zadanie 1

Dane: $P(A \cap B) = \frac{1}{4}$

$P(A^c) = \frac{1}{3} \iff P(A) = \frac{2}{3}$

$P(B) = \frac{1}{2}$

korzystając ze wzoru:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{11}{12}$

zadanie 5

x $2-x$

zadanie 3

$X \sim B(n_1, p)$
$Y \sim B(n_2, p)$

$Z = X + Y$, chcemy dostać $Z \sim B(n_1 + n_2, p)$

czyli $P(Z=k) = \binom{n_1+n_2}{k} p^k (1-p)^{(n_1+n_2)-k}$

$$P(Z=k) = \sum_{i=0}^{k} P(X=i, Y=k-i) = \sum_{i=0}^{k} \binom{n_1}{i} p^i (1-p)^{n_1-i} \cdot \binom{n_2}{k-i} p^{k-i} (1-p)^{n_2-k+i} =$$

$$= \sum_{i=0}^{k} \binom{n_1}{i} \binom{n_2}{k-i} p^k (1-p)^{(n_1+n_2)-k} =$$

$$= p^k (1-p)^{(n_1+n_2)-k} \cdot \sum_{i=0}^{k} \binom{n_1}{i} \binom{n_2}{k-i} = \binom{n_1+n_2}{k} p^k (1-p)^{(n_1+n_2)-k}$$

zadanie 4

$Z = X + Y$, chcemy dostać $Z \sim Poisson(\lambda_1 + \lambda_2)$

$X \sim Poisson(\lambda_1)$

$Y \sim Poisson(\lambda_2)$

czyli $P(Z=k) = e^{-(\lambda_1+\lambda_2)} \cdot \frac{(\lambda_1+\lambda_2)^k}{k!}$

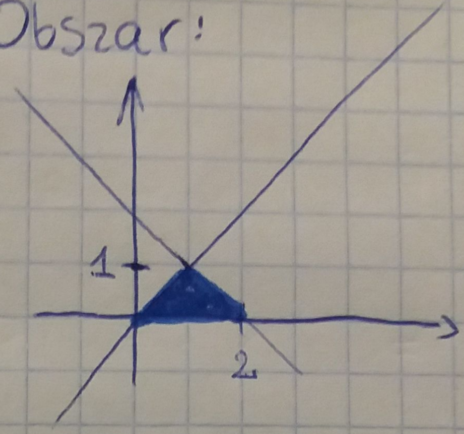
$$P(Z=k) = \sum_{i=0}^{k} P(X=i, Y=k-i) = \sum_{i=0}^{k} e^{-\lambda_1} \frac{\lambda_1^i}{i!} e^{-\lambda_2} \cdot \frac{\lambda_2^{k-i}}{(k-i)!} =$$

$$= e^{-(\lambda_1+\lambda_2)} \sum_{i=0}^{k} \frac{\lambda_1^i \lambda_2^{k-i}}{i!\,(k-i)!} = e^{-(\lambda_1+\lambda_2)} \cdot \frac{1}{k!} \sum_{i=0}^{k} \binom{n}{i} \lambda_1^i \lambda_2^{k-i} =$$

$$a) = e^{-(\lambda_1+\lambda_2)} \frac{(\lambda_1+\lambda_2)^k}{k!} \;\blacksquare$$

zadanie 5

Obszar:



$f(x,y) = 3xy$

dla x,y z danego obszaru

$f(x,y) \geq 0$

oraz

$$\int_0^1 \int_0^x 3xy \, dy\, dx + \int_1^2 \int_0^{2-x} 3xy \, dy\, dx = \frac{3}{8} + \frac{5}{8} = \underline{1}$$

Policzmy $f_1(x)$:

- dla $x < 1$: $f_1(x) = \int_0^x 3xy \, dy = \frac{3xy^2}{2}\Big|_0^x = \frac{3x^3}{2}$

- dla $x \geq 1$: $f_1(x) = \int_0^{2-x} 3xy \, dy = \frac{3x(x-2)^2}{2}$

Policzmy $f_2(y)$:

$$f_2(y) = \int_y^{2-y} 3xy \, dx = \frac{3yx^2}{2}\Big|_y^{2-y} = \frac{3y(y-2)^2}{2} - \frac{3y^3}{2} = 6y(1-y)$$

Zadanie 6

$f_2(y) = 6y(1-y)$ dla $y \in [0,1]$ nieujemna, oraz:

$$\int_0^1 6y(1-y)\,dy = 6\left(\int_0^1 y\,dy - \int_0^1 y^2\,dy\right) =$$

$$= 6\left(\frac{y^2}{2}\Big|_0^1 - \frac{y^3}{3}\Big|_0^1\right) = 6\left(\frac{1}{2} - \frac{1}{3}\right) = \underline{1}$$

$$EY = \int_0^1 y \cdot f_2(y)\,dy = 6\int_0^1 y^2(1-y)\,dy = 6\left(\int_0^1 y^2\,dy - \int_0^1 y^3\,dy\right) =$$

$$= 6\left(\frac{y^3}{3}\Big|_0^1 - \frac{y^4}{4}\Big|_0^1\right) = 6\left(\frac{1}{3} - \frac{1}{4}\right) = 1/2$$

X, Y niezależne, jeśli:

$$\forall_{x,y}\ f(x,y) = f_1(x)\cdot f_2(y)\quad , f_1(x) = \begin{cases} \frac{3}{2}x^3, & x<1 \\ \frac{3x(x-2)^2}{2}, & x \geq 1 \end{cases}$$

$x = y = \frac{1}{2}$:

$3xy = \frac{3}{4}$

$$f_1(x)\cdot f_2(y) = \frac{3}{16}\cdot\frac{6}{4} = \frac{3}{4}\cdot\frac{6}{13} \neq 3xy$$

, czyli zmienne są zależne.