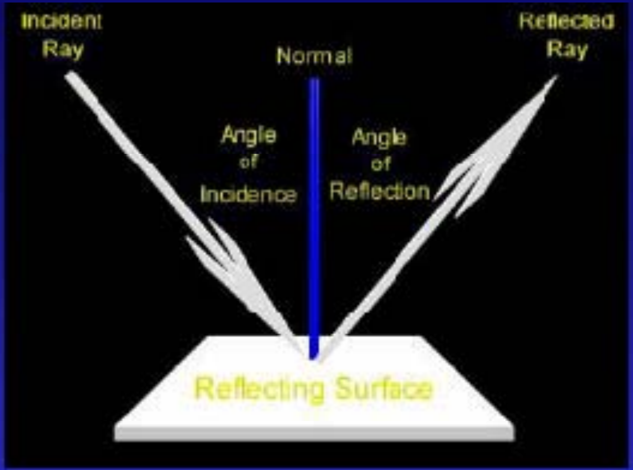
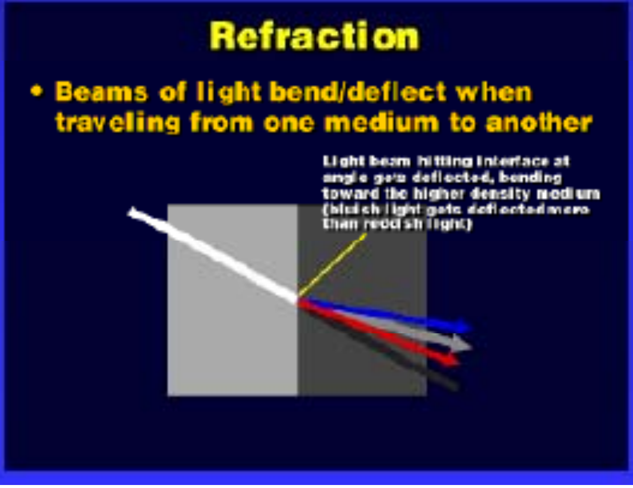
Chpt.9 Reflection & Refraction of HPW

反射



折射

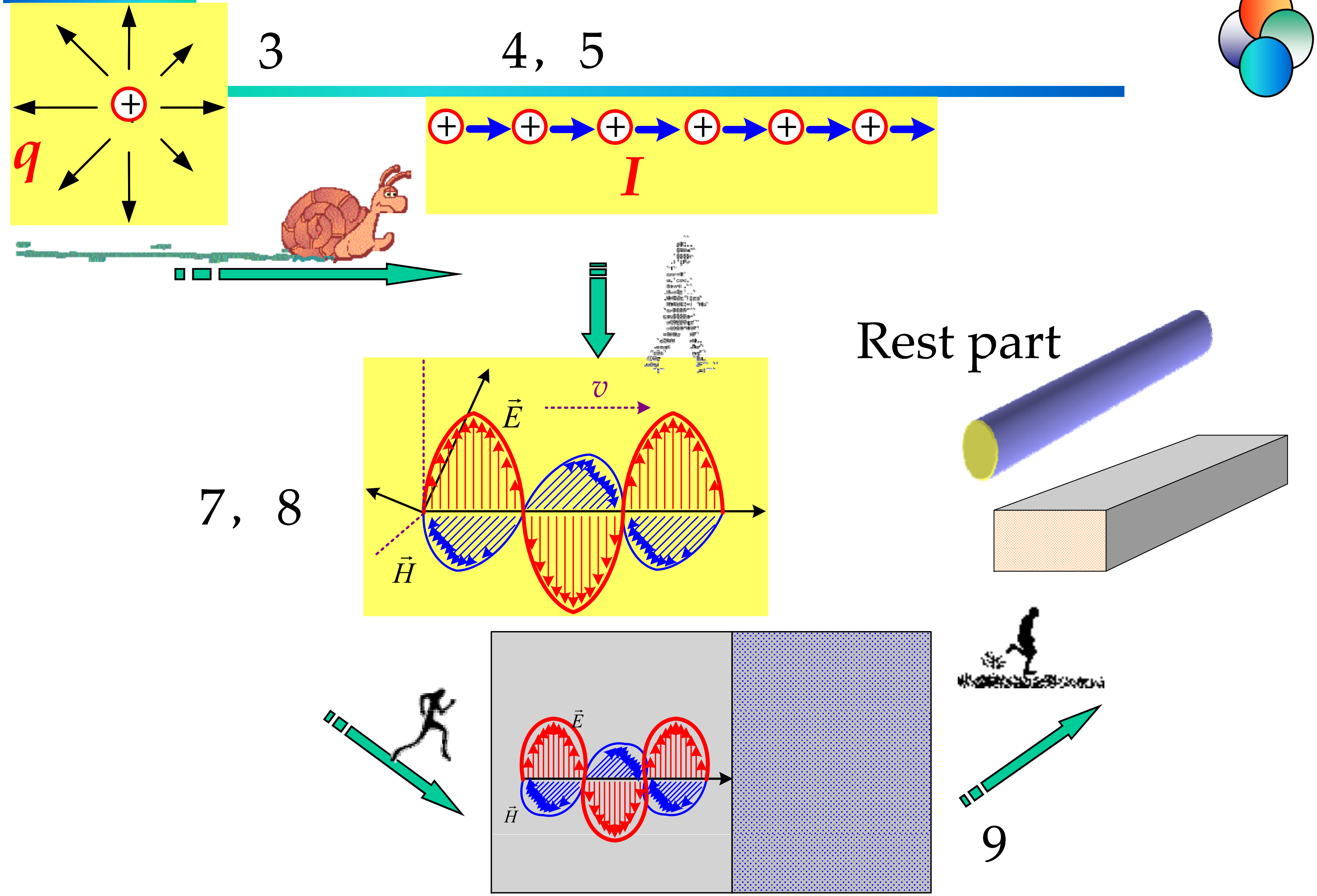


Refraction

- Beams of light bend/deflect when traveling from one medium to another

Light beam hitting interface at angle gets deflected, bending toward the higher density medium (bluish light gets deflected more than reddish light)

Up to now, we have gone so long and so far.



3

4，5

Rest part

7，8

9

Chpt.9 Reflection & Refraction of HPW

- 第8章学习了无限媒质或介质中的波
- 假设波可以无限长时间地传到无限远……
- To far and far away, for ever and ever
- 这是各类行波。
- 一旦波遇到了边界（实际情况往往如此）……有反射……有折射……
- 第9章学习有限导体和介质中的波
- 平面波在两种物质边界上的反射与折射

声明

- 本章涉及的媒质和介质——
  - 依然**均匀、线性、各向同性**
- 本章涉及的媒质和介质——
  - 只限于**非磁性**的
  - μ近似等于μ_0
- **研究**波的入射、反射、折射的**依据**——
  - 依然是**边界条件**
- 本章各小节的内容和思路相似，且整章都可看作是第8章的例题(在边界上的应用举例)

电磁场的边界条件

矢量形式

$$\vec{a}_n \bullet \left(\vec{D}_1 - \vec{D}_2\right) = \rho_{S_{FC}}$$

$$\vec{a}_n \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\vec{a}_n \bullet \left(\vec{B}_1 - \vec{B}_2\right) = 0$$

$$\vec{a}_n \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_{ST}$$

标量形式

$$D_{1n} - D_{2n} = \rho_{S_{FC}}$$

$$E_{1t} - E_{2t} = 0$$

$$B_{1n} - B_{2n} = 0$$

$$H_{1t} - H_{2t} = J_{ST}$$

理想导体，的边界条件

矢量形式

$$\vec{a}_n \bullet \vec{D}_1 = \rho_{S_{FC}}$$

$$\vec{a}_n \times \vec{E}_1 = 0$$

$$\vec{a}_n \bullet \vec{B}_1 = 0$$

$$\vec{a}_n \times \vec{H}_1 = \vec{J}_{ST}$$

标量形式

$$D_{1n} = \rho_{S_{FC}}$$

$$E_{1t} = 0$$

$$B_{1n} = 0$$

$$H_{1t} = J_{ST}$$

理想介质，的边界条件

矢量形式

$$\vec{a}_n \bullet \left(\vec{D}_1 - \vec{D}_2\right) = 0$$

$$\vec{a}_n \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\vec{a}_n \bullet \left(\vec{B}_1 - \vec{B}_2\right) = 0$$

$$\vec{a}_n \times \left(\vec{H}_1 - \vec{H}_2\right) = 0$$

标量形式

$$D_{1n} - D_{2n} = 0$$

$$E_{1t} - E_{2t} = 0$$

$$B_{1n} - B_{2n} = 0$$

$$H_{1t} - H_{2t} = 0$$

Chpt.9 Reflection & Refraction of HPW

Contents

- **Normal incidence on surface of perfect conductor**
- Normal incidence on surface of perfect dielectric
- Oblique incidence on surface of perfect conductor
- Oblique incidence on surface of perfect dielectric

9.1 Normal incidence on surface of perfect conductor

Incident wave in region 1

$$\vec{E}^{+} = \vec{a}_x E_0^{+} e^{j(\omega t - kz)}$$

$$\vec{H}^{+} = \vec{a}_z \times \vec{E}^{+} / \eta_1$$

$$\vec{\boldsymbol{H}}^{+} = \vec{a}_y H_y^{+} = \vec{a}_y \frac{E_0^{+}}{\eta} \mathrm{e}^{\mathrm{j}(\omega t - kz)}$$

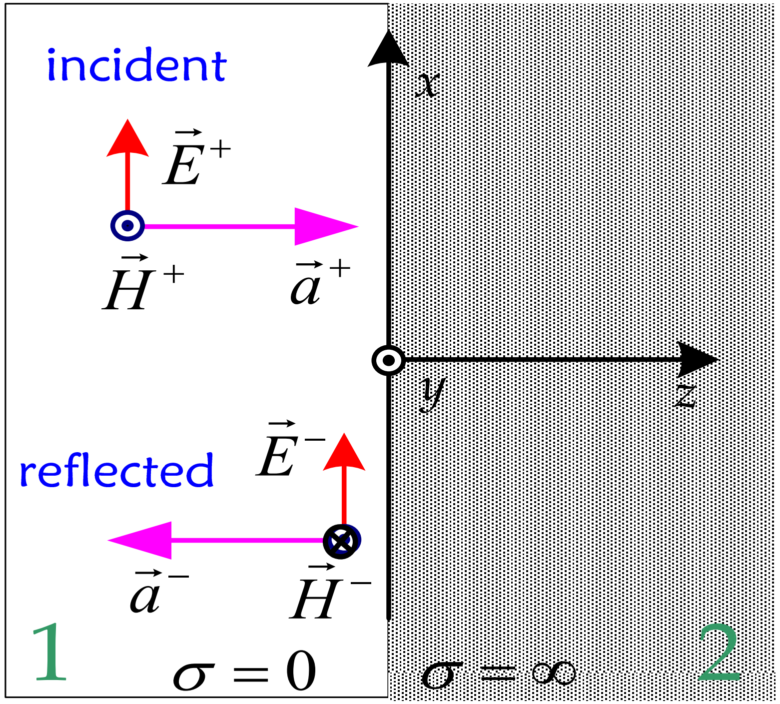
Reflected wave in region 1

假设

$$\vec{\boldsymbol{E}}^{-} = \vec{a}_x E_0^{-} \mathrm{e}^{\mathrm{j}(\omega t + kz)}$$

$$\vec{H}^{-} = (-\vec{a}_z) \times \vec{E}^{-} / \eta$$

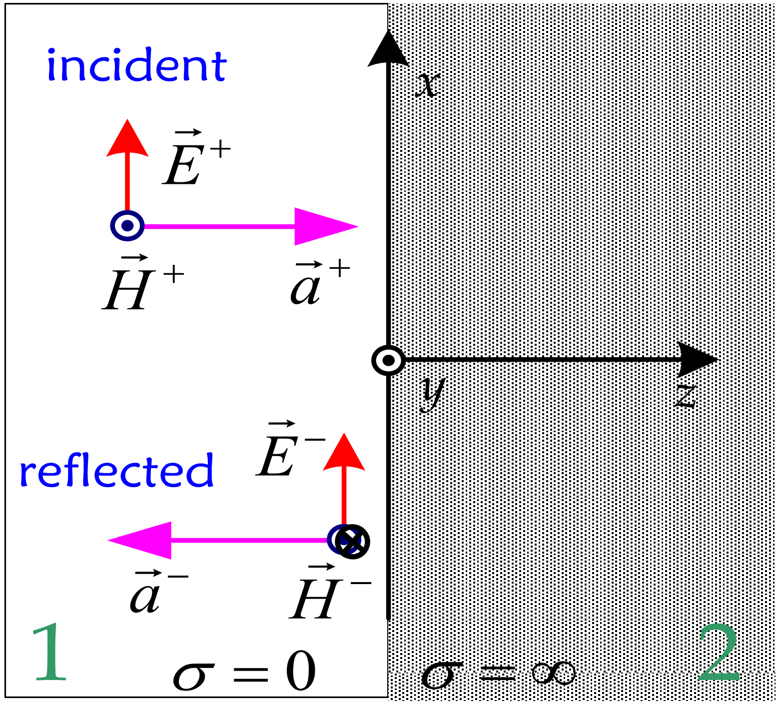
$$\vec{\boldsymbol{H}}^{-} = \vec{a}_y H_0^{-} \mathrm{e}^{\mathrm{j}(\omega t + kz)} = -\vec{a}_y (E_0^{-} / \eta) \mathrm{e}^{\mathrm{j}(\omega t + kz)}$$



