北京郵電大學
Beijing University of Posts and Telecommunications

# Chapter 6

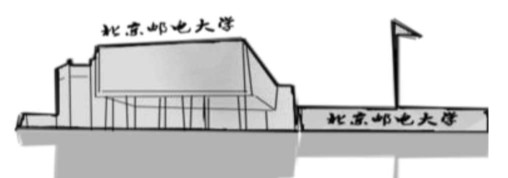
# Bandpass Transmission of Digital Signals

**School of Information and Communication Engineering**

**Beijing University of Posts and Telecommunications**

# Bandpass Transmission of Digital Signals

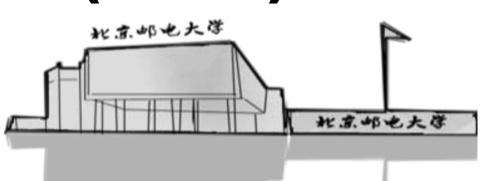
- Introduction
- Sinusoidal carrier modulation of binary digital signal
- Quadrature phase shift keying
- M-ary digital modulation



# Introduction

- **Digital modulation**
  - **Binary signal modulation and M-ary signal modulation schemes**
  - **Linear modulation and nonlinear modulation**
  - **Memoryless modulation and memory modulation**

- **Modulation schemes**
  - **Amplitude Shift Keying(ASK)**
  - **Frequency Shift Keying(FSK)**
  - **Phase Shift Keying(PSK)**
  - **Quadrature Amplitude Modulation(QAM)**



# Bandpass Transmission of Digital Signals

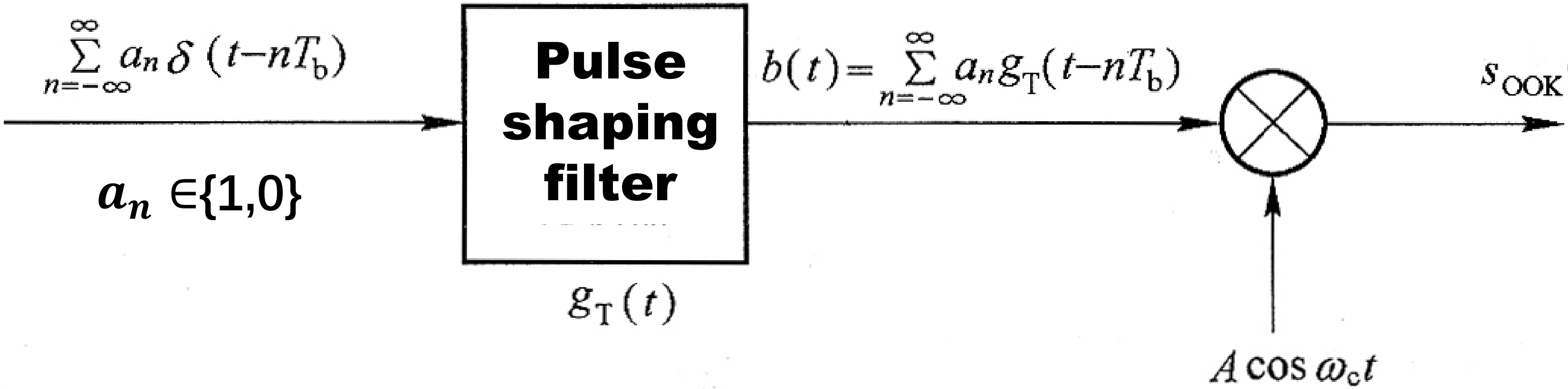
- Introduction
- Sinusoidal carrier modulation of binary digital signals
- Quadrature phase shift keying
- M-ary digital modulation

# Sinusoidal Carrier Modulation of Binary Digital Signals

- On-Off Keying(OOK/2ASK)
- Binary frequency shift keying (2FSK)
- Binary phase shift keying (2PSK)
- Carrier synchronization
- Differential Phase Shift Keying(DPSK)

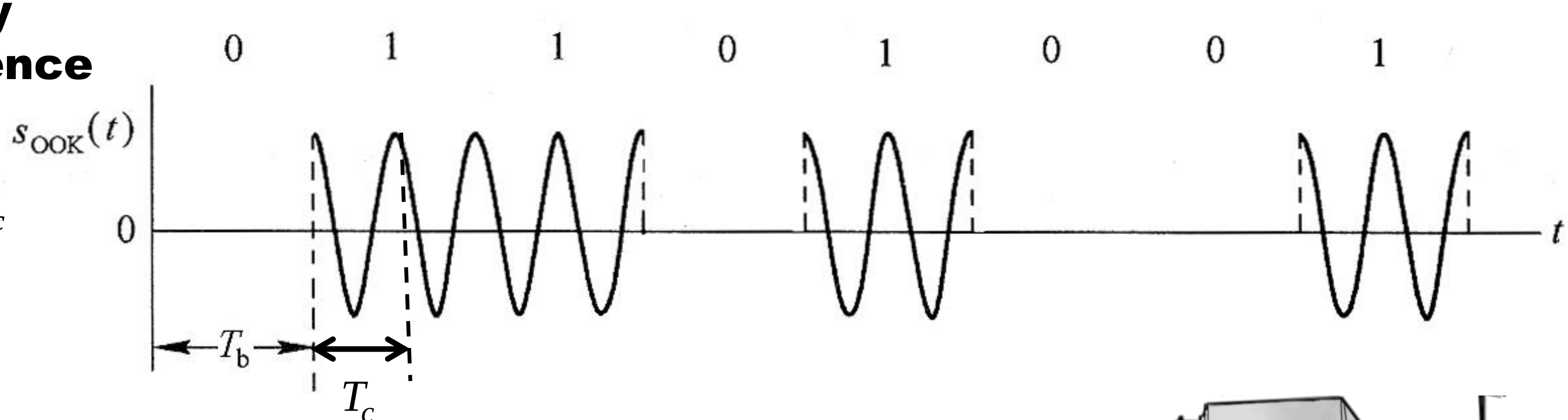
# On-Off Keying(OOK/2ASK)

- **OOK signal**




**Binary Sequence**

$$T_b = 2T_c$$



# OOK/2ASK

- **OOK signal**

$$s_{OOK}(t) = Ab(t)\cos\omega_c t = A\left[\sum_n a_n g_T(t - nT_b)\right]\cos\omega_c t,\ \ a_n \in \{0,1\}$$

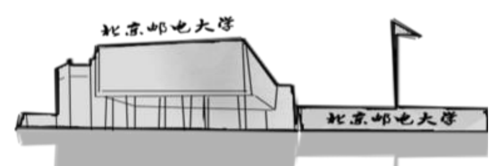
$$S_{OOK}(t) = \begin{cases} S_1(t) = A\cos\omega_c t & \text{'On'} \\ S_2(t) = 0 & \text{'Off'} \end{cases} \qquad 0 \le t \le T_b$$

$$s_1(t) = A\cos 2\pi f_c t \cdot rect\left(\frac{t}{T_b} - \frac{1}{2}\right), s_{1L}(t) = A \cdot rect\left(\frac{t}{T_b} - \frac{1}{2}\right)$$

$$s_2(t) = 0$$

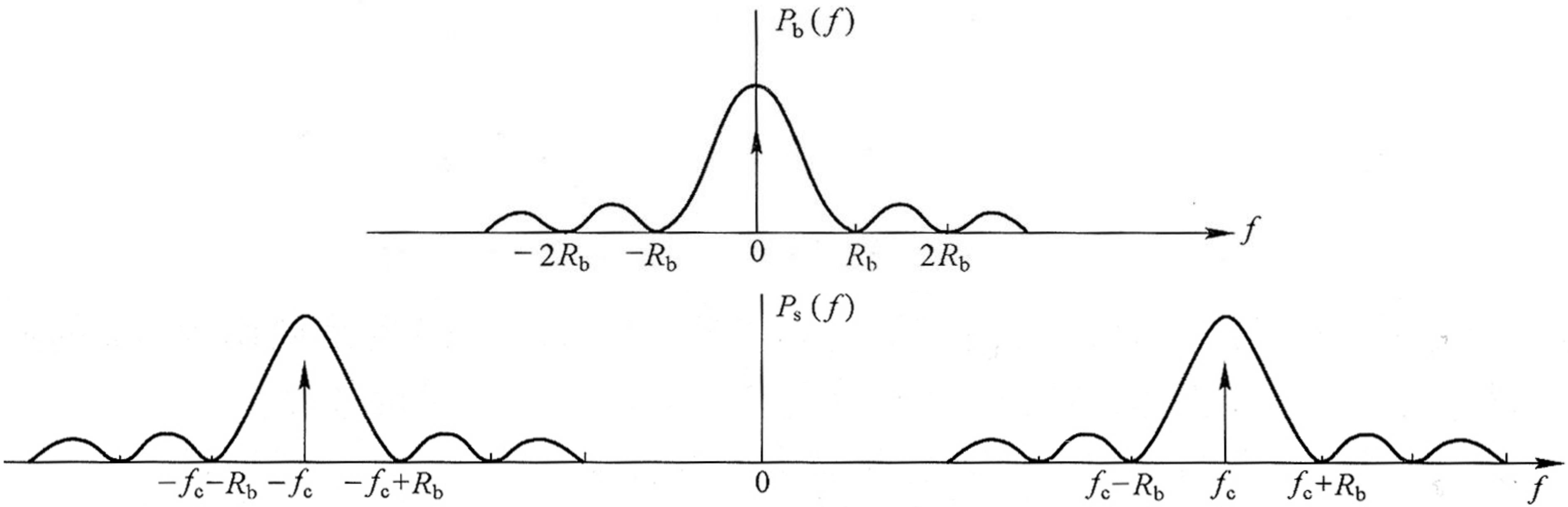
- **PSD of OOK signal**

$$P_s(f) = \frac{A^2}{4}\left[P_b(f - f_c) + P_b(f + f_c)\right]$$



# OOK/2ASK

- **PSD of OOK signal**




$$B = 2W = 2R_b = \frac{2}{T_b}$$

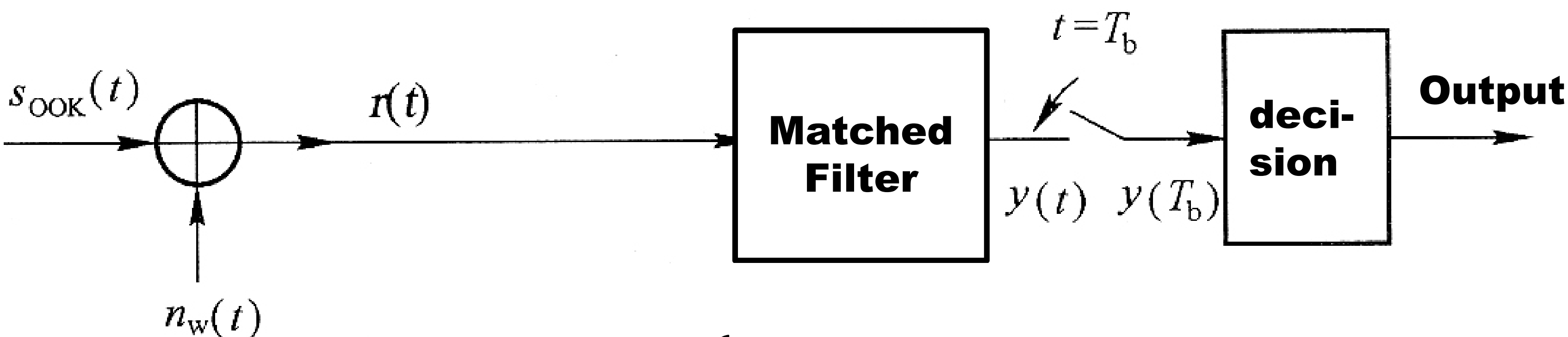
***W*** **~ Bandwidth of digital baseband signal *b*(*t*)**

