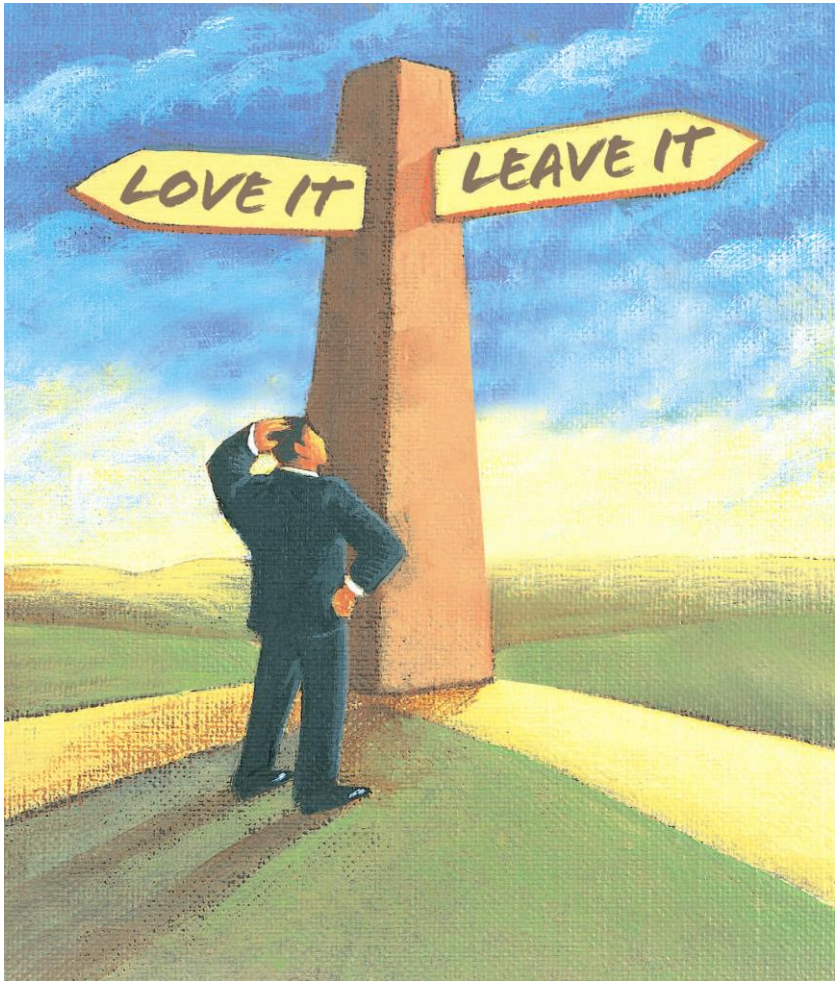
Universität Konstanz

Mean Squared Error

Mainly based on: *Z. Wang, A. Bovik,*
Mean Squared Error: Love it or leave it.
IEEE Signal processing Magazine, 2009



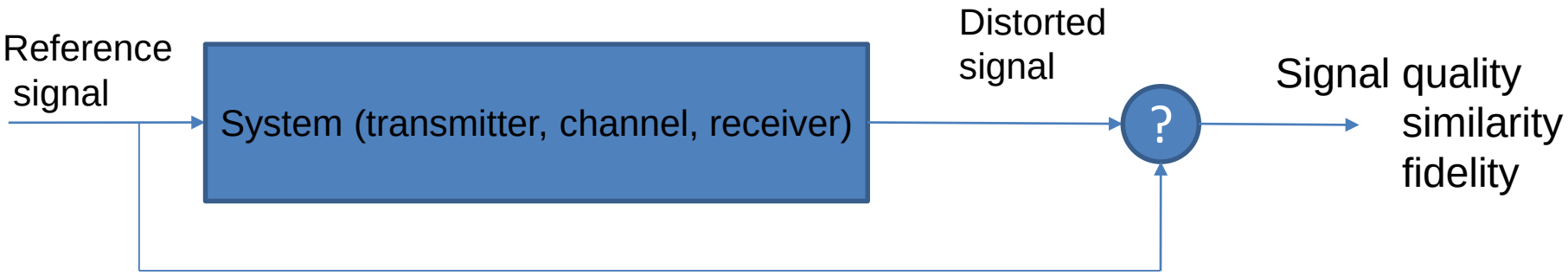
Igor Zingman,
Department of Computer and Information Science,
University of Konstanz

Universität Konstanz

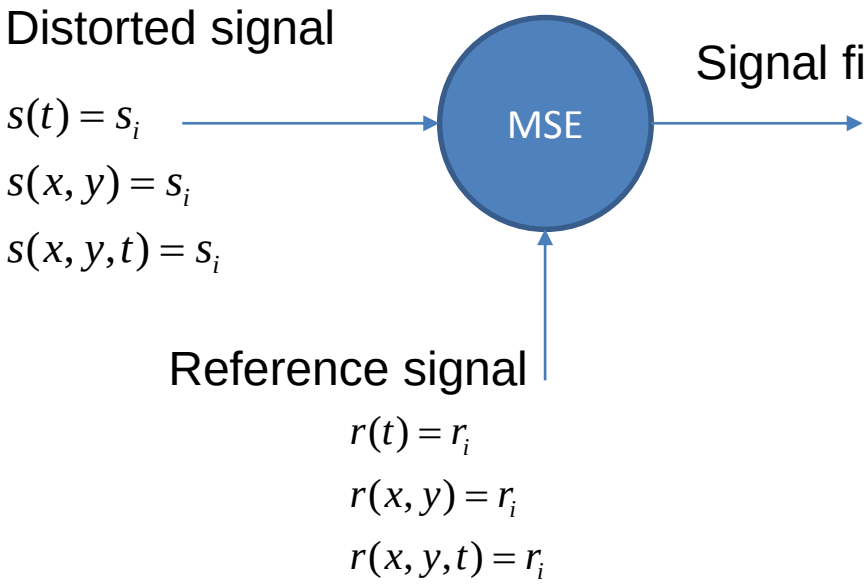
Agenda

- Assessment of signal Fidelity
- MSE, advantages and disadvantages
- SSIM and its advantages
- CW-SSIM and its advantages
- VIF
- Fidelity measures for Pattern Recognition

Assessment of Signal Fidelity



Mean Squared Error (MSE) Criterion



$$MSE(s,r) = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2$$

Peak Signal - to - Noise Ratio:

