

Tutorial: Linear Algebra

Andy Watson

Question 1

Consider the pair of vectors:

$$\psi_0 = (1,1) \quad \psi_1 = \sqrt{\frac{1}{2}}(1,-1)$$

By calculating relevant inner products and norms, identify whether or not these vectors form an orthogonal or an orthonormal set.

Sketch these two vectors on a diagram to confirm your answer

Question 2

Two sets of functions are given by:

$$\{\Psi_1\}=[(2,0),(a,2)]$$

$$\{\Psi_2\}=[(a,-1/8),(0,b)]$$

State the condition required for these two sets to be a Dual Basis and determine the corresponding values of a and b .

Question 3

The matrix A is:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Find an orthonormal basis of the NULL space of A.
- Find the RANK of A.
- Find an orthonormal basis of the ROW space of A.

Question 4

Vectors v_1 and v_2 span the space V .

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Do v_1 and v_2 form an orthonormal basis for V ?
If they do not, then find an orthonormal basis for V .

Question 5 Part i)

$$v_1 = \frac{1}{3\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}, \quad v_2 = \frac{1}{3} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad v_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Show that these three vectors are an orthonormal basis for \mathbb{R}^3

Question 5 Part ii)

Vector $x = [1 \ 1 \ 1]'$

Write x as a linear combination of v_1 , v_2 and v_3 .