Total E-Field

incident $\vec{E}^{+}=\vec{a}_x E_0^{+} e^{j(\omega t-kz)}$

reflected $\boldsymbol{E}^{-}=\vec{a}_x E_0^{-} \mathrm{e}^{\mathrm{j}(\omega t+kz)}$



total

$$\vec{E}_1=\vec{E}^{+}+\vec{E}^{-}$$

$$=\vec{a}_x\left(E_0^{+}e^{-jkz}+E_0^{-}e^{+jkz}\right)e^{jwt}$$

Boundary conditions: Z=0的平面上

$$E_{1t}\left.\right|_{z=0}=E_{2t}\left.\right|_{z=0}=E_0^{+}+E_0^{-}=0$$

$$\therefore E_0^{+}=-E_0^{-}$$

$$\vec{E}_1=-\vec{a}_x \mathrm{j}2E_0^{+}\sin(kz)\cdot\mathrm{e}^{\mathrm{j}\omega t}$$

Total M-Field

incident $\vec{\boldsymbol{H}}^{+} = \vec{a}_y (E_0^+ / \eta) \mathrm{e}^{\mathrm{j}(\omega t - kz)}$

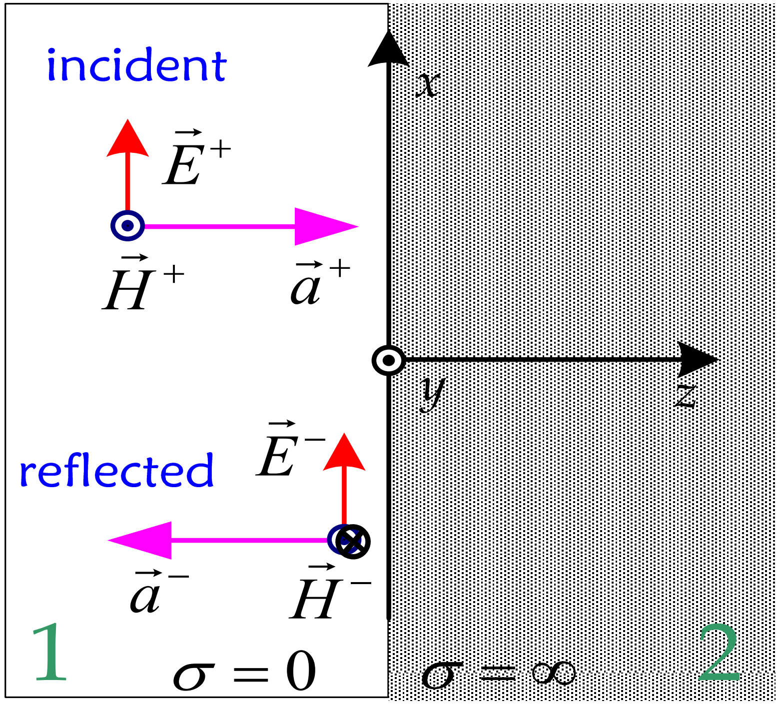
reflected $\vec{\boldsymbol{H}}^{-} = -\vec{a}_y (E_0^- / \eta) \mathrm{e}^{\mathrm{j}(\omega t + kz)}$

total

$$\vec{\boldsymbol{H}}_1 = \vec{\boldsymbol{H}}^{+} + \vec{\boldsymbol{H}}^{-}$$

$$= \vec{a}_y \left(\frac{E_0^+}{\eta} \mathrm{e}^{-\mathrm{j}kz} + \frac{E_0^+}{\eta} \mathrm{e}^{\mathrm{j}kz} \right) \mathrm{e}^{\mathrm{j}\omega t}$$

$$= \vec{a}_y 2 \frac{E_0^+}{\eta} \cos(kz) \cdot \mathrm{e}^{\mathrm{j}\omega t} \;\; = \vec{a}_y 2 H_0^+ \cos(kz) \mathrm{e}^{\mathrm{j}\omega t}$$



Total M-Field

$$\vec{E}_1 = -\vec{a}_x \mathrm{j} 2 E_0^+ \sin(kz) \cdot \mathrm{e}^{\mathrm{j}\omega t}$$

$$\vec{\boldsymbol{H}}_1 = \vec{a}_y 2 \frac{E_0^+}{\eta} \cos(kz) \cdot \mathrm{e}^{\mathrm{j}\omega t}$$

合成波场量的实数表达式为:

$$E_x = \mathrm{Re}\left(-2\mathrm{j}E_0^+ \sin(kz) \cdot \mathrm{e}^{\mathrm{j}\omega t}\right) = 2E_0^+ \sin kz \sin \omega t$$

$$H_y = \mathrm{Re}\left(2\frac{E_0^+}{\eta} \cos(kz) \cdot \mathrm{e}^{\mathrm{j}\omega t}\right) = 2\frac{E_0^+}{\eta} \cos(kz) \cos \omega t$$

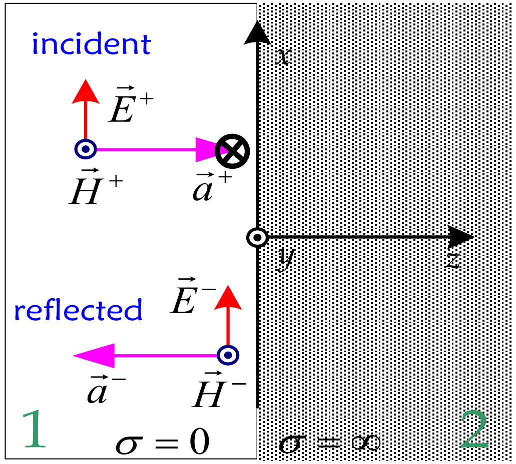
At boundary

$$\boxed{\vec{E}_1 = -\vec{a}_x \mathrm{j} 2 E_0^+ \sin(kz)\cdot \mathrm{e}^{\mathrm{j}\omega t}}$$

$$\boxed{\vec{\boldsymbol{H}}_1 = \vec{a}_y 2\frac{E_0^+}{\eta}\cos(kz)\cdot \mathrm{e}^{\mathrm{j}\omega t}}$$

$$\vec{E}_1 |_{z=0} = 2E_0^+ \sin\left(\beta_1 z |_{z=0}\right)\sin(\omega t) = 0$$

$$\vec{H}_1 |_{z=0} = \frac{2E_0^+}{\eta_1}\cos\left(\beta_1 z |_{z=0}\right)\cos(\omega t) = \frac{2E_0^+}{\eta_1}\cos(\omega t)$$



Induced surface current

$$\vec{J}_{S_{FC}} = \vec{a}_n \times \vec{H}_1 |_{z=0} = \left(-\vec{a}_z\right)\times \vec{a}_y \ldots = \vec{a}_x \ldots$$

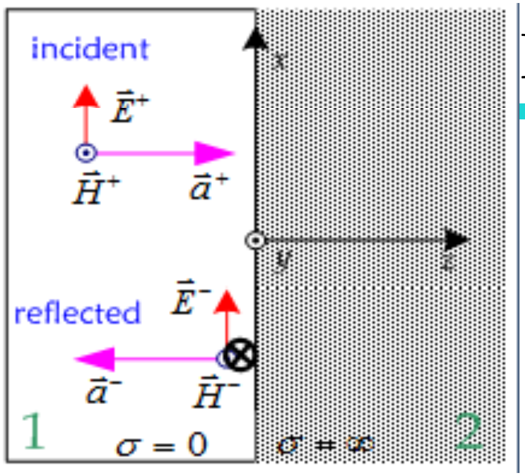
$$\vec{J}_{S_{FC}} = \vec{a}_x \frac{2E_0^+}{\eta_1}\cos(\omega t)$$

Average Poyting's vector

$$\vec{P}_{av} = \vec{S}_{av} = \frac{1}{2} R_e\left(\vec{E}\times \vec{H}^*\right) = 0$$

$$\vec{E}\times \vec{H}^* = \left[\vec{a}_x\left(-j2E_0^+ \sin\left(\beta_1 z\right)\right)\right]\times\left[\vec{a}_y\left(\frac{2E_0^+}{\eta_1}\cos\left(\beta_1 z\right)\right)\right]$$