# OOK/2ASK

- The optimal reception with AWGN : matched filter
- The optimal reception through bandpass channel and with AWGN
- Noncoherent demodulation:
  The optimal reception of OOK signal with random carrier phase and AWGN

# OOK/2ASK

- **Optimal reception through AWGN channel**




$$r(t)=s_{OOK}(t)+n_w(t)=\begin{cases} s_1(t)+n_w(t), & \text{“On”} \\ n_w(t), & \text{“Off”} \end{cases} \quad 0\le t\le T_b$$

$$h(t)=s_1(T_b-t)=A\cos 2\pi f_c(T_b-t), \qquad 0\le t\le T_b$$

$$=A\cos 2\pi f_c(T_b-t)\,\mathrm{rect}\left(\frac{t}{T_b}-\frac{1}{2}\right)$$

$$\boxed{h_e(t)=\frac{1}{2}h_L(t)=\frac{A}{2}e^{-j2\pi f_c T_b}\cdot \mathrm{rect}\left(\frac{t}{T_b}-\frac{1}{2}\right)}$$

# OOK/2ASK

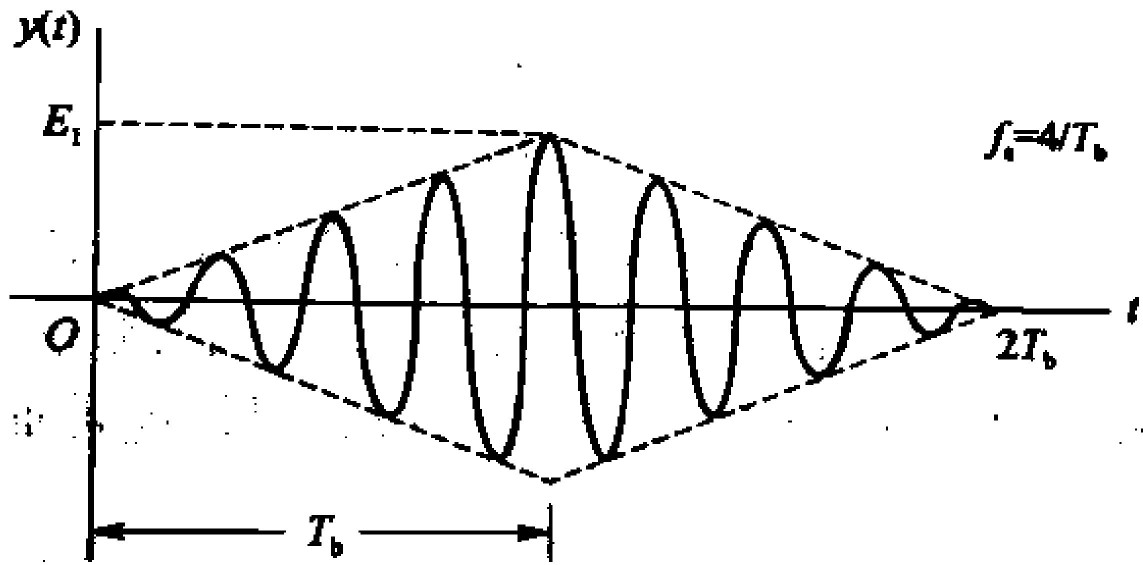
## Optimal reception through AWGN channel

### When "1" was transmitted

$$r(t) = s_1(t) + n_w(t)$$

$$y(t) = \int_0^t r(\tau) h(t-\tau) d\tau$$

$$= \int_0^t s_1(\tau) s_1\left[T_b - (t-\tau)\right] d\tau + \int_0^t n_w(\tau) s_1\left[T_b - (t-\tau)\right] d\tau$$



# OOK/2ASK

- **Equivalent baseband analysis**

$$s_{1,L}(t)=A\cdot\mathrm{rect}\left(\frac{t}{T_b}-\frac{1}{2}\right)$$

$$h_e(t)=\frac{1}{2}h_L(t)=\frac{A}{2}e^{-j2\pi f_c T_b}\cdot\mathrm{rect}\left(\frac{t}{T_b}-\frac{1}{2}\right)$$

$$y_L(t)=\frac{A^2T_b}{2}e^{-j2\pi f_c T_b}q(t-T_b)$$

$$y(t)=\mathrm{Re}\{y_L(t)e^{j2\pi f_c t}\}=E_1q(t-T_b)\cos(2\pi f_c t-2\pi f_c T_b)$$

$$\boxed{E_1=A^2T_b/2}$$

$$\boxed{q(t)=\begin{cases}1-\dfrac{|t|}{T_b}, & |t|\leqslant T_b\\ 0, & |t|>T_b\end{cases}}$$

# OOK/2ASK

- **At the sampling moment t = Tb**

$$y(T_b)=\int_0^{T_b} s_1^2(\tau)\,d\tau+\int_0^{T_b} n_w(\tau)\,s_1(\tau)\,d\tau=E_1+Z$$

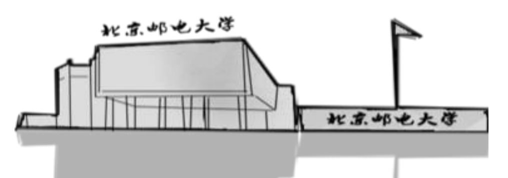
- **Generally**

$$E_1=\int_0^{T_b} s_1^2(\tau)\,\mathrm{d}\tau=\frac{A^2}{2}T_b,$$

$$Z=\int_0^{T_b} n_w(\tau)\,s_1(\tau)\,\mathrm{d}\tau$$ **Gaussian**

$$y(T_b)=aE_1+Z,\ a=\begin{cases}1, s_1(t)\ \text{transmitted}\\ 0, s_2(t)\ \text{transmitted}\end{cases}$$

$$E[Z]=0,\sigma^2=D[Z]=\frac{N_0}{2}E_1$$



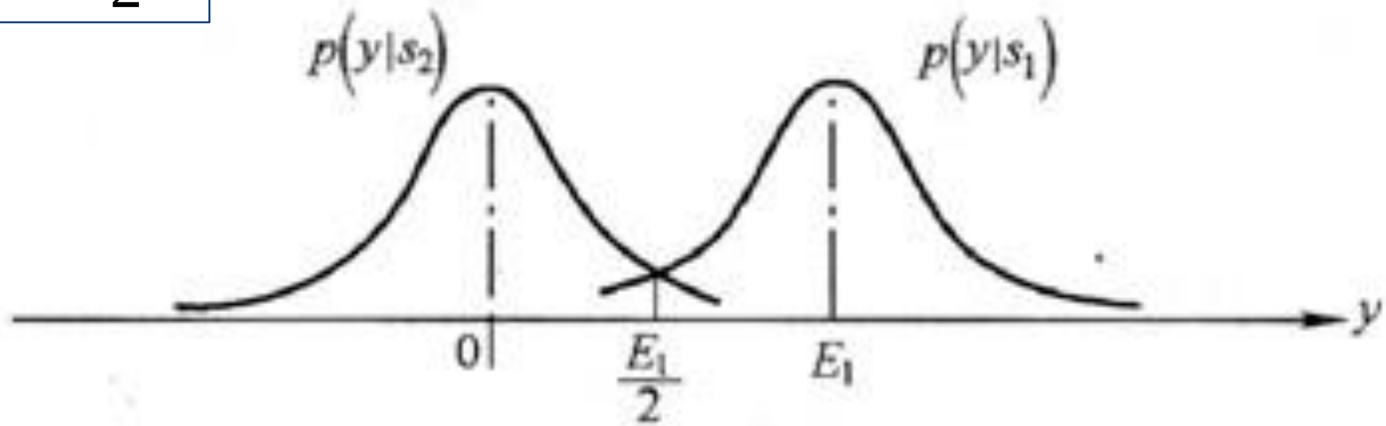
# OOK/2ASK

- **Decision threshold：** $V_T = E_1/2$

$$P(e|s_1) = P(Z < V_T) = P\left(Z < \frac{E_1}{2}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_1}{4N_0}}\right)$$

$$P(e|s_2) = P(Z > V_T) = P\left(Z > \frac{E_1}{2}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_1}{4N_0}}\right)$$

$$\boxed{A_F = \frac{E_1}{2}, \sigma^2 = \frac{N_0 E_1}{2}}$$



# OOK/2ASK

- **Optimal reception through AWGN channel**

**Average BER**

$$P_b = P(s_1)\cdot P(e|s_1) + P(s_2)\cdot P(e|s_2)$$

$$= \frac{1}{2} erfc\left(\sqrt{\frac{E_1}{4N_0}}\right)$$

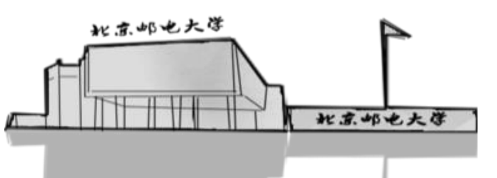
$$= \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\text{where } E_b = \frac{1}{2}(E_1 + E_2)$$

$$= \frac{1}{2}\left(A^2T_b/2 + 0\right)$$

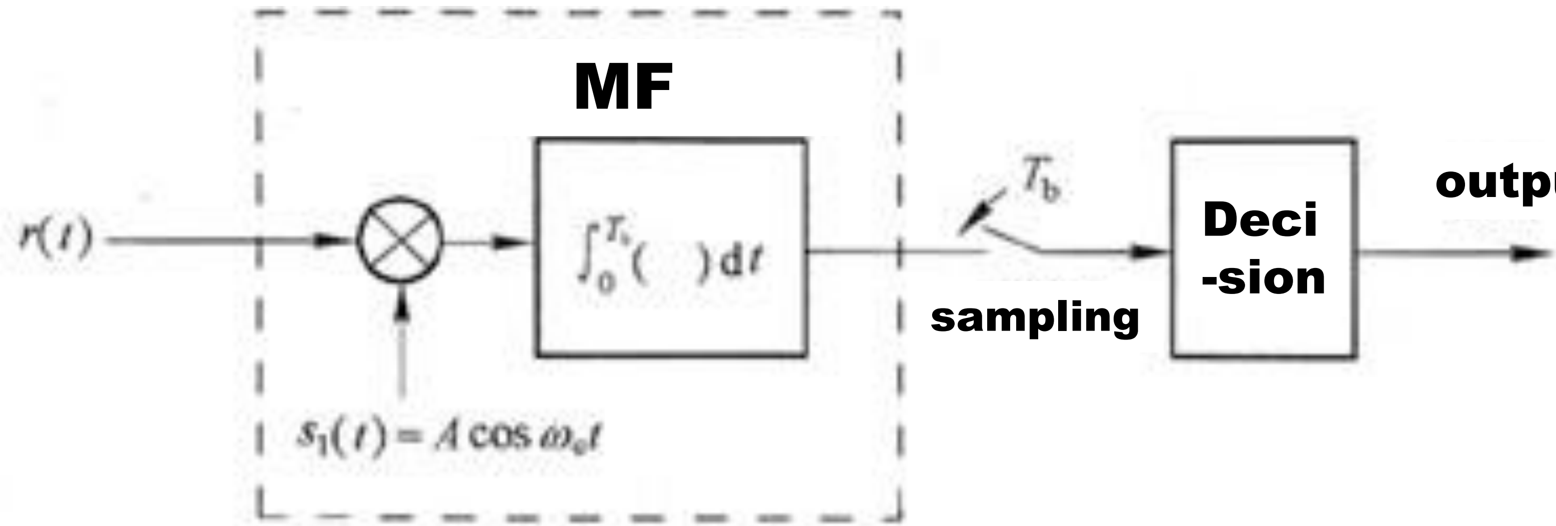
$$= \frac{A^2T_b}{4} \quad \square \text{ average bit energy}$$



