7.17 In a source-free, dielectric medium the electric field intensity is given as $\vec{\mathbf{E}} = C\cos\alpha x\cos(\omega t - \beta z)\vec{\mathbf{a}}_y$ V/m, where C is the amplitude and α and β are constant quantities. Determine (a) the magnetic field intensity and (b) the electric flux density.

<u>Exercise 7.17</u> $\vec{E} = C\cos(\alpha x)\cos(\omega t - \beta z)\vec{a}_y$, since $\vec{D} = \epsilon\vec{E}$

$\vec{D} = \epsilon C\cos(\alpha x)\cos(\omega t - \beta z)\vec{a}_y$ and $\nabla\cdot\vec{D} = 0$

From $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, we have

$$-\frac{\partial\vec{B}}{\partial t} = \beta C\cos(\alpha x)\sin(\omega t - \beta z)\vec{a}_x - \alpha C\sin(\alpha x)\cos(\omega t - \beta z)\vec{a}_z$$

or $\vec{B} = -\frac{\beta C}{\omega}\cos(\alpha x)\cos(\omega t - \beta z)\vec{a}_x + \frac{\alpha C}{\omega}\sin(\alpha x)\sin(\omega t - \beta z)\vec{a}_z$ T

Thus, $\vec{H} = -\frac{\beta C}{\omega\mu}\cos(\alpha x)\sin(\omega t - \beta z)\vec{a}_x + \frac{\alpha C}{\omega\mu}\sin(\alpha x)\sin(\omega t - \beta z)\vec{a}_z$ A/m

And $\nabla\cdot\vec{B} = 0$

7.29 If the electric field intensity in a source-free, dielectric medium is given as $\vec{\mathbf{E}} = E_0[\sin(\alpha x - \omega t) + \sin(\alpha x + \omega t)]\vec{\mathbf{a}}_y$ V/m, determine the magnetic field intensity using Maxwell's equation from Faraday's law. What is the displacement current density in the medium?

<u>Problem 7.29</u> $\vec{E} = [E_0\sin(\alpha x - \omega t) + E_0\sin(\alpha x + \omega t)]\vec{a}_y = 2E_0\sin(\alpha x)\cos\omega t\,\vec{a}_y$

$\vec{D} = 2\epsilon E_0\sin(\alpha x)\cos\omega t\,\vec{a}_y \Rightarrow \frac{\partial\vec{D}}{\partial t} = -2\omega\epsilon E_0\sin(\alpha x)\sin\omega t\,\vec{a}_y$ A/m^2

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow \frac{\partial\vec{B}}{\partial t} = -2\alpha E_0\cos\alpha x\cos\omega t\,\vec{a}_z$

Thus, $\vec{B} = -\frac{2\alpha}{\omega}E_0\cos\alpha x\sin\omega t\,\vec{a}_z$ and $\vec{H} = -\frac{2\alpha}{\omega\mu}E_0\cos\alpha x\sin\omega t\,\vec{a}_z$ A/m

7.30 If the magnetic field intensity in a source-free, dielectric medium is given as $\vec{\mathbf{H}} = H_0[\cos(\alpha x - \omega t) + \cos(\alpha x + \omega t)]\vec{\mathbf{a}}_z$ A/m, determine the electric field intensity using Maxwell's equation from Ampère's law. What is the displacement current density in the medium?

Problem 7.30 $\vec{H} = H_0[\cos(\alpha x - \omega t) + \cos(\alpha x + \omega t)]\vec{a}_z$

$$= 2H_0 \cos(\alpha x)\cos\omega t\ \vec{a}_z \quad A/m$$

$$\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H} = 2\alpha H_0 \sin(\alpha x)\cos\omega t\ \vec{a}_y \Rightarrow$$

$$\vec{D} = \frac{2\alpha}{\omega} H_0 \sin(\alpha x)\sin\omega t\ \vec{a}_y \quad C/m^2$$

and $$\vec{E} = \frac{2\alpha}{\omega\epsilon} H_0 \sin(\alpha x)\sin\omega t\ \vec{a}_y \quad V/m$$

7.33 If $\vec{\mathbf{E}} = E_0\cos(\omega t - \beta z)\vec{\mathbf{a}}_x$ V/m in a dielectric medium, show that the electric energy density is equal to the magnetic energy density. Also compute (a) the Poynting vector, (b) the average power density, and (c) the time-average values of the energy densities.

Problem 7.33 $\vec{E} = E_0\cos(\omega t - \beta z)\vec{a}_x$, $\vec{D} = \epsilon\vec{E} \Rightarrow \nabla\cdot\vec{D} = 0$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla\times\vec{E} = -\beta E_0 \sin(\omega t - \beta z)\vec{a}_y \Rightarrow \vec{B} = \frac{\beta E_0}{\omega}\cos(\omega t - \beta z)\vec{a}_y$$

$$\vec{H} = \frac{\beta E_0}{\omega\mu}\cos(\omega t - \beta z)\vec{a}_y \qquad \nabla\cdot\vec{B} = 0$$