Standing wave --- no energy transmitted

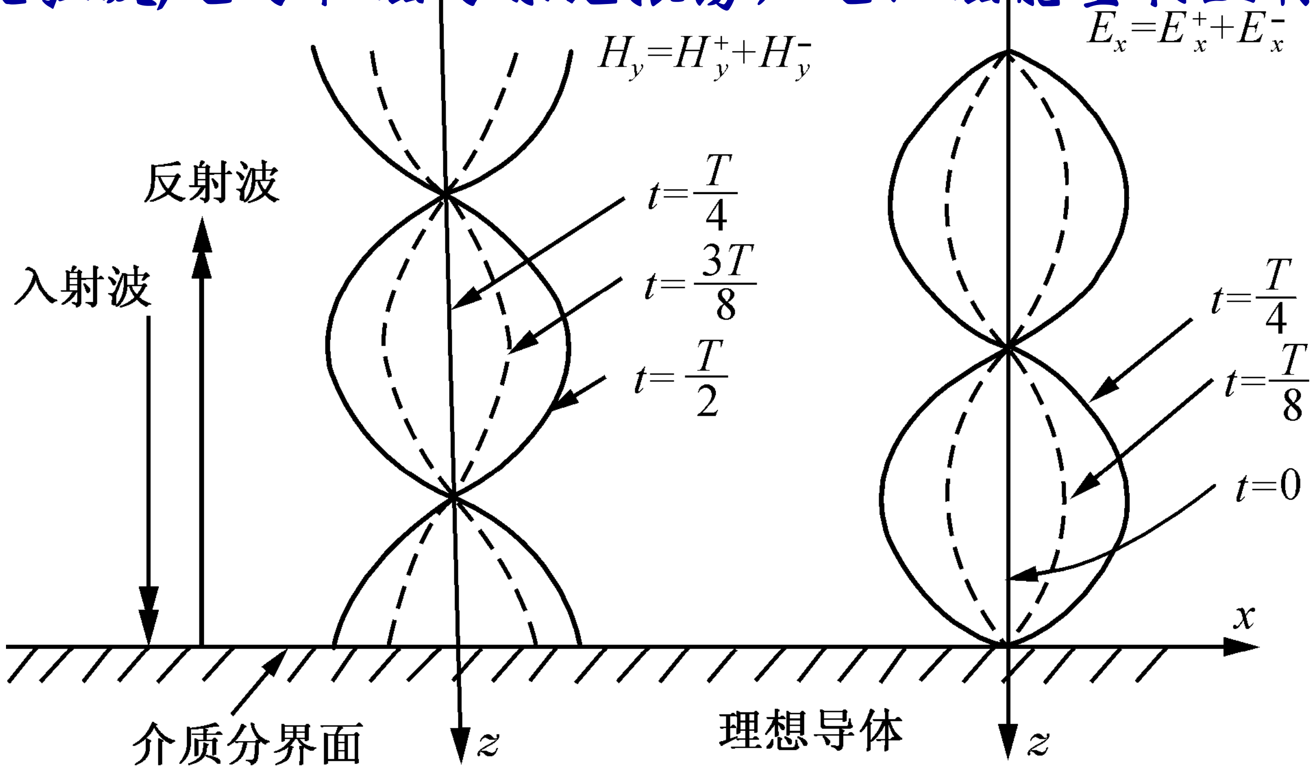


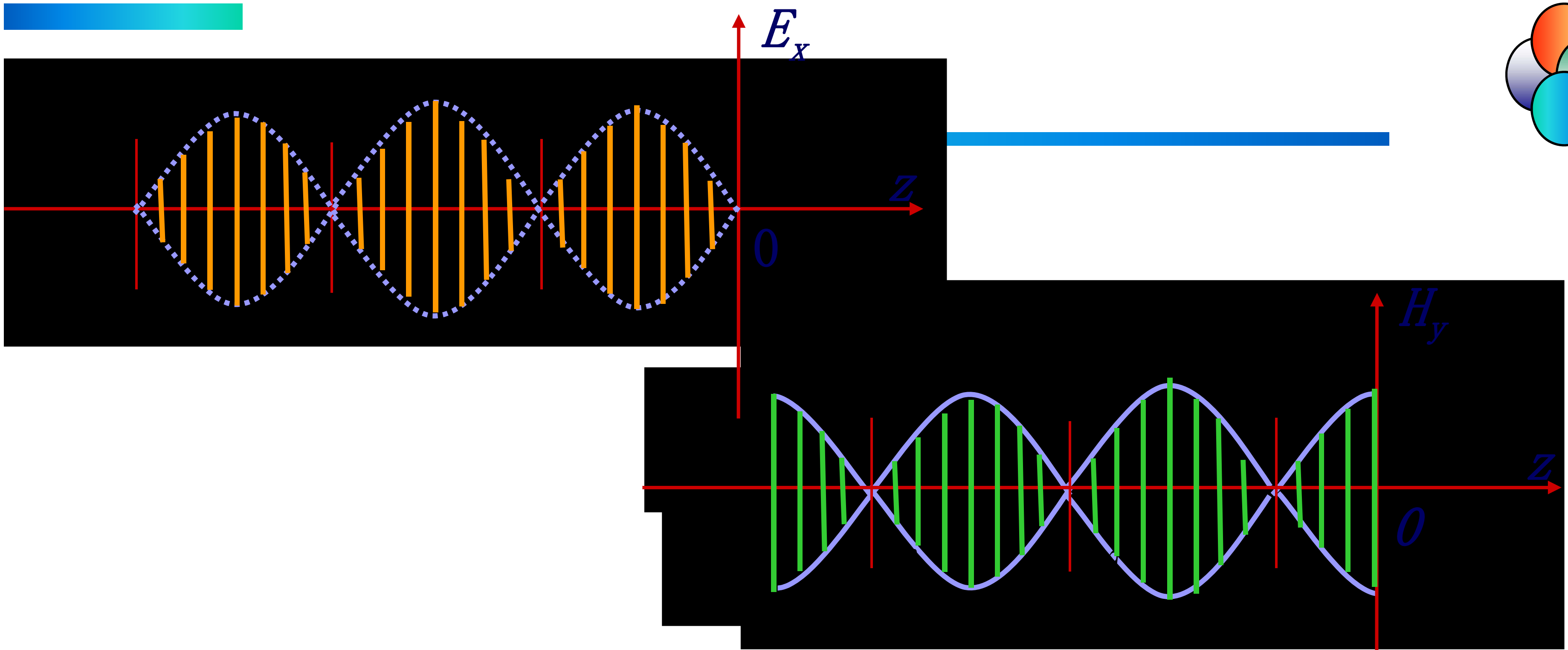
Phase-diff. btwn total E&M-fields is $\pi/2$.

$$E_x = 2E_0^+ \sin kz \sin \omega t$$

$$H_y = 2\frac{E_0^+}{\eta}\cos(kz)\cos \omega t$$

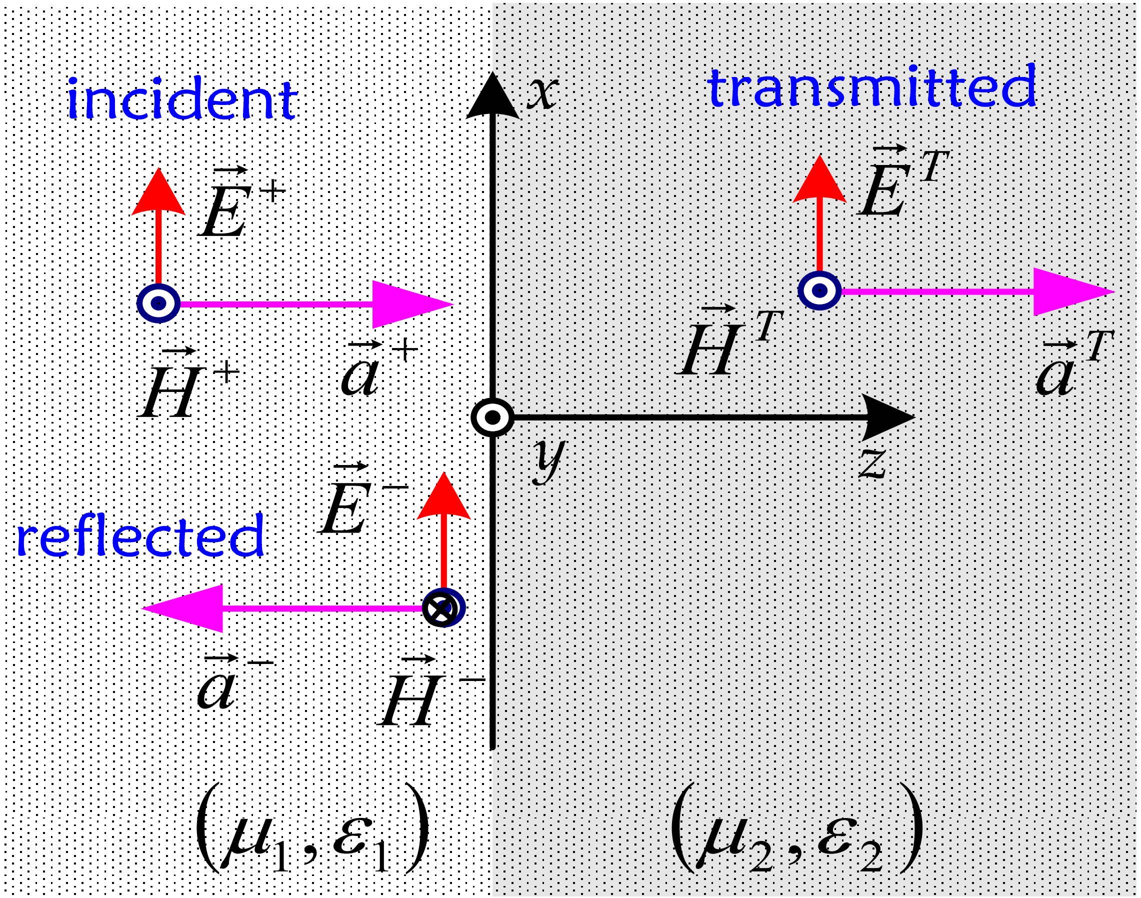
合成波为纯驻波,电场和磁场原地振荡，电、磁能量相互转化





A standing wave, also known as a stationary wave, is a wave that remains in a constant position. This phenomenon can occur because the medium is moving in the opposite direction to the wave, or it can arise in a stationary medium as a result of interference between two waves traveling in opposite directions. In the second case, for waves of equal amplitude traveling in opposing directions, there is on average no net propagation of energy.

9.2 Normal incidence on surface of perfect dielectric



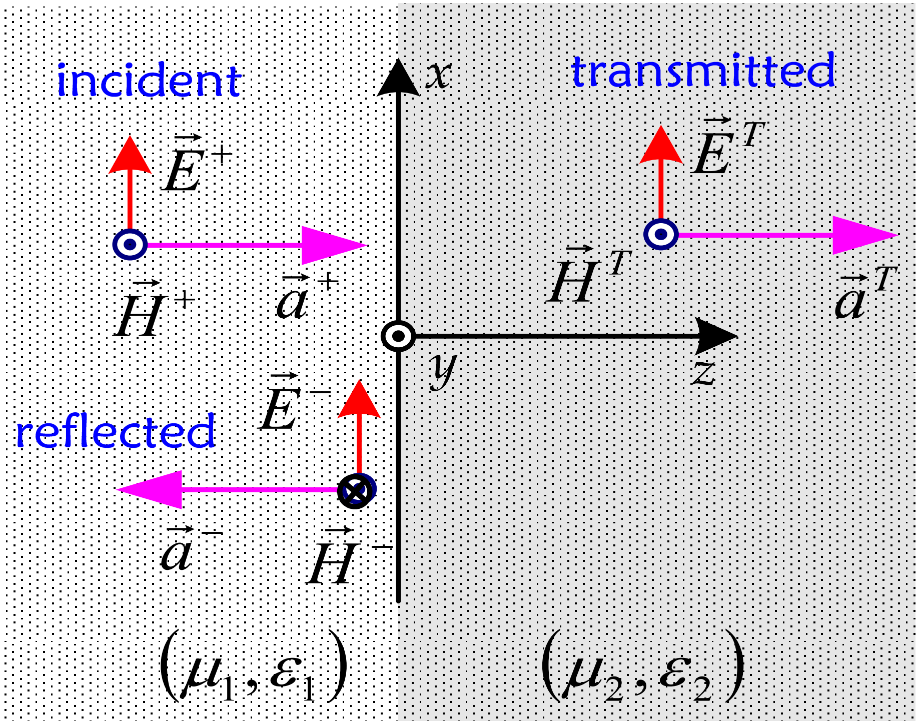
Incident Wave

assume

$$\vec{\boldsymbol{E}}^{+} = \vec{a}_x E_x^{+} = \vec{a}_x E_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

$$\vec{\boldsymbol{H}}^{+} = \vec{a}_y H_y^{+} = \vec{a}_y H_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

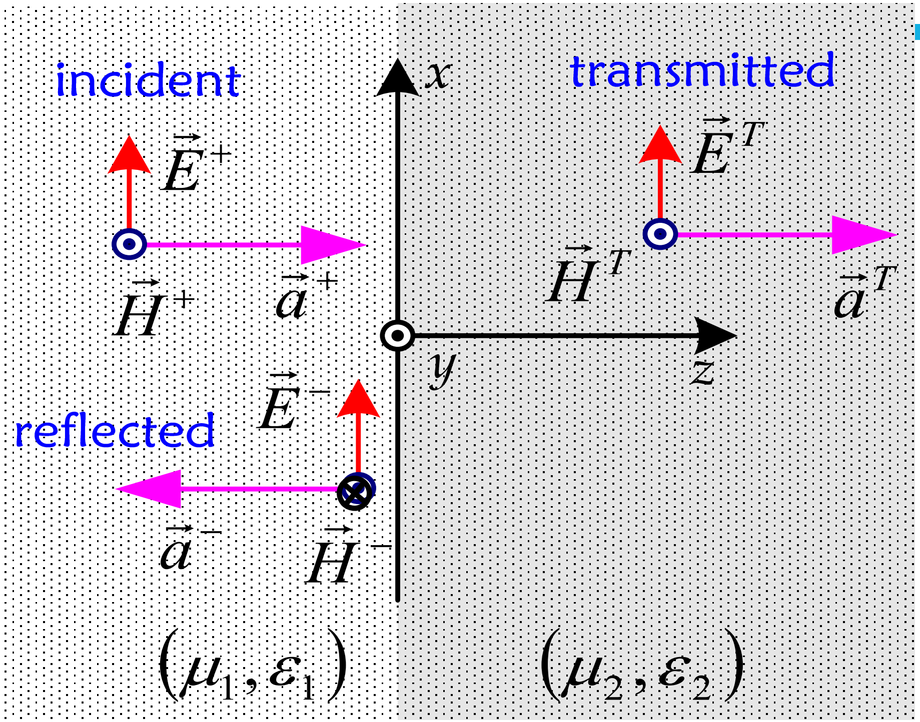
$$= \vec{a}_y \frac{E_0^{+}}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$



$k_1 = \omega\sqrt{\mu_0 \varepsilon_1}$ Propagation const. in region 1

$\eta_1 = \sqrt{\mu_0 / \varepsilon_1}$ Intrinsic impedance in region 1

Reflected Wave



$$\boxed{\vec{\boldsymbol{E}}^{-} = \vec{a}_x E_0^{-} \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)}}$$

