NYU TANDON SCHOOL OF ENGINEERING

Robot Perception

Optical Flow & Tracking

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Overview

- Optical flow & Tracking
  - + Basics definitions
  - ++ KLT
  - ++ Mean-shift
  - + Correlation filter
- CNNs for Tracking
  - + Supervised
  - + Self-supervised

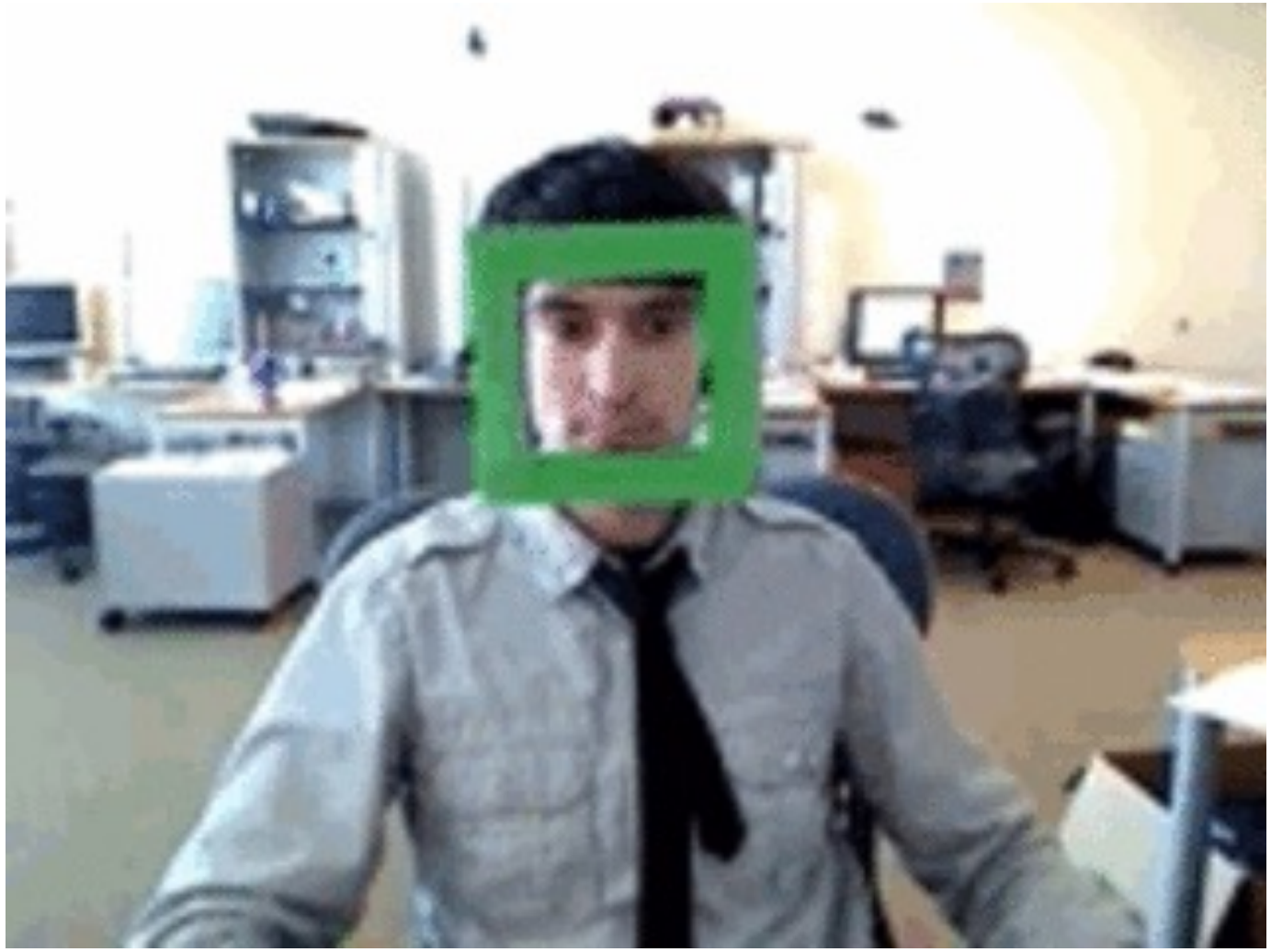
- *: know how to code
- ++: know how to derive
- +: know the concept

References

- Sz: Ch 7.1.5, 9.4

- Baker, Simon, and Iain Matthews. "Lucas-kanade 20 years on: A unifying framework." *International journal of computer vision*56.3 (2004): 221-255.
- Comaniciu, Dorin, and Peter Meer. "Mean shift: A robust approach toward feature space analysis." *IEEE Transactions on Pattern Analysis & Machine Intelligence* 5 (2002): 603-619.
- Feichtenhofer, Christoph, Axel Pinz, and Andrew Zisserman. "Detect to track and track to detect." In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 3038-3046. 2017.
- Wang, Xiaolong, Allan Jabri, and Alexei A. Efros. "Learning correspondence from the cycle-consistency of time." In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 2566-2576. 2019.

Tracking an Object



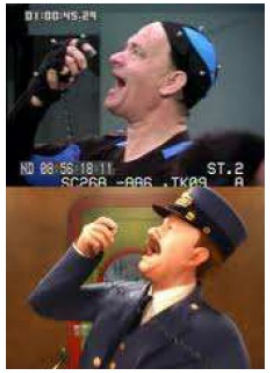
Gif from https://www.learnopencv.com/object-tracking-using-opencv-cpp-python/

Tracking Applications

- monitoring, assistance, surveillance, control, defense
- robotics, autonomous car driving, rescue
- measurements: medicine, sport, biology, meteorology
- **human computer interaction**
- **augmented reality**
- **film production and postproduction: motion capture, editing**
- **management of video content: indexing, search**
- **action and activity recognition**
- image stabilization
- mobile applications

Slides from Jiri Matas 2016

Tracking Applications



Slides from Jiri Matas 2016

Tracking Definition

- [Forsyth and Ponce, Computer Vision: A modern approach, 2003]
  - “Tracking is the problem of generating an inference about the motion of an object given a sequence of images.”
- Another definition
  - Establishing point-to-point correspondences in consecutive frames of an image sequence
- Yet another definition
  - Given an initial estimate of its position, locate X in a sequence of images
  - X can be a region, an interest point, or an object

Slides from Jiri Matas 2016

Tracking Definition

Given an initial estimate of the pose and state of X :

In all images in a sequence, (in a causal manner)

1. *estimate the pose and state of X*
2. *(optionally) update the model of X*

- Pose: any geometric parameter (position, scale, …)
- State: appearance, shape/segmentation, visibility, articulations
- Model update: essentially a semi-supervised learning problem
  - a priori information (appearance, shape, dynamics, …)
  - labeled data (“track this”) + unlabeled data = the sequences
- Causal: for estimation at T, use information from time $t \leq T$

Slides from Jiri Matas 2016

Two Types of Tracking

Short-term Trackers:

- primary objective: "where is X?" = precise estimation of pose
- secondary: be fast; don't lose track
- evaluation methodology: frame number where failure occurred
- examples: Lucas Kanade tracker, mean-shift tracker

Long-term Tracker-Detectors:

- primary objective: unsupervised learning of a detector, since *every (short-term) tracker fails, sooner or later* (disappearance from the field of view, full occlusion)
- avoid the "*first failure means lost forever*" problem
- close to online-learned detector, but assumes and exploits the fact that a sequence with temporal pose/state dependence is available
- evaluation methodology: precision/recall, false positive/negative rates (i.e. like detectors)
- note: the detector part may help even for short-term tracking problems, provides robustness to fast, unpredictable motions.

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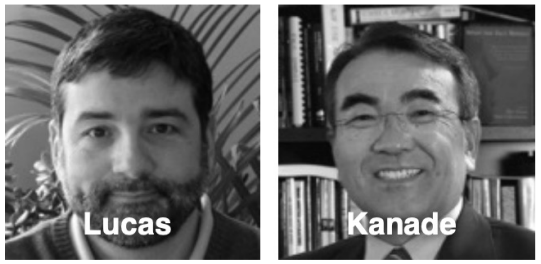
KLT Tracker



An Iterative Image Registration Technique
with an Application to Stereo Vision.

1981

History of the

Kanade-Lucas-Tomasi (KLT) Tracker

Detection and Tracking of Feature Points.

1991

The original KLT algorithm

Good Features to Track.

1994

Slides from Kris Kitani

KLT Tracker

Kanade-Lucas-Tomasi

How should we track them from frame to frame?

Lucas-Kanade

Method for aligning (tracking) an image patch

How should we select features?

Tomasi-Kanade

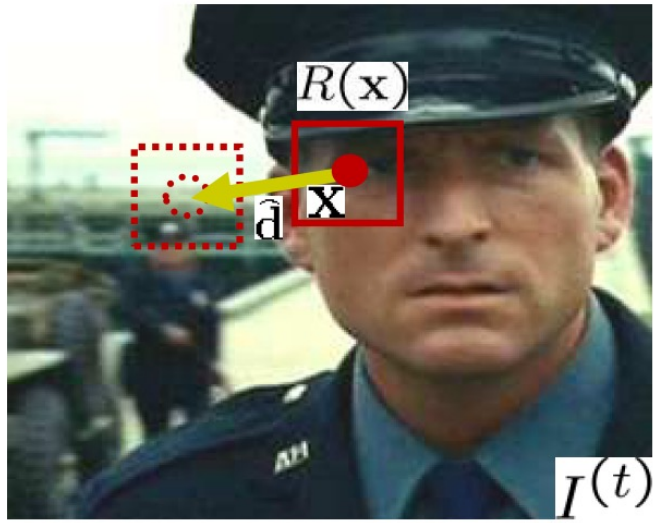
Method for choosing the best feature (image patch) for tracking

Slides from Kris Kitani

KLT Tracker

– Problem: tracking "key points" (SIFT, SURF, STAR, RIFF, FAST), or random image patches, as long as possible
  - Input: detected/chosen patches
  - Output: *tracklets* of various life-spans

slide credit: Patrick Perez



$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}} \underbrace{\sum_{\mathbf{p}\in R(\mathbf{x})} |I^{(t+1)}(\mathbf{p}+\mathbf{d}) - I^{(t)}(\mathbf{p})|^2}_{\text{SSD}}$$

Slides from Jiri Matas 2016

KLT Tracker

– Problem: tracking "key points" (SIFT, SURF, STAR, RIFF, FAST), or random image patches, as long as possible
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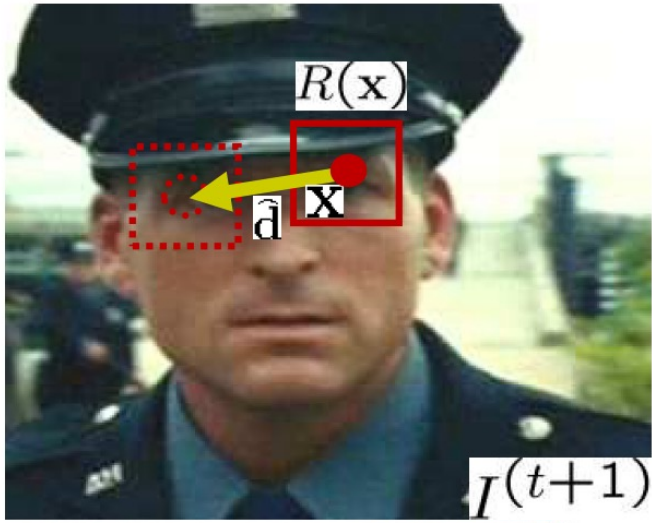


$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}} \underbrace{\sum_{\mathbf{p}\in R(\mathbf{x})} |I^{(t+1)}(\mathbf{p}+\mathbf{d}) - I^{(t)}(\mathbf{p})|^2}_{\text{SSD}}$$

Slides from Jiri Matas 2016

KLT Tracker

$$f(x) = f(a) + f'(a)(x-a) + R_2$$

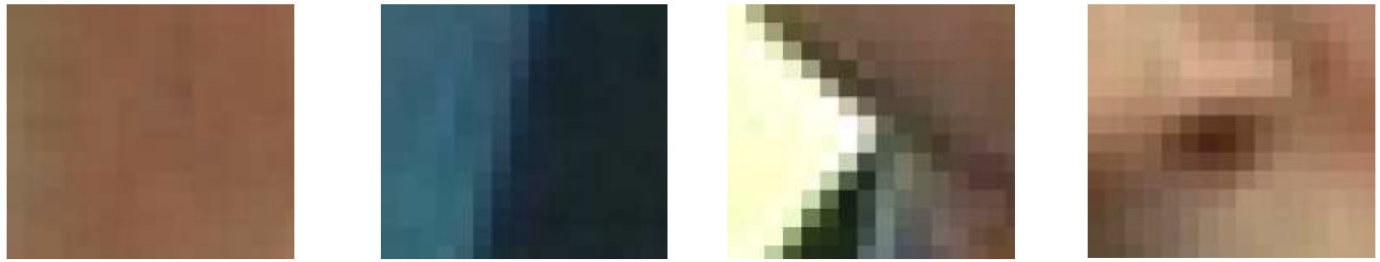
– First assuming small displacement: 1st-order Taylor expansion inside SSD

$$\hat{d} = \underset{d}{\operatorname{argmin}} \sum_{p \in R(x)} \left| I^{(t+1)}(p) + \nabla I^{(t+1)}(p)^T d - I^{(t)}(p) \right|^2$$

$$\hat{d} = -\left(\sum_{p \in R(x)} \nabla I^{(t+1)}(p) \nabla I^{(t+1)}(p)^T \right)^{-1} \sum_{p \in R(x)} \nabla I^{(t+1)}(p) [I^{(t+1)}(p) - I^{(t)}(p)]$$

