3D Graphics Programming Tools

Geometric Primitives

Agenda

- Geometric primitives
  - Points, vectors
- Operators on these primitives
  - Dot product, cross product, norm

3D point

What is a point?

A point specifies a location in space

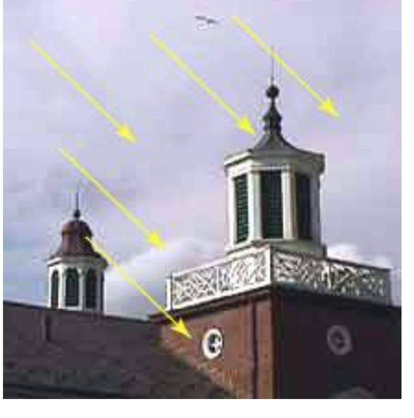
- Represented by three coordinates
- Infinitely small

(x,y,z)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

3D vector

Look at the picture of the building. The scene is being illuminated by the sun.
In the scene, sunlight is coming in diagonally from off scene to the left.

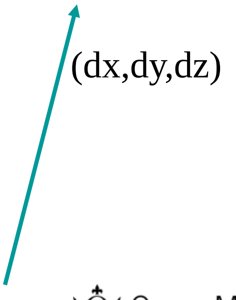


The sun rays could be represented by 3D vectors.

3D vector

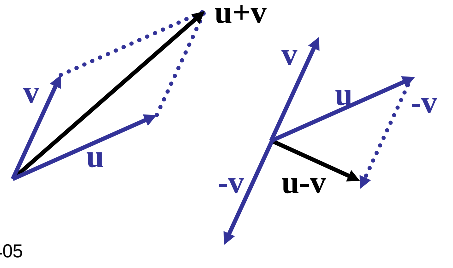
- A 3D vector specifies a direction and a magnitude
  - Represented by three coordinates
  - Magnitude |v| = sqrt (dx dx + dy dy + dz dz)
  - Has no location

If u = (1, -3, 2)
What is the magnitude of u ?



Vector addition / subtraction

- Vector addition/subtraction
  - operation **u + v**, with:
  - Identity **0** : **v** + **0** = **v**
  - Inverse **-** : **v** + (-**v**) = **0**
- Addition
  - uses the parallelogram rule



Vector space

- Vectors define a vector space
  - They support <u>vector-vector addition</u>
    - Commutative and associative
    - Possess identity and inverse
  - They support <u>scalar-vector multiplication</u>
    - Associative, distributive
    - Possess identity

The vector space contains only two types of objects: scalars (real numbers) and vectors.

If u = (2, 5, 6) and v = (-2, 7, 1)
What is u + v ?

What is 6u ?

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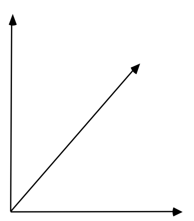
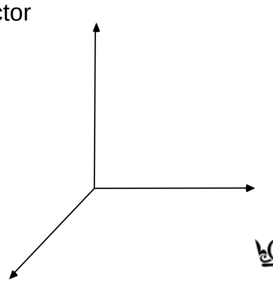
Affine spaces

- Vector spaces
  - lack position and distance
  - have magnitude and direction <u>but no location</u>

The affine space adds a third element: the point.

- A point and three vectors define a 3-D coordinate system
  - Combine the point and vector primitives
  - Permits describing vectors relative to a common location
  - Point-point subtraction yields a vector





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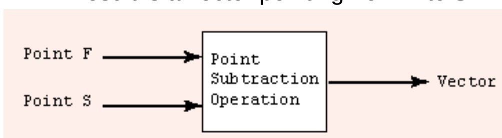
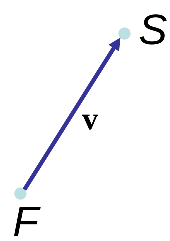
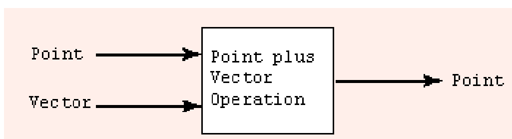
Points + Vectors

- Points support these operations
  - Point-point subtraction: ***S* - *F* = v**
    - Result is a vector pointing from *F* to *S*





  - Vector-point addition: ***F* + v = *S***
    - Result is a new point



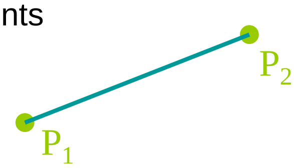
  - Note that the addition of two points is not defined

3D line segment

- Linear path between two points

- Use a linear combination of two points
  - Parametric representation:

    $\mathbf{P = P_1 + t\,(P_2 - P_1)}$ $(0 \le t \le 1)$



Show that $\mathbf{P_1 + t\,(P_2 - P_1)}$, $(0 \le t \le 1)$ yields a new point **P**.