By comparison

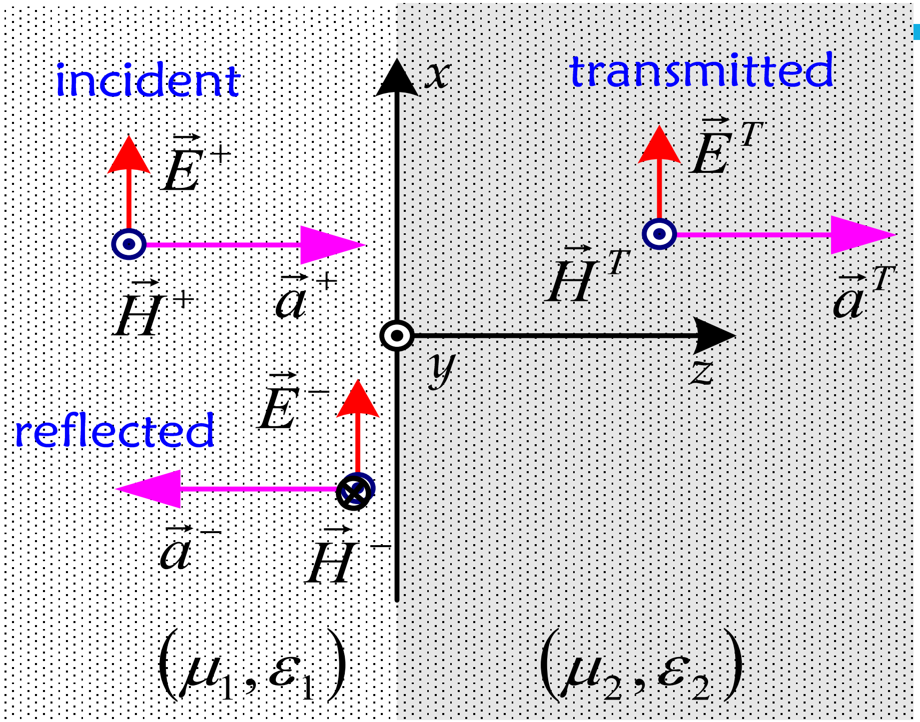
$$\vec{\boldsymbol{E}}^{+} = \vec{a}_x E_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

By comparison

$$\boxed{\begin{aligned}\vec{\boldsymbol{H}}^{-} &= \vec{a}_y H_0^{-} \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)} \\ &= -\vec{a}_y \frac{E_0^{-}}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)}\end{aligned}}$$

$$\begin{aligned}\vec{\boldsymbol{H}}^{+} &= \vec{a}_y H_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)} \\ &= \vec{a}_y \frac{E_0^{+}}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}\end{aligned}$$

Transmitted Wave



$$\boxed{\vec{\boldsymbol{E}}^T = \vec{a}_x E_0^T \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)}}$$

By comparison

$$\vec{\boldsymbol{E}}^+ = \vec{a}_x E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

$$\boxed{\begin{aligned}\vec{\boldsymbol{H}}^T &= \vec{a}_y H_0^T \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)} \\ &= \vec{a}_y \frac{E_0^T}{\eta_2} \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)}\end{aligned}}$$

By comparison

$$\begin{aligned}\vec{\boldsymbol{H}}^+ &= \vec{a}_y H_0^+ \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)} \\ &= \vec{a}_y \frac{E_0^+}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}\end{aligned}$$

Reflection Coefficient of E-Field

$$R = \left.\frac{E_0^-}{E_0^+}\right|_{z=0}$$

$$\vec{\boldsymbol{E}}^+ = \vec{a}_x E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

$$\vec{\boldsymbol{E}}^- = \vec{a}_x E_0^- \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)}$$

$$\vec{\boldsymbol{E}}^T = \vec{a}_x E_0^T \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)}$$

$$\vec{\boldsymbol{H}}^+ = \vec{a}_y \frac{E_0^+}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

$$\vec{\boldsymbol{H}}^- = -\vec{a}_y \frac{E_0^-}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)}$$

$$\vec{\boldsymbol{H}}^T = \vec{a}_y \frac{E_0^T}{\eta_2} \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)}$$

Due to E_{1t}=E_{2t} $E_{1t} = E_0^+ + E_0^- = \underline{E_{2t}}$

Due to H_{1t}=H_{2t} $H_{1t} = H_0^+ + H_0^- = \frac{E_0^+}{\eta_1} - \frac{E_0^-}{\eta_1} = H_{2t} = \frac{E_0^T}{\eta_2} = \frac{E_{2t}}{\underline{\eta_2}}$

so

$$E_{2t} = E_0^+ + E_0^- = \frac{\eta_2}{\eta_1}(E_0^+ - E_0^-)$$

$$R = \frac{E_0^-}{E_0^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission Coefficient of E-Field

$$T = \left.\frac{E_0^T}{E_0^+}\right|_{z=0}$$

$$T = \frac{E_0^T}{E_0^+} = \frac{E_{2t}}{E_0^+} = \frac{E_{1t}}{E_0^+} = \frac{E_0^+ + E_0^-}{E_0^+} = 1 + R$$

$$T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$T = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\xrightarrow[\eta = \sqrt{\frac{\mu}{\varepsilon}}]{\mu \approx \mu_0}$$

$$R = \left.\frac{E_0^-}{E_0^+}\right|_{z=0}$$

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$R = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

Reflection & Transmission Coefficient of M-Field

$$\vec{E}^{+} = \vec{a}_x E_0^{+} e^{j(\omega t - k_1 z)}$$

$$\vec{E}^{-} = \vec{a}_x E_0^{-} e^{j(\omega t + k_1 z)}$$

$$\vec{E}^{T} = \vec{a}_x E_0^{T} e^{j(\omega t - k_2 z)}$$

$$\vec{H}^{+} = \vec{a}_y \frac{E_0^{+}}{\eta_1} e^{j(\omega t - k_1 z)}$$

$$\vec{H}^{-} = -\vec{a}_y \frac{E_0^{-}}{\eta_1} e^{j(\omega t + k_1 z)}$$

$$\vec{H}^{T} = \vec{a}_y \frac{E_0^{T}}{\eta_2} e^{j(\omega t - k_2 z)}$$

$$R_H = \frac{H_0^-}{H_0^+} = \frac{-E_0^- / \eta_1}{E_0^+ / \eta_1} = -\frac{E_0^-}{E_0^+} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$|R| = |R_H|$$

$$T_H = \frac{H_0^T}{H_0^+} = \frac{E_0^T / \eta_2}{E_0^+ / \eta_1} = \frac{\eta_1}{\eta_2} \frac{E_0^T}{E_0^+} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

A Comparison

$$\vec{\boldsymbol{E}}^{+} = \vec{a}_x E_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

$$\vec{\boldsymbol{E}}^{-} = \vec{a}_x E_0^{-} \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)}$$

$$\vec{\boldsymbol{E}}^{T} = \vec{a}_x E_0^{T} \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)}$$

$$\vec{\boldsymbol{H}}^{+} = \vec{a}_y \frac{E_0^{+}}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t - k_1 z)}$$

$$\vec{\boldsymbol{H}}^{-} = -\vec{a}_y \frac{E_0^{-}}{\eta_1} \mathrm{e}^{\mathrm{j}(\omega t + k_1 z)}$$

$$\vec{\boldsymbol{H}}^{T} = \vec{a}_y \frac{E_0^{T}}{\eta_2} \mathrm{e}^{\mathrm{j}(\omega t - k_2 z)}$$

$$R_H = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$R_H = -R$$

A transposition from η_1 & η_2

$$T_H = \frac{2\eta_1}{\eta_1 + \eta_2}$$

$$T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$T_H = \left(\frac{\eta_1}{\eta_2}\right) T$$

Relation of Power Flow Densities

$$R = \left.\frac{E_0^-}{E_0^+}\right|_{z=0}$$

$$R_H = \frac{H_0^-}{H_0^+} = -R$$

$$\begin{cases} S_{\text{av}}^{-} = R^2 S_{\text{av}}^{+} \\ S_{\text{av}}^{T} = \left(\frac{\eta_1}{\eta_2}\right) T^2 \cdot S_{\text{av}}^{+} \end{cases}$$

$$1 - R^2 = \left(\frac{\eta_1}{\eta_2}\right) T^2$$

$$S_{\text{av}}^{+} = S_{\text{av}}^{-} + S_{\text{av}}^{T}$$

Energy is conservative.

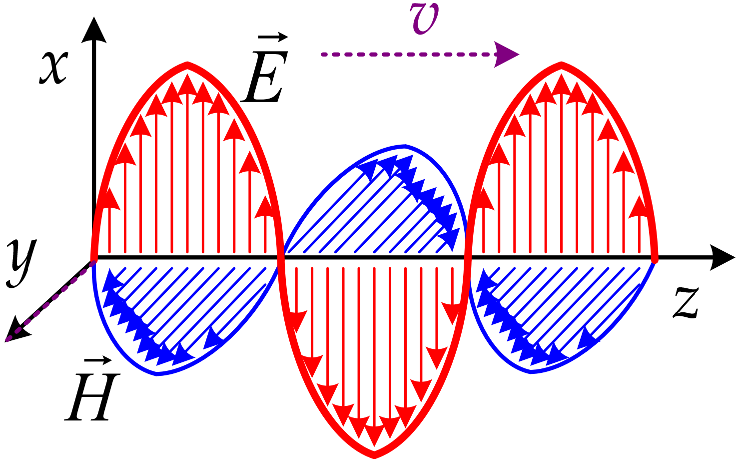
9.3 Oblique incidence on surface of perfect conductor

Wave along z-axis

$$\vec{E} = \vec{a}_x E_m^+ e^{-jkz}$$

$$\vec{H} = \vec{a}_y \frac{1}{\eta} \cdot E_m^+ \cdot e^{-jkz}$$

$$\vec{H} = \frac{1}{\eta} \vec{a}_z \times \vec{E}$$



Wave along x-axis

$$\vec{E} = \vec{a}_y E_m^+ e^{-jkx}$$

$$\vec{H} = \vec{a}_z \frac{1}{\eta} \cdot E_m^+ \cdot e^{-jkx}$$

$$\vec{H} = \frac{1}{\eta} \vec{a}_x \times \vec{E}$$

Wave along arbitrary direction

$$\vec{E} = \vec{a}_E E_m^+ e^{-j\vec{k} \bullet \vec{r}}$$

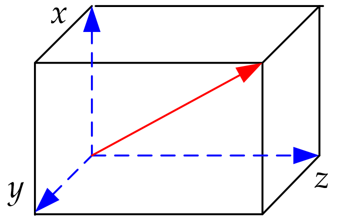
$$\vec{H} = \vec{a}_H \frac{1}{\eta} \cdot E_m^+ \cdot e^{-j\vec{k} \bullet \vec{r}}$$

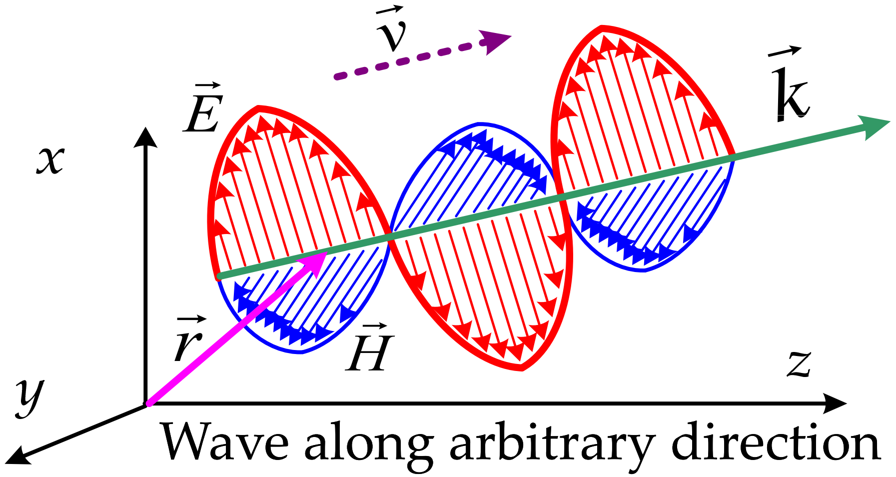
$$\vec{H} = \frac{1}{\eta} \vec{a}_k \times \vec{E} \qquad \vec{k} = \vec{a}_k k$$