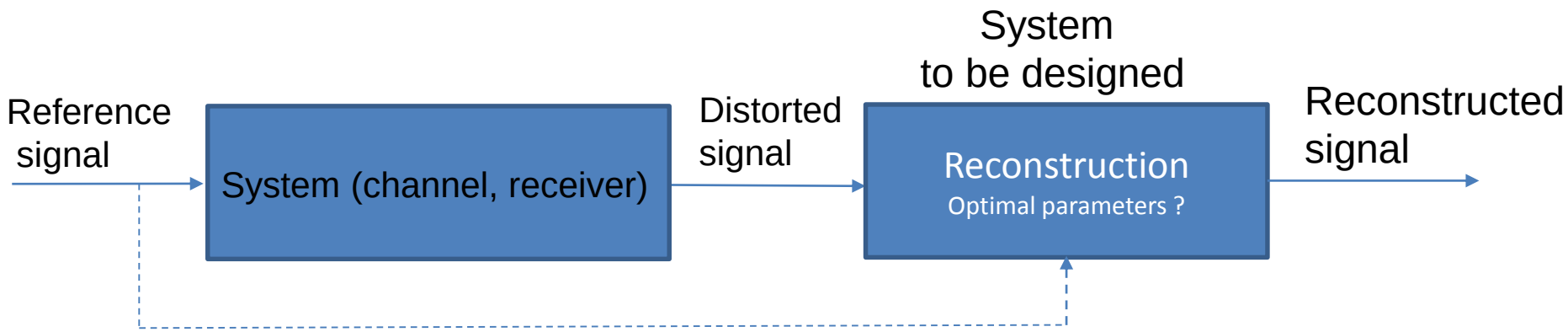
$$PSNR = 10\log_{10}\frac{L^2}{MSE},$$

L – dynamic range

(for example 255 for 8 bit images)

Mean Squared Error (MSE) Criterion

- MSE is not only used measure the quality/fidelity of signals, but also as design criteria to optimize signal processing algorithms



Why do we love the MSE?

- Simple
  - Parameter free
  - Inexpensive to compute
  - Memoryless (evaluated at each sample independently)

$$MSE(s,r) = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2$$

- Valid distance metric in N-dimensional Euclidean space
  - Nonnegativity: $d(x,y) \geq 0$
  - Identity $d(x,y) = 0$ if and only if $x = y$
  - Symmetry: $d(x,y) = d(y,x)$
  - Triangular inequality: $d(x,z) \leq d(x,y) + d(y,z)$

Why do we love the MSE?

$$MSE(s,r) = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2$$

- Clear physical meaning – energy of the error signal
  - Energy is preserved after any orthogonal linear transformation (e.g. Fourier transform)
- Easy to optimize (with respect to system parameters to be designed
  - Convexity
  - Symmetry
  - Differentiability

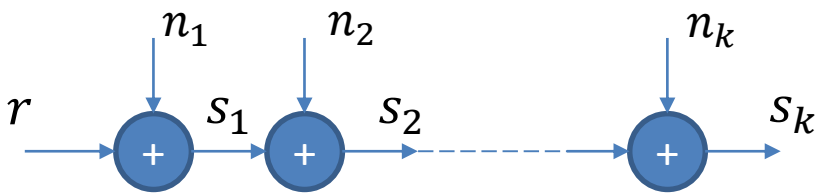
Why do we love the MSE?

In statistical framework the sample average $MSE(r,s) = \frac{1}{N}\sum_{i=1}^{N}(r_i - s_i)^2$ is replaced by statistical expectation:

$$MSE(r,s) = E\{error^2\} = E\left\{(r-s)^2\right\}$$

Why do we love the MSE?

- Additive for independent sources of distortions $s_k = r + \sum_{i=1}^{K} n_i$ for $k = 1,2,\ldots,K$



  - Contribution from each source of distortion may be analyzed independently

$$MSE(r, s_K) = E\left\{(r - s_K)^2\right\} = E\left\{\left(\sum_{k=1}^{K} n_k\right)^2\right\} = \sum_{k=1}^{K} E\left\{n_k^2\right\}$$
$$= MSE(r, s_1) + MSE(s_1, s_2) + \ldots + MSE(s_{K-1}, s_K)$$

Why do we love the MSE?

- MSE is a convention
  - It has been extensively employed for optimizing and assessing a wide variety of signal processing applications (e.g. filter design, signal compression, denoising, reconstruction, classification..)
  - New algorithms can be comparatively evaluated using standard MSE/PSNR

What wrong with the MSE

- Given all the above mentioned attractive properties does the MSE really measure signal fidelity?

- Unfortunately, the converse appears true when MSE is used to predict human perception of image fidelity

Universität Konstanz

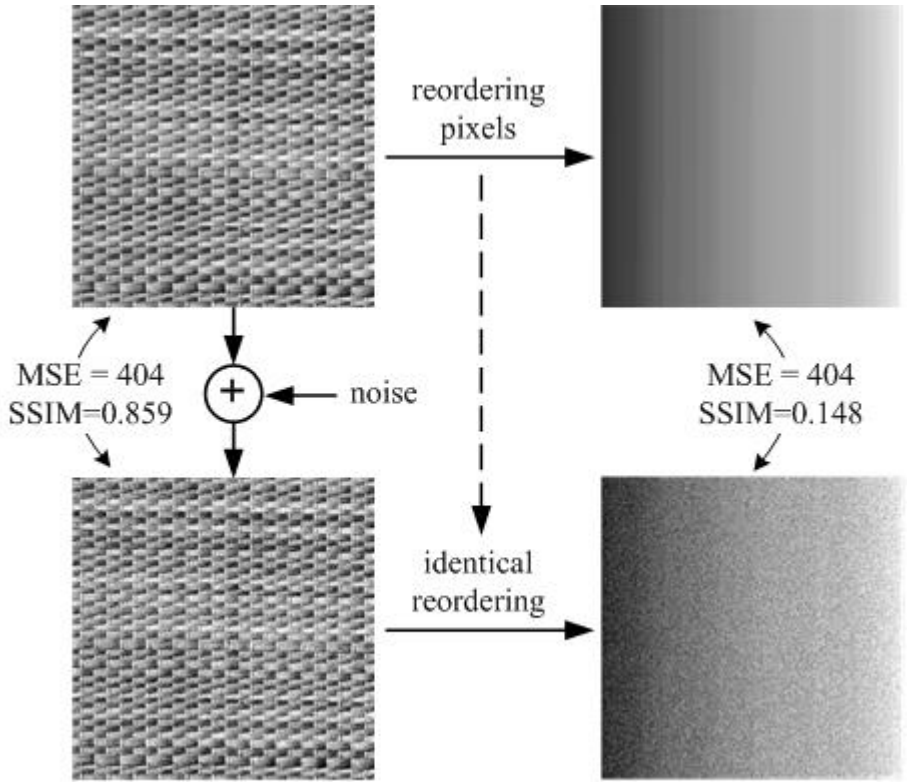
What wrong with the MSE

a. Reference
b. Mean contrast stretch
c. Luminance shift
d. Gaussian noise
e. Impulsive noise
f. JPEG compression
g. Blurring
h. Zooming out
i. Spatial shift to the right
j. Spatial shift to the left
k. Rotation counter-clockw.
l. Rotation clockwise

