

7.19. the wavelength of a uniform linear polarization plane wave in the air is 60m, $\vec{E} = \vec{e}_x \cos(\omega t) \text{ V/m}$ at the place where it is 1 meters below sea level as the wave enters into the sea along the z axis and propagates down vertically. Find the instantaneous \vec{E} and \vec{H} , the phase velocity and the wavelength at any point below the sea level. For the sea water, $\sigma = 4 \text{ S/m}$ and $\epsilon_r = 80, \mu_r = 1$.

Solution:

$$\frac{\sigma}{\omega \epsilon} = 180 \gg 1$$

So the sea is a good conductor.

$$\alpha = \beta \approx \sqrt{\frac{1}{2} \omega \mu \sigma} = \sqrt{\frac{1}{2} \omega \mu_r \mu_0} = 8.9 \text{ rad/m};$$

$$\vec{V}_p = \frac{\omega}{\beta} \vec{e}_z \approx 3.35 \times 10^6 \vec{e}_z \text{ m/s};$$

$$\lambda = \frac{2\pi}{\beta} \approx 0.71 \text{ m};$$

$$\text{For } : \vec{E} = \vec{e}_x E_0 e^{-\alpha z} \cos(\omega t - \beta z + \varphi_0),$$

$$\text{when } z=1 : \vec{E} = \vec{e}_x E_0 e^{-\alpha z} \cos(\omega t - \beta + \varphi_0) = \vec{e}_x \cos \omega t;$$

$$E_0 = e^\alpha \text{ V/m};$$

$$\text{so } : \varphi_0 = \beta \text{ rad};$$

$$\eta_c = \sqrt{\frac{\omega \mu}{\sigma}} e^{j\frac{\pi}{4}} = \pi e^{j\frac{\pi}{4}} (\Omega);$$

$$\vec{H} = \frac{1}{\eta_c} \vec{e}_k \times \vec{E} = \vec{e}_z 2.32 \times 10^3 e^{j(\pi \times 10^7 t - 8.9z + 8.1)};$$

$$\vec{E} = \vec{e}_x 7.3 \times 10^3 e^{j(\pi \times 10^7 t - 8.9z + 8.9)};$$

7.20 The conductivity of sea water is $\sigma=4\text{S/m}$ and $\epsilon_r=8$, find the attenuation constant, wavelength and wave impedance of the electromagnetic wave in the sea with frequencies of 10kHz, 1MHz, 10MHz and 1GHz.

Solution:

$$(1) \quad w = 2\pi f \frac{\sigma}{\omega \epsilon} = \frac{4}{2\pi f \epsilon} \gg 1$$

$$\alpha = \sqrt{\pi f \mu \sigma} = 0.126 \pi$$

$$\lambda = \frac{2\pi}{\beta} = 15.87m$$

$$\eta = \sqrt{\frac{\mu}{\epsilon_r}} = 0.032\pi(1+j)\Omega$$

$$(2) \frac{\sigma}{w\epsilon} = \frac{4}{2\pi f \epsilon} \gg 1$$

$$\alpha = \sqrt{\pi f \mu \sigma} = 0.126 \pi$$

$$\lambda = \frac{2\pi}{\beta} = 1.587m$$

$$\eta = \sqrt{\frac{\mu}{\epsilon_r}} = 0.316\pi(1+j)\Omega$$

$$(3) \frac{\sigma}{w\epsilon} = \frac{4}{2\pi f \epsilon} \gg 1$$

$$\alpha = \sqrt{\pi f \mu \sigma} = 4\pi$$

$$\lambda = \frac{2\pi}{\beta} = 0.5m$$

$$\beta = \sqrt{\frac{\mu}{\epsilon_r}} = \pi(1+j)\Omega$$

$$(4) \frac{\sigma}{w\epsilon} \approx 1$$

$$\begin{aligned} \alpha &= w \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right]} \\ &= 2\pi f \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right]} = 24.65\pi \end{aligned}$$

$$\beta = w \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right]} = 12.8\pi \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = 0.03m$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \left(1 - j \frac{\sigma}{w\epsilon} \right)^{-0.5} = \frac{42}{\sqrt{1 - 0.39j}} \Omega$$