For good conditioning, patch must be textured/structured enough:

- Uniform patch: no information
- Contour element: aperture problem (one dimensional information)
- Corners, blobs and texture: best estimate



[Lucas and Kanda 1981][Tomasi and Shi, CVPR'94]

$$\underset{\beta}{\text{minimize}} \left\| Y - X\beta \right\|^2$$

$$\hat{\beta} = \left(X^{\mathsf{T}} X \right)^{-1} X^{\mathsf{T}} Y$$

Slides from Jiri Matas 2016

KLT Tracker

– Translation is usually sufficient for small fragments, but:
  • Perspective transforms and occlusions cause drift and loss
– Two complementary options
  • Kill tracklets when minimum SSD too large
  • Compare as well with *initial patch under affine transform (warp)* assumption

slide credit:
Patrick Perez

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}} \sum_{\mathbf{p}\in R_t} |I^{(t+1)}(\mathbf{p}+\mathbf{d}) - I^{(t)}(\mathbf{p})|^2$$

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}} \sum_{\mathbf{p}\in R_0} |I^{(t+1)}(\mathbf{w}[\mathbf{p}]) - I^{(0)}(\mathbf{p})|^2$$

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KLT Tracker

- cost function: sum of squared intensity differences between template and window
- optimization technique: gradient descent
- model learning: no update

- attractive properties:
  - fast
  - easily extended to image-to-image transformations with multiple parameters

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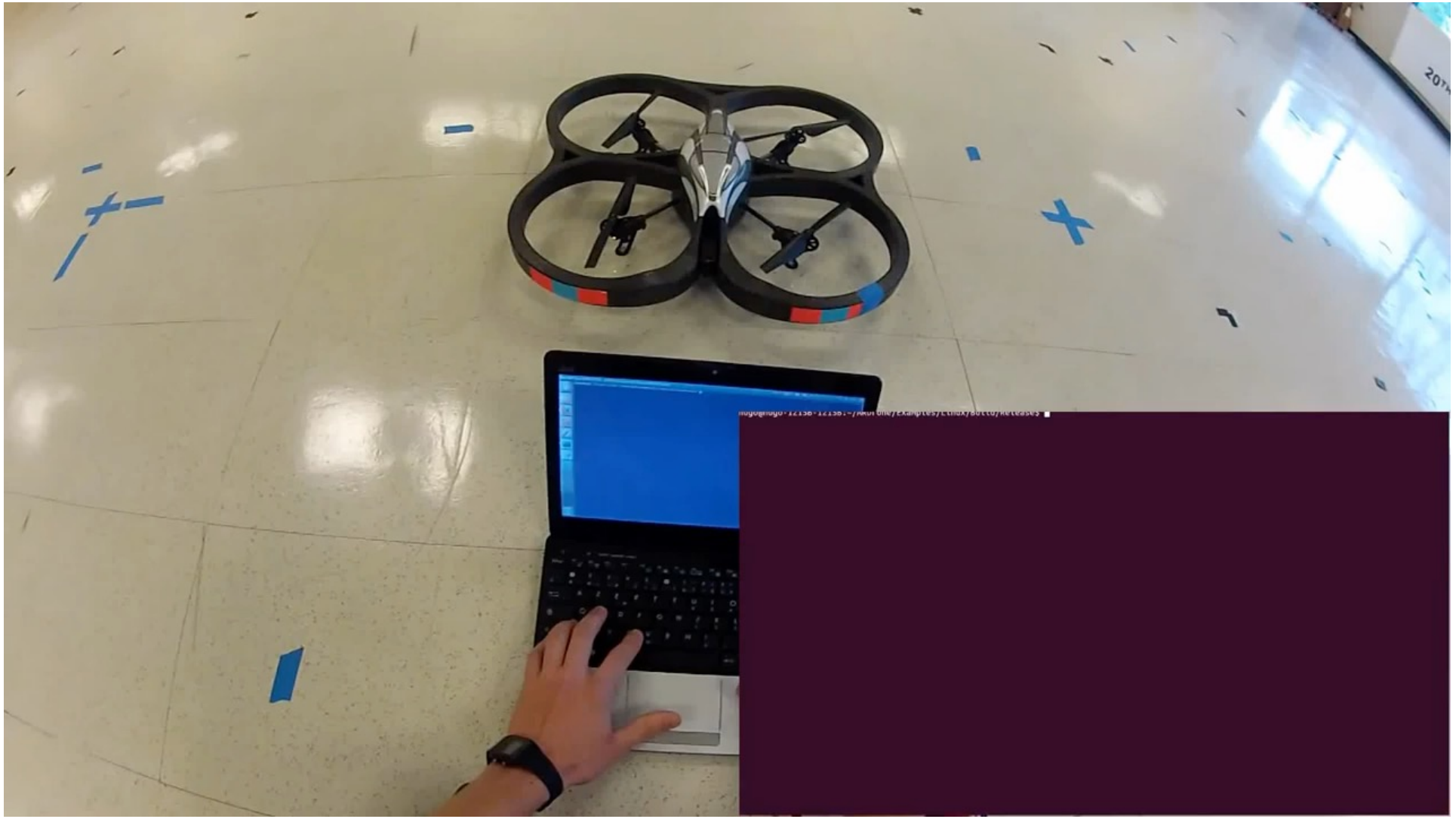
Optical Flow in Robotics



https://youtu.be/C95bngCOv9Q

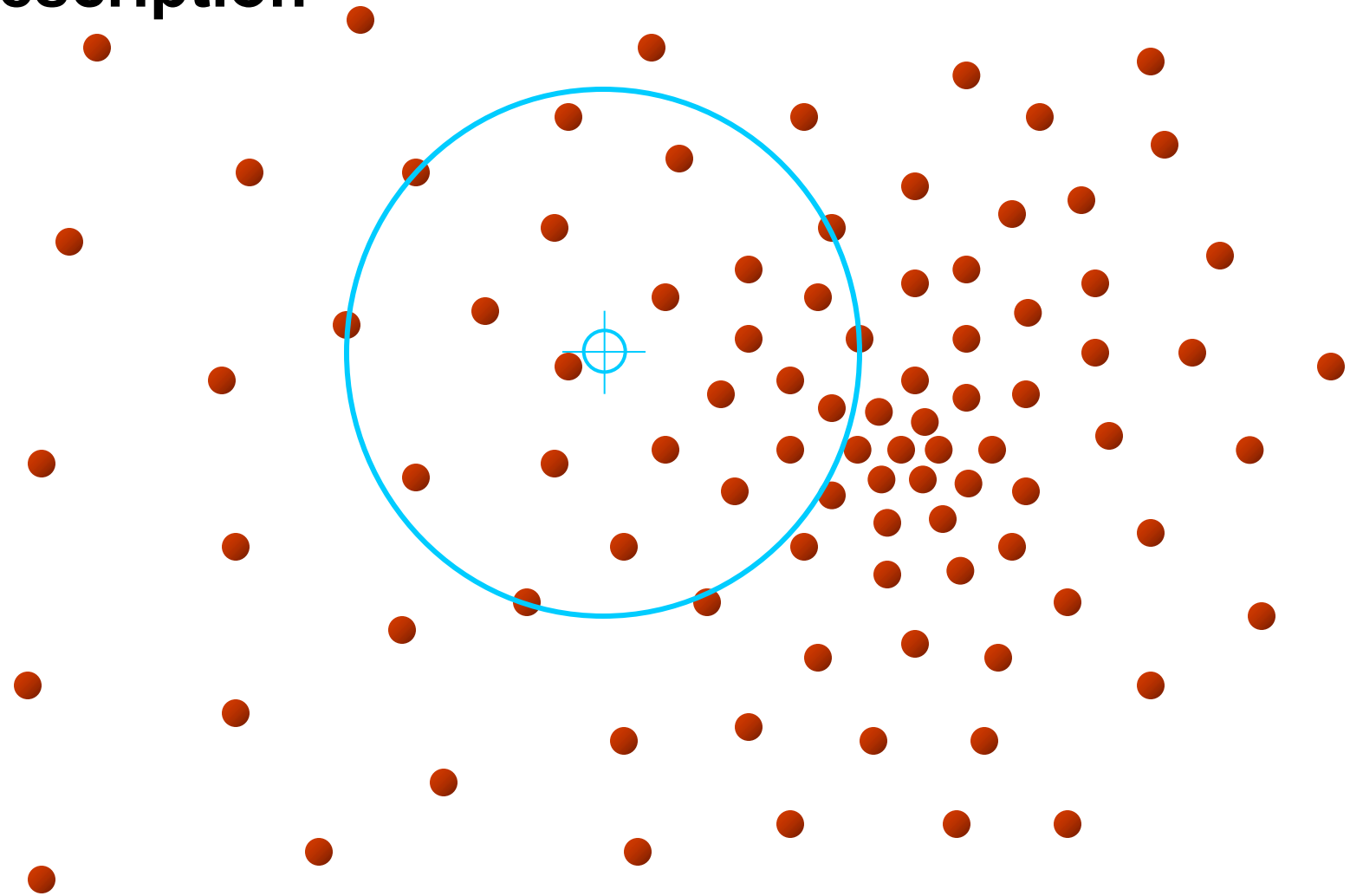
Mean Shift Theory

Comaniciu, Dorin, Visvanathan Ramesh, and Peter Meer.
"Real-Time Tracking of Non-Rigid Objects Using Mean Shift."

Proceedings IEEE Conference on Computer Vision and Pattern Recognition. CVPR 2000. Vol. 2. IEEE, 2000.