Implicit assumptions are bad

$$MSE(s,r) = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2$$

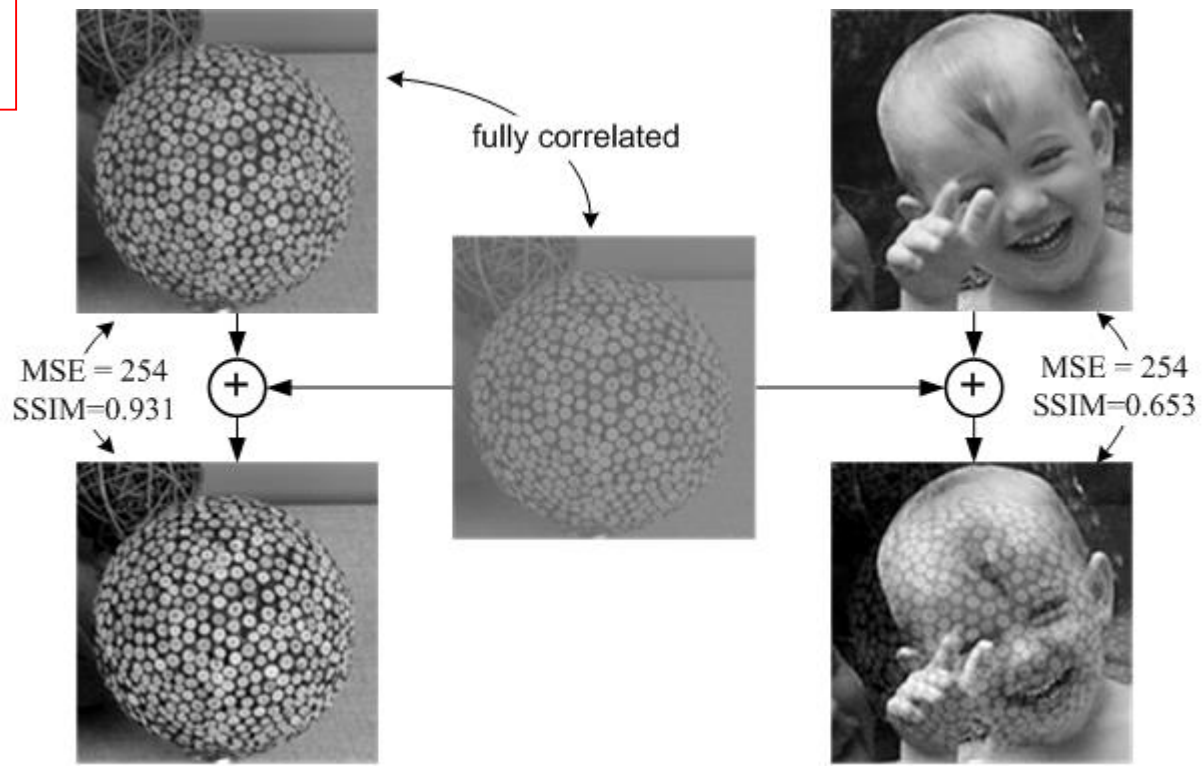
1. Signal fidelity is independent of relations between signal samples



Implicit assumptions are bad

$$MSE(s,r) = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2$$

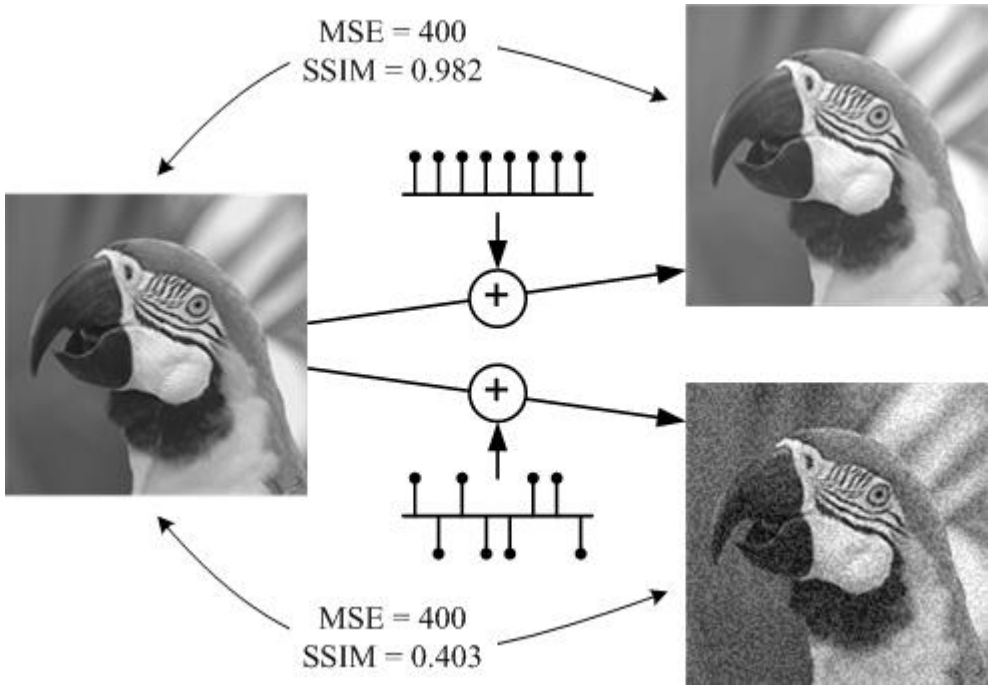
2. Signal fidelity is independent of any relationship between the original signal and the error signal



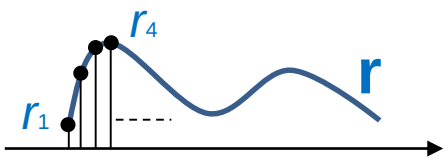
Implicit assumptions are bad

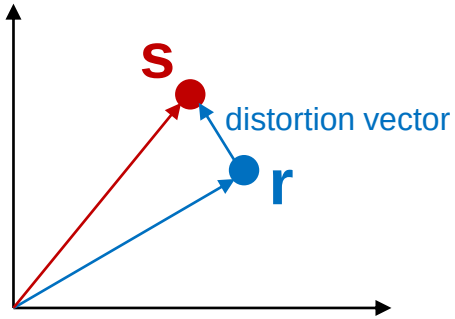
$$MSE(s,r) = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2$$