Wave Vector $\vec{k}$

$$\vec{E} = \vec{a}_E E_m^+ e^{-j\vec{k}\bullet\vec{r}}$$

$$\vec{H} = \vec{a}_H \frac{1}{\eta} \cdot E_m^+ \cdot e^{-j\vec{k}\bullet\vec{r}}$$





Wave along arbitrary direction

$$\vec{\boldsymbol{k}} = \vec{\boldsymbol{e}}_x k_x + \vec{\boldsymbol{e}}_y k_y + \vec{\boldsymbol{e}}_z k_z = \vec{\boldsymbol{e}}_x k \cos\alpha + \vec{\boldsymbol{e}}_y k \cos\beta + \vec{\boldsymbol{e}}_z k \cos\gamma$$

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

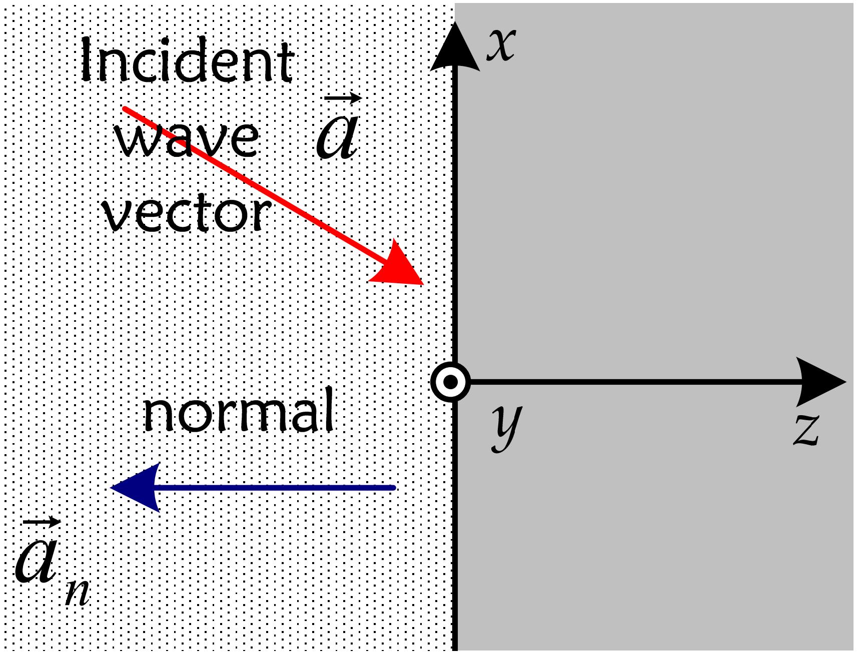
$$\vec{\boldsymbol{k}} \cdot \vec{r} = (\vec{\boldsymbol{e}}_x k_x + \vec{\boldsymbol{e}}_y k_y + \vec{\boldsymbol{e}}_z k_z) \cdot (\vec{\boldsymbol{e}}_x x + \vec{\boldsymbol{e}}_y y + \vec{\boldsymbol{e}}_z z)$$

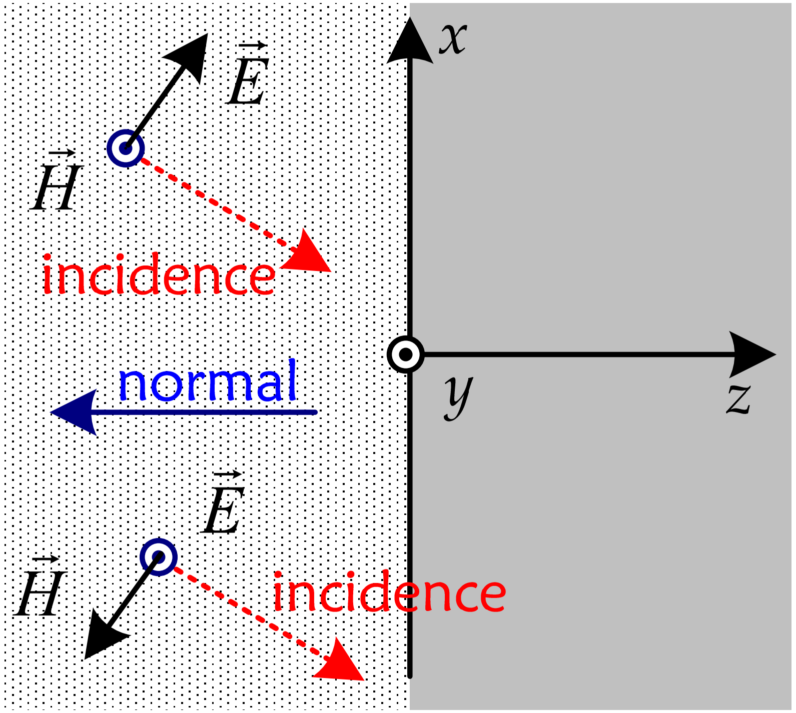
$$= k_x x + k_y y + k_z z$$

Classification of Oblique Incidence

Incident Plane is determined by **wave vector** & **normal vector**.

- **parallel** polarized incidence: E lies in incident plane
- **normal** polarized incidence: E is normal to incident plane





What kind of incidence?

Normal Polarized Oblique Incidence

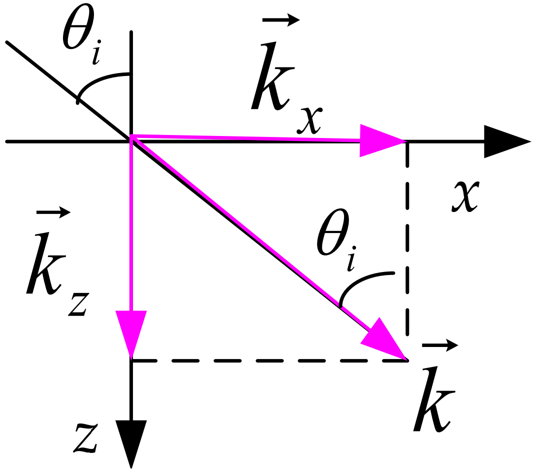
Incident Wave

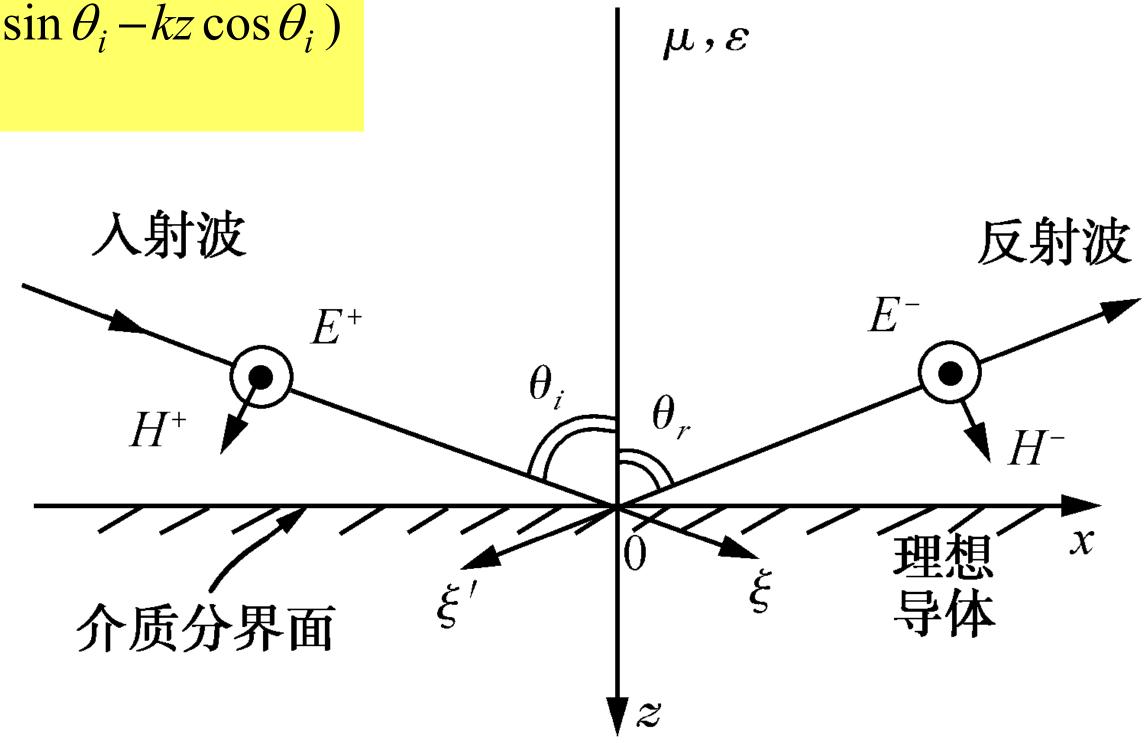
$$\vec{E}^{+} = \vec{e}_y E_y^{+} = \vec{e}_y E_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - \vec{k} \bullet \vec{r})}$$

$$\vec{k} \cdot \vec{r} = k_x x + k_y y + k_z z$$

$$= \vec{e}_y E_0^{+} \exp\{\mathrm{j}(\omega t - k_x x - k_z z)\}$$

$$= \vec{e}_y E_0^{+} \mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta_i - kz\cos\theta_i)}$$





Normal Polarized Oblique Incidence

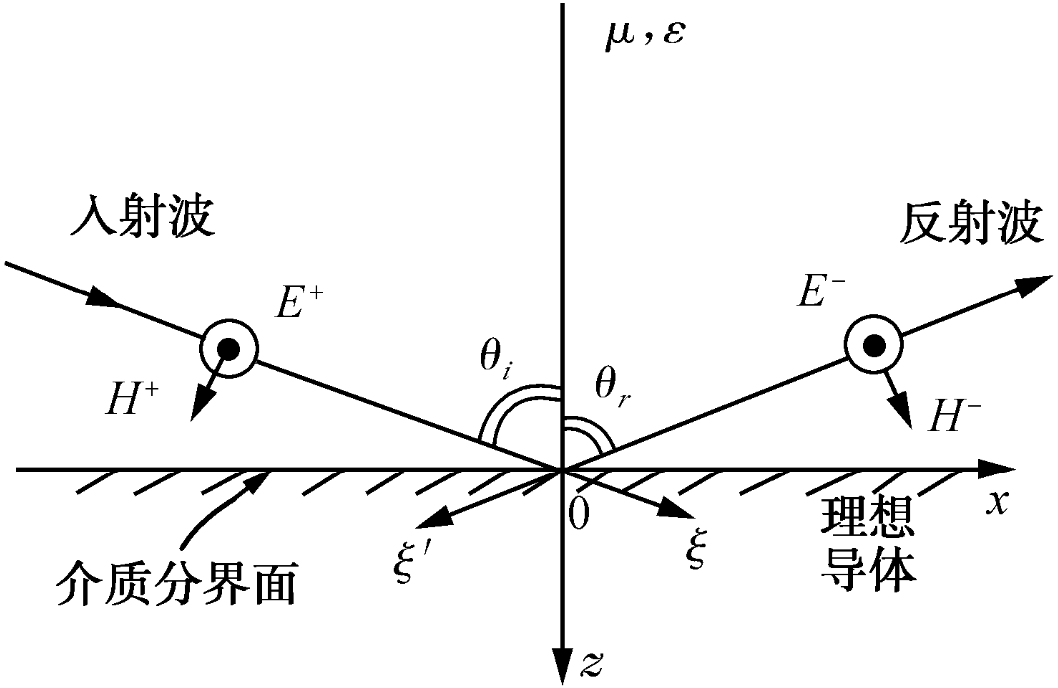
Reflected Wave

$$\vec{E}^{-} = \vec{e}_y E_y^{-}$$

$$= \vec{e}_y E_0^{-} \mathrm{e}^{\mathrm{j}(\omega t + \vec{k}^{-} \bullet \vec{r})}$$

$$= \cdots$$?