Won the Best Paper Award CVPR 2000

Intuitive Description

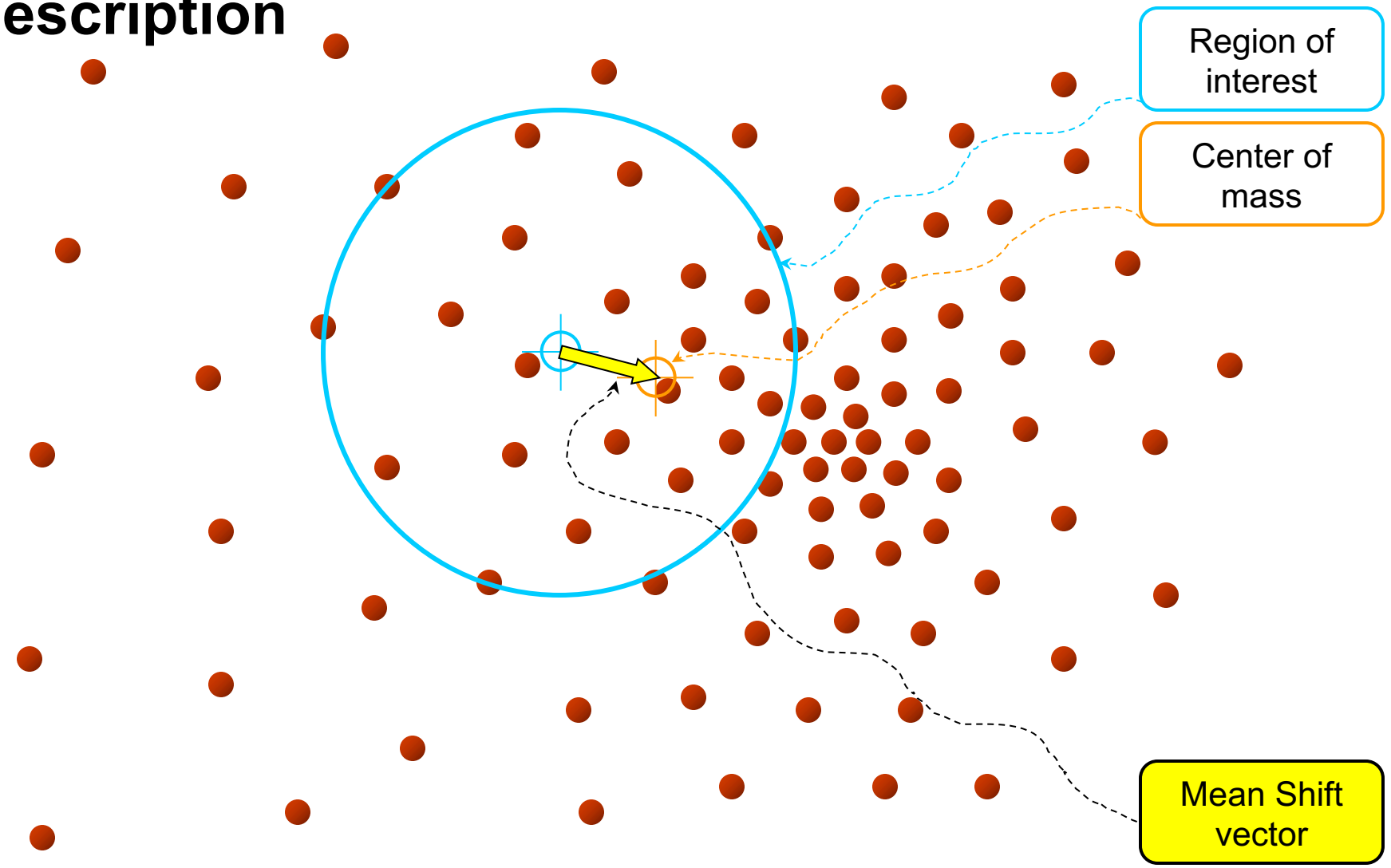


Objective : Find the densest region

Distribution of identical billiard balls

Slides from Ukrainitz & Sarel

Intuitive Description



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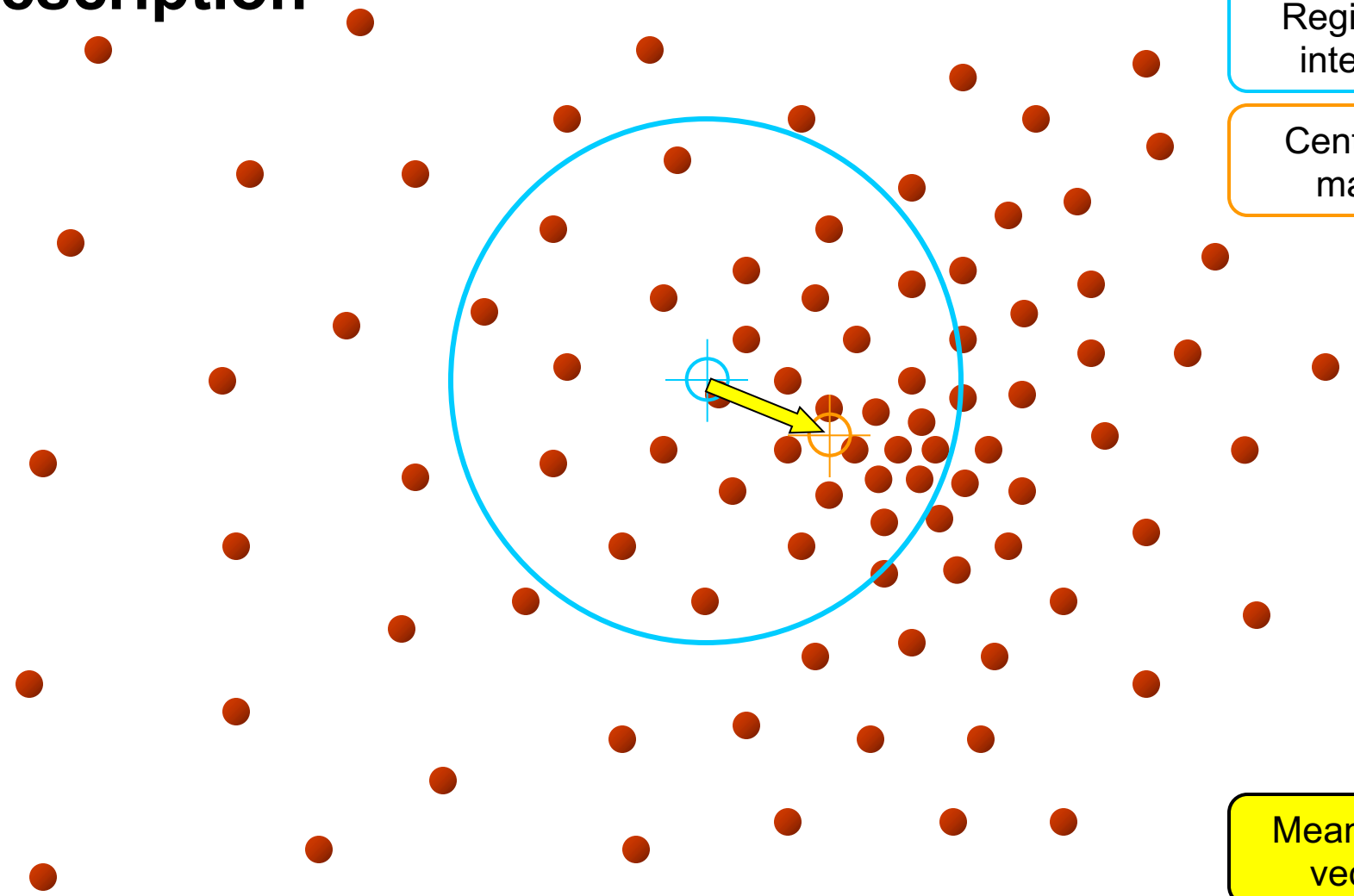

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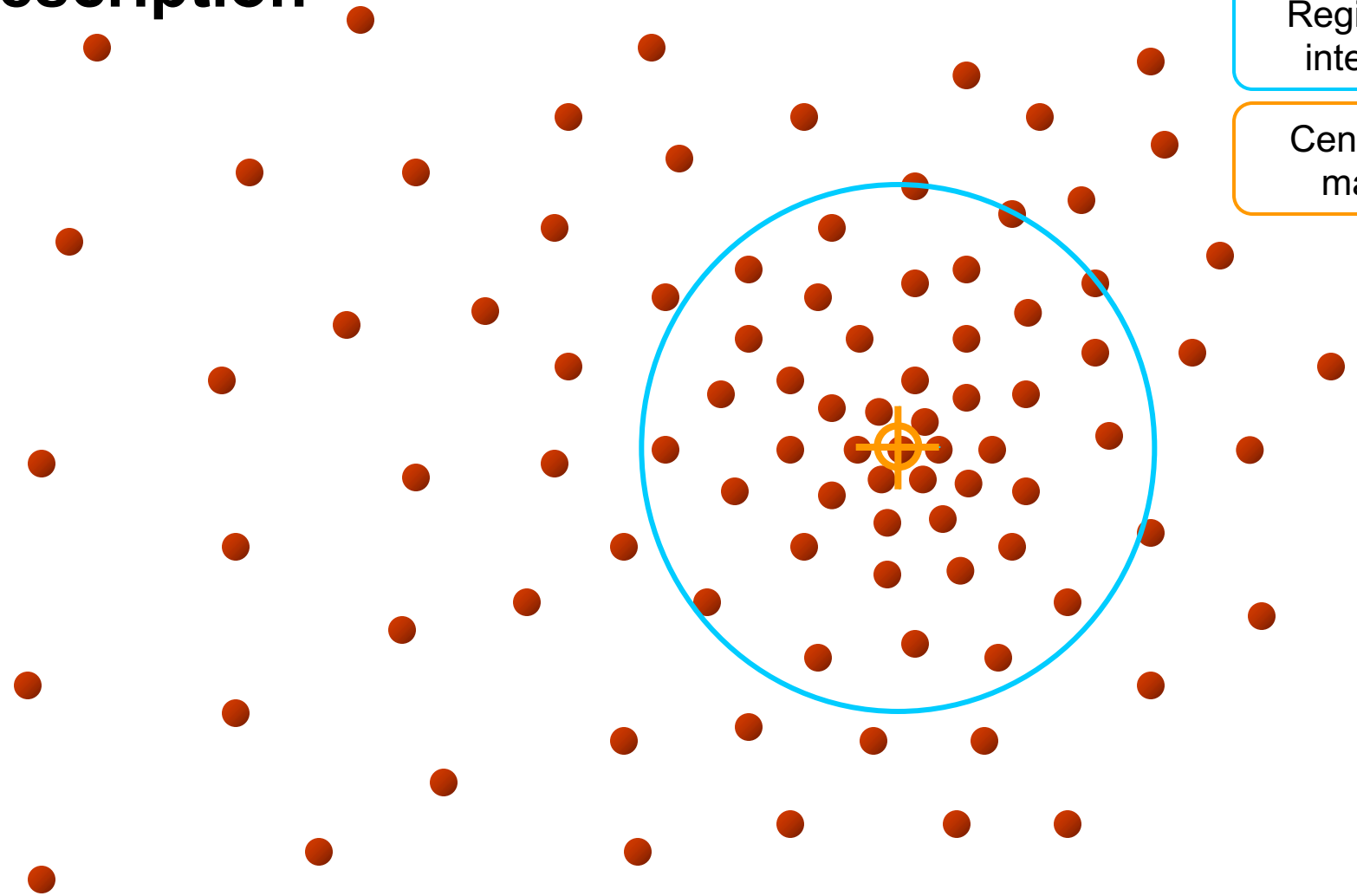

Objective : Find the densest region

Distribution of identical billiard balls

Slides from Ukrainitz & Sarel

Mean Shift Vector

- Given:
  - Data points and approximate location of the mean of this data:

- Task:
  - Estimate the exact location of the mean of the data by determining the shift vector from the initial mean.

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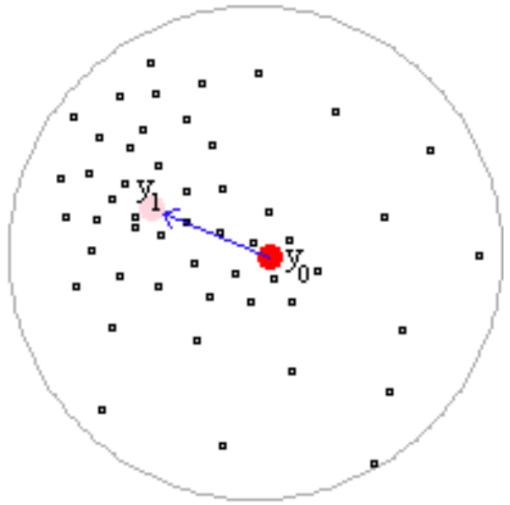
Mean Shift Vector

- An example



$$M_h(\mathbf{y}) = \left[\frac{1}{n_x} \sum_{i=1}^{n_x} \mathbf{x}_i \right] - \mathbf{y}_0$$

Mean shift vector always points towards the direction of the maximum increase in the density.

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Mean Shift (Weighted)

$$M_h(\mathbf{y}_0) = \left[\frac{\sum_{i=1}^{n_x} w_i(\mathbf{y}_0)\mathbf{x}_i}{\sum_{i=1}^{n_x} w_i(\mathbf{y}_0)} \right] - \mathbf{y}_0$$

n_x : number of points in the kernel
$\mathbf{y}_0$: initial mean location
$\mathbf{x}_i$: data points
h : kernel radius

Weights are determined using kernels (masks):
Uniform, Gaussian or Epanechnikov

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Properties of Mean Shift

- Mean shift vector has the direction of the gradient of the density estimate.

- It is computed iteratively for obtaining the maximum density in the local neighborhood.

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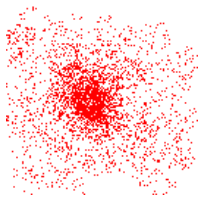
What is Mean Shift ?