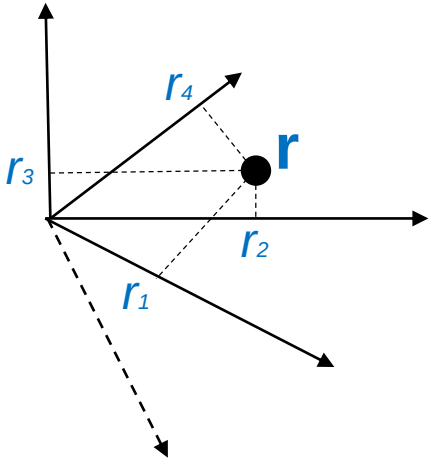
3. Spatial fidelity is independent of the signs of the error signal samples



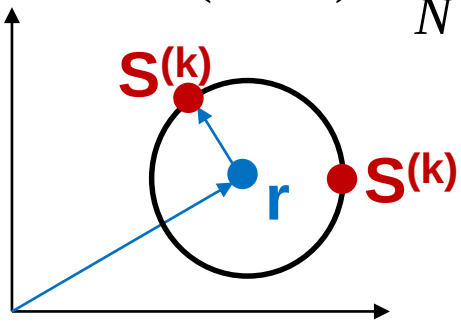
Observing MSE in a signal space







$$MSE(\mathrm{s}^{(k)},\mathrm{r}) = \frac{1}{N}\sum_{i=1}^{N}(s_i^{(k)} - r_i)^2 = \text{constant}$$



Observing MSE in a signal space

(a) MSE=0, SSIM=1
CW-SSIM=1

(b) MSE=306, SSIM=0.928
CW-SSIM=0.938

(c) MSE=309, SSIM=0.987
CW-SSIM=1.000

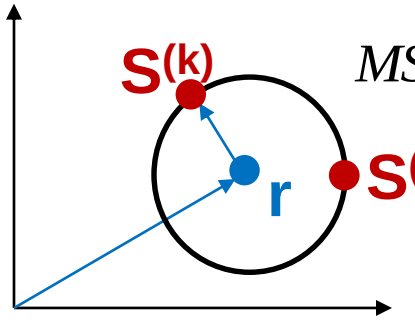
(d) MSE=309, SSIM=0.576
CW-SSIM=0.814

(e) MSE=313, SSIM=0.730
CW-SSIM=0.811

(f) MSE=309, SSIM=0.580
CW-SSIM=0.633

(g) MSE=308, SSIM=0.641
CW-SSIM=0.603

a. Reference
b. Mean contrast stretch
c. Luminance shift
d. Gaussian noise
e. Impulsive noise
f. JPEG compression
g. Blurring



$$MSE(\mathrm{S}^{(k)}, \mathrm{r}) = \text{constant}$$

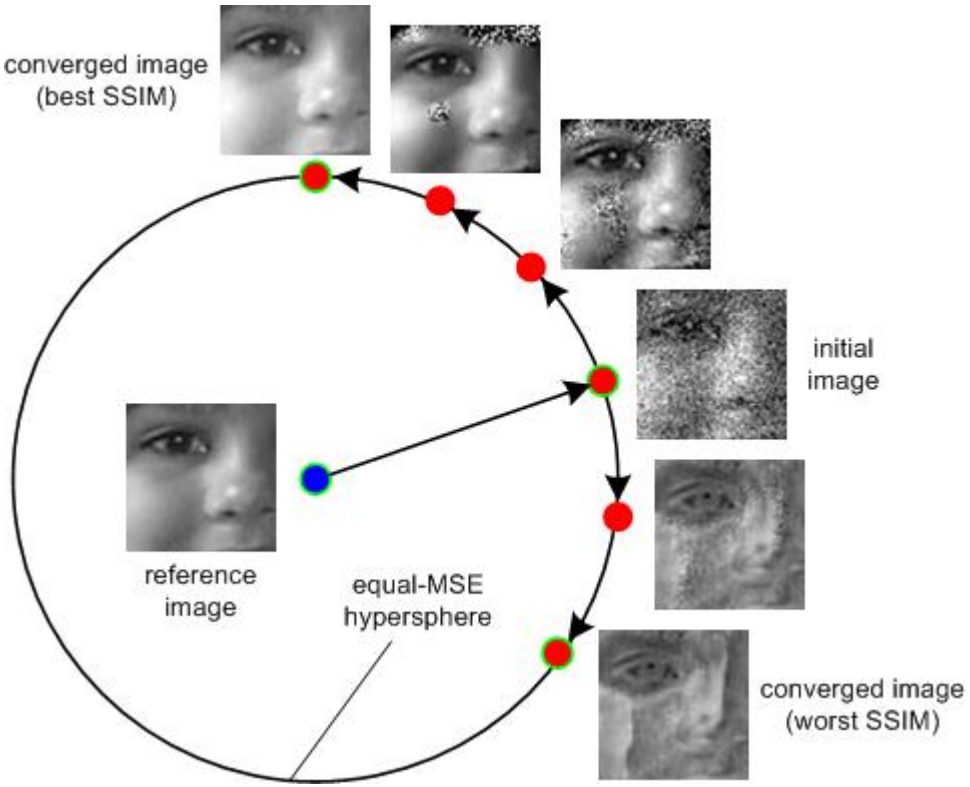
- The length of distortion vector does not suffice, directions are also important

Universität Konstanz

Observing MSE in a signal space

Failing of the MSE:

- Various synthesized images with fixed MSE exhibit astonishingly different image fidelities
- Images are automatically synthesized via optimization of SSIM measures along constant MSE



Structural Similarity Index (SSIM)[2,3]

