Queen Mary
University of London

EBU6018
Advanced Transform Methods

Tutorial: Haar Transform

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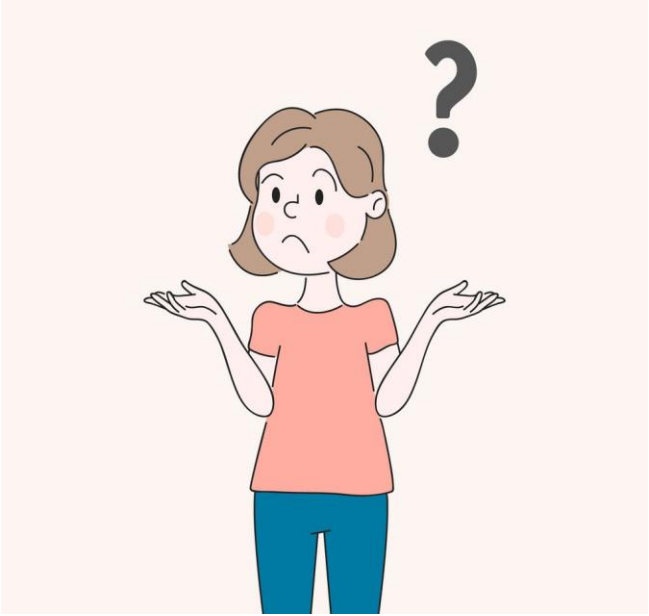
Haar Transform - Quiz

Question 1

Which of the following is true?

a. Haar transform is a non-orthogonal transform
b. Haar transform has fixed basis functions
c. Haar transform is slow
d. Haar transform is complex-valued

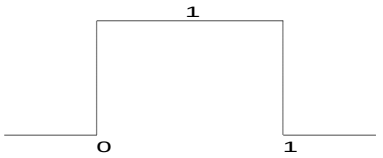


Haar Transform - Quiz

Question 2

What is the name of this function in terms of Haar transform?

a. Wavelet function
b. Transform function
c. Square function
d. Scaling function

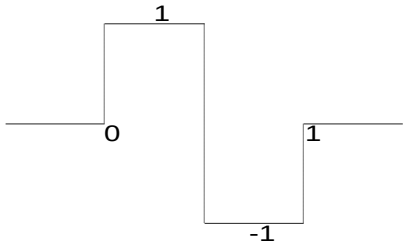


Haar Transform - Quiz

Question 3

Which one is **not** name of this function in terms of Haar transform?

a. Wavelet function
b. Mother wavelet
c. Daughter wavelet



Haar Transform - Quiz

$$H_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

4x4 Haar transform matrix

Question 4

Given the 4x4 Haar transform matrix, which one is the 4x4 inverse Haar transform matrix?

a. A
b. B
c. C
d. D

A

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ -\sqrt{2} & \sqrt{2} & 0 & 0 \\ 0 & 0 & -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

B

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & \sqrt{2} & 0 \\ 1 & 1 & -\sqrt{2} & 0 \\ 1 & -1 & 0 & \sqrt{2} \\ 1 & -1 & 0 & -\sqrt{2} \end{bmatrix}$$

C

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & -\sqrt{2} & 0 \\ 1 & 1 & \sqrt{2} & 0 \\ 1 & -1 & 0 & -\sqrt{2} \\ 1 & -1 & 0 & \sqrt{2} \end{bmatrix}$$

D

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & \sqrt{2} & 0 \\ 1 & 1 & -\sqrt{2} & 0 \\ 1 & -1 & 0 & -\sqrt{2} \\ 1 & -1 & 0 & \sqrt{2} \end{bmatrix}$$

Example 1

- Apply the Haar Transform to the 4-point input sequence:

$$S[n] = [2, 5\ -3, 7]$$

Example 2

$$\mathrm{H}_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

- For the normalized 4x4 Haar matrix show that

$$H_4 H_4^T = \mathrm{I}_4$$

Example 3

- Perform a Haar Transform on the 4-point input sequence :

$$S[n] = [1, 2, 3, 4]$$

- Reconstruct the input sequence using the inverse Haar transform.

Example 4

- Compute the **normalized** 8x8 Haar Transform Matrix

The unnormalized matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Example 5 – Part 1

- Perform Haar Transform on the 8-point input sequence:

$$[\,1, 1, 1, -1, -1, -1, 2, -2]$$

❖ Here is the 8x8 normalized Haar transform matrix:

$$\frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

Example 5 – Part 2

- Given the input sequence and the Haar transform output. Explain the meaning of each transform coefficient in terms of the input
  - From both time and frequency prospective
  - Visualize the input may help

$$\frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{4}{\sqrt{8}} \\ \frac{2\sqrt{2}}{\sqrt{8}} \\ -\frac{2\sqrt{2}}{\sqrt{8}} \\ 0 \\ \frac{4}{\sqrt{8}} \\ 0 \\ \frac{8}{\sqrt{8}} \end{bmatrix}$$

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