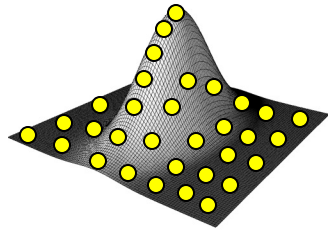
A tool for:
Finding modes in a set of data samples, manifesting an underlying probability density function (PDF) in R^N

PDF in feature space
- **Color space**
- **Scale space**
- **Actually, any feature space you can conceive**



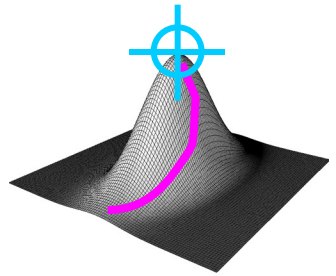
Data

Non-parametric Density Estimation



Discrete PDF Representation

Non-parametric Density **GRADIENT** Estimation (Mean Shift)

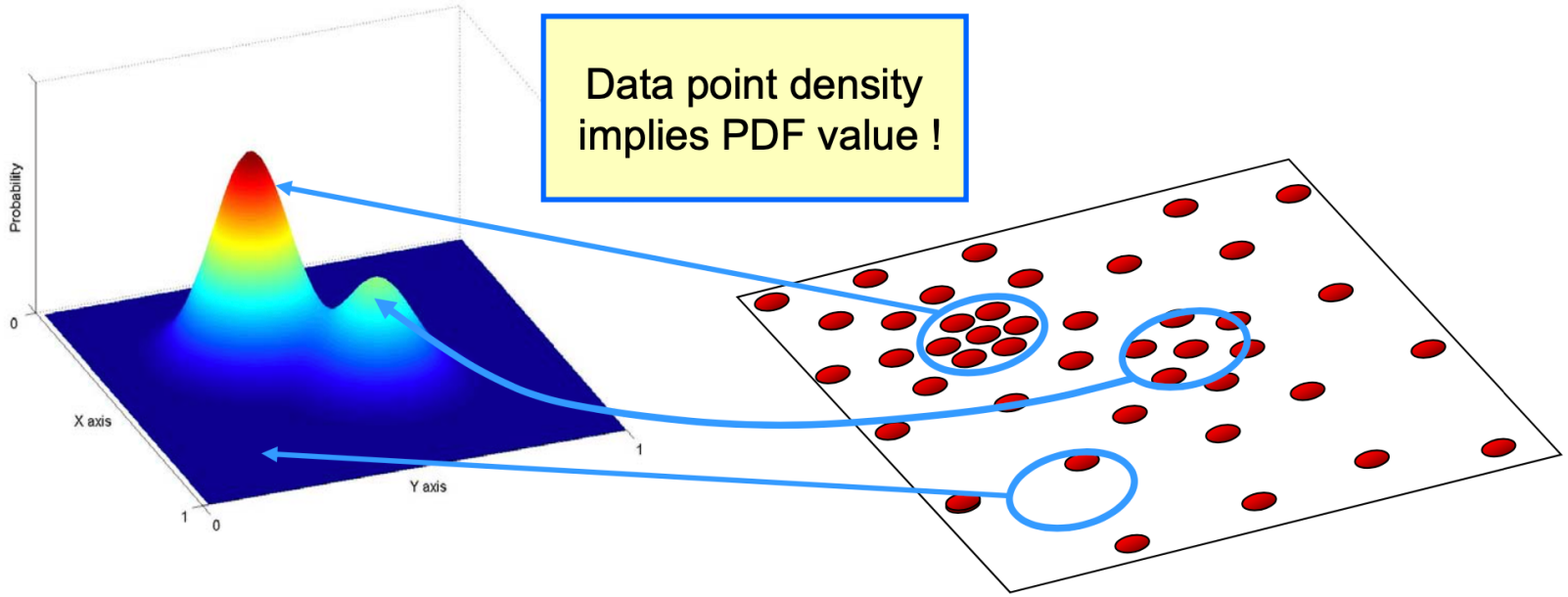


PDF Analysis

Slides from Ukrainitz & Sarel

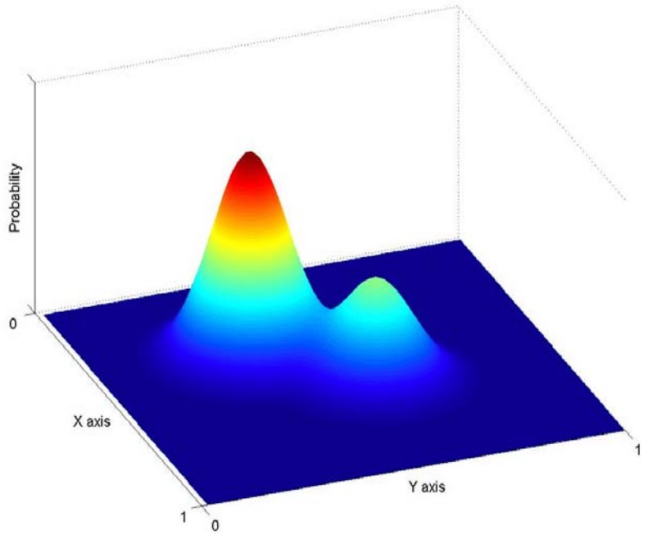
Non-Parametric Density Estimation

- Assumption : The data points are sampled from an underlying PDF

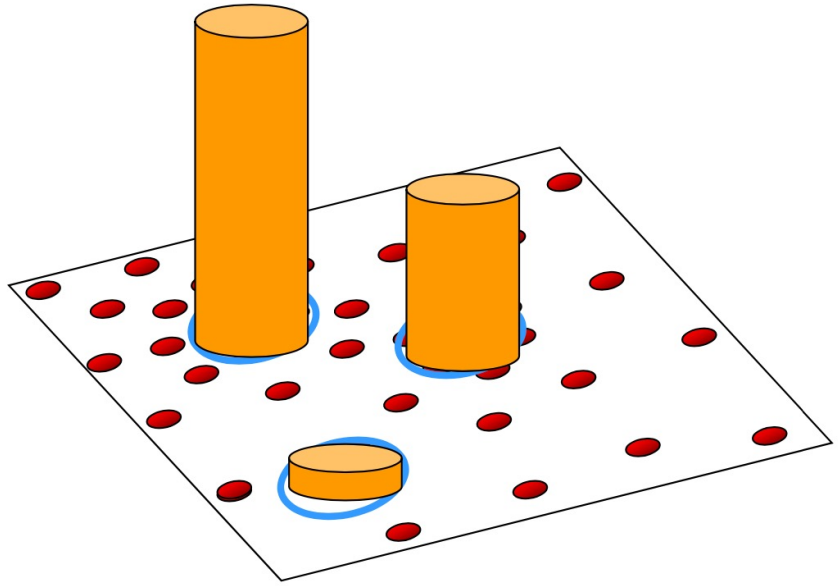


Slides from Ukrainitz & Sarel

Non-Parametric Density Estimation



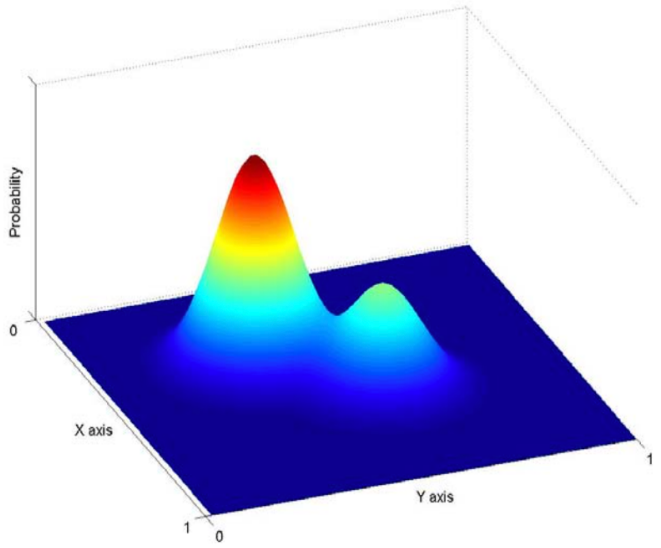
Assumed Underlying PDF



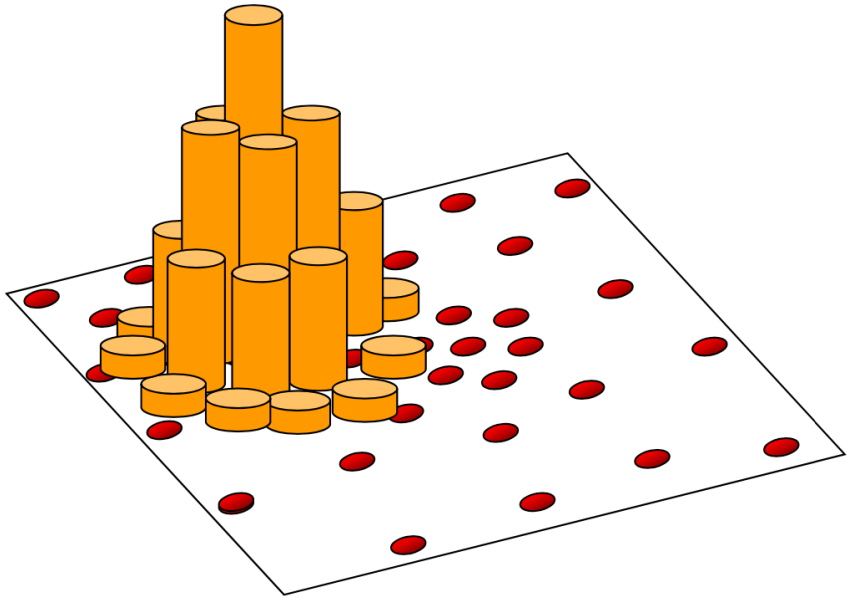
Real Data Samples

Slides from Ukrainitz & Sarel

Non-Parametric Density Estimation



Assumed Underlying PDF



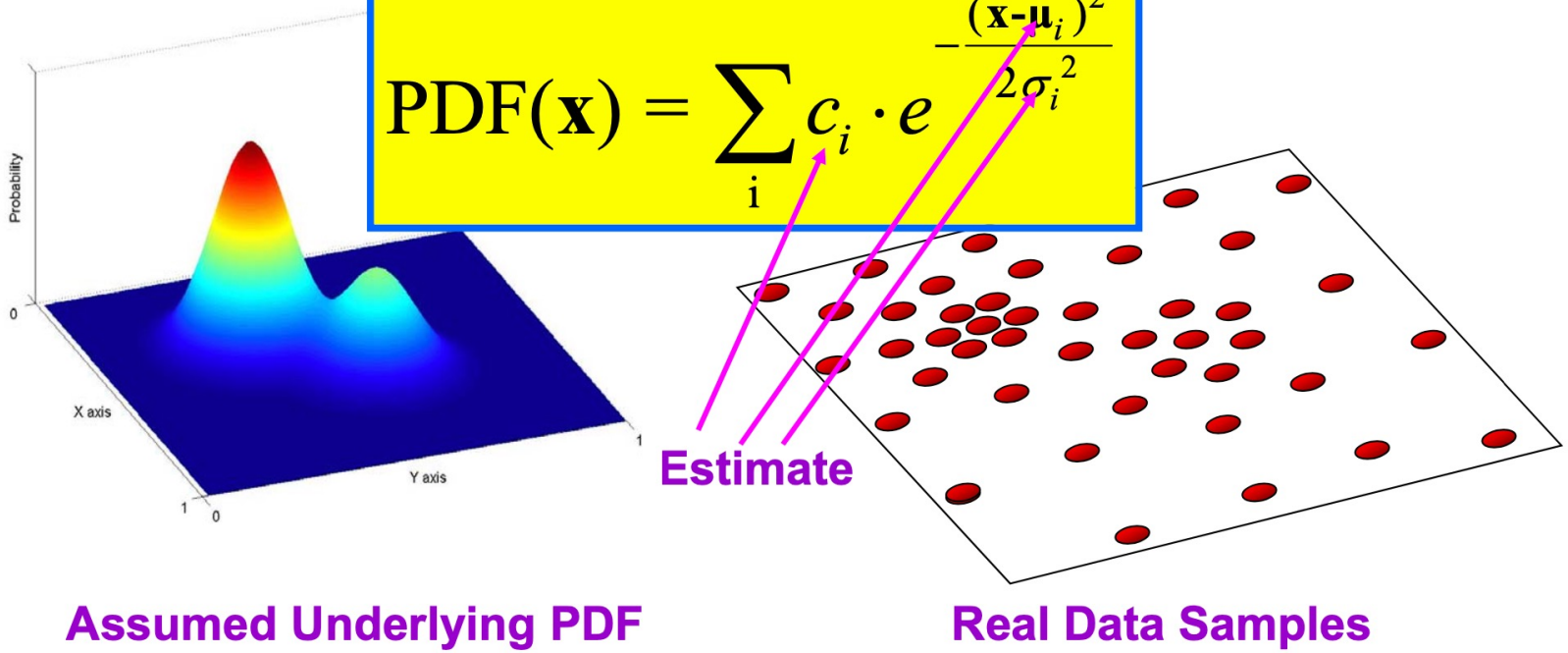
Real Data Samples

Slides from Ukrainitz & Sarel

Parametric Density Estimation

- Assumption: The data points are samples from an underlying PDF

$$\mathrm{PDF}(\mathbf{x}) = \sum_{i} c_i \cdot e^{-\frac{(\mathbf{x}-\boldsymbol{\mu}_i)^2}{2\sigma_i^2}}$$



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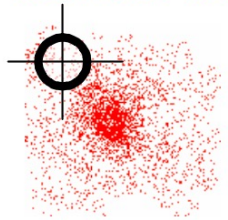
Kernel Density Estimation - Function Forms

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

A function of some finite number of data points $x_1 \ldots x_n$



Data

In practice one uses the forms:

$$K(\mathbf{x}) = c\prod_{i=1}^{d} k(x_i) \quad \text{or} \quad K(\mathbf{x}) = ck\left(\|\mathbf{x}\|\right)$$

