Study of Lincoln temperature

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Outline

Introduction

Data

- Retrieve from `www.ncdc.noaa.gov`.
- Containing daily information of minimum, maximum, and average temperature of some weather stations round Nebraska.
- I use data from station USW00014939 in year 2011.

Data

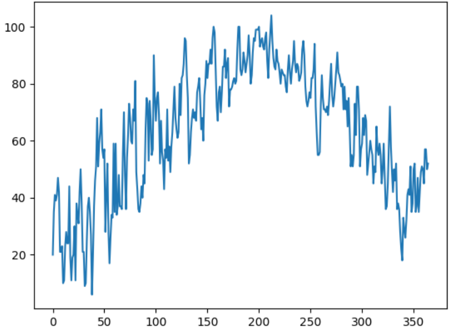

Figure: Daily mininum temperature in 2011

Method

I use two method:

- Best fit polynomimal, degree 1-8
- Piecewise interpolation (linear, quadratic, and cubic)

Best fit polynomial

- From data:

$$(x_i, y_i)$$

- Polynomial function:

$$f(x) = \sum_{i=0}^{k} a_i x^i = a_0 + ... + a_k * x^k$$

- Optimize function:

$$error = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

Best fit polynomial

- Setup matrices

$$A = \begin{bmatrix} 1 & \dots & x_1^k \\ \vdots & \ddots & \vdots \\ 1 & \dots & x_n^k \end{bmatrix}, x = \begin{bmatrix} a_0 \\ \vdots \\ a_k \end{bmatrix}, y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

Best fit polynomial

- Setup matrices

$$A = \begin{bmatrix} 1 & \dots & x_1^k \\ \vdots & \ddots & \vdots \\ 1 & \dots & x_n^k \end{bmatrix}, x = \begin{bmatrix} a_0 \\ \vdots \\ a_k \end{bmatrix}, y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

- We can find x by solving

$$A^T A x = A^T y$$

Linear spline

$$f_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i)$$

Quadratic spline

$$z_1 = 0$$

$$z_{i+1} = -z_i + 2\frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

$$f_i(x) = \frac{z_{i+1} - z_i}{2(x_{i+1} - x_i)}(x - x_i)^2 + z_i(x - x_i) + y_i$$

Cubic spline

INPUT $n; x_0, x_1, \ldots, x_n; a_0 = f(x_0), a_1 = f(x_1), \ldots, a_n = f(x_n)$.

OUTPUT a_j, b_j, c_j, d_j for $j = 0, 1, \ldots, n-1$.
(*Note:* $S(x) = S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$ *for* $x_j \le x \le x_{j+1}$.)

Step 1 For $i = 0, 1, \ldots, n-1$ set $h_i = x_{i+1} - x_i$.

Step 2 For $i = 1, 2, \ldots, n-1$ set

$$\alpha_i = \frac{3}{h_i}(a_{i+1} - a_i) - \frac{3}{h_{i-1}}(a_i - a_{i-1}).$$

Step 3 Set $l_0 = 1$; (*Steps 3, 4, and 5 and part of Step 6 solve a tridiagonal linear system using a method described in Algorithm 6.7.*)
$\mu_0 = 0$;
$z_0 = 0$.

Step 4 For $i = 1, 2, \ldots, n-1$
set $l_i = 2(x_{i+1} - x_{i-1}) - h_{i-1}\mu_{i-1}$;
$\mu_i = h_i / l_i$;
$z_i = (\alpha_i - h_{i-1} z_{i-1}) / l_i$.

Step 5 Set $l_n = 1$;
$z_n = 0$;
$c_n = 0$.

Step 6 For $j = n-1, n-2, \ldots, 0$
set $c_j = z_j - \mu_j c_{j+1}$;
$b_j = (a_{j+1} - a_j)/h_j - h_j(c_{j+1} + 2c_j)/3$;
$d_j = (c_{j+1} - c_j)/(3h_j)$.

Step 7 OUTPUT $(a_j, b_j, c_j, d_j$ for $j = 0, 1, \ldots, n-1)$;
STOP.