3D ray

- Line segment with one endpoint at infinity
  - Parametric representation:
    $P = P_1 + t\,V$ $(0 <= t < \infty)$

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Today's agenda

- Geometric primitives
  - Points, vectors
- Operators on these primitives
  - Dot product, cross product, norm

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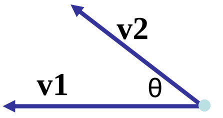
Euclidean spaces

- Distance function between points and vectors in affine space
  - Dot product
    - Euclidean affine space = affine space plus dot product
    - Permits the computation of distance and angles
  - The dot product (or *inner product)* of two vectors is a scalar

$\boxed{v_1 \cdot v_2 = x_1x_2 + y_1y_2 + z_1z_2}$ (in 3D)



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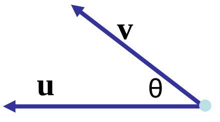
Dot product

- Useful for many purposes
  - Computing the length (Euclidean Norm) of a vector: **|v| = sqrt (v • v)**
  - Normalizing a vector, making it unit-length: $\hat{v} = v / |v|$
  - Computing the angle between two vectors: **u • v = |u| |v| cos(θ)**
  - Checking two vectors for orthogonality: **u • v = 0**



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Calculate the following dot products:

(2, 3, 1) . (0, 4, -1)

(2, 2, 2, 2) . (4, 1, 2, 1.1)

Normalise the following vector:

U = (1, -2, 0.5)

Find the angle between u = (3, 4) and v = (5, 2).

Dot product

- Is commutative

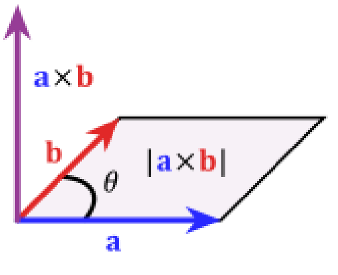
  u • v = v • u

- Is distributive with respect to addition

  u • (v + w) = u • v + u • w

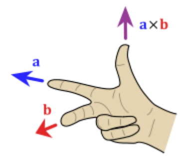
Cross product

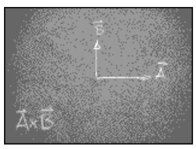
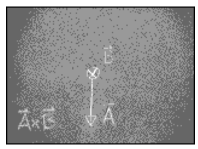
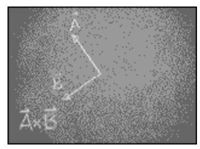
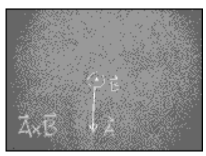
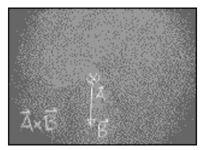
- The *cross product* or *vector product* of two vectors is a vector orthogonal to both
- The magnitude of the cross product can be interpreted as the unsigned area of the parallelogram having the two vectors as sides



Cross product right hand rule

- Right-hand rule dictates direction of cross product
  - Orient it such that your palm is at the beginning of A and your fingers point in the direction of A
  - Twist your hand about the A-axis such that B extends perpendicularly from your palm
  - As you curl your fingers to make a fist, your thumb will point in the direction of the cross product



Cross product

If $a = (a_x, a_y, a_z)$ and $b = (b_x, b_y, b_z)$

$a \times b = (a_y b_z - a_z b_y,\ a_z b_x - a_x b_z,\ a_x b_y - a_y b_x)$

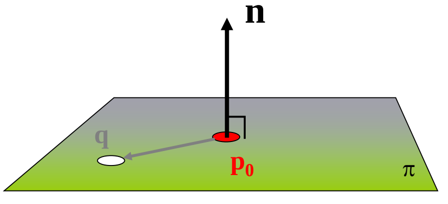
$|a \times b| = |a|\ |b| \sin\theta$ (θ < 180 degrees)

For a = (3, 0, 2) and b = (4, 1, 8), what is a x b?

How about b x a?

3D plane

- A linear combination of three points
  - Implicit representation:
    ax + by + cz + d = 0
  - n is the plane "normal"
    - Unit-length vector
    - Perpendicular to plane



A point **q** belongs to the plane if
$(\mathbf{q} - \mathbf{p}_0) \cdot \mathbf{n} = 0$

Find a normal vector to the plane that passes through the points (1, 0, 2), (2, 3, 0), and (1, 2, 4).