Same function on each dimension

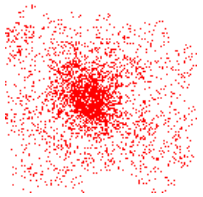
Function of vector length only

Slides from Ukrainitz & Sarel

Kernel Density Estimation - Various Kernels

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

A function of some finite number of data points $x_1 \ldots x_n$

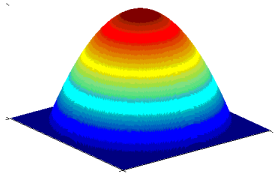


Data

Examples:

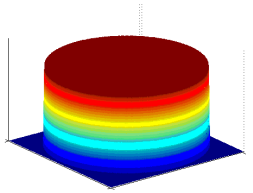
- Epanechnikov Kernel

$$K_E(\mathbf{x}) = \begin{cases} c\left(1 - \|\mathbf{x}\|^2\right) & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$



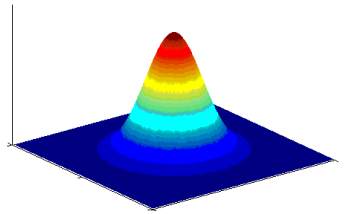
- Uniform Kernel

$$K_U(\mathbf{x}) = \begin{cases} c & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$



- Normal Kernel

$$K_N(\mathbf{x}) = c \cdot \exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right)$$

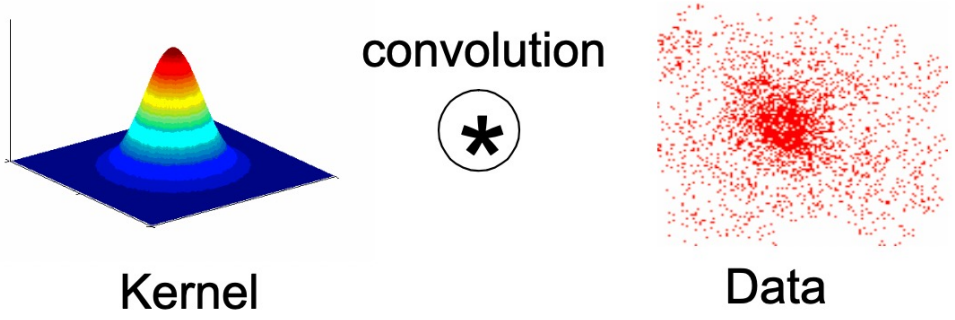


Slides from Ukrainitz & Sarel

Kernel Density Estimation - Key Idea

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

Superposition of kernels, centered at each data point is equivalent to convolving the data points with the kernel.



Slides from Ukrainitz & Sarel

Profile and Kernel

Radially symmetric Kernel

$$K(\mathrm{x}) = \mathrm{c}k(\|\mathrm{x}\|^2)$$

Profile

$$P(\mathrm{x}) = \frac{1}{n}\sum_{\mathrm{i}=1}^{\mathrm{n}} K(\mathrm{x}-\mathrm{x}_\mathrm{i}) = \frac{1}{n}\mathrm{c}\sum_{\mathrm{i}=1}^{\mathrm{n}} k(\|\mathrm{x}-\mathrm{x}_\mathrm{i}\|^2)$$

Slides from Ukrainitz & Sarel

Kernel Density *Gradient* Estimation

$$P(\mathrm{x}) = \frac{1}{n} \mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} k(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)$$

$$\nabla P(\mathrm{x}) = \frac{1}{n} \mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} \nabla k(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)$$

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{x} - \mathrm{x}_\mathrm{i}) k'(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)$$

Slides from Ukrainitz & Sarel

Kernel Density *Gradient* Estimation

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{x} - \mathrm{x}_\mathrm{i}) k'(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)$$

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{x}_\mathrm{i} - \mathrm{x}) g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)$$

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_\mathrm{i} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2) - \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)$$

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2) \left[\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_\mathrm{i} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)}{\sum_{\mathrm{i}=1}^{\mathrm{n}} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)} - \mathrm{x} \right] \qquad g(\mathbf{x}) = k'(\mathbf{x})$$

Slides from Ukrainitz & Sarel

Kernel Density *Gradient* Estimation

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2) \left[\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_\mathrm{i} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)}{\sum_{\mathrm{i}=1}^{\mathrm{n}} g(\|\mathrm{x} - \mathrm{x}_\mathrm{i}\|^2)} - \mathrm{x} \right]$$

$$\nabla P(\mathrm{x}) = \frac{1}{n} 2\mathrm{c} \sum_{\mathrm{i}=1}^{\mathrm{n}} g_i \left[\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_\mathrm{i} g_i}{\sum_{\mathrm{i}=1}^{\mathrm{n}} g_i} - \mathrm{x} \right]$$

Slides from Ukrainitz & Sarel

Kernel Density *Gradient* Estimation

$$\nabla P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} \nabla K(\mathbf{x} - \mathbf{x}_i)$$

Give up estimating the PDF !
Estimate **ONLY** the gradient

Using the Kernel form:

$$K(\mathbf{x} - \mathbf{x}_i) = ck\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)$$

Size of window

We get :

$$\nabla P(\mathbf{x}) = \frac{c}{n}\sum_{i=1}^{n} \nabla k_i = \frac{c}{n}\left[\sum_{i=1}^{n} g_i\right]\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right]$$

Slides from Ukrainitz & Sarel

$$g(\mathbf{x}) = -k'(\mathbf{x})$$

Computing The Mean Shift

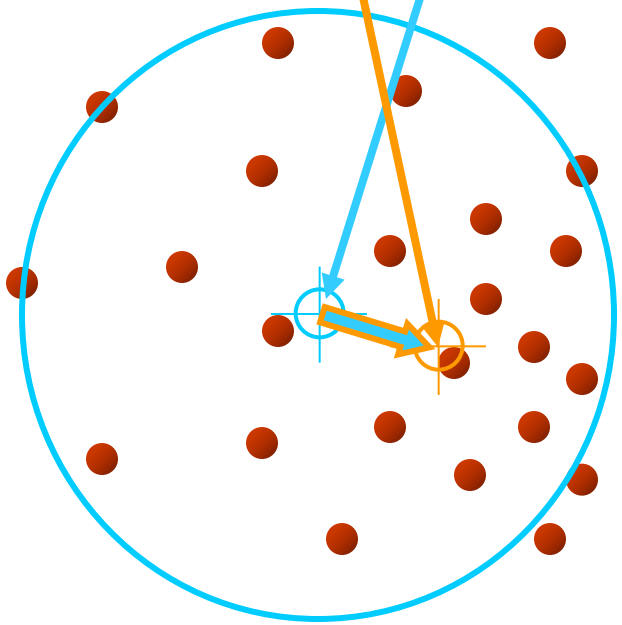
$$M_h(\mathbf{y}_0) = \left[\frac{\sum_{i=1}^{n_x} w_i(\mathbf{y}_0)\mathbf{x}_i}{\sum_{i=1}^{n_x} w_i(\mathbf{y}_0)}\right] - \mathbf{y}_0$$

n_x : number of points in the kernel
$\mathbf{y}_0$: initial mean location
$\mathbf{x}_i$: data points
h : kernel radius

Weights are determined using kernels (masks):
Uniform, Gaussian or Epanechnikov

$$\nabla P(\mathbf{x}) = \frac{c}{n}\sum_{i=1}^{n}\nabla k_i = \frac{c}{n}\left[\sum_{i=1}^{n} g_i\right]\left[\frac{\sum_{i=1}^{n}\mathbf{x}_i g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right]$$

Yet another Kernel density estimation !



Simple Mean Shift procedure:

- Compute mean shift vector

$$\mathbf{m}(\mathbf{x}) = \left[\frac{\sum_{i=1}^{n}\mathbf{x}_i g\left(\frac{\|\mathbf{x}-\mathbf{x}_i\|^2}{h}\right)}{\sum_{i=1}^{n} g\left(\frac{\|\mathbf{x}-\mathbf{x}_i\|^2}{h}\right)} - \mathbf{x}\right]$$

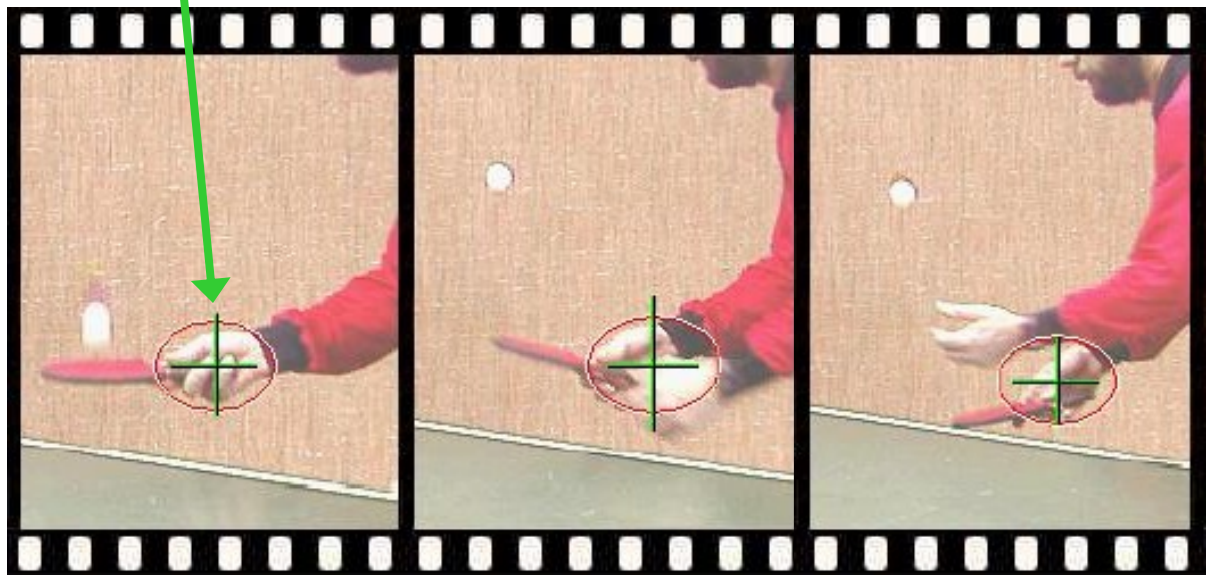
- Translate the Kernel window by **m(x)**

$$g(\mathbf{x}) = -k'(\mathbf{x})$$

Slides from Ukrainitz & Sarel

Mean-Shift Object Tracking - General Framework: Target Representation

Choose a reference model in the current frame → Choose a feature space → Represent the model in the chosen feature space