# OOK/2ASK

- **Optimal reception through AWGN channel**

**Coherent demodulation: equivalent to MF reception**

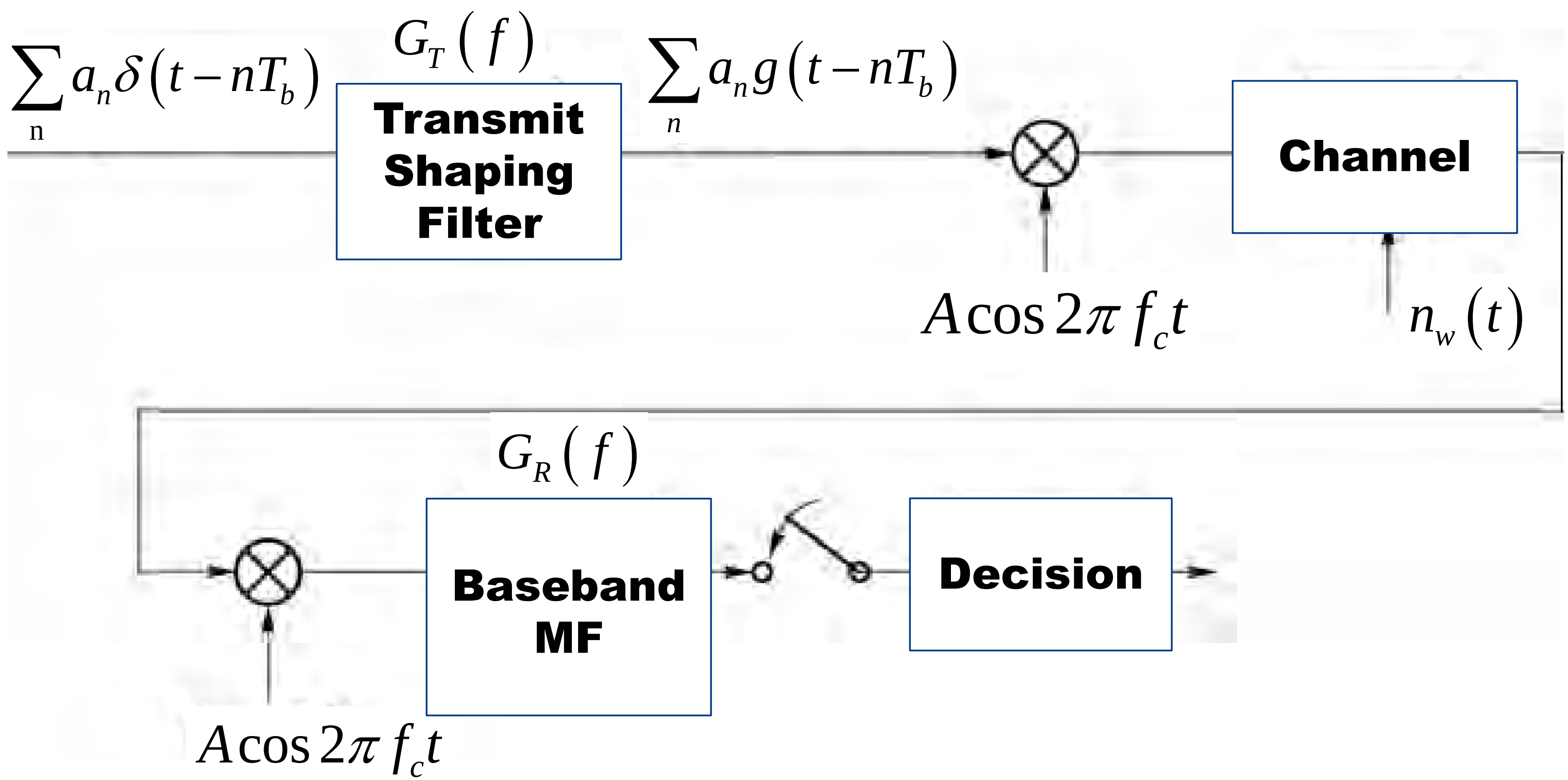



$$y(T_b)=\int_0^t r(\tau)h(t-\tau)\,d\tau\Big|_{t=T_b}=\int_0^t r(\tau)s_1(T_b-t+\tau)\,d\tau\Big|_{t=T_b}$$

$$=\int_0^{T_b} r(\tau)s_1(\tau)\,d\tau$$

# OOK/2ASK

- **Equivalent optimal reception**



# OOK/2ASK

- **For optimal reception through ideal bandpass channel + AWGN**

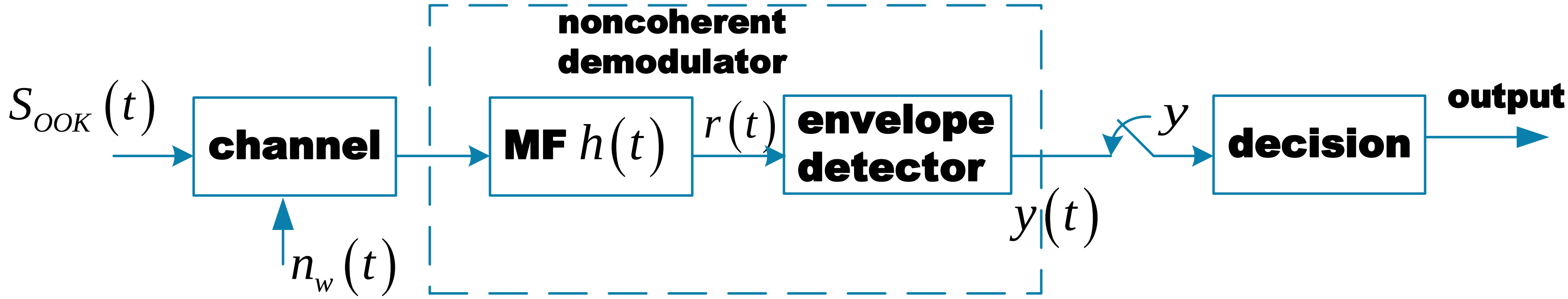
$$G_T(f) = G_R(f) = \sqrt{X_{\text{rcos}}(f)}$$

$$B = 2W = (1+\alpha)R_s$$

# OOK/2ASK

## For noncoherent reception

- **With random carrier phase after transmitted through AWGN channel**




$$s(t)=\begin{cases} s_1(t)=A\cos\omega_c t, \\ s_2(t)=0, \end{cases} \quad 0\le t\le T_b, \qquad r(t)=\begin{cases} A\cos(\omega_c t+\phi)*h(t)+n(t), \\ n(t), \end{cases} \quad 0\le t\le T_b$$

$$s_{1,L}(t)=A e^{j\phi}\cdot \operatorname{rect}\left(\frac{t}{T_b}-\frac{1}{2}\right)$$

$$h(t)=A\cos\left[2\pi f_c(T_b-t)\right],\quad 0\le t\le T_b \quad ;h_e(t)=\frac{A}{2}e^{-j2\pi f_c T_b}\operatorname{rect}\left(\frac{t}{T_b}-\frac{1}{2}\right)$$

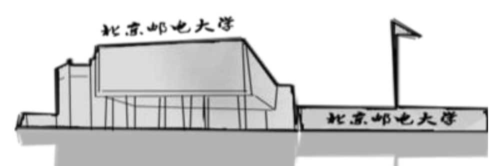
$$y_L(t)=E_1 e^{j\phi} e^{-j2\pi f_c T_b} q(t-T_b)$$

# OOK/2ASK

- **At the moment $t=T_b$, and $f_c$ is times of $1/T_b$**

$$y_L(T_b) = \underbrace{a}_{=0 \text{ or } 1}E_1e^{j\phi} + \underbrace{(n_c + jn_s)}_{\text{noted as } Z'} = \left[aE_1 + \underbrace{(n_c + jn_s)}_{\text{noted as } Z}e^{-j\phi}\right]e^{j\phi}$$

- ***where** $n_c$* **and** $n_s$ **are i.i.d. Gaussian variables with 0 means and identical variances $N_0E_1/2$.**
- $Z$ **and** $Z'$ **are i.d. complex Gaussian variables.**



# OOK/2ASK noncoherent reception

- **Decision variable:**

$$v = \left| y_L(T_b) \right| = \left| aE_1 + (n_c + jn_s)e^{-j\phi} \right|$$

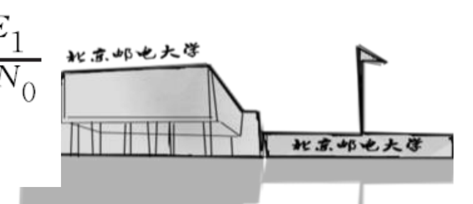
**Decision threshold for equal probability transmission:** $V_{\mathrm{T}} = E_1/2$

**When $s_2(t)$ transmitted (Off)**

$$v = \left| (n_c + jn_s)e^{-j\phi} \right| = \left| n_c + jn_s \right|$$ **~Rayleigh distribution**

$$p(v|s_2) = \frac{v}{\sigma^2}\mathrm{e}^{-\frac{v^2}{2\sigma^2}} \qquad \sigma^2 = \frac{N_0 E_1}{2}$$

$$P(e|s_2) = P\left(v > \frac{E_1}{2}\right) = \int_{\frac{E_1}{2}}^{\infty} \frac{v}{\sigma^2}\mathrm{e}^{-\frac{v^2}{2\sigma^2}}\mathrm{d}v = \mathrm{e}^{-\frac{E_1}{4N_0}}$$



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# OOK/2ASK noncoherent reception

**When $s_1(t)$ is transmitted (On)**

$$v = \left| E_1 + (n_c + jn_s) e^{-j\phi} \right| = \left| E_1 + Z' \right|$$

**With high SNR**

$$v \approx \mathrm{Re}\left\{ E_1 + (n_c + jn_s) e^{-j\phi} \right\} = E_1 + n_c \cos\phi + n_s \sin\phi$$

$$= \mathrm{Re}\left\{ E_1 + Z' \right\} = E_1 + \mathrm{Re}\left\{ Z' \right\}$$

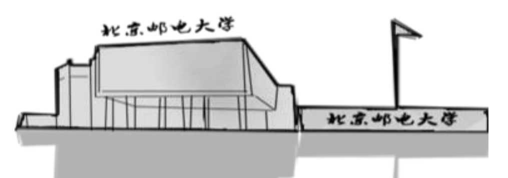
**Z ’: Gaussian random variable with 0 mean and variance $N_0E_1/2$.**

$$P(e|s_1) = P\left(v < \frac{E_1}{2}\right) \approx P\left(\mathrm{Re}\{Z'\} < -\frac{E_1}{2}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_1}{4N_0}}\right)$$