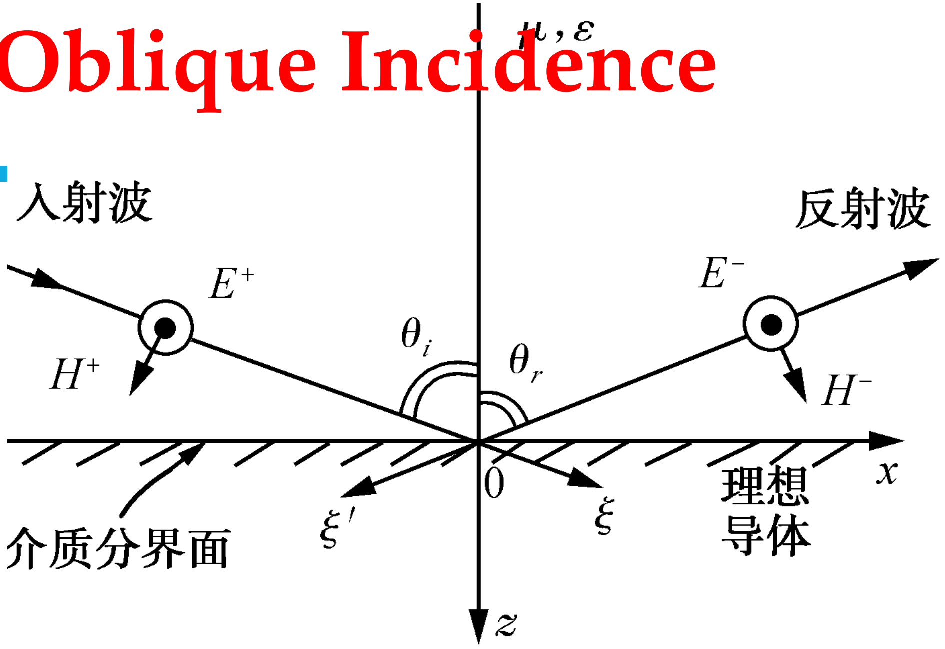
同一种介质k的大小相同



$$\vec{E}^{-} = \vec{e}_y E_0^{-} \mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta_r + kz\cos\theta_r)}$$

Normal Polarized Oblique Incidence

Total E-Field



$$\vec{E}^+ = \vec{e}_y E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta_i - kz\cos\theta_i)}$$

$$\vec{E}^- = \vec{e}_y E_0^- \mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta_r + kz\cos\theta_r)}$$

$$E_y = E_y^+ + E_y^-$$

$$= E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - k\sin\theta_i x - k\cos\theta_i z)} + E_0^- \mathrm{e}^{\mathrm{j}(\omega t - k\sin\theta_r x + k\cos\theta_r z)}$$

在z=0平面上应用边界条件 $$E_0^+ \mathrm{e}^{-\mathrm{j}k\sin\theta_i x} + E_0^- \mathrm{e}^{-\mathrm{j}k\sin\theta_r x} = 0$$

E-field is tangential and thus continuous at boundary z=0.

Law of reflection $\theta_i = \theta_r = \theta$ $\qquad E_0^- = -E_0^+$

Normal Polarized Oblique Incidence

M-Field

$$\vec{\boldsymbol{H}}^{+}=\vec{\boldsymbol{e}}_{x}H_{x}^{+}+\vec{\boldsymbol{e}}_{z}H_{z}^{+}$$

where $H_{x}^{+}=-H_{0}^{+}\mathrm{e}^{\mathrm{j}(\omega t-k\sin\theta_{i}x-k\cos\theta_{i}z)}\cdot\cos\theta_{i}$

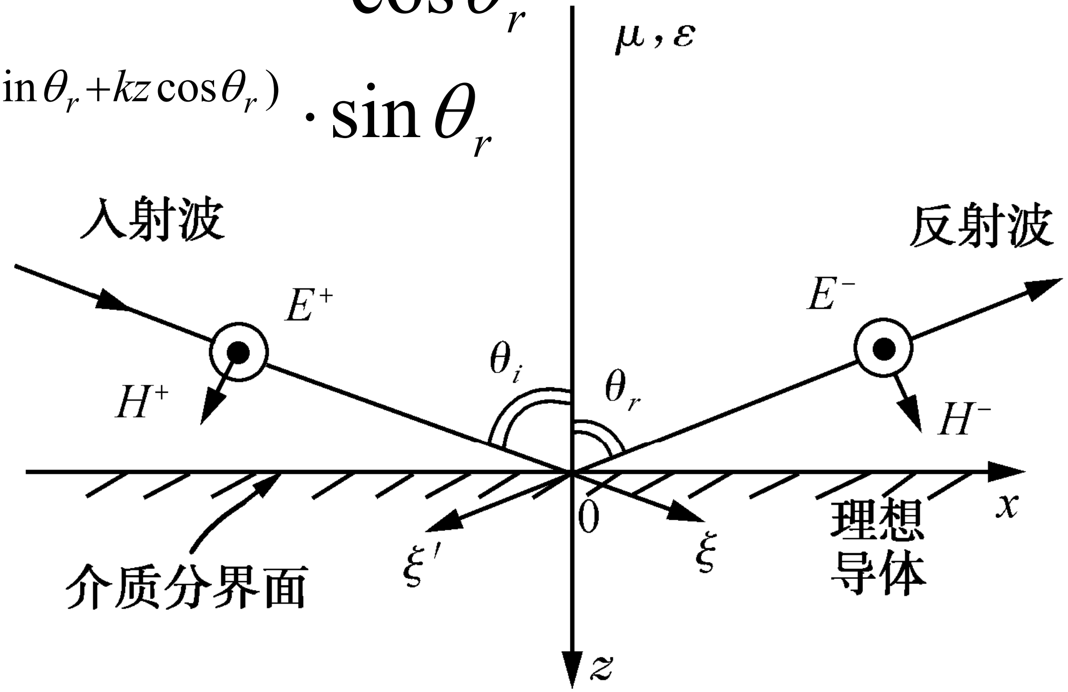
$$H_{z}^{+}=H_{0}^{+}\mathrm{e}^{\mathrm{j}(\omega t-k\sin\theta_{i}x-k\cos\theta_{i}z)}\cdot\sin\theta_{i}$$

$\theta_i=\theta_r=\theta$

$$H_{0}^{+}=-H_{0}^{-}$$

Similarly $H_{x}^{-}=H_{0}^{-}\mathrm{e}^{\mathrm{j}(\omega t-kx\sin\theta_{r}+kz\cos\theta_{r})}\cdot\cos\theta_{r}$

$$H_{z}^{-}=H_{0}^{-}\mathrm{e}^{\mathrm{j}(\omega t-kx\sin\theta_{r}+kz\cos\theta_{r})}\cdot\sin\theta_{r}$$

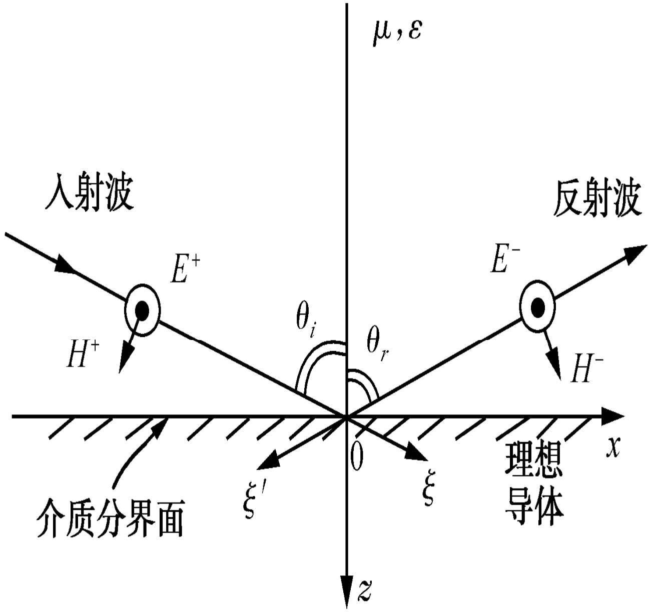


$$E_y = E_0^+ \mathrm{e}^{\mathrm{j}(\omega t - k\sin\theta_i x - k\cos\theta_i z)} + E_0^- \mathrm{e}^{\mathrm{j}(\omega t - k\sin\theta_r x + k\cos\theta_r z)}$$

$$E_y = -\mathrm{j}2E_0^+ \sin(kz\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta)}$$

$$H_z = -\mathrm{j}\frac{2E_0^+}{\eta}\sin\theta\sin(kz\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta)}$$

$$H_x = \frac{-2E_0^+}{\eta}\cos\theta\cos(kz\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta)}$$



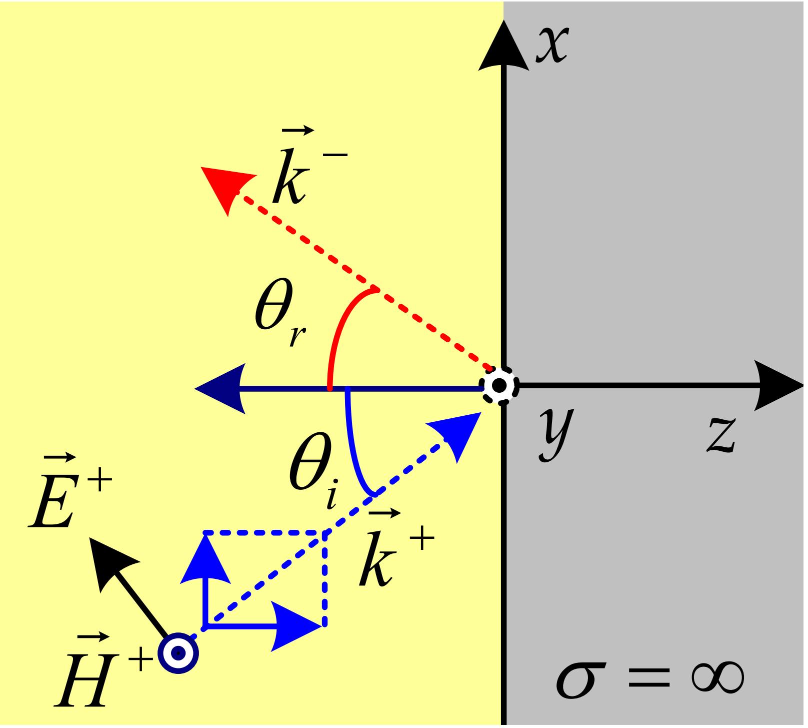
E_y与H_z的时间相位相同，
因此构成沿x方向有功率传输。

沿x方向为行波

E_y与H_x有90°的时间相位差，
二者叉乘无实部
因此沿z方向没有功率传输。 沿z方向是驻波

Only E lies in transverse (横向) plane, hence the name TE-wave.

Parallel Polarized Oblique Incidence



$$\vec{E}^+ = \vec{E}_0^+ \cdot e^{-j\vec{k}^+ \bullet \vec{r}}$$

$$\vec{E}^- = \vec{E}_0^- \cdot e^{-j\vec{k}^- \bullet \vec{r}}$$

$$\vec{r} = \{x, y, z\}$$

$$\vec{k}^+ = \vec{a}^+ k = \vec{a}_x (k \cdot \sin\theta_i) + \vec{a}_z (k \cdot \cos\theta_i)$$

$$\vec{k}^- = \vec{a}^- k = \vec{a}_x (k \cdot \sin\theta_r) - \vec{a}_z (k \cdot \cos\theta_r)$$

At boundary $\vec{E}_1 |_{z=0} = 0$

Total fields: $\vec{E}_1 = \vec{E}^+ + \vec{E}^-$

$$\vec{H}_1 = \vec{H}^+ + \vec{H}^- = \frac{1}{\eta_1} \vec{a}^+ \times \vec{E}^+ + \frac{1}{\eta_1} \vec{a}^- \times \vec{E}^-$$