Figure: Cubic Spline Solver

Graph - Degree 1 polynomial

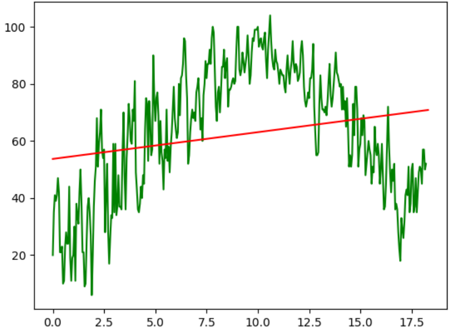

Figure: Degree 1 Polynomial

Graph - Degree 2 polynomial

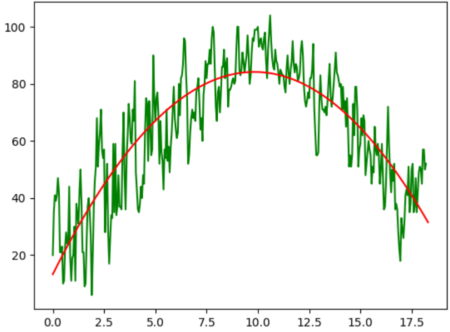

Figure: Degree 2 Polynomial

Graph - Degree 3 polynomial

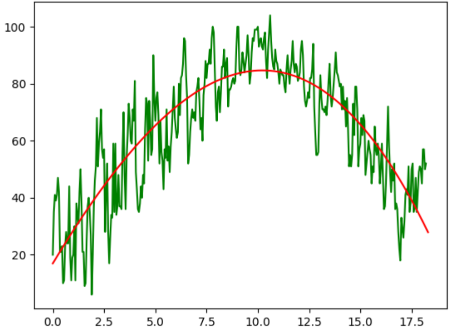

Figure: Degree 3 Polynomial

Graph - Degree 4 polynomial

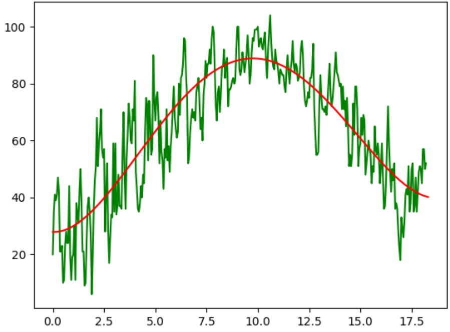

Figure: Degree 4 Polynomial

Graph - Degree 5 polynomial

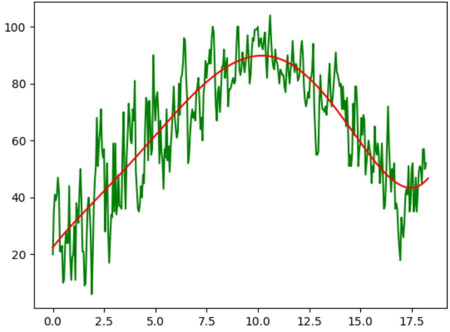

Figure: Degree 5 Polynomial

Graph - Degree 6 polynomial

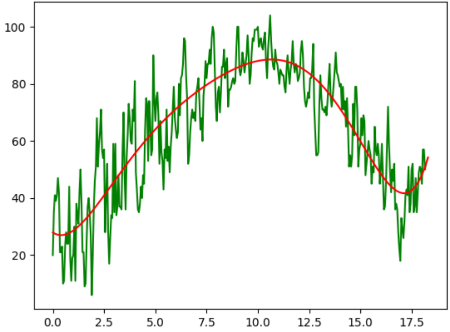

Figure: Degree 6 Polynomial

Graph - Degree 7 polynomial

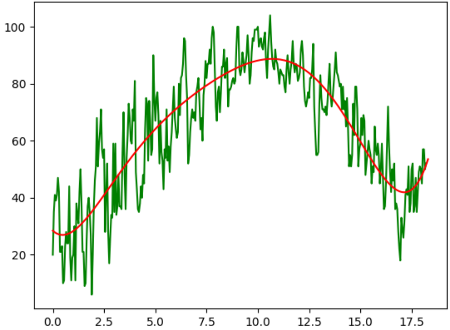

Figure: Degree 7 Polynomial

Graph - Degree 8 polynomial

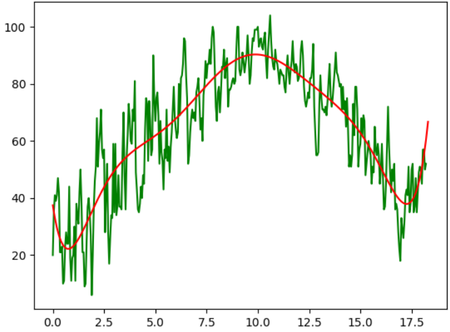

Figure: Degree 8 Polynomial

Graph - Degree 9 polynomial

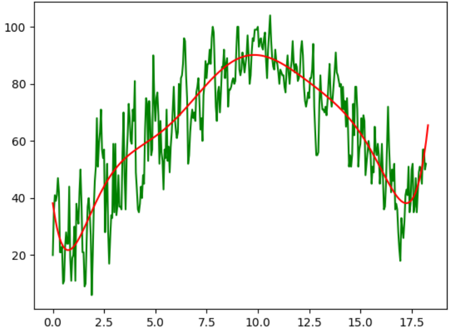

Figure: Degree 9 Polynomial

Graph - Linear Spline