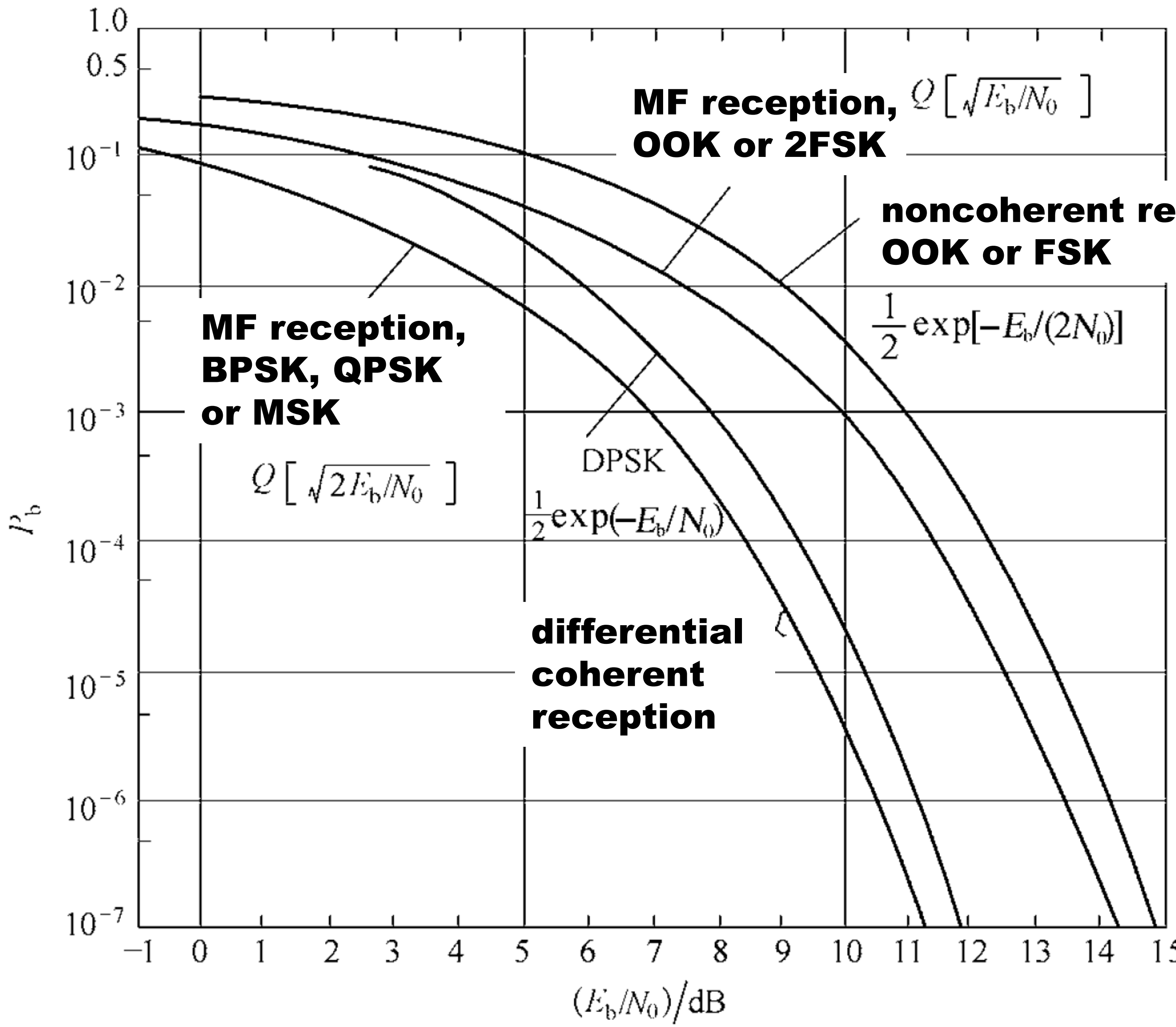
$$P_{\mathrm{b}} = \frac{1}{2}P(e|s_1) + \frac{1}{2}P(e|s_2) \approx \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{E_1}{4N_0}}\right) + \frac{1}{2}\mathrm{e}^{-\frac{E_1}{4N_0}} \approx \frac{1}{2}\mathrm{e}^{-\frac{E_\mathrm{b}}{2N_0}}$$

**Optimal noncoherent receiver**

$$E_b = \frac{E_1}{2}$$



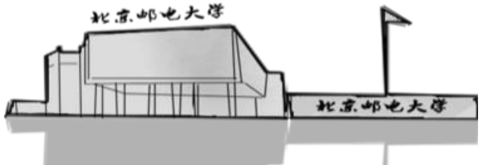
# OOK/2ASK



MF reception, OOK or 2FSK
$Q\left[\sqrt{E_b/N_0}\right]$
noncoherent reception, OOK or FSK
$\frac{1}{2}\exp[-E_b/(2N_0)]$
MF reception, BPSK, QPSK or MSK
$Q\left[\sqrt{2E_b/N_0}\right]$
DPSK
$\frac{1}{2}\exp(-E_b/N_0)$
differential coherent reception
$P_b$
$(E_b/N_0)/\mathrm{dB}$
1.0
0.5
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
−1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

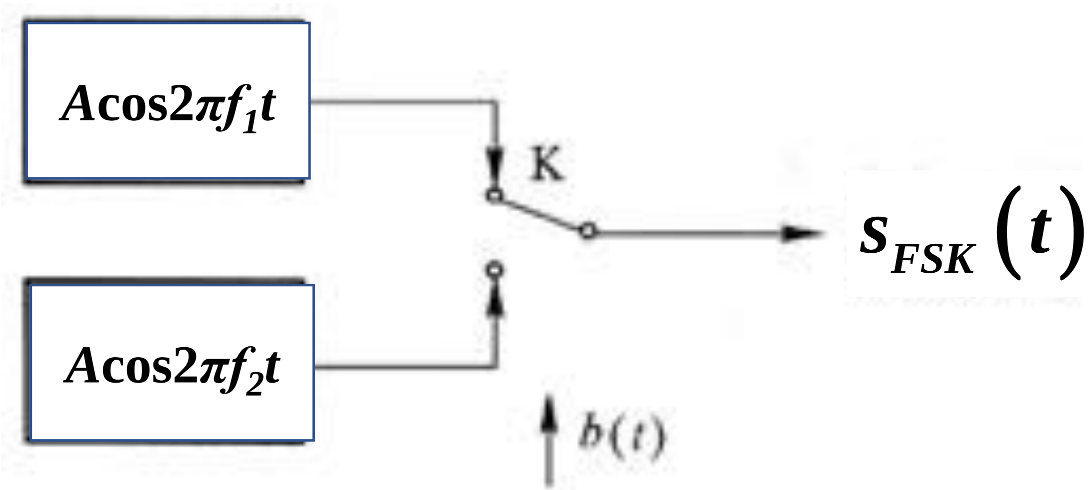
# Sinusoidal Carrier Modulation of Binary Digital Signals

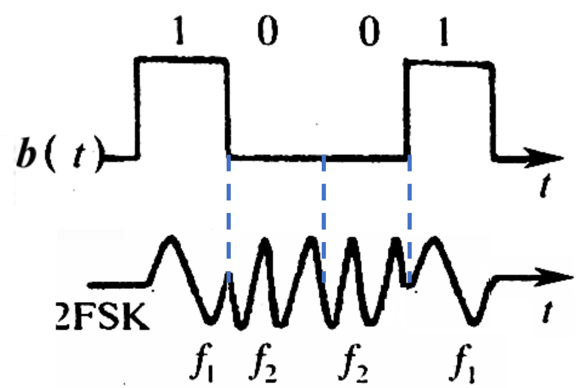
- On-Off Keying(OOK/2ASK)
- Binary frequency shift keying (2FSK)
- Binary phase shift keying (2PSK)
- Carrier synchronization
- Differential Phase Shift Keying(DPSK)



# Binary Frequency Shift Keying (2FSK)

## ●2FSK signal with discontinuous phases

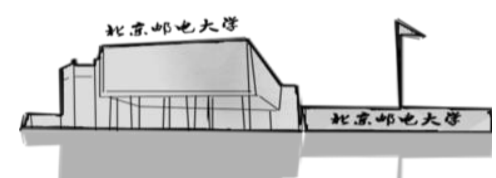



$$s_{FSK}(t)=\begin{cases} s_1(t)=A\cos 2\pi f_1 t, \\ s_2(t)=A\cos 2\pi f_2 t, \end{cases} \qquad 0\le t\le T_b$$

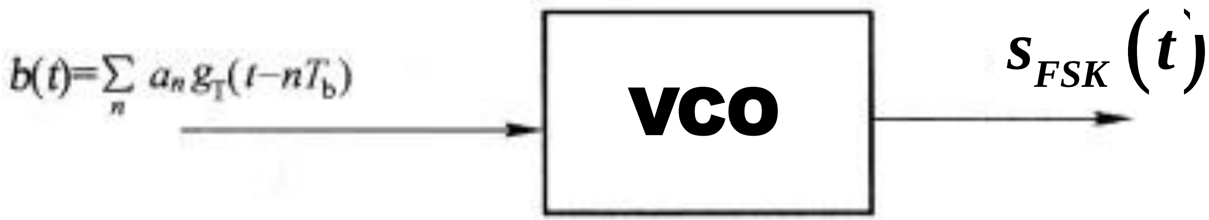
**Define:** $f_c=\dfrac{f_1+f_2}{2}$, $\Delta f=\dfrac{f_1-f_2}{2}$ $\boxed{E_1=E_2=\dfrac{A^2T_b}{2}, E_b=\dfrac{A^2T_b}{2}}$

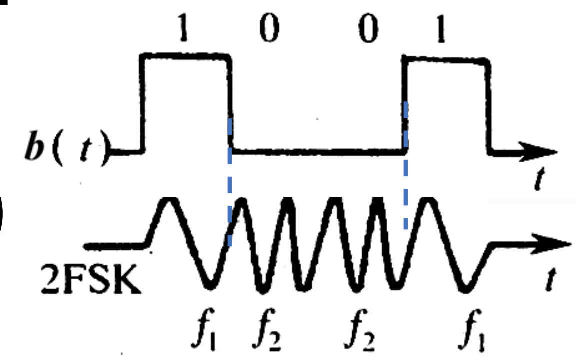
$$\Rightarrow \begin{cases} s_1(t)=A\cos 2\pi\left(f_c+\Delta f\right)t, \\ s_2(t)=A\cos 2\pi\left(f_c-\Delta f\right)t, \end{cases} \qquad 0\le t\le T_b$$