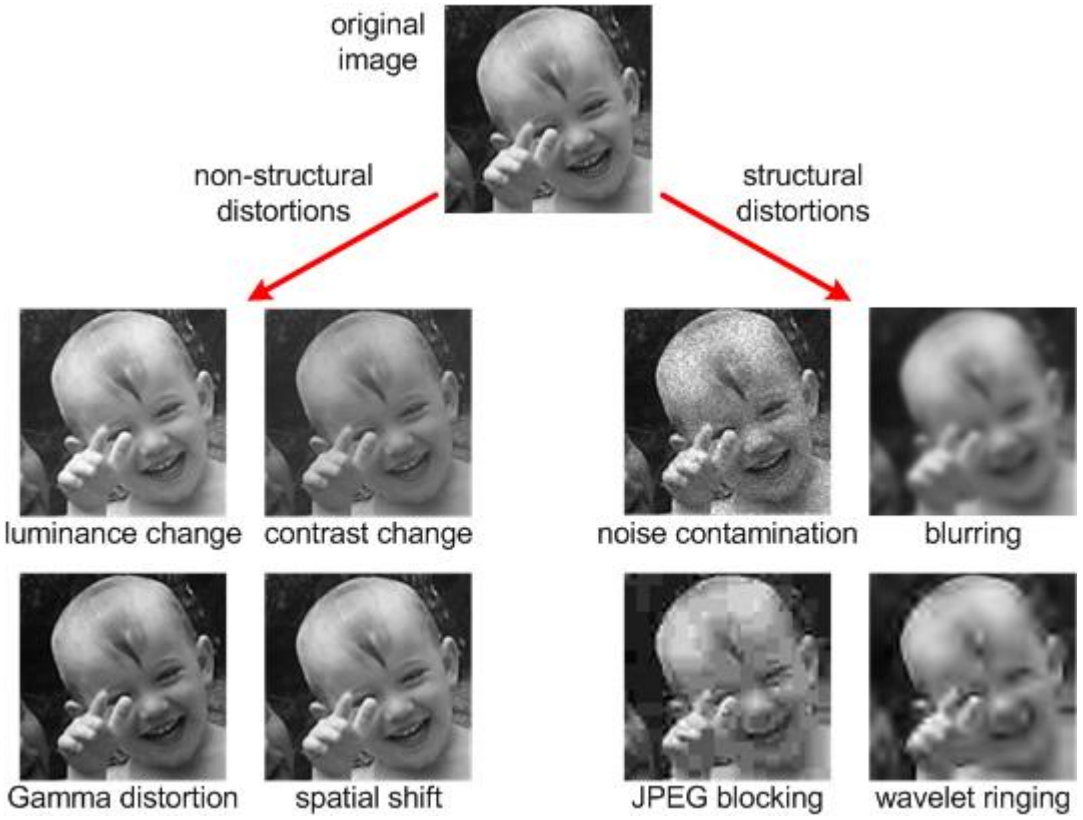
Good Image fidelity should have

- high sensitivity

to structural distortions

- low sensitivity to

non-structural distortions



Structural Similarity Index (SSIM)

Local SSIM computed within a neighborhood:

$$SSIM(r,s) = \text{Luminance}(r,s) \cdot \text{Contrast}(r,s) \cdot \text{Structure}(r,s)$$

$$= \frac{2\mu_r\mu_s + C_1}{\mu_r^2 + \mu_s^2 + C_1} \cdot \frac{2\sigma_r\sigma_s + C_2}{\sigma_r^2 + \sigma_s^2 + C_2} \cdot \frac{\text{cov}(r,s) + C_3}{\sigma_r\sigma_s + C_3}$$

Properties:

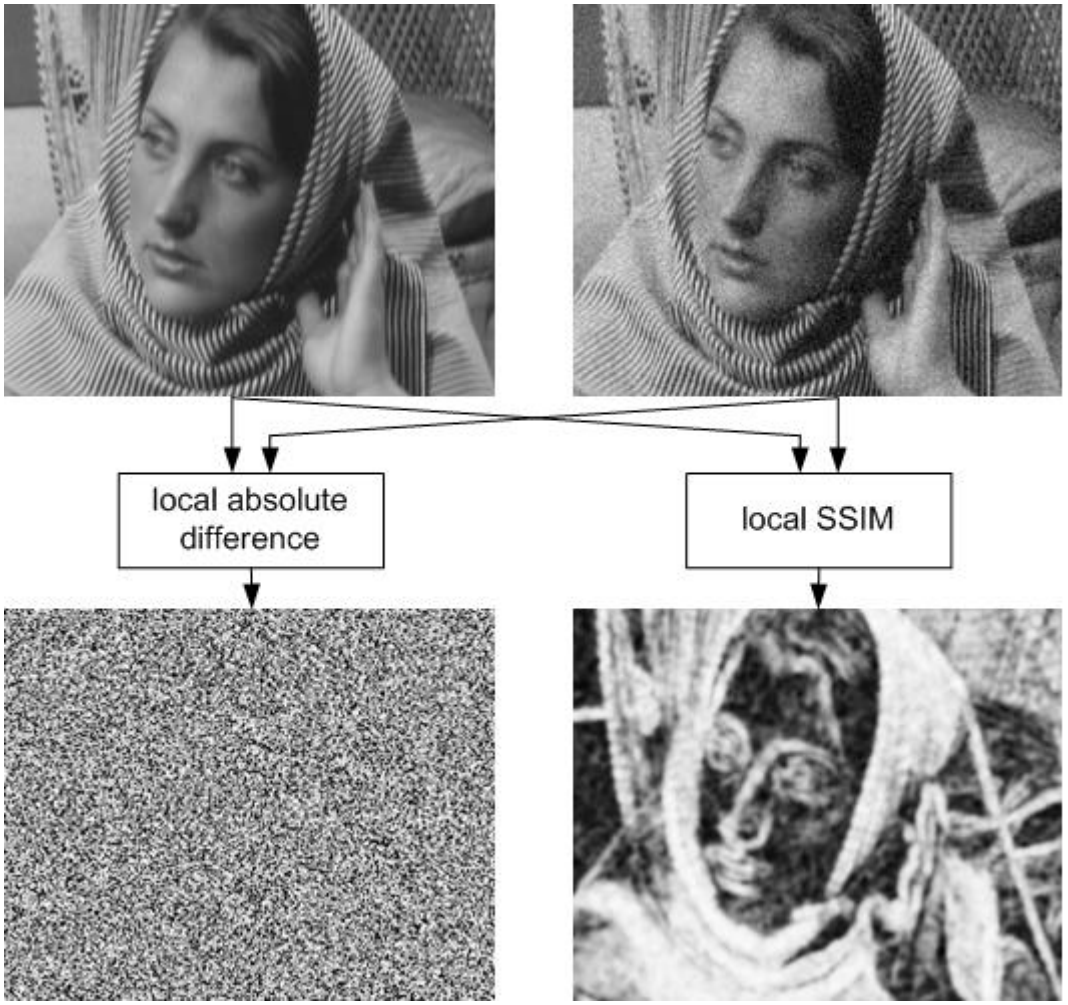
a. $SSIM(r,s) = SSIM(s,r)$

b. $-1 \le SSIM(r,s) \le 1$

c. $SSIM(r,s) = 1 \text{ iff } x = y$

Structural Similarity Index (SSIM)

- Signal samples are not equally important to signal fidelity based on SSIM



Structural Similarity Index (SSIM)

