Introductory mini projects 2020

```
assignement ::= expression .
expression ::= term | term ("+" | "-") term
term ::= factor | factor ("*" | "/") factor
factor ::= ["-"] (identifier | integerConstant)
```

Figure 1: Input grammar for Project 1.

Project 1: Parsing and Abstract Syntax Trees.

The intention of this project is to use the C programming language in order to create a parser for the grammar shown in Figure 1. The grammar describes arithmetic expressions between identifiers and integer constants. Identifiers can be any non-null alphanumeric expressions with without spaces. The arithmetic operations use the conventional operator precedence from mathematics. The parser must be able to store the parsed information as a binary Abstract Syntax Tree (AST).

The input to the parser should be the path of the input filename, provided as the first argument of the C binary. The output of the C program must be the structure of the generated AST printed in the standard output of the program, or an error message in case that the input does not include a valid expression.

Notes: You can use any preferable parsing technique, including the use of parsing software packages such as flex/bison (though not recommended). You can change the grammar of Figure 1 to another equivalent representation. You are free to choose the form of the printing of the AST, including the order of traversal of the tree (Depth-First, Breadth-First, etc.). The deliverable is an archive file with all the necessary source code files and a Makefile for their compilation. Feel free to also include any example input files.

Project 2: State estimation of a pendulum.

In this project, we examine the state estimation of a vertical pendulum. Figure 2 shows a damped pendulum with angle of displacement θ from the resting point, acceleration due to gravity G, length of the rod L, and damping constant B with units of kg s⁻¹, M the mass of the pendulum's bob, and t the elapsed time. Equation 1 describes the dynamics of the oscillation:

$$\frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2} + \frac{B}{M} \frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{G}{L} \sin(\theta) = 0. \tag{1}$$

The goal of state estimation in an embedded system attached to the pendulum bob is to estimate the true value of θ from noisy measurements of the angular speed $\frac{d\theta}{dr}$. We define the state of the example system as the vector of the angle θ and the derivative $\frac{d\theta}{dr}$. The Z axis of the gyroscope on the pendulum's mass measures $\frac{d\theta}{dr}$. The goal of the project

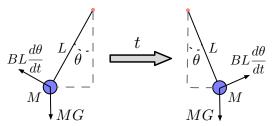


Figure 2: Damped pendulum with angle of displacement θ from the resting point, acceleration due to gravity G, rod length L, damping constant B and drag force $BL\frac{\mathrm{d}\theta}{\mathrm{d}t}$ [2].

is to generate a Kalman filter which tracks the state using measurements from a gyroscope attached to the mass of the pendulum. The Kalman filter [1] is one method to estimate the state of a dynamic system from noisy measurements of signals that are related to the state being estimated. We will use the extended Kalman filter since Equation 1 includes the sin function, making it nonlinear. To design the filter, we must specify the *process* and *measurement* models of the pendulum.

For the *process* model of the pendulum we convert Equation 1 to two state prediction equations: one for the angular displacement and one for the angular speed. We assume a small amount of time Δt between two successive cycles of the filter and use Δt , to linearize the prediction equations. Equations 2 and 3 summarize the *process* model of the pendulum.

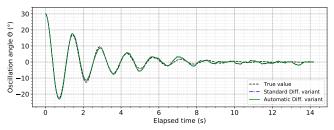
$$\theta_k = \theta_{k-1} + \omega_{k-1} \Delta t. \tag{2}$$

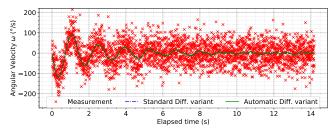
$$\omega_k = \omega_{k-1} + \frac{B}{M}\omega_{k-1}\Delta t - \frac{G}{L}\sin(\theta_{k-1})\Delta t.$$
 (3)

The deliverable of the project is to evaluate state estimation of a pendulum with drag, length equal to $0.5\,\mathrm{m}$ and damping factor equal to $0.8\,\mathrm{kg}\,\mathrm{s}^{-1}$. The initial angle displacement should be 30° and the measurement noise variance equal to $0.8\,\mathrm{rad}^2/\mathrm{s}^2$.

We provide an input csv file (pendulum_input.csv) with trace from the simulation of the aforementioned pendulum. The csv file includes the following columns:

- **Timestamp:** Timestamp of a measurement.
- **displacement_truth:** The actual displacement *theta* of the pendulum with respect to its vertical axis at the specific timestamp.
- **speed_noisy:** The noisy measurement of *dtheta* that the filter samples from the gyroscope attached on the mass of the pendulum.
- **speed_truth:** The actual speed *dtheta* of the mass of the pendulum at the specific timestamp.
- acc_truth: The actual acceleration of the mass of the pendulum at the specific timestamp.





(a) Estimation of oscillation angle over time.

(b) Estimation of angular velocity over time.

Figure 3: State estimation of a pendulum with drag. Initial displacement is 30 $^{\circ}$, measurement noise variance 0.8 rad $^2/s^2$, and length 0.5 m.

You should develop a program in any language of preference which must be able to parse the input trace file and implement an Extended Kalman filter that predicts the state vector, i.e., the displacement (theta) and speed (dtheta) values of the oscillation of the pendulum. The output of the program should be a csv file named pendulum_ekf_result.csv, with columns time, theta, dtheta, theta_cov, dtheta_cov, where theta_cov and dtheta_cov are the covariances of theta and dtheta, respectively. We provide a python script (plot.py), which takes

as input both the csv files and produces the plots of oscillation angle and angular velocity over time as Figure 3 shows.

References

- [1] Rudolph Kalman. E. 1960. a new approach to linear filtering and prediction problems. *Transactions of the ASME–Journal of Basic Engineering*, 82:35–45, 1960.
- [2] Pirooz Mohazzabi and Siva P Shankar. Damping of a simple pendulum due to drag on its string. *Journal of Applied Mathematics and Physics*, 5(01):122, 2017.