# 2FSK

## ● 2FSK signal with continuous phase




$$f_1=\frac{1}{T_s},\quad f_2=\frac{2}{T_s}$$

$$s_{FSK}(t)=A\cos\left[2\pi f_c t+2\pi k_f\int_{-\infty}^{t} b(\tau)d\tau\right]=\mathrm{Re}\left[v(t)e^{j2\pi f_c t}\right]$$

complex envelope: $v(t)=Ae^{j\theta(t)}$

phase: $\theta(t)=2\pi k_f\int_{-\infty}^{t} \underline{b(\tau)}d\tau$

**Bipolar NRZ sequence**

# 2FSK

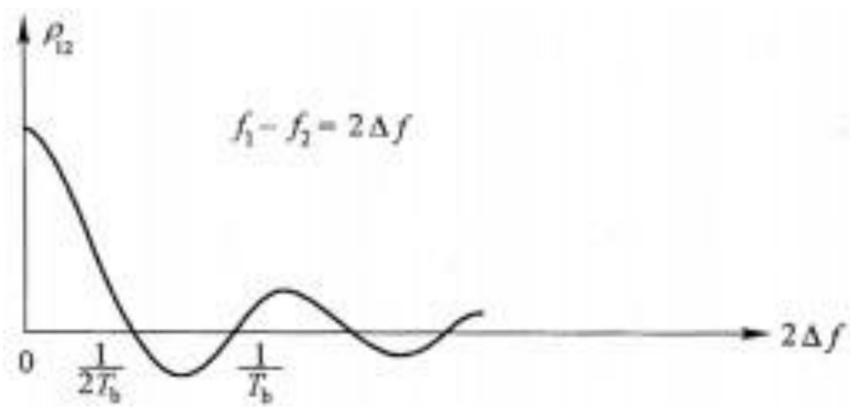
- **Nominal correlation coefficient of $s_1(t)$ and $s_2(t)$**

$$\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_0^{T_b} s_1(t) \cdot s_2(t) dt = \frac{1}{E_b} \int_0^{T_b} s_1(t) \cdot s_2(t) dt$$

$$= \frac{2}{T_b} \int_0^{T_b} [\cos 2\pi ( f_c + \Delta f ) t \cdot \cos 2\pi ( f_c - \Delta f ) t] dt$$

$$= \frac{1}{T_b} \int_0^{T_b} [\cos 4\pi \Delta f t + \cos 4\pi f_c t] dt$$

$$= \mathrm{sinc}(4\Delta f T_b) + \mathrm{sinc}(4 f_c T_b)$$




$$f_c \gg \frac{1}{T_b} \Rightarrow \mathrm{sinc}(4 f_c T_b) \approx 0 \Rightarrow \rho_{12} = \mathrm{sinc}(4\Delta f T_b)$$

when $\rho_{12} = 0$, the minimum frequency interval

$$2\Delta f = | f_1 - f_2 | = \frac{1}{2T_b}$$

# 2FSK

- **PSD of 2FSK signals**
  - **The average PSD of continuous phase 2FSK signal: the sidelobes decrease by the law of $1/f^4$.**
  - **The average PSD of discontinuous phase 2FSK signal: the sidelobes decrease by $1/f^2$.**
- **Bandwidth of 2FSK signals**

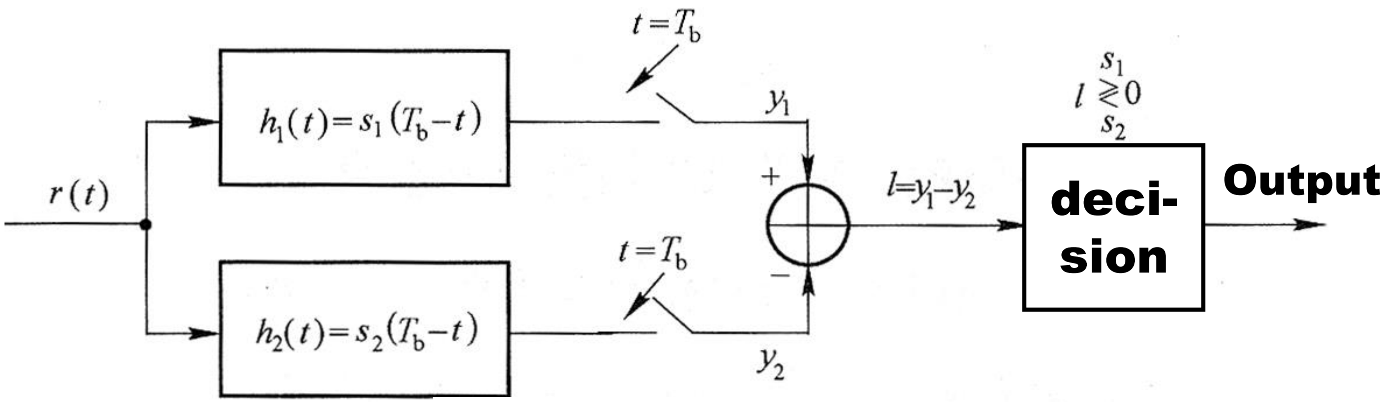
$$B_{FSK} = 2\Delta f + 2W$$

**Suppose we adopt the PSD main lobe width as the bandwidth *W* of digital baseband signal, we have $W = R_b$,**

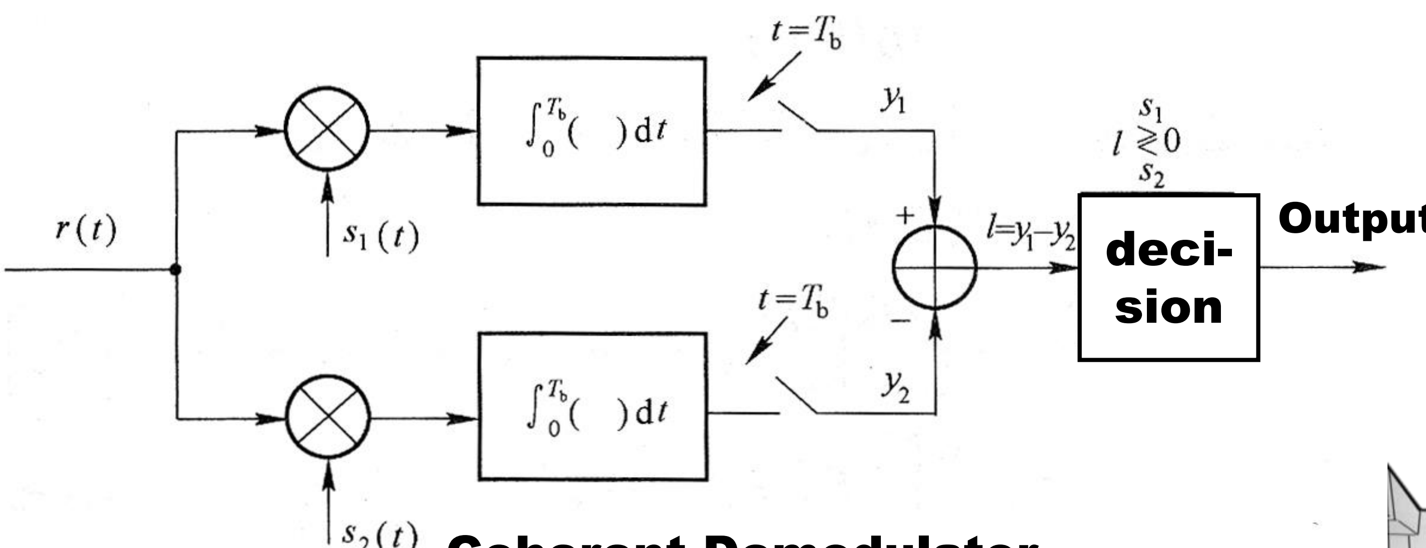
**$B_{FSK} = 2\Delta f + 2R_b$**

# 2FSK

- **Optimal reception of orthogonal 2FSK signal through AWGN channel**




**Matched Filter**




**Coherent Demodulator**

# 2FSK

suppose $s_1(t)$ and $s_2(t)$ are orthogonal,

$$s_1(t)\ \text{transmitted:}\quad \begin{cases} y_1 = E_b + Z_1, & Z_1 = \int_0^{T_b} n_w(t)s_1(t)dt \\ y_2 = Z_2, & Z_2 = \int_0^{T_b} n_w(t)s_2(t)dt \end{cases}$$

$$D(Z_1) = \frac{N_0E_1}{2} = \frac{N_0E_b}{2},\quad D(Z_2) = \frac{N_0E_2}{2} = \frac{N_0E_b}{2}$$

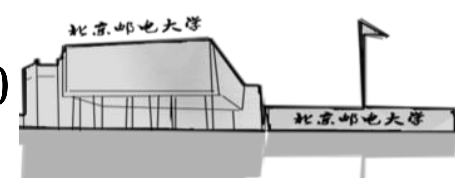
$$s_2(t)\ \text{transmitted:}\quad \begin{cases} y_1 = Z_1, \\ y_2 = E_b + Z_2, \end{cases}$$

$$\boxed{\begin{aligned} y(T_b) &= \begin{cases} y_1 = aE_b + Z_1 \\ y_2 = (1-a)E_b + Z_2 \end{cases}, \\ a &= \begin{cases} 1, s_1(t)\, transmitted \\ 0, s_2(t)\, transmitted \end{cases} \end{aligned}}$$

$$\Rightarrow l = dE_b + Z = y_1 - y_2 \underset{S_2}{\overset{S_1}{\gtrless}} V_T,$$

$$d = \begin{cases} 1, s_1(t)\ \text{transmitted} \\ -1, s_2(t)\ \text{transmitted} \end{cases},\quad Z = Z_1 - Z_2, D[Z] = N_0E_b$$

if $P(s_1) = P(s_2) = \frac{1}{2}$, then the optimal threshold $V_T = 0$



# 2FSK

- **Distribution of *l***

$$E[l|s_1]=E[(y_1-y_2)|s_1]=E[(E_b+Z_1-Z_2)|s_1]=E_b$$

$$D[l|s_1]=E\left\{\left[\left(l|s_1\right)-E\left(l|s_1\right)\right]^2\right\}=E\left\{[Z_1-Z_2]^2\right\}$$

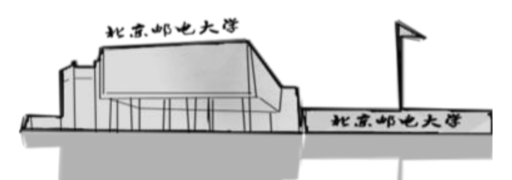
$$Cov(Z_1,Z_2)=E\left\{\left[Z_1-E(Z_1)\right]\left[Z_2-E(Z_2)\right]\right\}=E\left[Z_1Z_2\right]$$

***Z*$_1$ and *Z*$_2$ are i.i.d.**

$$=\int_0^{T_b}\int_0^{T_b}E\left[n_w(t_1)n_w(t_2)\right]s_1(t_1)s_2(t_2)dt_1dt_2$$

$$=\int_0^{T_b}\int_0^{T_b}\frac{N_0}{2}\delta\left(t_1-t_2\right)s_1(t_1)s_2(t_2)dt_1dt_2$$

$$=\int_0^{T_b}s_1(t_1)s_2(t_1)\frac{N_0}{2}dt_1=0$$



# 2FSK

$$\therefore\ D[l|s_1] = E(Z_1^2) + E(Z_2^2) = \frac{N_0 E_b}{2} + \frac{N_0 E_b}{2} = N_0 E_b$$

$$p(l|s_1) = \frac{1}{\sqrt{2\pi N_0 E_b}} \exp\left[-\frac{(l-E_b)^2}{2N_0 E_b}\right]$$

also， $$p(l|s_2) = \frac{1}{\sqrt{2\pi N_0 E_b}} \exp\left[-\frac{(l+E_b)^2}{2N_0 E_b}\right]$$

average BER

$$P_b = P(s_1)P(e|s_1) + P(s_s)P(e|s_s) \overset{P(s_1)=P(s_2),\,P(e|s_1)=P(e|s_2)}{=} P(e|s_1)$$