… Current frame → …

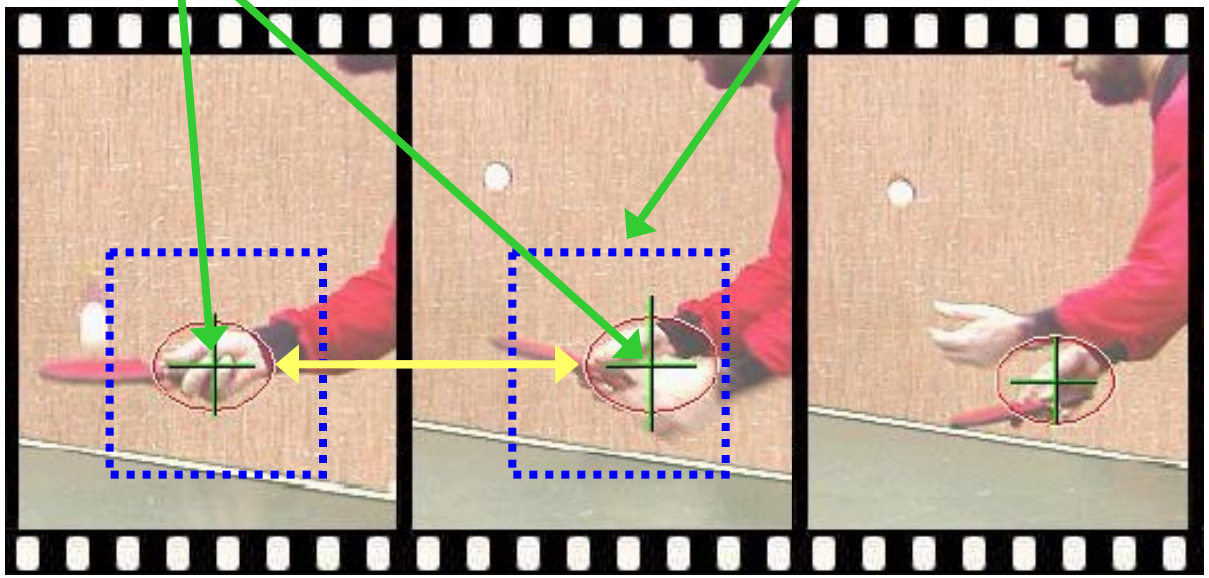
Slides from Ukrainitz & Sarel

Mean-Shift Object Tracking - General Framework: Target Localization

Start from the position of the model in the current frame

Search in the model's neighborhood in next frame

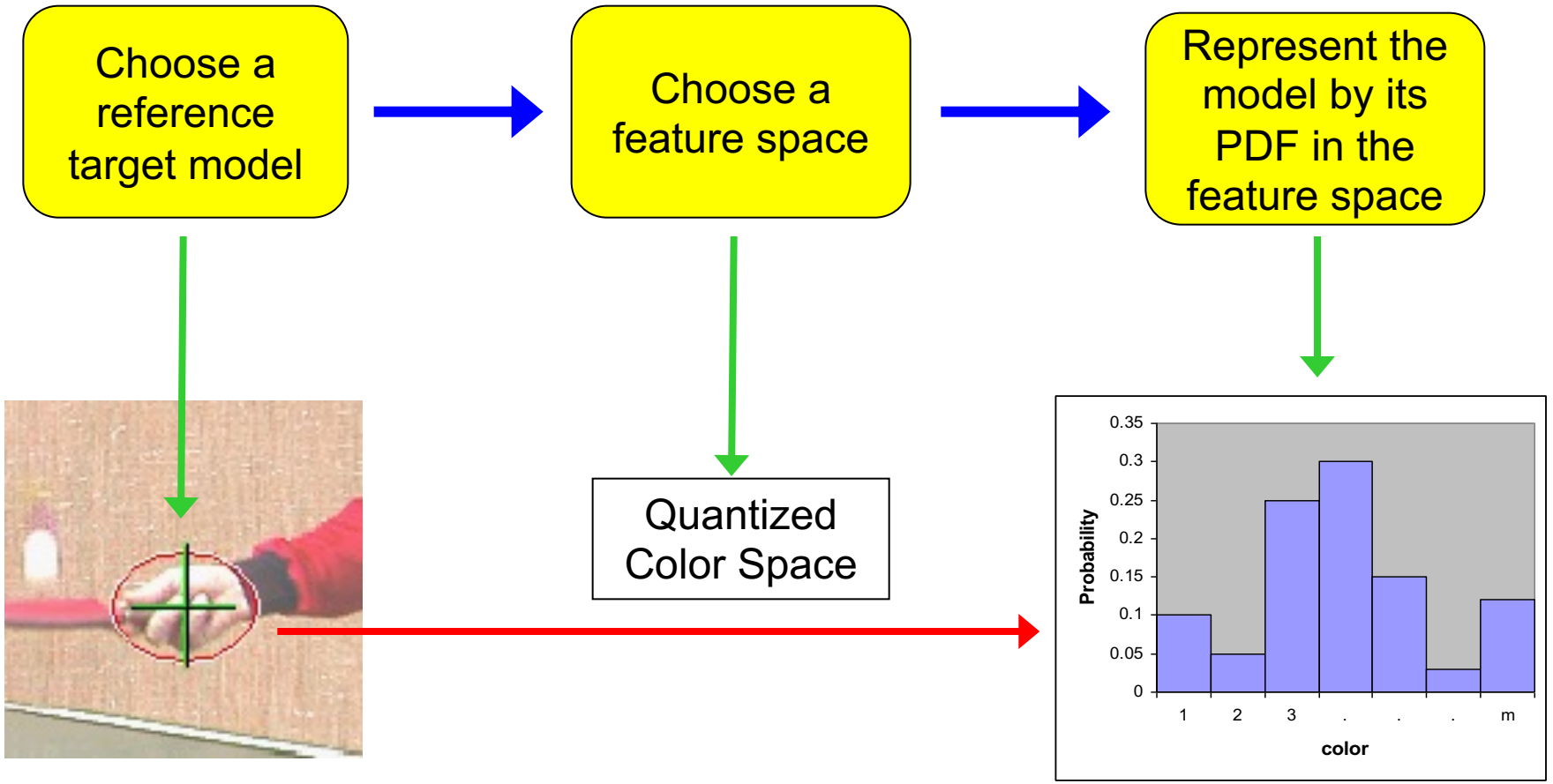
Find best candidate by maximizing a similarity func.



… Current frame …

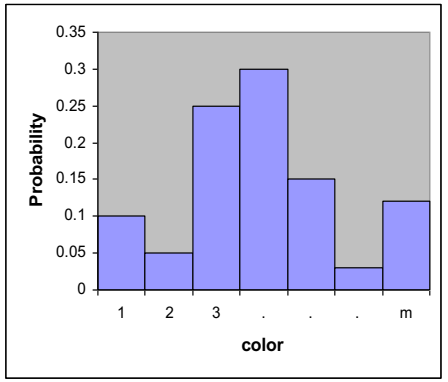
Repeat the same process in the next pair of frames

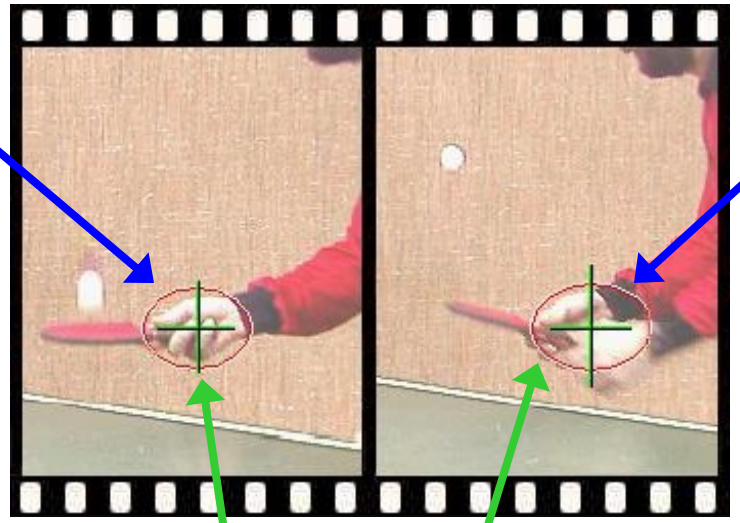
Mean-Shift Object Tracking - Target Representation



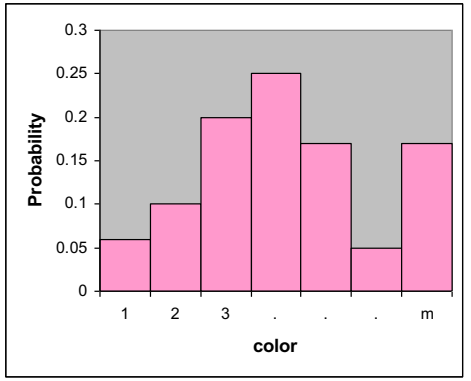
Mean-Shift Object Tracking - PDF Representation

Target Model
(centered at 0)





Target Candidate
(centered at y)



$$\vec{q} = \{q_u\}_{u=1..m} \qquad \sum_{u=1}^{m} q_u = 1$$

$$\vec{p}(y) = \{p_u(y)\}_{u=1..m} \qquad \sum_{u=1}^{m} p_u = 1$$

Similarity Function:

$$f(y) = f[\vec{q}, \vec{p}(y)]$$

Mean-Shift Object Tracking - Similarity Function

Target model: $\vec{q} = (q_1, \ldots, q_m)$

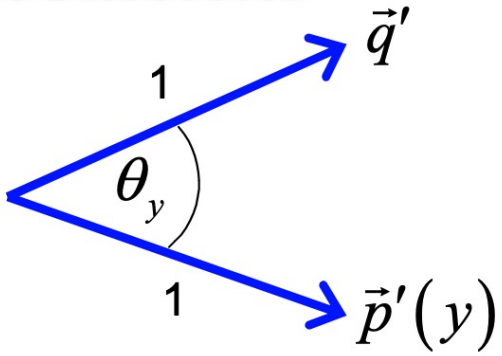
Target candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Similarity function: $f(y) = f[\vec{p}(y), \vec{q}] = ?$

The Bhattacharyya Coefficient

$$\vec{q}' = \left(\sqrt{q_1}, \ldots, \sqrt{q_m}\right)$$

$$\vec{p}'(y) = \left(\sqrt{p_1(y)}, \ldots, \sqrt{p_m(y)}\right)$$



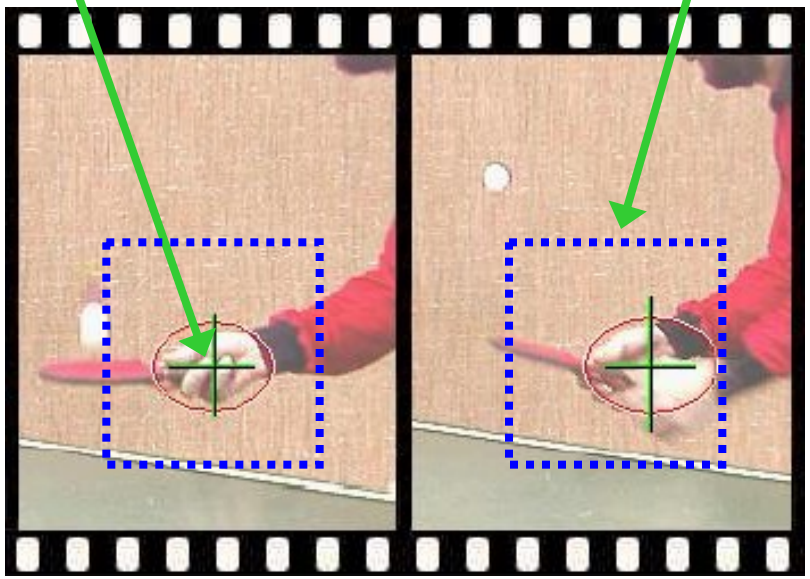
$$f(y) = \cos\theta_y = \frac{p'(y)^T q'}{\|p'(y)\| \cdot \|q'\|} = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Mean-Shift Object Tracking - Target Localization Algorithm

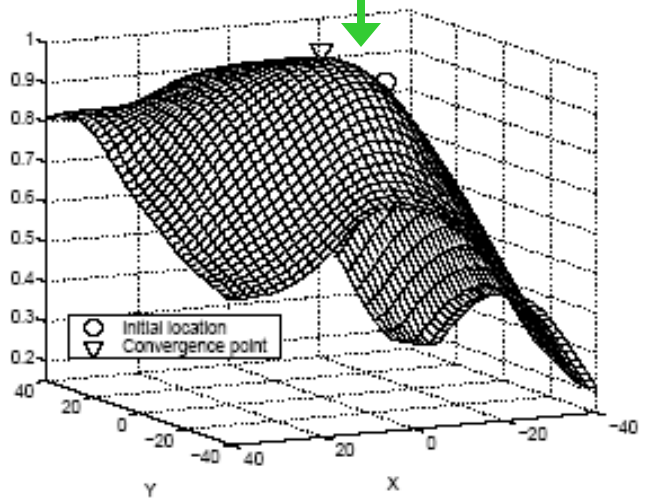
Start from the position of the model in the current frame

Search in the model's neighborhood in next frame

Find best candidate by maximizing a similarity func.



$\vec{q}$ $\vec{p}(y)$



$f\left[\vec{p}(y), \vec{q}\right]$

Mean-Shift Object Tracking - Approximating the Similarity Func.

$$f(y)=\sum_{u=1}^{m}\sqrt{p_u(y)q_u}$$

Model location: y_0

Candidate location: y

Linear approx. (around y_0)

$$f(y)\approx \underbrace{\frac{1}{2}\sum_{u=1}^{m}\sqrt{p_u(y_0)q_u}}_{\text{Independent of } y} + \underbrace{\frac{1}{2}\sum_{u=1}^{m}p_u(y)\sqrt{\frac{q_u}{p_u(y_0)}}}$$

$$p_u(y)=C_h\sum_{b(x_i)=u}k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$$

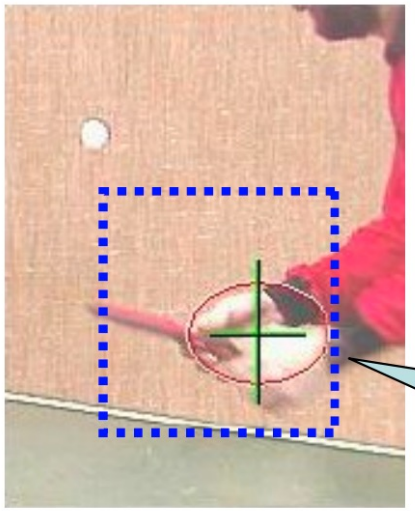
$$\frac{C_h}{2}\sum_{i=1}^{n}w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$$

Density estimate! (as a function of y)