Total Field

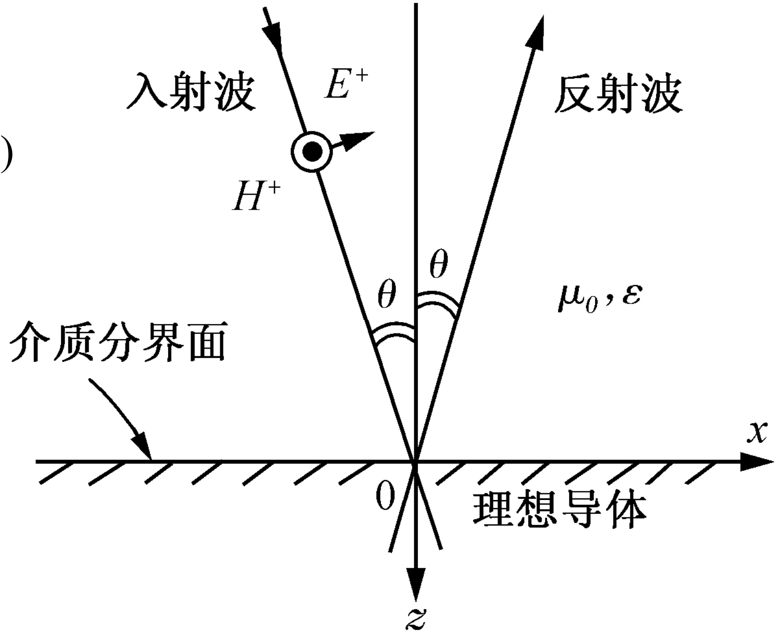
$$E_z = 2E_0^+ \sin\theta \cos(kz\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta)}$$

$$= -2H_0^+ \eta \sin\theta \cos(\underline{kz}\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - \underline{kx}\sin\theta)}$$

$$E_x = -\mathrm{j}2E_0^+ \cos\theta \sin(kz\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - kx\sin\theta)}$$

$$= -\mathrm{j}2H_0^+ \eta \cos\theta \sin(\underline{kz}\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - \underline{kx}\sin\theta)}$$

$$H_y = 2H_0^+ \cos(\underline{kz}\cos\theta)\mathrm{e}^{\mathrm{j}(\omega t - \underline{kx}\sin\theta)}$$



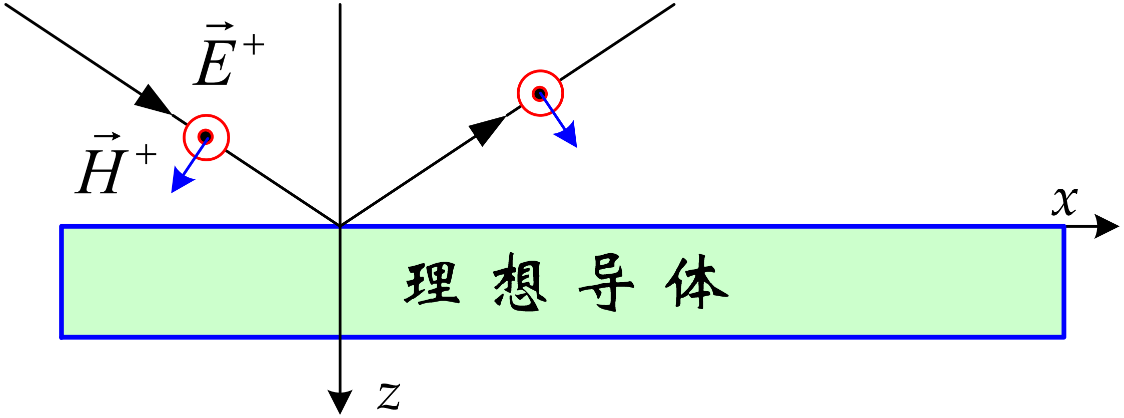
Standing wave along z-axis

Traveling wave along x-axis

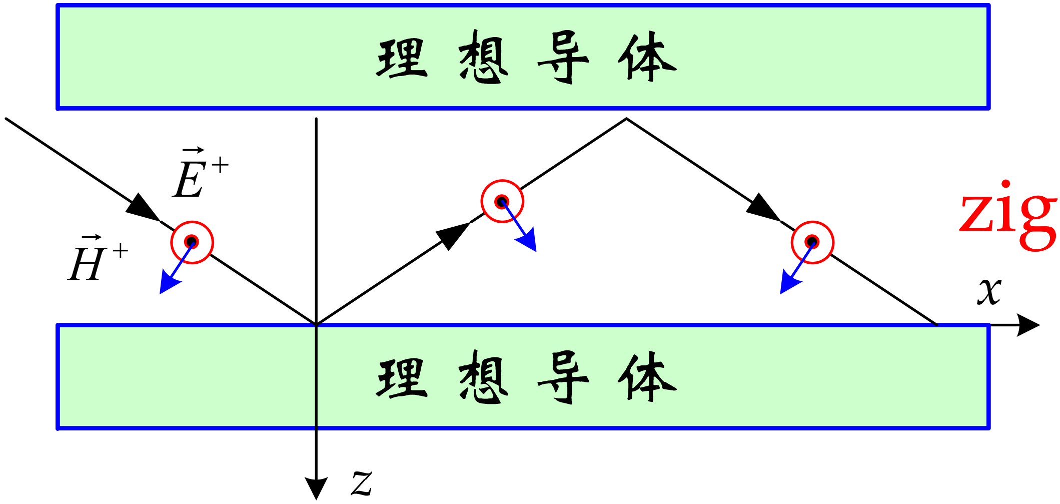
Only M-field lies in transverse plane, hence the name TM-wave.

平面波对理想导体斜入射

沿x方向，只有电场是横向的，称为横电波(TE波).

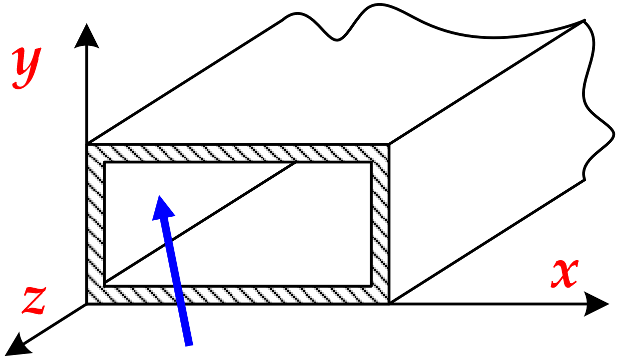


若在反射波区域
再加一导体……？



zigzag

若是任意斜入射，又如何？



9.4 Oblique incidence on surface of perfect dielectric

Contents

- Incident, Reflected & Transmitted Rays
  - Snell’s Law
- Reflection Coefficient & Transmission Coefficient
  - Fresnel Formula
- **Total Reflection（全反射）**
  - **Critical Angle**
- **Total Transmission（全折射）**
  - **Brewster Angle**

在介质分界面上某一点,第一种介质中的入射波和反射波的切向磁场之和,即H_{1t},必然等于同一点的介质2中的(即折射波的)切向磁场H_{2t}。即$H_{1t}=H_{2t}$。但是处在介质1一侧的入射波和反射波,以及处在介质2一侧的折射波都是沿x方向传播的,由于不仅在介质分界面上某一点满足$H_{1t}=H_{2t}$,**而且,在介质分界面上任何点都应满足$\boldsymbol{H_{1t}=H_{2t}}$。这就要求入射波、反射波、折射波沿$x$方向应以相同的速度传播,或者说这三个波沿传播方向具有相同的相移常数。**

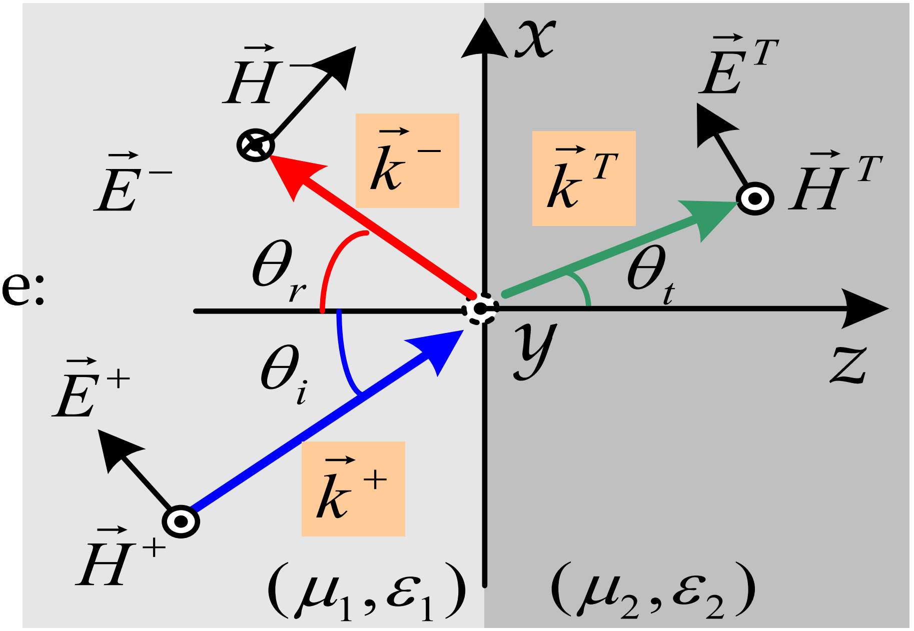
三个波沿x方向的相移常数分别为:入射波的为$k_1\sin\theta_i$,反射波的为$k_1\sin\theta_r$,折射波的为$k_2\sin\theta_T$,三者相等,即

$$k_1\sin\theta_i = k_1\sin\theta_r = k_2\sin\theta_T$$

9.4.1 Snell's Law

- Assume the boundary is an infinite plane surface. Due to continuation of k_x on the boundary surface, we must have:

$$n_1 k_0 \sin\theta_i = n_1 k_0 \sin\theta_r = n_2 k_0 \sin\theta_t$$



$$k_0^2 = \omega^2 \mu_0 \varepsilon_0$$

$$n = \sqrt{\mu_r \varepsilon_r}$$

$$\therefore \theta_i = \theta_r \qquad \sin\theta_i = \frac{n_2}{n_1}\sin\theta_t$$

$$k_x^+ = k_x^- = k_x^T$$

- It is Snell's Law. Furthermore, the incident ray, reflected ray and transmitted ray must lies on the same plane.

Snell's Law

$$\theta_i = \theta_r \qquad \sin\theta_i = \frac{n_2}{n_1}\sin\theta_t$$

- Furthermore, the incident ray, reflected ray and transmitted ray must lies on the same plane.

$$n = \sqrt{\mu_r \varepsilon_r}$$

Index of the Material (折射率)

Assuming $\mu_1 \approx \mu_2 \approx \mu_0$

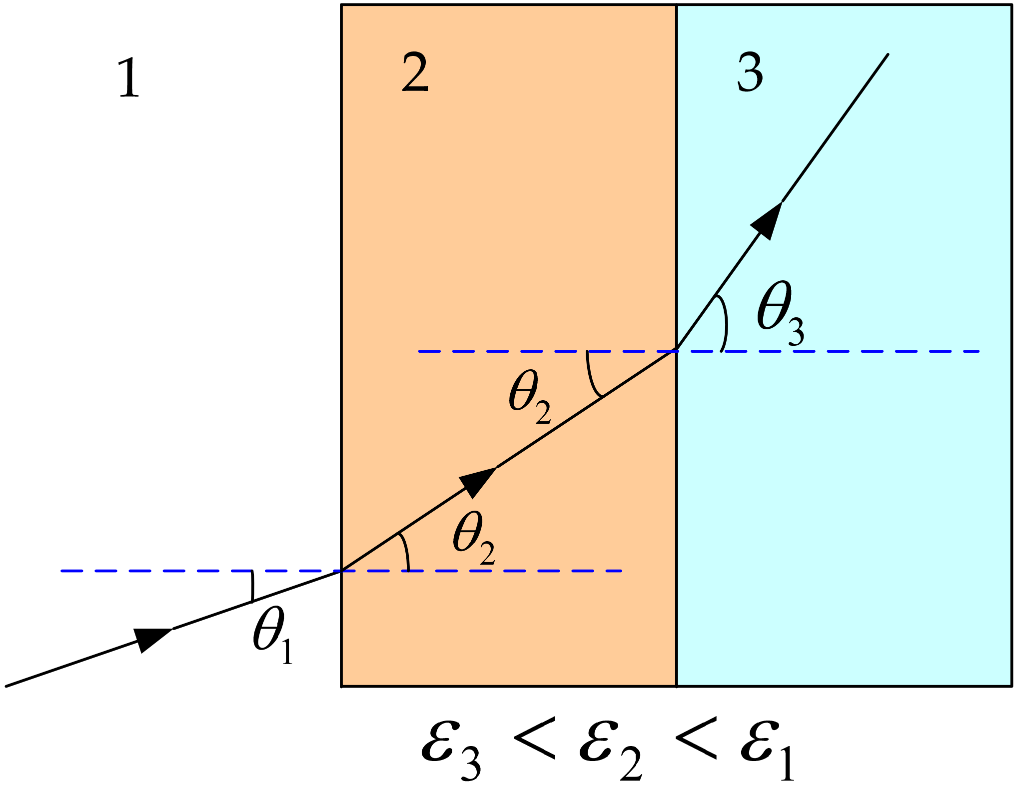
$$\sin\theta_i = \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}}\sin\theta_t$$

Then we have

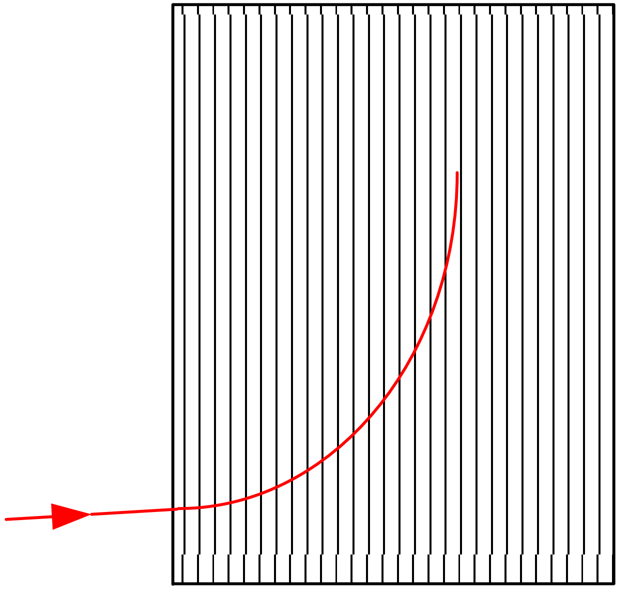
$$\theta_i = \theta_r \qquad \sin\theta_i = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\sin\theta_t$$

Bend the Light!