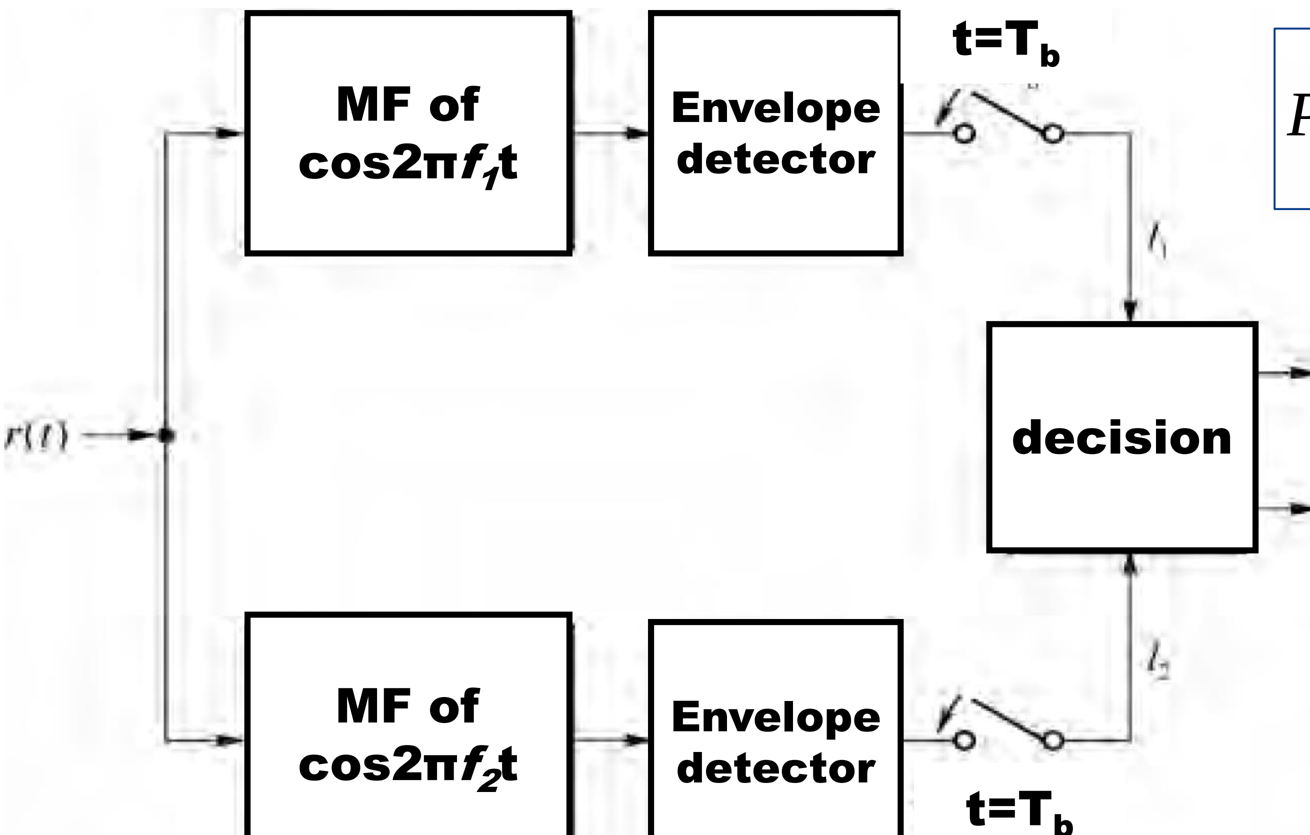
$$= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi N_0 E_b}} \exp\left[-\frac{(l-E_b)^2}{2N_0 E_b}\right] \mathrm{d}l = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

# 2FSK

- **The optimal reception of orthogonal 2FSK signal with random carrier phase through AWGN channel**

$$\begin{cases} s_1(t) = A\cos 2\pi f_1 t, \\ s_2(t) = A\cos 2\pi f_2 t, \end{cases} \quad 0 \le t \le T_b \Rightarrow \begin{cases} A\cos(2\pi f_1 t + \theta) + n_w(t), \\ A\cos(2\pi f_2 t + \theta) + n_w(t), \end{cases} \quad 0 \le t \le T_b$$

$$P_b = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$



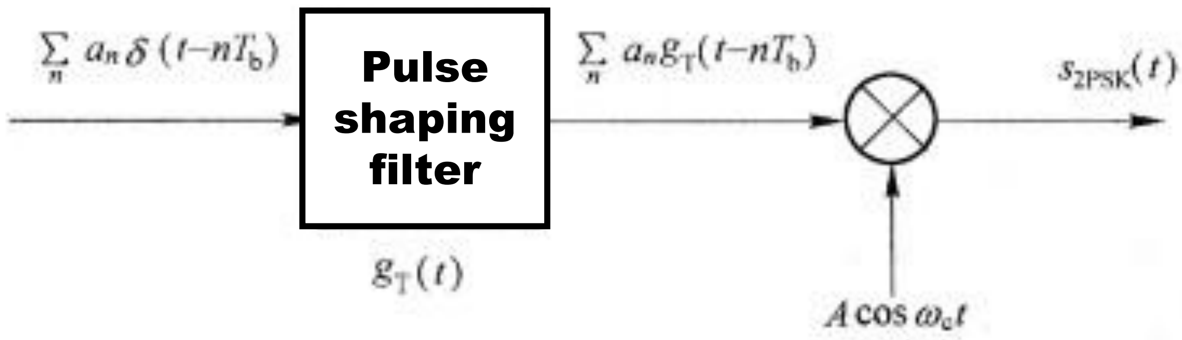

**1 for** $l_1 > l_2$

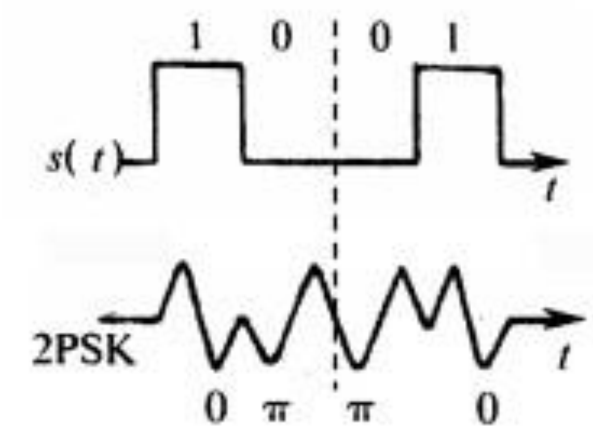
**0 for** $l_1 < l_2$

# Sinusoidal Carrier Modulation of Binary Digital Signals

- On-Off Keying(OOK/2ASK)
- Binary frequency shift keying (2FSK)
- Binary phase shift keying (2PSK)
- Carrier synchronization
- Differential Phase Shift Keying(DPSK)

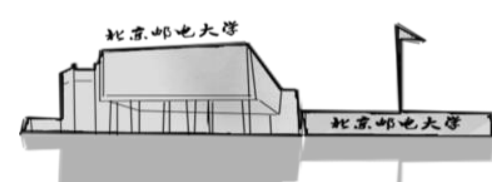
# BPSK




$$s_{2PSK}(t) = A\left[\sum_n a_n g_T(t-nT_b)\right]\cos\omega_c t \quad \text{where } a_n \in \{+1,-1\}$$

$$= \begin{cases} s_1(t) = A\cos\omega_c t, & \text{for '1'} \\ s_2(t) = \text{-}A\cos\omega_c t = A\cos(\omega_c t+\pi), & \text{for '0'} \end{cases}$$



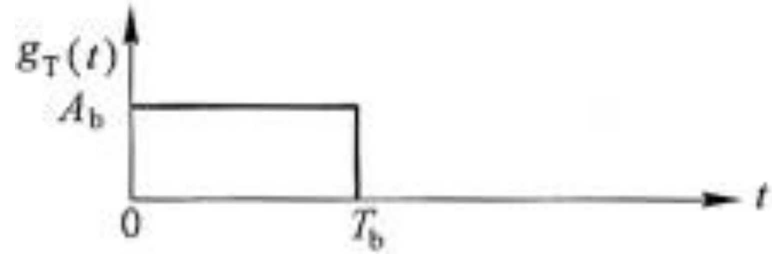
# BPSK

- **The average PSD of BPSK signal**

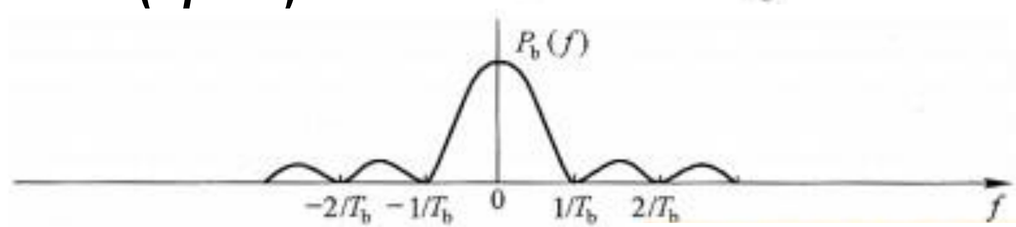
$$P_{2PSK}(f) = \frac{A^2}{4}\left[P_b(f - f_c) + P_b(f + f_c)\right]$$

$$\because m_a = 0,\ \sigma_a^2 = 1$$

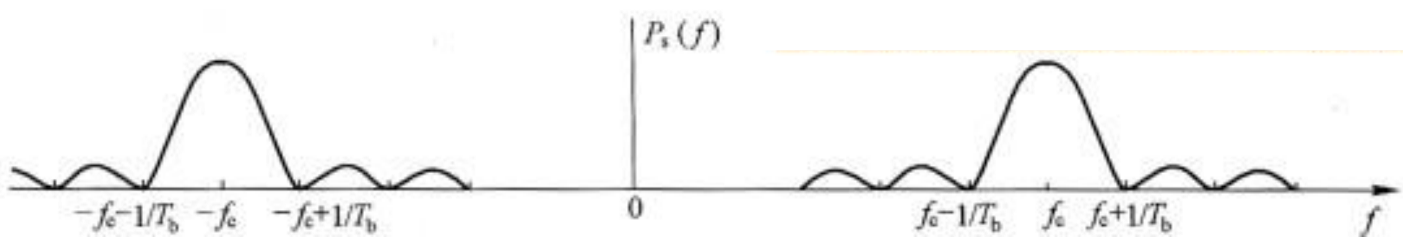
$$\therefore P_b(f) = T_b \mathrm{sinc}^2(fT_b)$$