From $\nabla\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$ we obtain $\beta^2 = \omega^2\mu\epsilon$

$$w_e = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\epsilon E_0^2\cos^2(\omega t - \beta z) \Rightarrow \langle w_e\rangle = \frac{1}{T}\int_0^T w_e\,dt = \frac{1}{4}\epsilon E_0^2 \quad J/m^3$$

$$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\mu H^2 = \frac{\beta^2}{2\omega^2\mu}E_0^2\cos^2(\omega t - \beta z) \Rightarrow \langle w_m\rangle = \frac{1}{T}\int_0^T w_m\,dt = \frac{1}{4}\frac{\beta^2}{\omega^2\mu}E_0^2 = \frac{1}{4}\epsilon E_0^2$$

$$\langle\vec{S}\rangle = \frac{1}{T}\int_0^T(\vec{E}\times\vec{H})\,dt = \frac{\beta}{\omega\mu}E_0^2\frac{1}{T}\int_0^T\cos^2(\omega t - \beta z)\,dt\ \vec{a}_z = \frac{1}{2}\frac{\beta}{\omega\mu}E_0^2\vec{a}_z \quad W/m^2$$

7.39 The electric field intensity in a source-free dielectric medium is given as $\vec{\mathbf{E}} = E\cos(\omega t - ax - kz)\vec{\mathbf{a}}_y$ V/m. Find the corresponding $\vec{\mathbf{H}}$ field. What is the necessary condition for these fields to exist? Determine the time-average values of electric energy density, magnetic energy density, and the Poynting vector.

Problem 7.39 $\vec{E} = E_o \cos(\omega t - ax - kz)\,\vec{a}_y$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow \quad \frac{\partial B_x}{\partial t} = \frac{\partial}{\partial z}E_y = kE_o \sin(\omega t - ax - kz) \Rightarrow$

$B_x = -\frac{kE_o}{\omega}\cos(\omega t - ax - kz)$

and $\quad \frac{\partial}{\partial t}B_z = -\frac{\partial}{\partial x}E_y = -aE_o \sin(\omega t - ax - kz) \Rightarrow B_z = \frac{aE_o}{\omega}\cos(\omega t - ax - kz)$

$\nabla\cdot\vec{B} = 0 \qquad \nabla\cdot\vec{D} = 0 \qquad$ Source-free: $\Rightarrow \vec{J} = 0$

$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \Rightarrow \quad \nabla\times\vec{B} = \mu\epsilon\frac{\partial\vec{E}}{\partial t} \Rightarrow$

$-(a^2+k^2)\frac{E_o}{\omega}\sin(\omega t - ax - kz)\,\vec{a}_y = -\mu\epsilon\omega E_o \sin(\omega t - ax - kz)\,\vec{a}_y$

or $\qquad \omega^2\mu\epsilon = a^2 + k^2 \qquad$ [Condition for the fields to exist]

$w_e = \frac{1}{2}\epsilon E^2 = \frac{1}{2}\epsilon E_o^2\cos^2(\omega t - ax - kz) \Rightarrow \langle w_e\rangle = \frac{1}{4}\epsilon E_o^2$

$w_m = \frac{1}{2}\frac{B^2}{\mu} = \frac{E_o^2}{2\mu}\left(\frac{a^2+k^2}{\omega^2}\right)\cos^2(\omega t - ax - kz) \Rightarrow \langle w_m\rangle = \frac{E_o^2(a^2+k^2)}{4\mu\omega^2} = \frac{1}{4}\epsilon E_o^2$

$\vec{S} = \vec{E}\times\vec{H} = \frac{E_o^2}{\omega\mu}\cos^2(\omega t - ax - kz)\left[a\vec{a}_x + k\vec{a}_z\right]$

$$\langle\vec{S}\rangle = \frac{E_o^2}{2\omega\mu}(a\,\vec{a}_x + k\,\vec{a}_z)$$

7.24 The magnetic flux density is typically 0.04 mT near the surface of the earth. What is the magnetic energy density? If the radius of the earth is approximated as 6400 km, and the magnetic flux density is assumed constant up to an altitude equal to the earth's radius, what is the total magnetic energy stored in the region above the earth's surface?

Problem 7.24 $\qquad B = 0.04\text{ mT} \Rightarrow w_m = \frac{1}{2}\frac{(0.04\times 10^{-3})^2}{4\pi\times 10^{-7}} = 636.62\ \mu J/m^3$

$$W = 636.62\times 10^{-6}\times\frac{4\pi}{3}\left[12.8^3 - 6.4^3\right]10^{12} = 4.89\times 10^{12}\ J$$

7.37 The conductivity of seawater is approximately 0.4 mS/m and its dielectric constant is 81. Determine the frequency at which the magnitude of the displacement current density is equal to the magnitude of the conduction current density. Comment on the electric behavior of seawater at very low and very high frequencies.

Problem 7.37 $J_c = \sigma E$ $J_d = \omega \epsilon E$ $\Rightarrow$ $J_d/J_c = \frac{\omega\epsilon}{\sigma}$

For $J_d/J_c = 1$, $\omega\epsilon = \sigma$ $\Rightarrow$ $f = \frac{\sigma}{2\pi\epsilon}$

Sea water: $\epsilon = 81\epsilon_0$ $\sigma = 0.4 \times 10^{-3}$ S/m $\Rightarrow$ $f = 88.889$ kHz

when $f \ll 88.889$ kHz $J_c \gg J_d$ conductor

and when $f \gg 88.889$ kHz, $J_d \gg J_c$ poor conductor