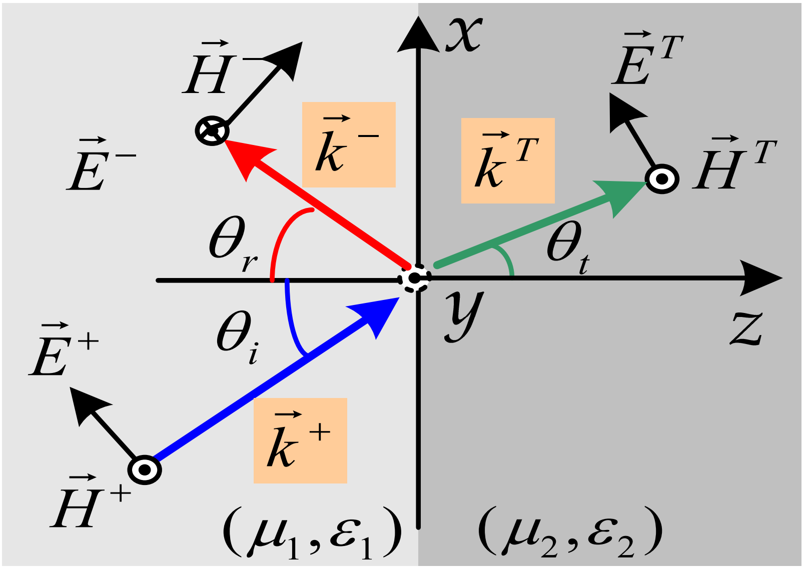
$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$$



$\varepsilon_3 < \varepsilon_2 < \varepsilon_1$



9.4.2. Reflection & Transmission Coefficients



在理想介质边界z=0的切向上:

入射波

$$E_x^+ = E_{x0}^+ \mathrm{e}^{\mathrm{j}(\omega t - k_1 \sin \theta x)}$$

$$H_y^+ = H_{y0}^+ \mathrm{e}^{\mathrm{j}(\omega t - k_1 \sin \theta x)}$$

反射波

$$E_x^- = E_{x0}^- \mathrm{e}^{\mathrm{j}(\omega t - k_1 \sin \theta x)}$$

$$H_y^- = -H_{y0}^- \mathrm{e}^{\mathrm{j}(\omega t - k_1 \sin \theta x)}$$

折射波

$$E_x^T = E_{x0}^T \mathrm{e}^{\mathrm{j}(\omega t - k_2 \sin \theta_T x)}$$

$$H_y^T = H_{y0}^T \mathrm{e}^{\mathrm{j}(\omega t - k_2 \sin \theta_T x)}$$

$$E_{x0}^+ + E_{x0}^- = E_{x0}^T$$

$$H_{y0}^+ - H_{y0}^- = H_{y0}^T$$

$$Z_{z1} = \frac{E_x^+}{H_y^+} = \frac{E^+ \cos \theta_i}{H^+} = \eta_1 \cos \theta_i$$

$$Z_L = \frac{E_x^T}{H_y^T} = \frac{E^T \cos \theta_T}{H^T} = \eta_2 \cos \theta_T$$

应用边界条件和Snell's Law可得: **Fresnel's Formula**

P---Parallel

R+1=T

$$R_P = \frac{E_x^-}{E_x^+} = \frac{E_{x0}^-}{E_{x0}^+} = \frac{Z_L - Z_{z1}}{Z_L + Z_{z1}}$$

$$T_P = \frac{E_x^T}{E_x^+} = \frac{E_{x0}^T}{E_{x0}^+} = \frac{2Z_L}{Z_L + Z_{z1}}$$

$$\sin\theta_T / \sin\theta_i = \sqrt{\varepsilon_1} / \sqrt{\varepsilon_2}$$

$$\cos\theta_T = \frac{n_1}{n_2}\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta}$$

$$R_P = \frac{-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}$$

$$T_P = \frac{2\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}$$

同理：对于垂直极化平面波 Fresnel's Formula

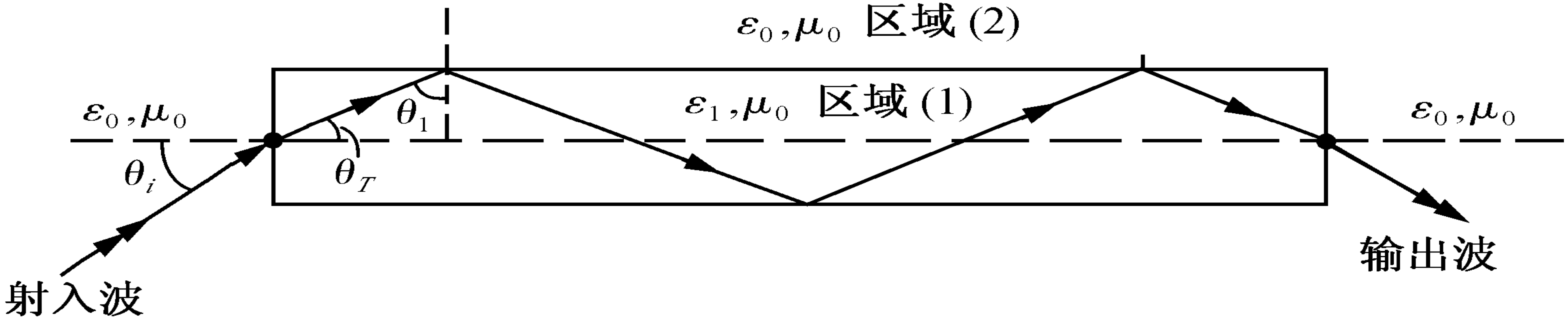
$$R_{(N)}=\frac{\cos\theta_i-\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)-\sin^2\theta_i}}{\cos\theta_i+\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)-\sin^2\theta_i}}$$

N----Normal

R+1=*T*

$$T_{(N)}=\frac{2\cos\theta_i}{\cos\theta_i+\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)-\sin^2\theta_i}}$$

下面研究全反射和全折射

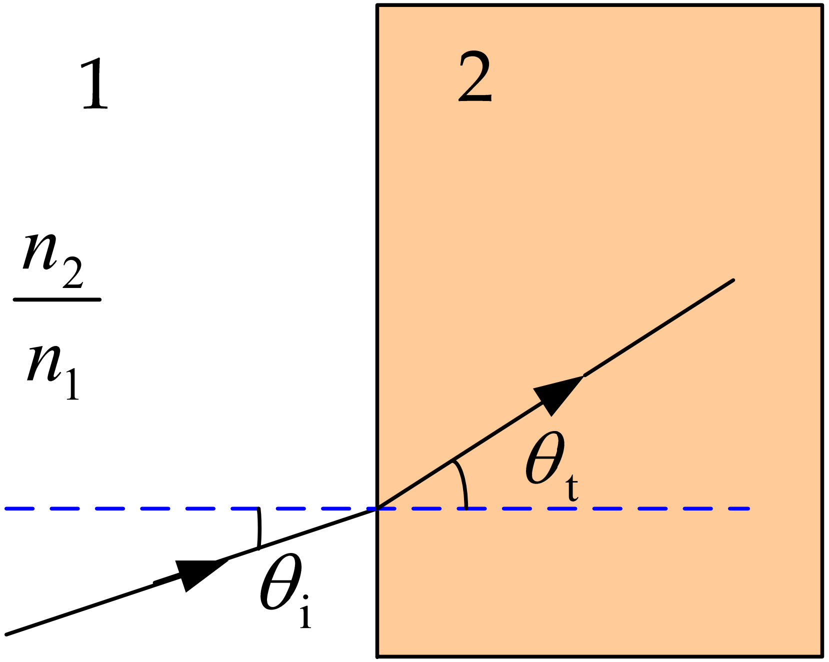


- 研究全反射和全折射对通信技术的意义：
- 它们是电磁波在波导中传输的基本原理。
- 以光纤通信为例：
  - 光信号在进入光纤时需要全折射
  - 光信号在光纤中传输时需要全反射

9.4.3. total reflection & total transmission

Condition of total reflection (from ray optics point of view):

$\theta_t = 90^o$



$$\sin\theta_i = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\sin 90^o = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} = \frac{n_2}{n_1}$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\frac{n_2}{n_1} = \text{Critical Angle}$$

Condition of total reflection (from wave theory's point of view):

$|R|=1$ or $T=0$

$$T_P = \frac{2\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}} = 0$$

$$\sin\theta_i = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} = \frac{n_2}{n_1}$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$R_P = \frac{-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}$$

$$\theta_c = \sin^{-1}\frac{n_2}{n_1} = \text{Critical Angle}$$

垂直极化也
可以

When $\theta_i > \theta_c$, what happens?

$$\sin\theta_c = \frac{n_2}{n_1}$$

- In this case, $\sin\theta_i > \sin\theta_c$, i.e. $\sin\theta_i > (n_2/n_1)$, then

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i}$$

$$= \pm j\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i - 1} = \text{imaginary}$$

- ***Its physical meaning is that the dielectric ② stores energy but not transmit energy.***
- The corresponding impedance is a reactance(电抗).
- Therefore $|R|=1$, & $\theta_i \in [\theta_c, 90^o]$ → Total reflection.

A Summary: Condition of total reflection

- Incident angle θ_i lies in interval of $[\theta_c, 90^o]$.
- If incident wave goes from region 1 to region 2,

$$\varepsilon_2 < \varepsilon_1 \qquad \sin\theta_c = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} = \frac{n_2}{n_1}$$

$$n_2 < n_1$$

9.4.4. Total Transmission & Brewster Angle

Condition: R=0 or ∣T∣=1

For normal polarized incidence

$$R_{(N)}=\frac{\cos\theta_i-\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)-\sin^2\theta_i}}{\cos\theta_i+\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)-\sin^2\theta_i}}=0$$

$$T_{(N)}=\frac{2\cos\theta_i}{\cos\theta_i+\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)-\sin^2\theta_i}}=\pm1$$

These 2 equations have no solution.
Therefore,
normal-polarized plane wave is unable to transmit totally from dielectric ① to dielectric ②.

垂直极化平面波无论怎么入射都不可能形成全折射！

Condition: R=0 or $|T|$=1

For parallel polarized incidence

$$R_P = \frac{-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}} = 0$$

$$T_P = \frac{2\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}} = \pm 1$$

Solving these 2 equations,
we have

$$\tan\theta_i = \sqrt{\varepsilon_2}/\sqrt{\varepsilon_1}$$

$$\text{or} \quad \tan\theta_i = n_2/n_1$$

$$\therefore \theta_i = \theta_B = tg^{-1} n_2/n_1$$

θ_B is named as Brewster Angle.

- E8.22,E8.25
- P8.27,P8.30