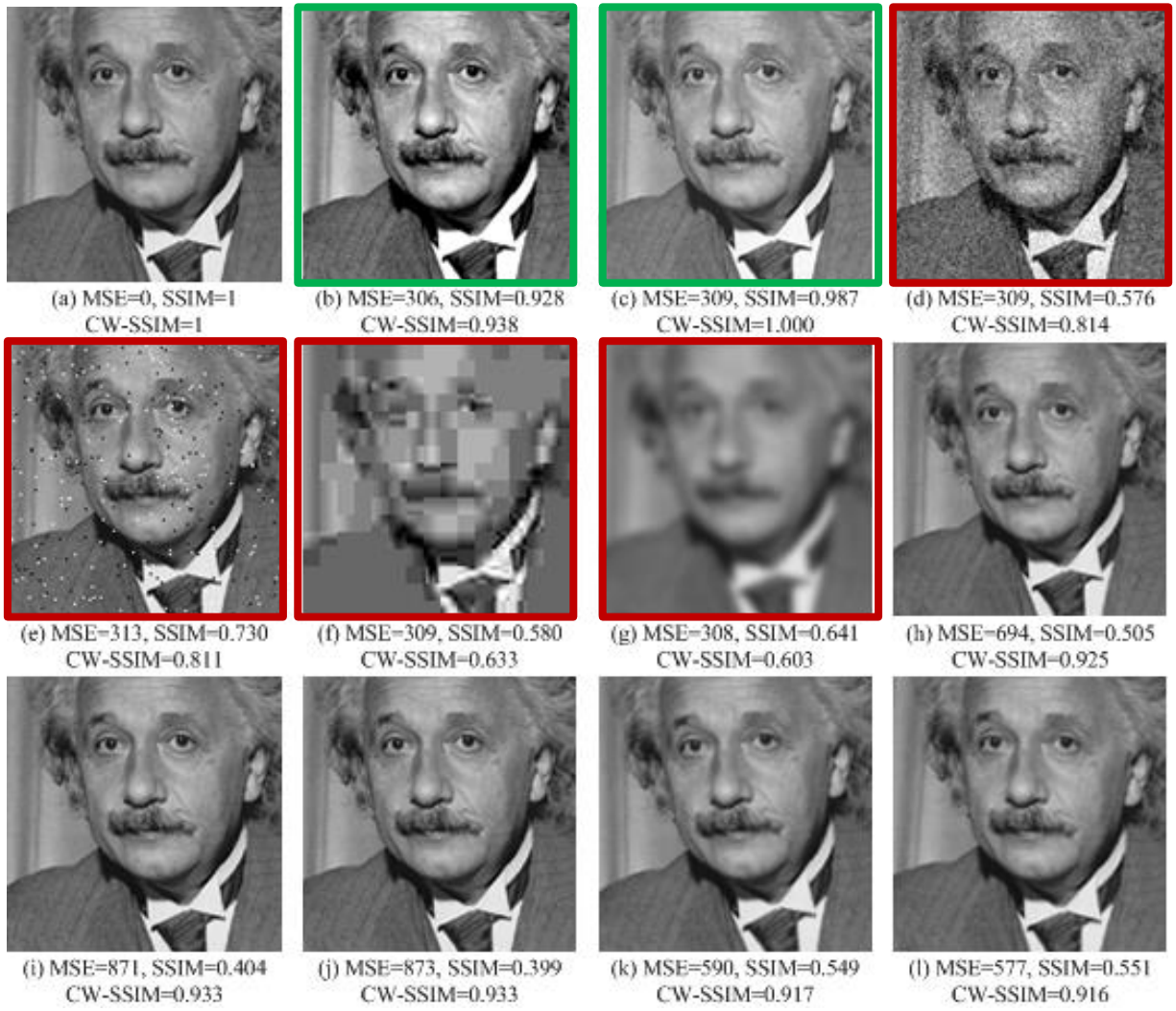
Global SSIM computed for the whole image

- Mean SSIM: $$SSIM(\mathrm{r},\mathrm{s}) = \frac{1}{M}\sum_{j=1}^{M} SSIM(r_j, s_j)$$

- Adaptive space variant weighting[4] : $$SSIM(\mathrm{r},\mathrm{s}) = \frac{\sum_{j=1}^{M} w_j SSIM(r_j, s_j)}{\sum_{j=1}^{M} w_j}$$

Saliency map *w*

Structural Similarity Index (SSIM)



a. Reference
b. Mean contrast stretch
c. Luminance shift
d. Gaussian noise
e. Impulsive noise
f. JPEG compression
g. Blurring
h. Zooming out
i. Spatial shift to the right
j. Spatial shift to the left
k. Rotation counter-clockw.
l. Rotation clockwise

Structural Similarity Index (SSIM)

- Drawback of SSIM is its sensitivity to geometric distortions, e.g.
  - translations
  - rotations
  - scaling

Structural Similarity Index (SSIM)

a. Reference
b. Mean contrast stretch
c. Luminance shift
d. Gaussian noise
e. Impulsive noise
f. JPEG compression
g. Blurring
h. Zooming out
i. Spatial shift to the right
j. Spatial shift to the left
k. Rotation counter-clockw.
l. Rotation clockwise

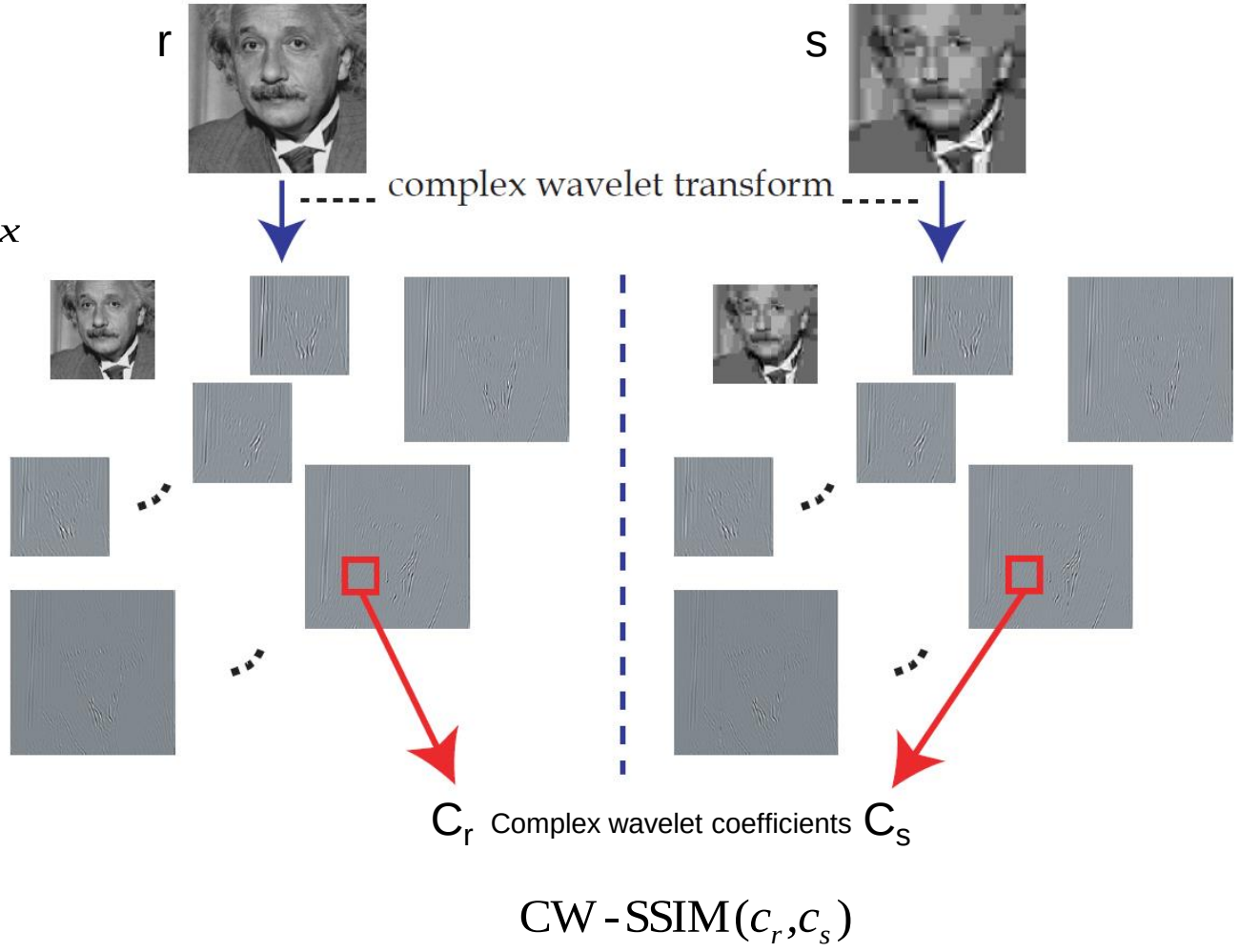
Complex Wavelet SSIM (CW-SSIM)