$$B = 2W = 2R_b = \frac{2}{T_b}$$



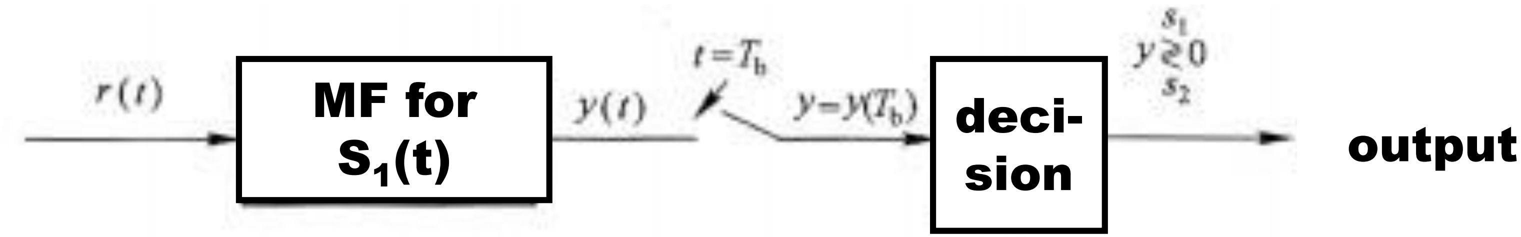

$$W = 1/T_b$$



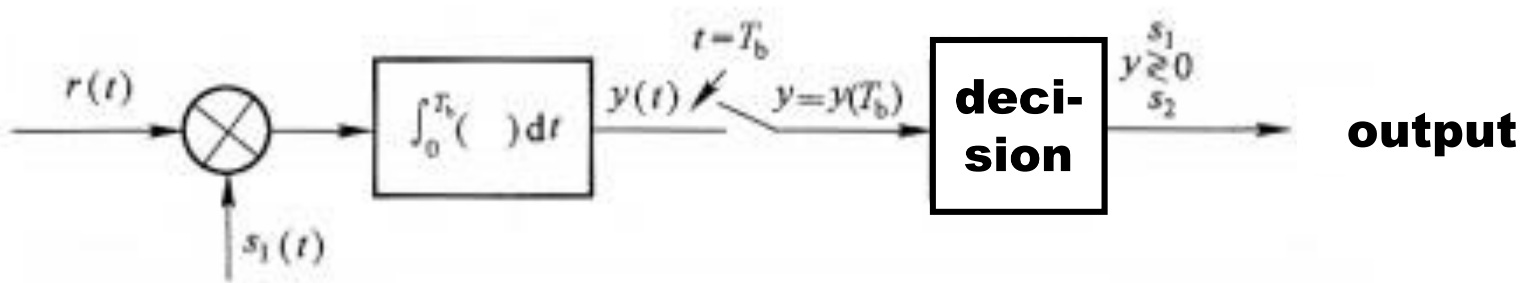

**Continuous spectrum**

# BPSK

- **Optimal reception of 2PSK signal through AWGN channel**




**MF demodulator**



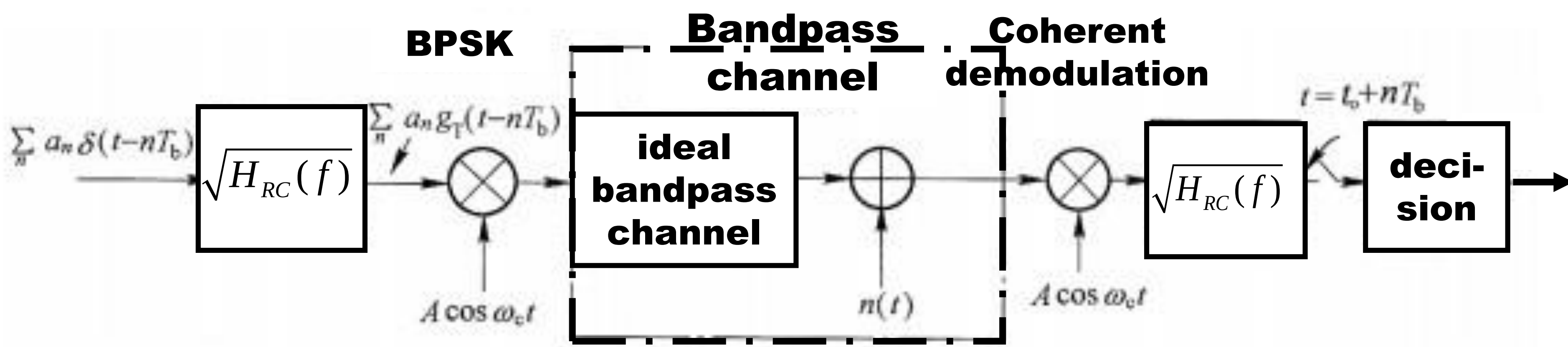

**Coherent demodulator**

$$s_1: \; y(T_b) = E_b + Z$$
$$s_2: \; y(T_b) = -E_b + Z$$

$$\xRightarrow{P(s_1)=P(s_2)=1/2} V_T = 0 \Longrightarrow P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

# BPSK

- **Optimal reception of 2PSK signal through ideal bandpass channel and AWGN channel**

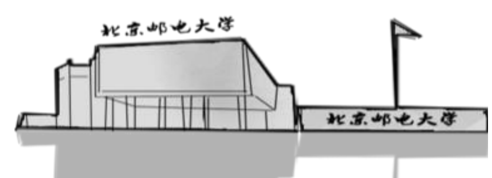



**Average BER of equal probability transmission:**

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

# Sinusoidal Carrier Modulation of Binary Digital Signals

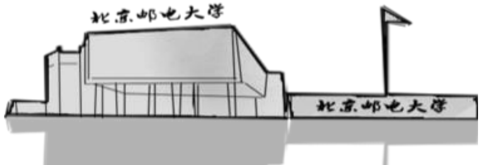
- On-Off Keying(OOK/2ASK)
- Binary frequency shift keying (2FSK)
- Binary phase shift keying (2PSK)
- Carrier synchronization
- Differential Phase Shift Keying(DPSK)



# Carrier synchronization

- Square Loop Method

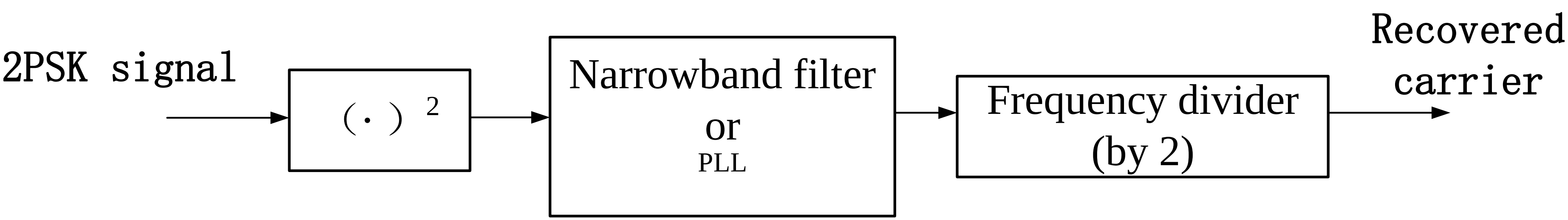
- COSTAS Loop Method



# Square Loop Method

$$s_{2PSK}(t) = b(t)\cos\omega_c t$$

$$s_{2PSK}^2(t) = b^2(t)\cos^2\omega_c t$$




- **Input of frequency divider:**

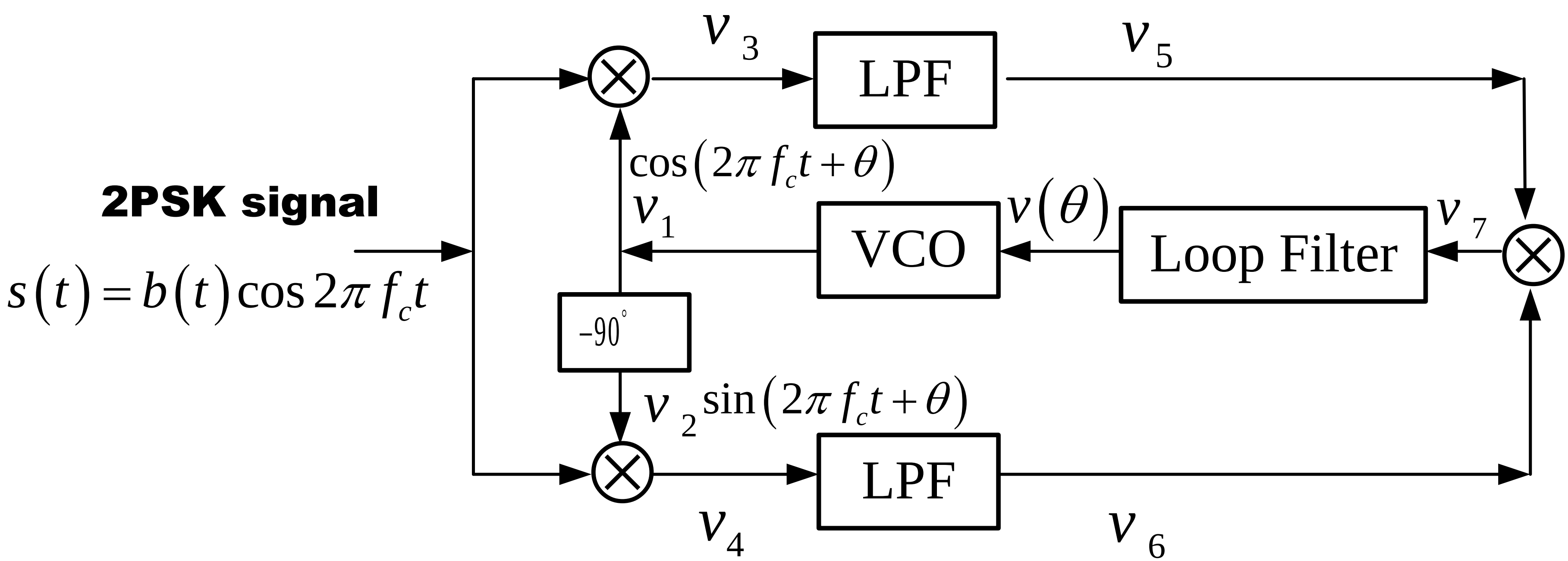
$$\cos 2\omega_c t = \cos(4\pi f_c t + 2\pi)$$

- **Output of frequency divider:**

$$\cos\omega_c t, \quad \text{or} \quad \cos(2\pi f_c t + \pi) = -\cos(2\pi f_c t)$$

**Phase Ambiguity might exist .**

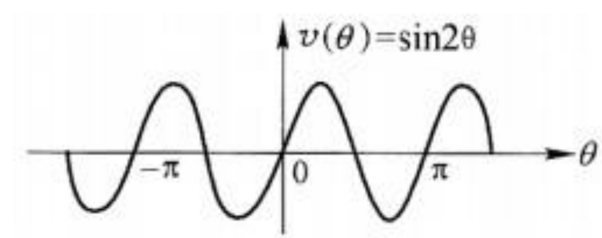
# COSTAS Loop Method




$$v_5(t)=b(t)\cos\omega_c t\cos(\omega_c t+\theta)\Big|_{LPF}=\frac{1}{2}b(t)\cos\theta$$

$$v_6(t)=b(t)\cos\omega_c t\sin(\omega_c t+\theta)\Big|_{LPF}=\frac{1}{2}b(t)\sin\theta$$

$$v_7(t)=v_5(t)v_6(t)=\frac{1}{8}b^2(t)\sin 2\theta \overset{\theta \ll 1}{\approx} \frac{1}{4}b^2(t)\theta$$




$$\theta=n\pi, v(\theta)=0$$

**Phase Ambiguity might also exist.**

$v(\theta)$ is propotional to $\theta$, it controls the frequency of the *VCO*.