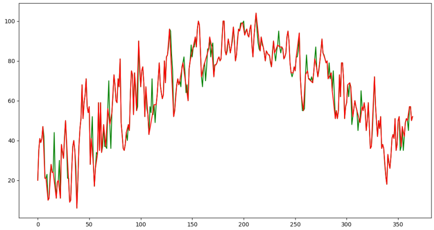

Figure: Linear Spline

Graph - Quadratic Spline

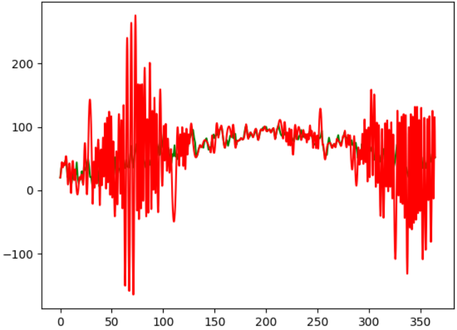

Figure: Quadratic Spline

Graph - Cubic Spline

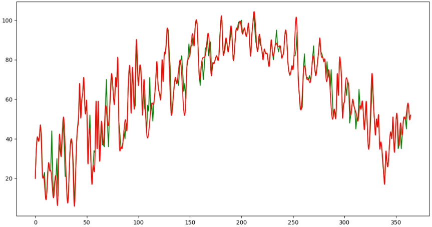

Figure: Cubic Spline

Analysis

Sample size	Spline			Polynomial								
	Linear	Quadratic	Cubic	Linear	2	3	4	5	6	7	8	9
330	2.33	39.48	2.69	21.62	11.29	10.93	10.55	10.51	10.51	9.81	9.81	9.67
300	3.24	52.97	3.77	21.63	11.29	10.94	10.56	10.52	10.52	9.82	9.83	9.69
270	4.02	66.16	4.69	21.64	11.30	10.94	10.57	10.53	10.54	9.84	9.84	9.70
240	4.79	67.00	5.71	21.65	11.31	10.96	10.59	10.55	10.56	9.86	9.87	9.73
73	9.41	130.25	14.64	21.86	11.49	11.17	10.82	10.82	10.87	10.29	10.34	10.20
37	10.88	131.92	18.94	22.22	11.70	11.40	11.12	11.22	11.42	10.97	11.47	11.50

Figure: Average of sum square errors

Reference

Linear and Quadratic spline: Class slide.
Cubic spline: Numerical Analysis, Edition 10th.
Presentation template:
`https://gist.github.com/albarralnunez/5621664`