For 1D continuous case:

$$H(\sigma, p) = \int_{-\infty}^{\infty} h(x) w^*_{\sigma,p}(x) dx$$

$$w(x) = g(x)\exp(j\omega x)$$

$$w_{\sigma,p}(x) = \frac{1}{\sqrt{\sigma}} w\left(\frac{x-p}{\sigma}\right)$$



Complex Wavelet SSIM (CW-SSIM)

$$\text{CW-SSIM}(c_r, c_s) = \frac{2\sum_{i=1}^{N}|c_{r.i}||c_{s.i}| + K}{\sum_{i=1}^{N}|c_{r.i}|^2 + \sum_{i=1}^{N}|c_{s.i}|^2 + K} \cdot \frac{2\left|\sum_{i=1}^{N} c_{r,i}c_{s,i}^*\right| + K}{2\sum_{i=1}^{N}|c_{r,i}||c_{s,i}^*| + K}$$

Magnitude comparison

Phase comparison

maximum when magnitude of the coefficients $c_{r,i}$ and $c_{s,i}$ is equal for all i

maximum value one when phase difference between $c_{r,i}$ and $c_{s,I}$ is constant for all i

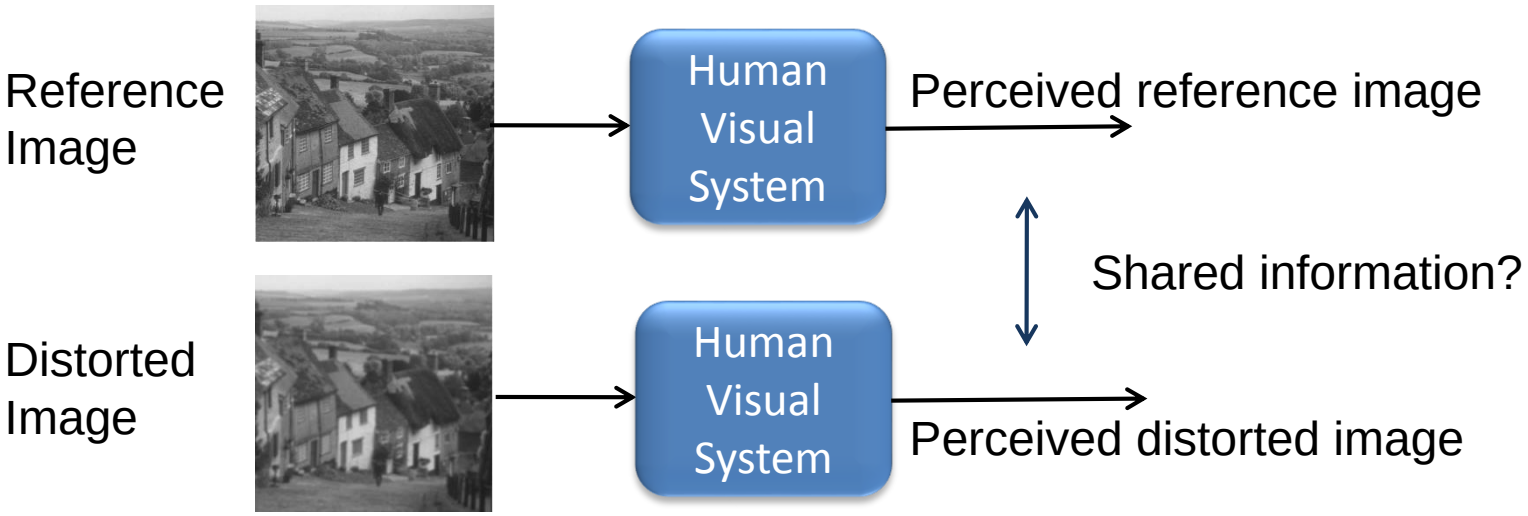
- CW-SSIM is robust with respect to ***luminance***, ***contrast*** and ***translation*** changes
- Leads to robustness to **small changes** in ***scaling*** and ***rotations*** since they can be locally approximated with translations
- Sensitive to structural distortion

Complex Wavelet SSIM (CW-SSIM)

a. Reference
b. Mean contrast stretch
c. Luminance shift
d. Gaussian noise
e. Impulsive noise
f. JPEG compression
g. Blurring
h. Zooming out
i. Spatial shift to the right
j. Spatial shift to the left
k. Rotation counter-clockw.
l. Rotation clockwise

Visual Information Fidelity (VIF)

