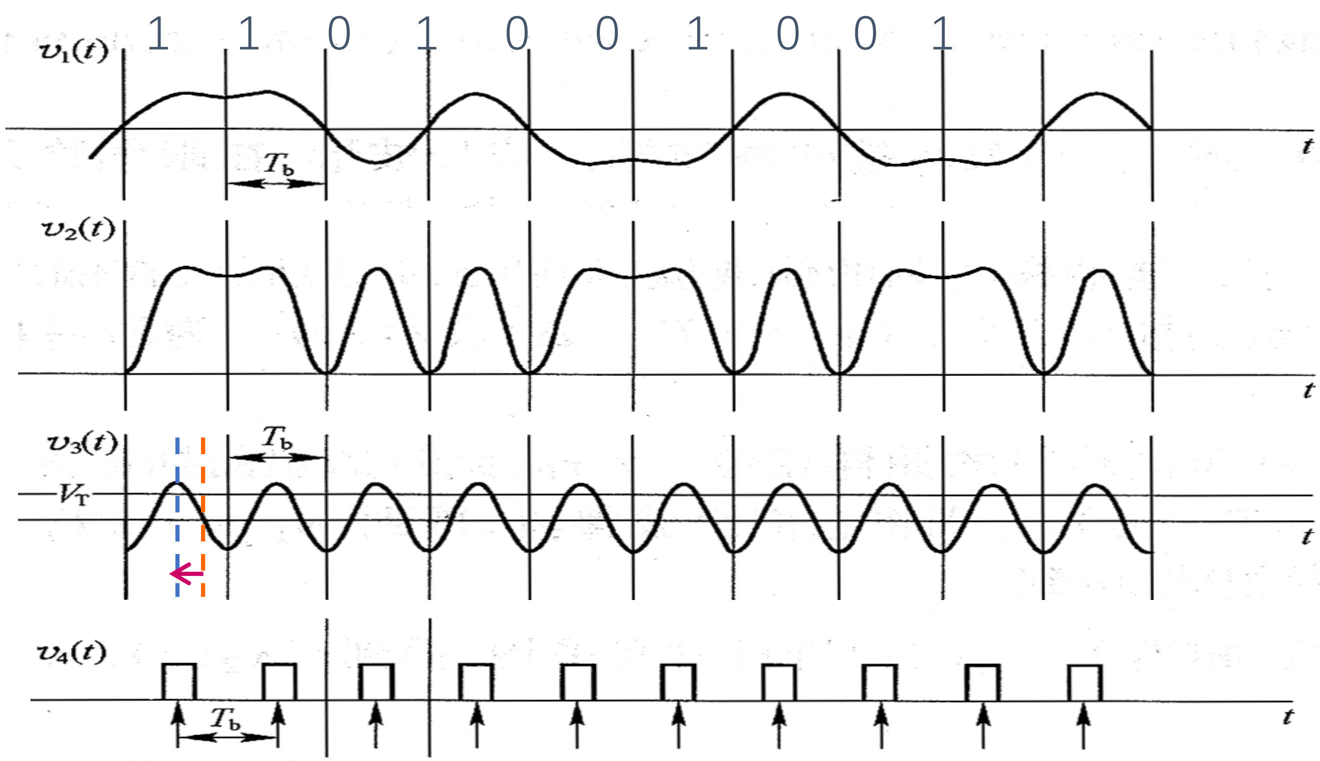
if $W=1/T_s\Leftrightarrow X(f)=0\ \ for\ |f|>1/T_s$ $\Rightarrow$ $$c_m=\begin{cases}Non\text{-}zero, & m=0,\pm1\\ Zero, & \text{otherwise}\end{cases}$$

$$E\left[s^2(t)\right]\xrightarrow{\text{narrowband filter}}\frac{\sigma_a^2}{T_s}R_e\left[c_1e^{j2\pi(t-\tau_0)/T_s}\right]=\frac{\sigma_a^2}{T_s}c_1\cos\frac{2\pi}{T_s}(t-\tau_0)$$