Mean-Shift Object Tracking - Maximizing the Similarity Function

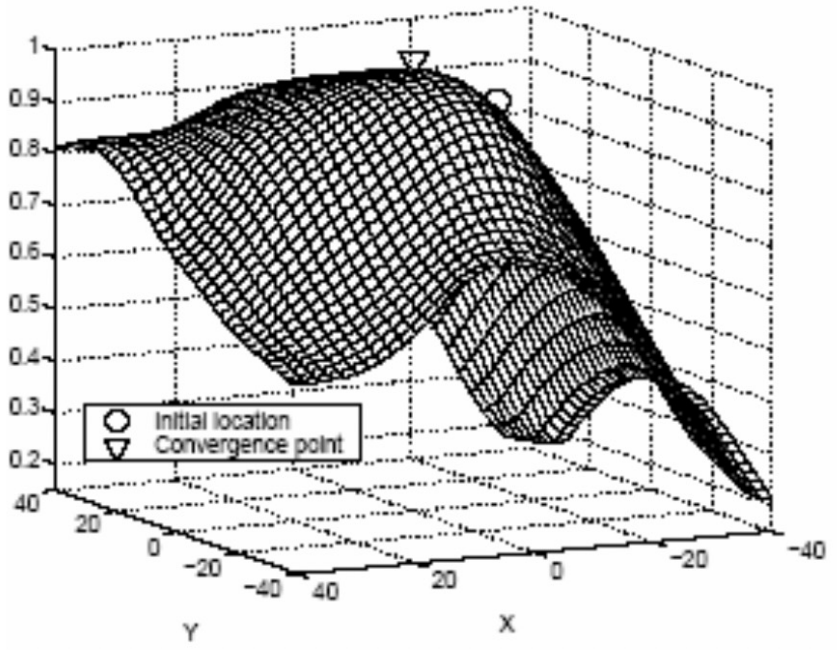
The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

Important Assumption:



The target representation provides sufficient discrimination

One mode in the searched neighborhood



Mean-Shift Object Tracking - Applying Mean-Shift

The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

Original Mean-Shift:

Find mode of $c\sum_{i=1}^{n} k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

Extended Mean-Shift:

Find mode of $c\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x w_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

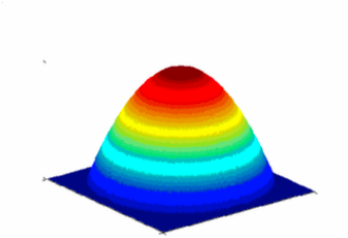
Slides from Ukrainitz & Sarel

Mean-Shift Object Tracking - Choosing the Kernels

A special class of radially symmetric kernels:

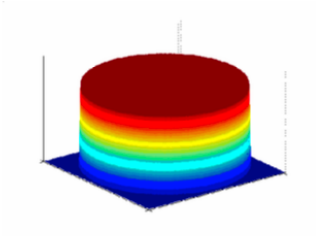
$$K(x) = ck\left(\|x\|^2\right)$$

Epanechnikov kernel:



$$k(x) = \begin{cases} 1-x & \text{if } \|x\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Uniform kernel:



$$g(x) = -k'(x) = \begin{cases} 1 & \text{if } \|x\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$y_1 = \frac{\sum_{i=1}^{n} x_i w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)} \rightarrow y_1 = \frac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i}$$

Likelihood Maximization Using Mean Shift Vector

Maximization of the likelihood of target and candidate depends on the weights:

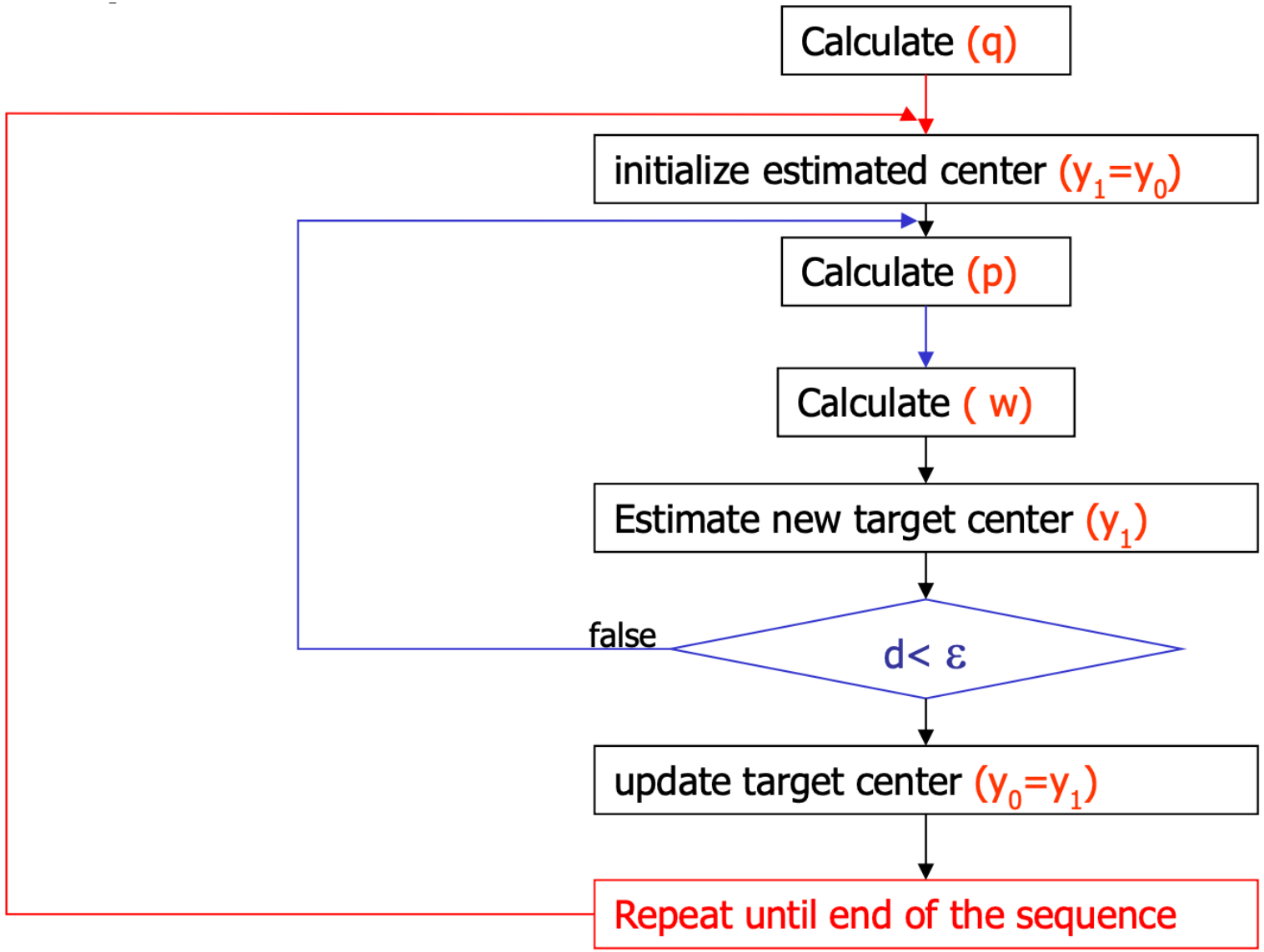
$$w_i(\mathbf{y}_o) = \sum_{u=1}^{m} \delta[S(\mathbf{x}_i) - u] \sqrt{\frac{q_u}{\hat{p}_u(\mathbf{y}_o)}}$$

Again, the mean shift vector M is:

$$M_h(\mathbf{y}_0) = \frac{\sum_{i=1}^{n_x} w_i(\mathbf{y}_0)\mathbf{x}_i}{\sum_{i=1}^{n_x} w_i(\mathbf{y}_0)} - \mathbf{y}_0$$

Thus, new target center is $\hat{\mathbf{y}} = \mathbf{y}_0 + M_h(\mathbf{y}_0)$

Algorithm


Calculate (q)
initialize estimated center (y_1=y_0)
Calculate (p)
Calculate (w)
Estimate new target center (y_1)
false
d< ε
update target center (y_0=y_1)
Repeat until end of the sequence

Mean-Shift Pros/Cons

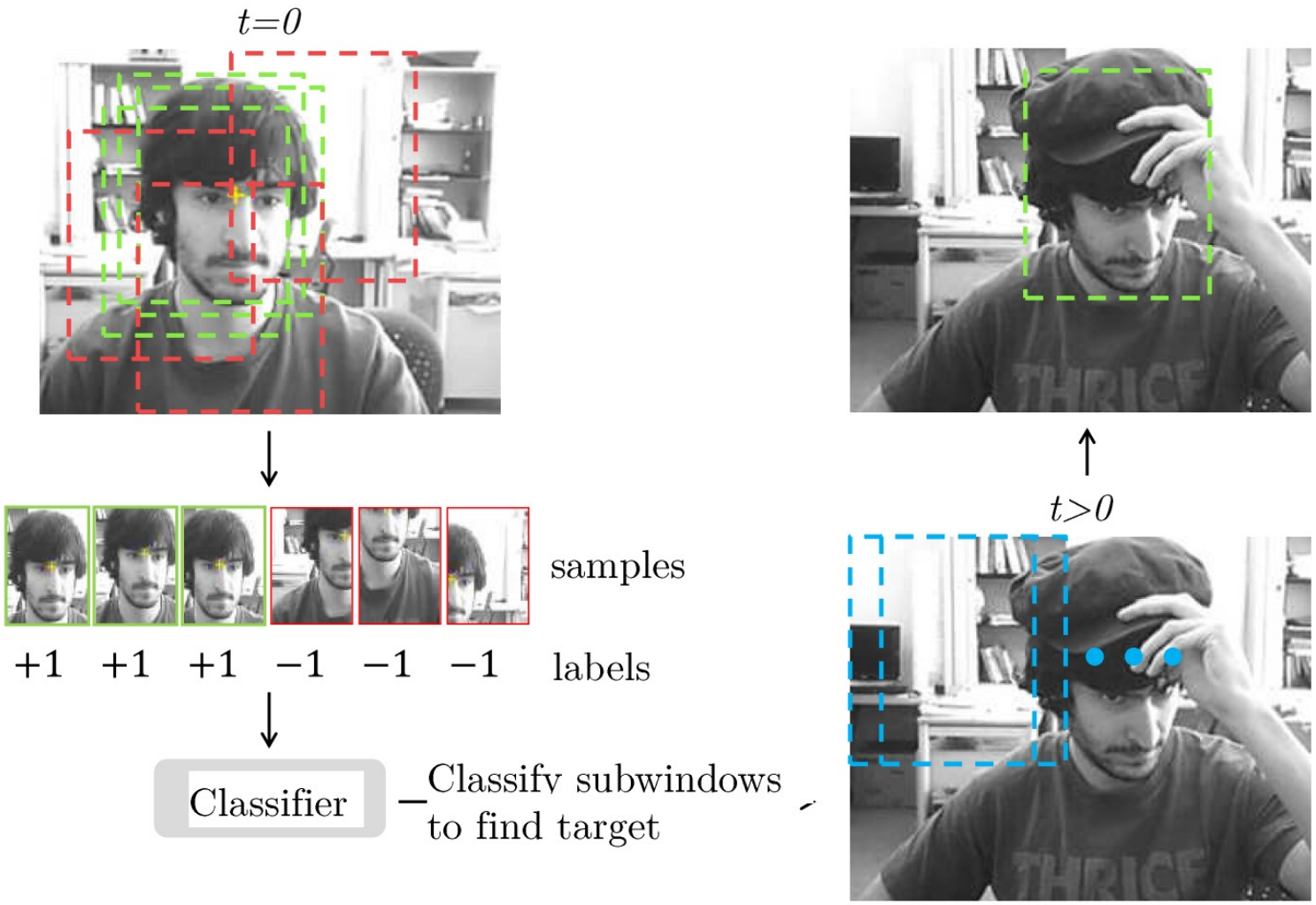
- Low computational cost (easily real-time)
- Surprisingly robust
  - Invariant to pose and viewpoint
  - Often no need to update reference color model
- Invariance comes at a price
  - Position estimate prone to fluctuation
  - Scale and orientation not well captured
  - Sensitive to color clutter (e.g., teamates in team sports)
- Local search by gradient descent
- Problems:
  - abrupt moves
  - occlusions

Slides from Jiri Matas 2016

Mean-Shift Example

https://youtu.be/RG5uV_h50b0

Discriminative Tracking: Tracking by Detection



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Connection to Correlation

- Let's have a linear classifier with weights $\mathbf{w}$

$$y = \mathbf{w}^T \mathbf{x}$$

- During tracking we want to evaluate the classifier at subwindows $\mathbf{x}_i$:

$$y_i = \mathbf{w}^T \mathbf{x}_i$$

- Then we can concatenate y_i into a vector $\mathbf{y}$ (i.e. response map)

- This is equivalent to **cross-correlation** formulation which can be computed efficiently in Fourier domain

$$\mathbf{y} = \mathbf{x} \circledast \mathbf{w}$$

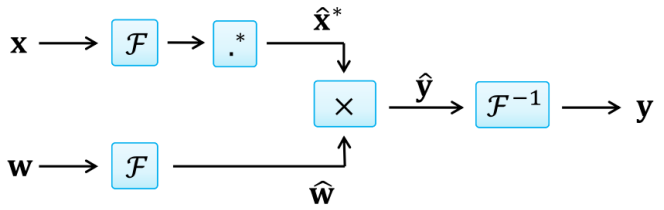
- Note: Convolution is related; it is the same as cross-correlation, but with the flipped image of $\mathbf{w}$ (P → d).