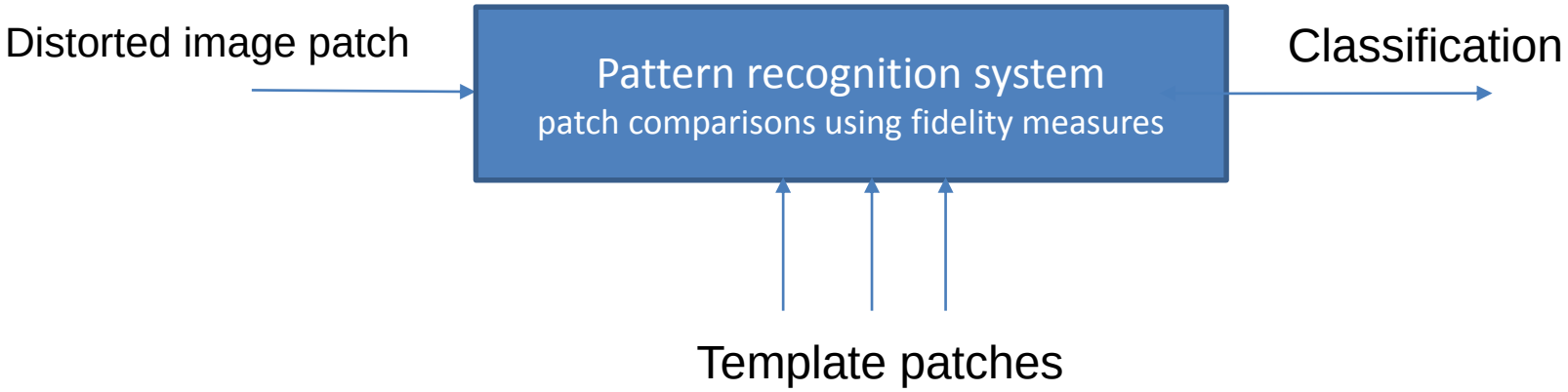
- Information theoretic approach



- H. Sheikh, A. Bovik, Image information and visual quality, *IEEE Trans. Image Processing, 2006*

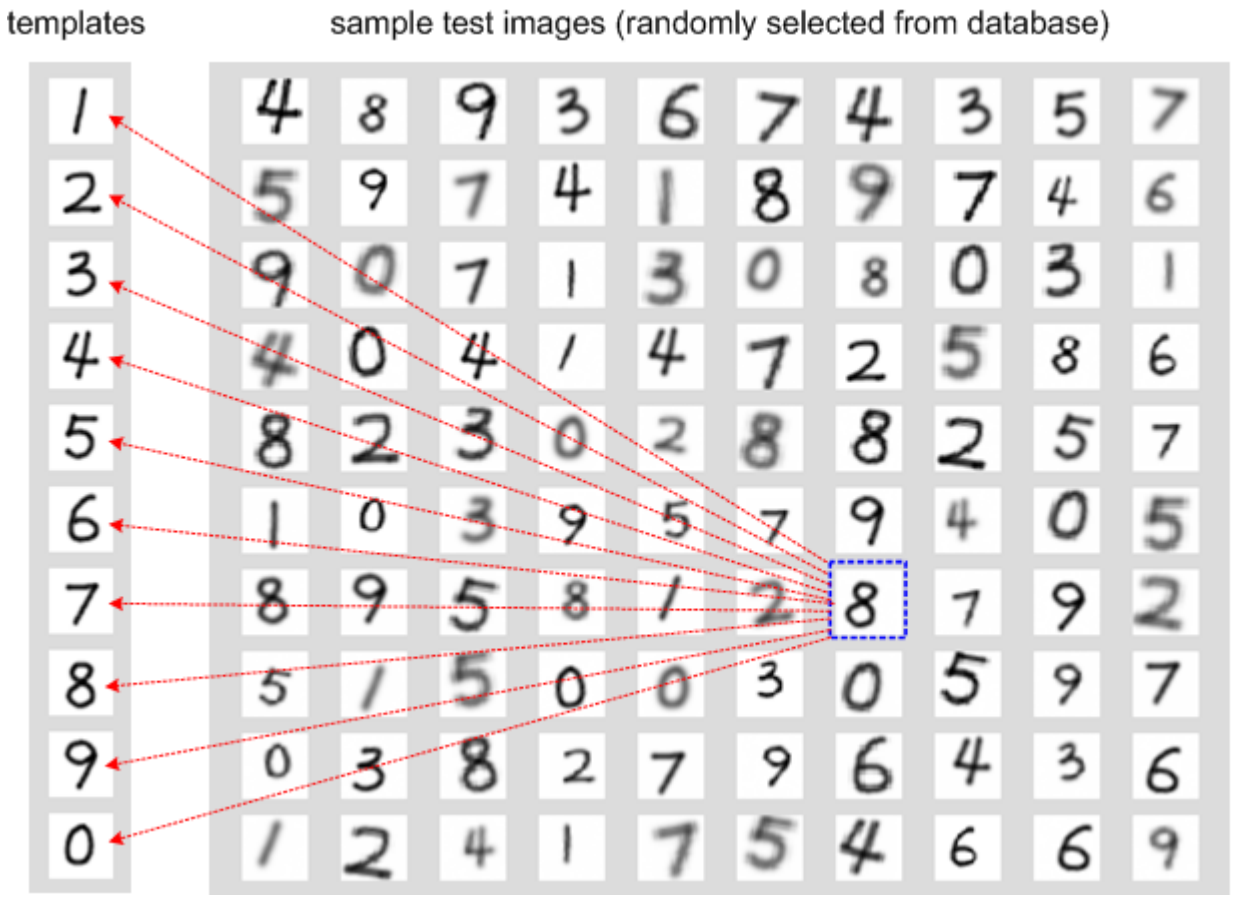
Universität Konstanz

Fidelity Measures for Pattern Recognition



Universität Konstanz

Character recognition[5]



recognition error rate (%)

digit	MSE	CW-SSIM
1	16.0	0
2	34.6	1.6
3	50.6	2.9
4	36.2	0
5	52.3	3.7
6	43.6	2.1
7	31.7	5.8
8	50.2	0.4
9	40.7	0
0	48.6	7.0
all	40.4	2.3

- Sample test images: shifted, scaled, rotated and blurred images
- Direct matching with 10 templates without any prior alignment/registration process

Final comments

- There are powerful alternatives to MSE
  - Especially useful in applications where perceptual criteria might be relevant
- Ideally the performance of signal processing algorithms might be compared to other algorithms using several fidelity criteria
- Alternative fidelity measures might be advantageous not only for signal quality measurements but also as a design criteria for optimizing signal processing algorithms

Universität Konstanz

Literature

1. Z. Wang, A. Bovik, Mean Squared Error: Love it or leave it, *IEEE Signal processing Magazine, 2009*
2. Z. Wang, A. Bovik, A universal Image quality Index, *IEEE Signal Processing Letters, 2002*
3. Z. Wang et. al, Image Quality Assessment: From Error Visibility to Structural Similarity, *IEEE Tran. Image Processing, 2004*
4. Z. Wang, Q. Li, Information Content Weighting for Perceptual Image quality Assessment, *IEEE Tran. Image Processing, 2011*
5. Z. Wang, E. Simoncelli, Translation insensitive image similarity in complex wavelet domain, *Proc. IEEE Int. Conf. Acoustic, Speech & Signal Processing, 2005*
6. M. Sampat et. al., Complex Wavelet Structural Similarity: A new Image Similarity Index, *IEEE Tran. Image Processing, 2009*
7. H. Sheikh, A. Bovik, Image Information and visual quality, *IEEE Tran. Image Processing, 2006*

Tank you for your attention!

Questions?