3.4 Two infinite planes with equal and opposite but uniform charge distributions are separated by a distance d. Find the electric field intensity above, below, and in the region between the planes.

Exercise 3.4 $\quad |\vec{E}_+| = |\vec{E}_-| = \frac{\rho_s}{2\epsilon_0}$

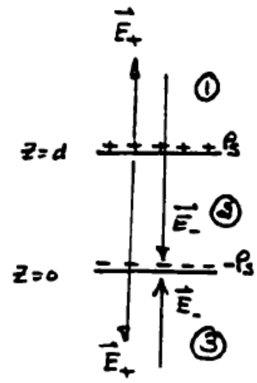
Region - 1 : $\vec{E} = 0$

Region - 3 : $\vec{E} = 0$

Region - 2 :

$$\vec{E}_2 = -\left[\frac{\rho_s}{2\epsilon_0} + \frac{\rho_s}{2\epsilon_0} \right] \vec{a}_z$$

$$= -\frac{\rho_s}{\epsilon_0} \vec{a}_z \quad V/m$$



3.7 The charge distribution within a spherical region bounded by radii a and b ($a < b$) is given as $\rho_v = k/r$, where k is a constant. Determine the electric field intensity everywhere in space. What is the total flux passing through a surface at $r = b$?

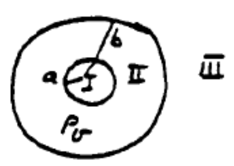
Exercise 3.7 $\quad \vec{E}_I = 0$

$$\oint \vec{E} \cdot \vec{ds} = 4\pi r^2 E_r \qquad a \le r \le b$$

$$Q = \int_a^r \frac{k}{r} r^2 dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi = 2\pi k (r^2 - a^2)$$

Region - II $\quad \vec{E}_{II} = \frac{k}{2\epsilon_0} \left(\frac{r^2 - a^2}{r^2} \right) \vec{a}_r$

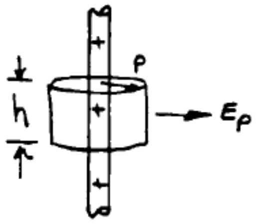
Region - III $\quad Q = 2\pi k (b^2 - a^2) \Rightarrow \vec{E}_{III} = \frac{k}{2\epsilon_0} \left[\frac{b^2 - a^2}{r^2} \right] \vec{a}_r$



3.8 A cylindrical conductor of radius a and of infinite length has a uniform charge distribution ρ_s over its surface. Compute the electric field intensity and the electric flux density everywhere in space. Calculate the flux passing through a cylindrical surface of radius b ($b > a$) and length ℓ.

Exercise 3.8 $\quad a \le \rho \le \infty$

$$Q = \int_s \rho_s \, ds = 2\pi a h \rho_s$$



$$\oint_s \vec{D}\cdot d\vec{s} = 2\pi\rho h D_\rho \Rightarrow D_\rho = \frac{a\rho_s}{\rho} \Rightarrow E_\rho = \frac{a\rho_s}{\rho\epsilon_0}$$

$$\psi = \int_s \vec{D}\cdot d\vec{s} = a\rho_s \int_0^{2\pi} \frac{1}{\rho}\rho \, d\phi \int_0^{\ell} dz = 2\pi a \ell \rho_s$$

3.10 Using (3.9) and vector operations, show that (a) $\vec{\mathbf{E}} = -\nabla V$, and (b) $\nabla \times \vec{\mathbf{E}} = 0$.

Exercise 3.10 $\quad \vec{E} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\vec{a}_r}{r^2}$ but $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{a}_r}{r^2}$

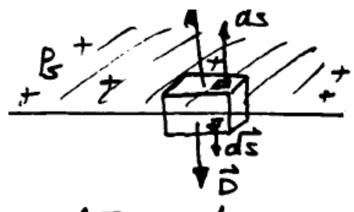
a) $\quad$ Thus, $\vec{E} = -\frac{Q}{4\pi\epsilon_0} \nabla(1/r) = -\nabla\left(\frac{Q}{4\pi\epsilon_0 r}\right) = -\nabla V$

b)

$$\nabla\times\vec{E} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & r\sin\theta\,\vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ E_r & 0 & 0 \end{vmatrix} = 0$$

3.21 Using Gauss's law, compute the electric field intensity and electric flux density at any point due to a uniform charge distribution on an infinite plane sheet of charge.

Problem 3.21



A: Area of Top and bottom surfaces.

$$\oint_s \vec{D}\cdot d\vec{s} = Q_{enc} \Rightarrow 2 D_z A = \rho_s A$$

$$D_z = \frac{\rho_s}{2}$$

$$E_z = \frac{\rho_s}{2\epsilon_0}$$