# Sinusoidal Carrier Modulation of Binary Digital Signals

- On-Off Keying(OOK/2ASK)
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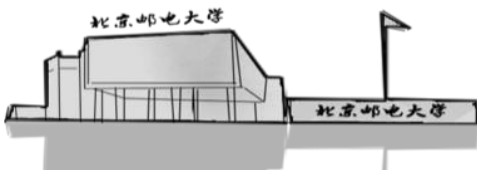
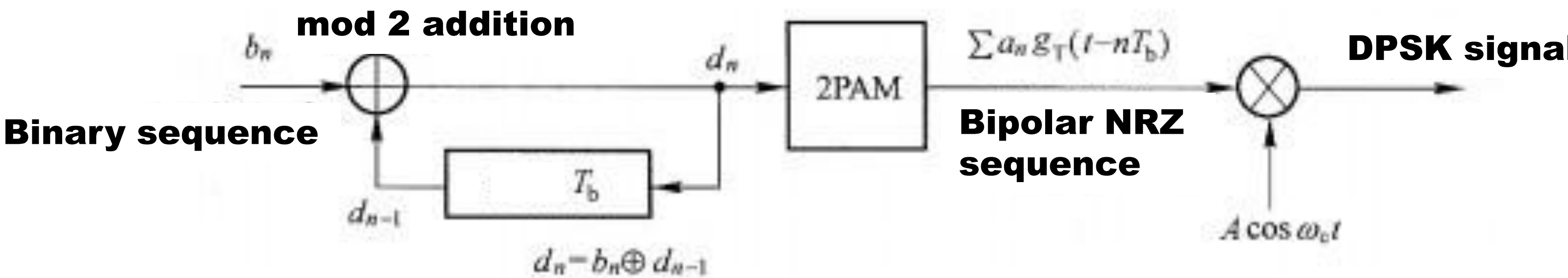
- Binary phase shift keying (2PSK)
- Carrier synchronization
- **Differential Phase Shift Keying(DPSK)**



# DPSK Modulation

- **An effective solution to the problem of phase ambiguity.**

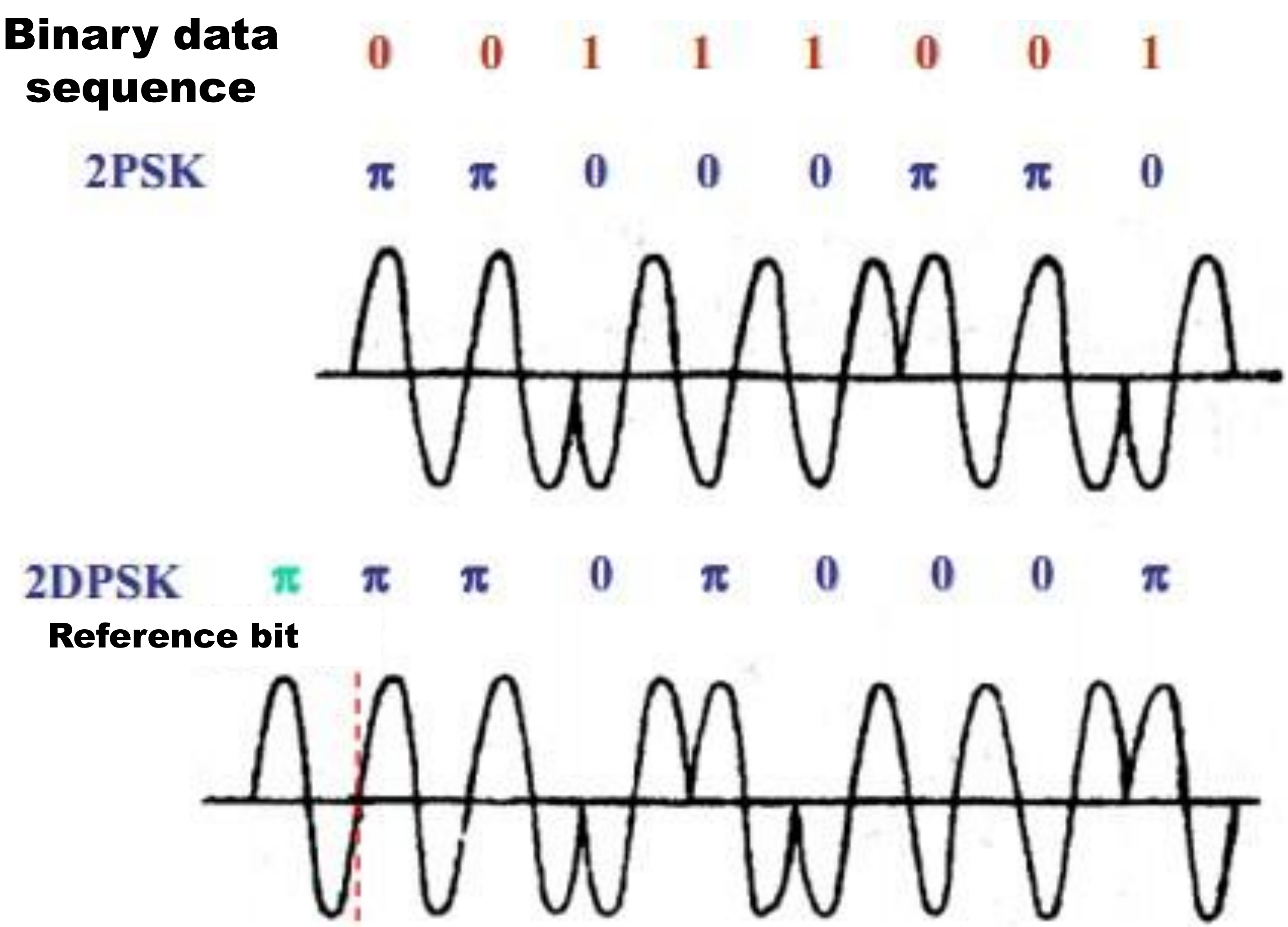
$$\Delta\theta = \theta_n - \theta_{n-1} = \begin{cases} \pi, & "1" \\ 0, & "0" \end{cases}$$




| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\{b_n\}$ | | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| $\{d_n\}$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| $\{a_n\}$ | -1 | -1 | -1 | +1 | -1 | +1 | +1 | +1 | -1 | -1 | +1 |
| $\{\theta_n\}$ | $\pi$ | $\pi$ | $\pi$ | 0 | $\pi$ | 0 | 0 | 0 | $\pi$ | $\pi$ | 0 |
| $\{\theta_n-\theta_{n-1}\}$ | | 0 | 0 | $\pi$ | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | 0 | $\pi$ |

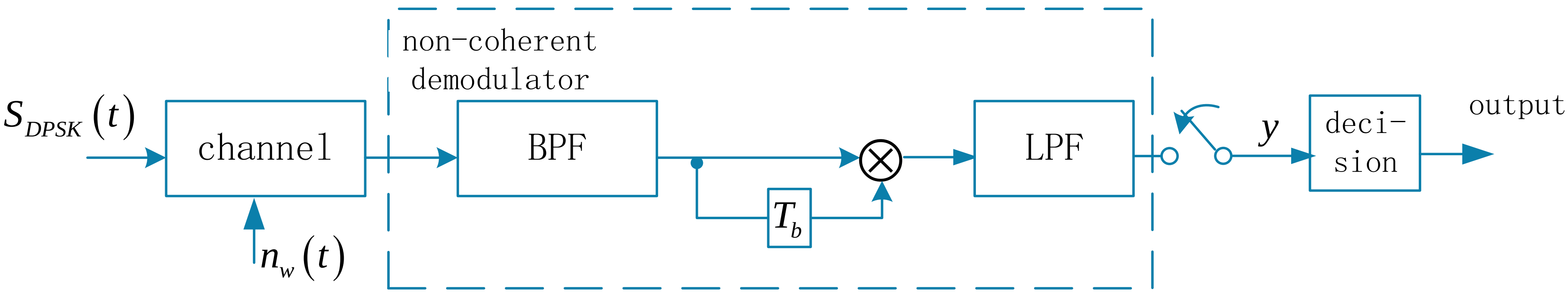
# DPSK Signal

- **DPSK signal waveform**



# Demodulation of DPSK Signal

- **Differential coherent demodulation (also non-coherent demodulation)**




$$s(t)s(t-T_b)=\cos(2\pi f_c t+\theta_n)\cdot\cos\left[2\pi f_c(t-T_b)+\theta_{n-1}\right]$$

$$=\left[\cos(2\pi f_c T_b+\theta_n-\theta_{n-1})+\cos(4\pi f_c t-2\pi f_c T_b+\theta_n+\theta_{n-1})\right]/2$$

$$f_c T_b=n,\quad \text{LPF} \xrightarrow{\text{LPF}} \frac{\cos(\theta_n-\theta_{n-1})}{2}$$

**Decision rule:** $\cos(\theta_n-\theta_{n-1}) \overset{0}{\underset{1}{\gtrless}} 0$

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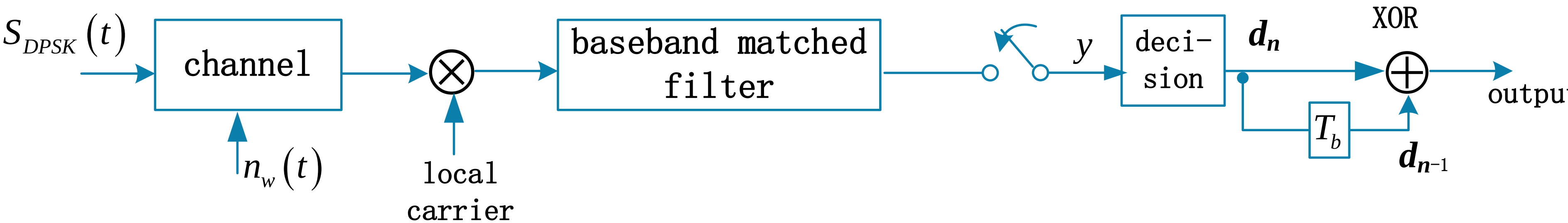
# Demodulation of DPSK Signal

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\{b_n\}$ | | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| $\{d_n\}$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| $\{a_n\}$ | -1 | -1 | -1 | +1 | -1 | +1 | +1 | +1 | -1 | -1 | +1 |
| $\{\theta_n\}$ | $\pi$ | $\pi$ | $\pi$ | 0 | $\pi$ | 0 | 0 | 0 | $\pi$ | $\pi$ | 0 |
| $\{\theta_n-\theta_{n-1}\}$ | | 0 | 0 | $\pi$ | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | 0 | $\pi$ |
| $\cos(\theta_n-\theta_{n-1})$ | | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 |
| $\{\hat{b}_n\}$ | | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |

# Demodulation of DPSK Signal

- **Coherent demodulation**




| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\{b_n\}$ | | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| $\{d_n\}$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| $\{\theta_n\}$ | $\pi$ | $\pi$ | $\pi$ | 0 | $\pi$ | 0 | 0 | 0 | $\pi$ | $\pi$ | 0 |
| $\{\theta'_n\}$ | 0 | 0 | 0 | $\pi$ | 0 | $\pi$ | $\pi$ | $\pi$ | 0 | 0 | $\pi$ |
| $\{d'_n\}$ | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\{\hat{b}_n\}$ | | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |

- **Even phase ambiguity exists, but no detection error is introduced.**

# Demodulation of DPSK Signal

- **Average BER analysis**
  - **with differential coherent demodulation (with an envelope matched filter as the LPF)**

$$P_b = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

  - **with coherent demodulation**

$$P_{cd} = P_c^2 + P_d^2 = \left(1 - P_b\right)^2 + P_b^2 = 1 - 2P_b + 2P_b^2$$

$$P_{ed} = 1 - P_{cd} = 2P_b - 2P_b^2 \approx 2P_b$$ **When $P_b$ is small enough**

  - **where $P_b$ is the average BER of BPSK, and $P_c$ is the average probability of correct detection.**

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# Demodulation of DPSK Signal