Symbol Synchronization

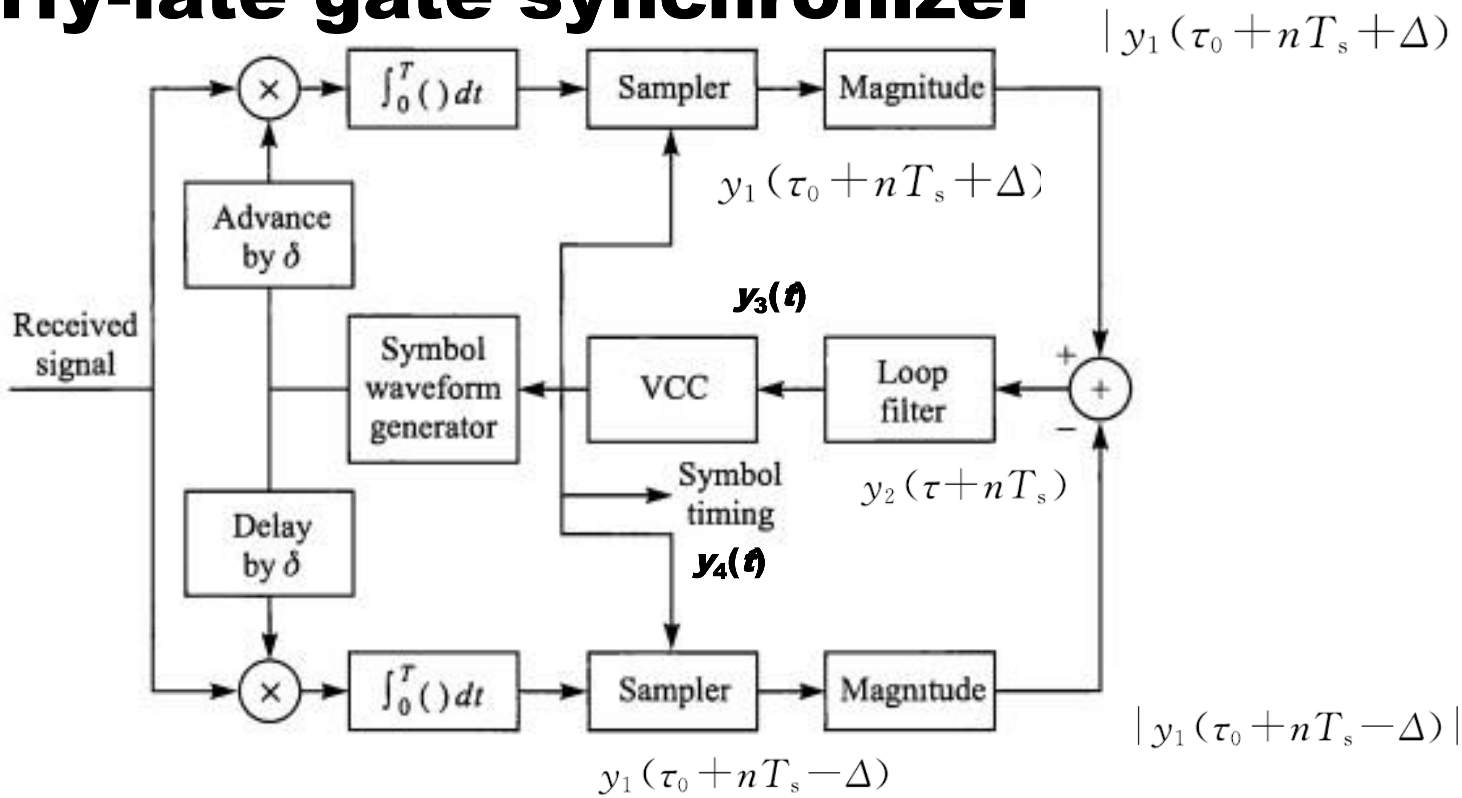
◻Line spectrum method

$$\frac{2\pi}{T_s}(t-\tau_0)=2\pi\cdot k+\frac{\pi}{2} \Rightarrow t=kT_s+\tau_0+\frac{T_s}{4}$$



Symbol Synchronization

Early-late gate synchronizer



$$y_2(\tau+nT_s)=|y_1(\tau+nT_s-\Delta)|-|y_1(\tau+nT_s+\Delta)|$$

if $\tau<\tau_0$ **(advanced)** , $y_3(t)<0$, ***fc*** **is decreased;**
if $\tau>\tau_0$ **(delayed)** , $y_3(t)>0$, ***fc*** **is increased;**
when: $\tau=\tau_0$, $y_3(t)=0$, ***fc*** **is maintained.**