The Convolution Theorem

"Cross-correlation is **equivalent** to an **element-wise product** in Fourier domain"

$$\mathbf{y} = \mathbf{x} \circledast \mathbf{w} \iff \hat{\mathbf{y}} = \hat{\mathbf{x}}^* \times \hat{\mathbf{w}}$$

- In practice:



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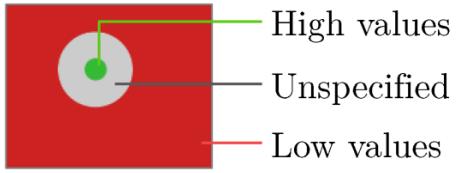
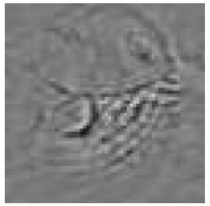
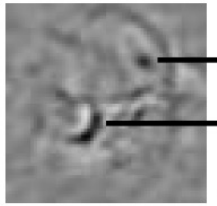
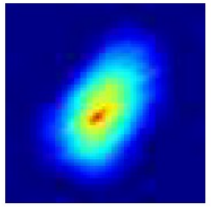
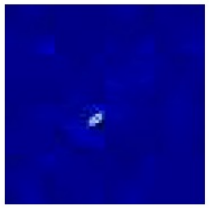
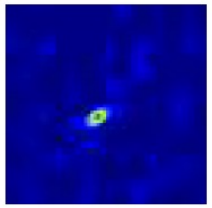
How to Decide Classifier Weights, w?

Training image: $\mathbf{x} =$ 

Objective

 $\circledast\ \mathbf{w} =$ 

	Naïve filter ($\mathbf{w} = \mathbf{x}$)	Sharp peak (UMACE)	Gaussian peak (GMACE)
Classifier ($\mathbf{w}$)			
Output ($\mathbf{w} * \mathbf{x}$)			

- A good compromise.
- Tiny details are ignored.
- focuses on **larger, more robust structures.**

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Minimum Output Sum of Squared Error (MOSSE) based Correlation Filter

Slides from Jiri Matas 2016

A Nice Summary for Tracking API in OpenCV

- https://www.learnopencv.com/object-tracking-using-opencv-cpp-python/

Deep Learning based Tracking

Supervised Tracking

- **Paper : Detect to Track and Track to Detect**

- A ConvNet architecture for simultaneous detection and tracking, using a multi-task objective for frame-based object detection and across-frame track regression

- Introduces correlation features that represent object co-occurrences across time to aid the ConvNet during tracking.

- Links the frame level detections based on our across-frame tracklets to produce high accuracy detections at the video level

https://arxiv.org/pdf/1710.03958.pdf

Backbone network : R-FCN

- R-FCNs are convolutional networks used for object detection.
- It is faster than traditional object detection network such as R-CNN as its region-based feature maps are independent of region of interests.
- The network is lighter because it is fully convolutional and has no FC layers.
- The output of the network is a set of bounding coordinates and class labels for detected objects.

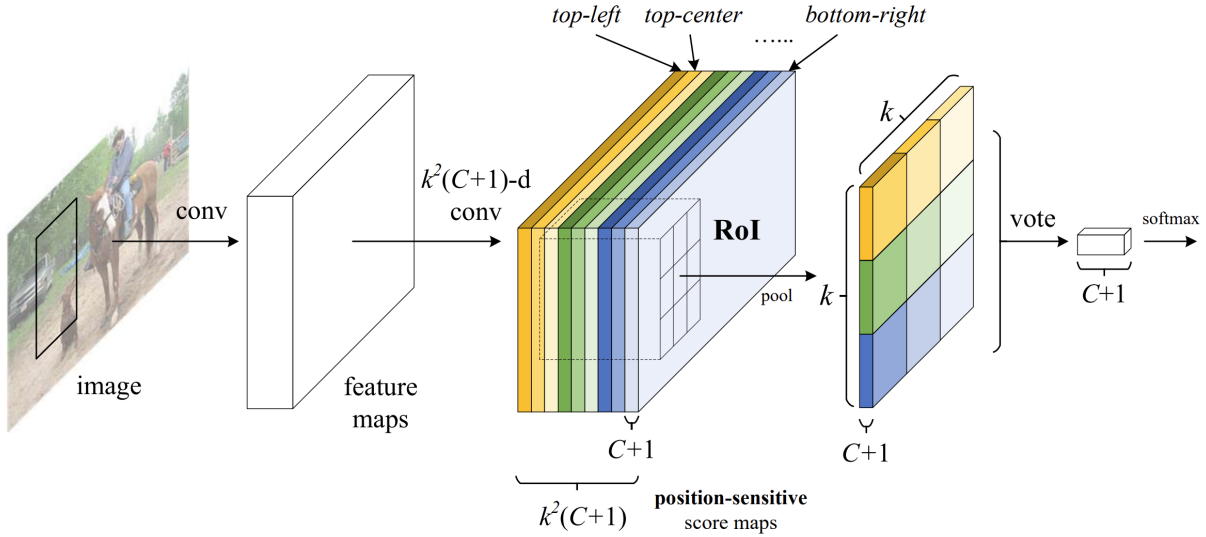
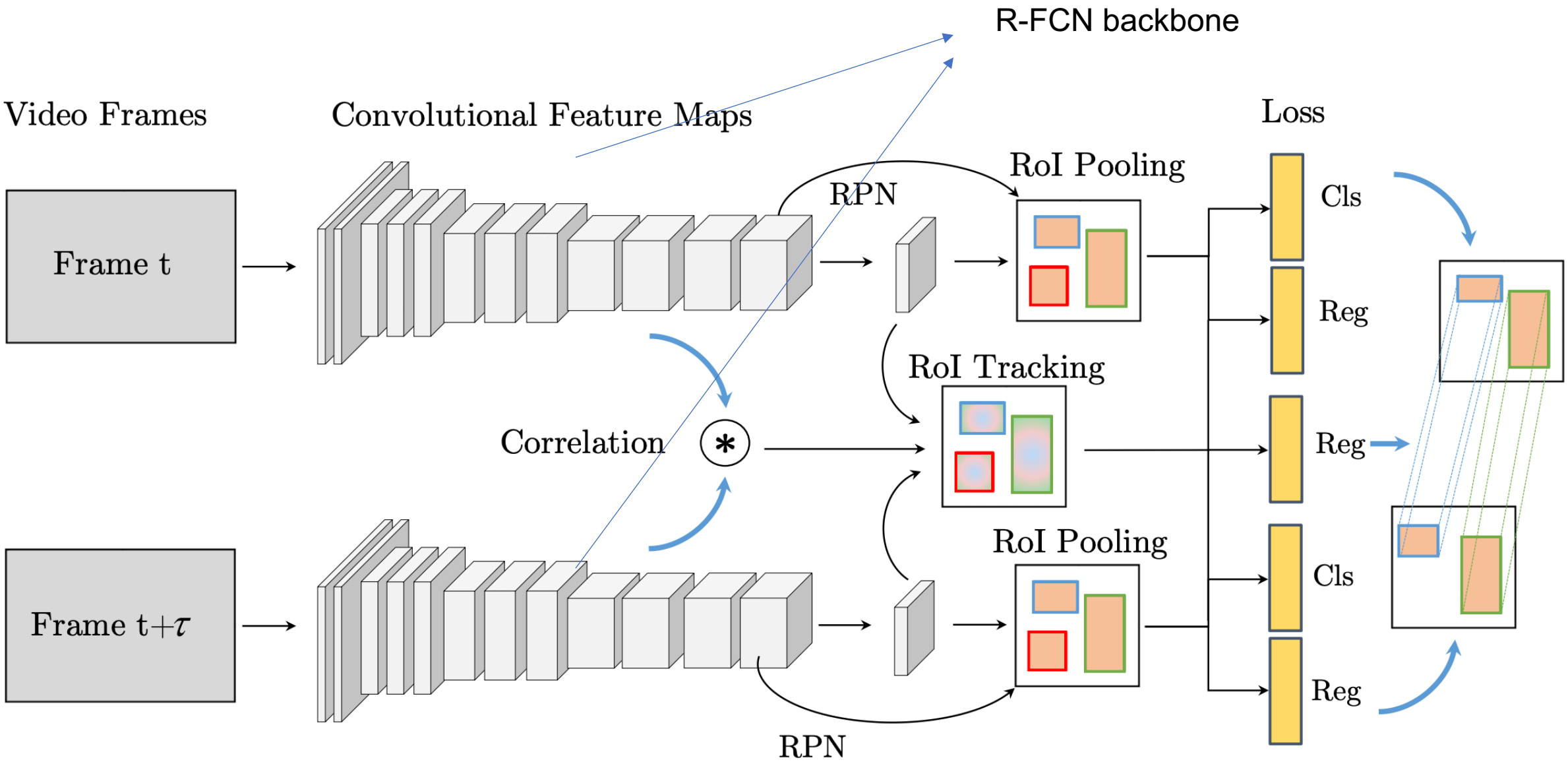


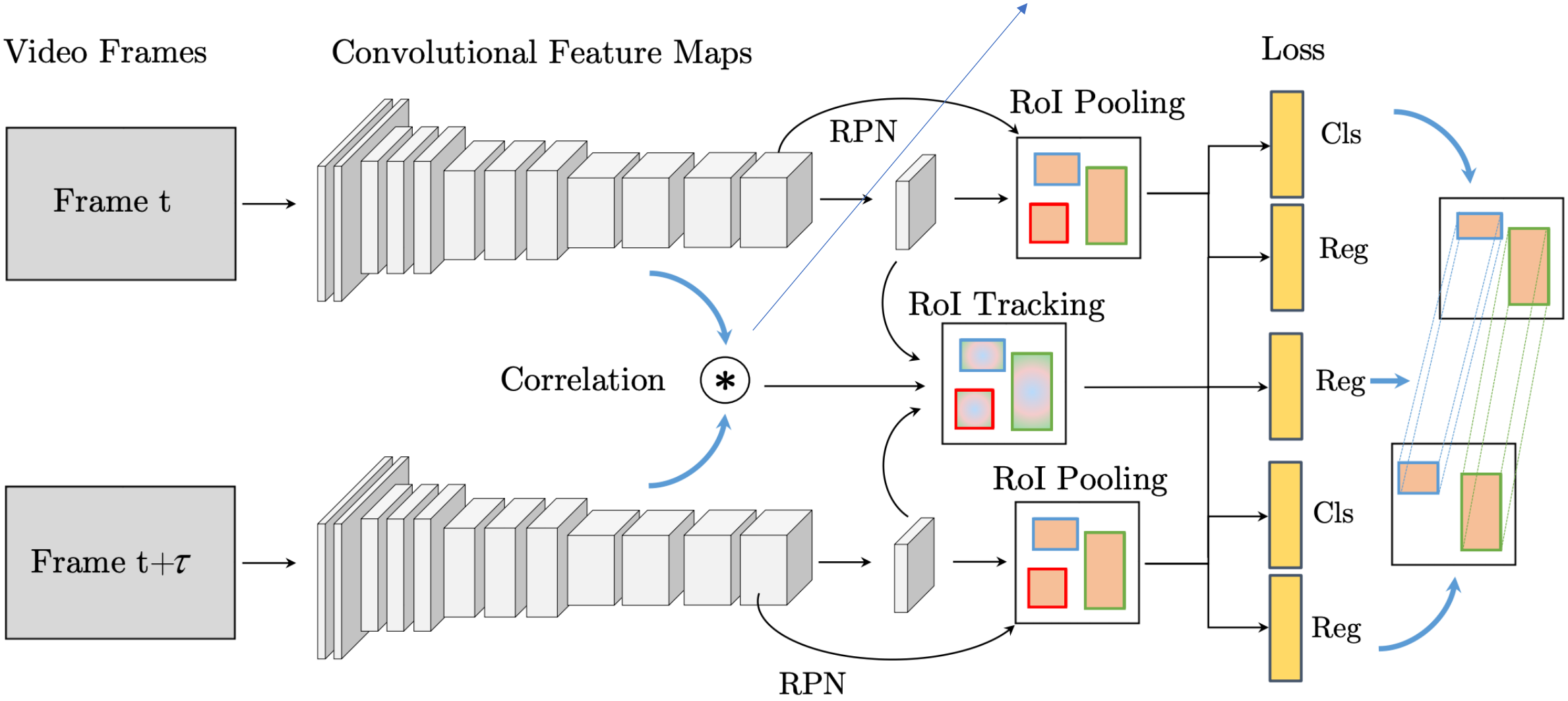
Figure 1: Key idea of **R-FCN** for object detection. In this illustration, there are $k \times k = 3 \times 3$ position-sensitive score maps generated by a fully convolutional network. For each of the $k \times k$ bins in an RoI, pooling is only performed on one of the k^2 maps (marked by different colors).

https://arxiv.org/pdf/1605.06409.pdf

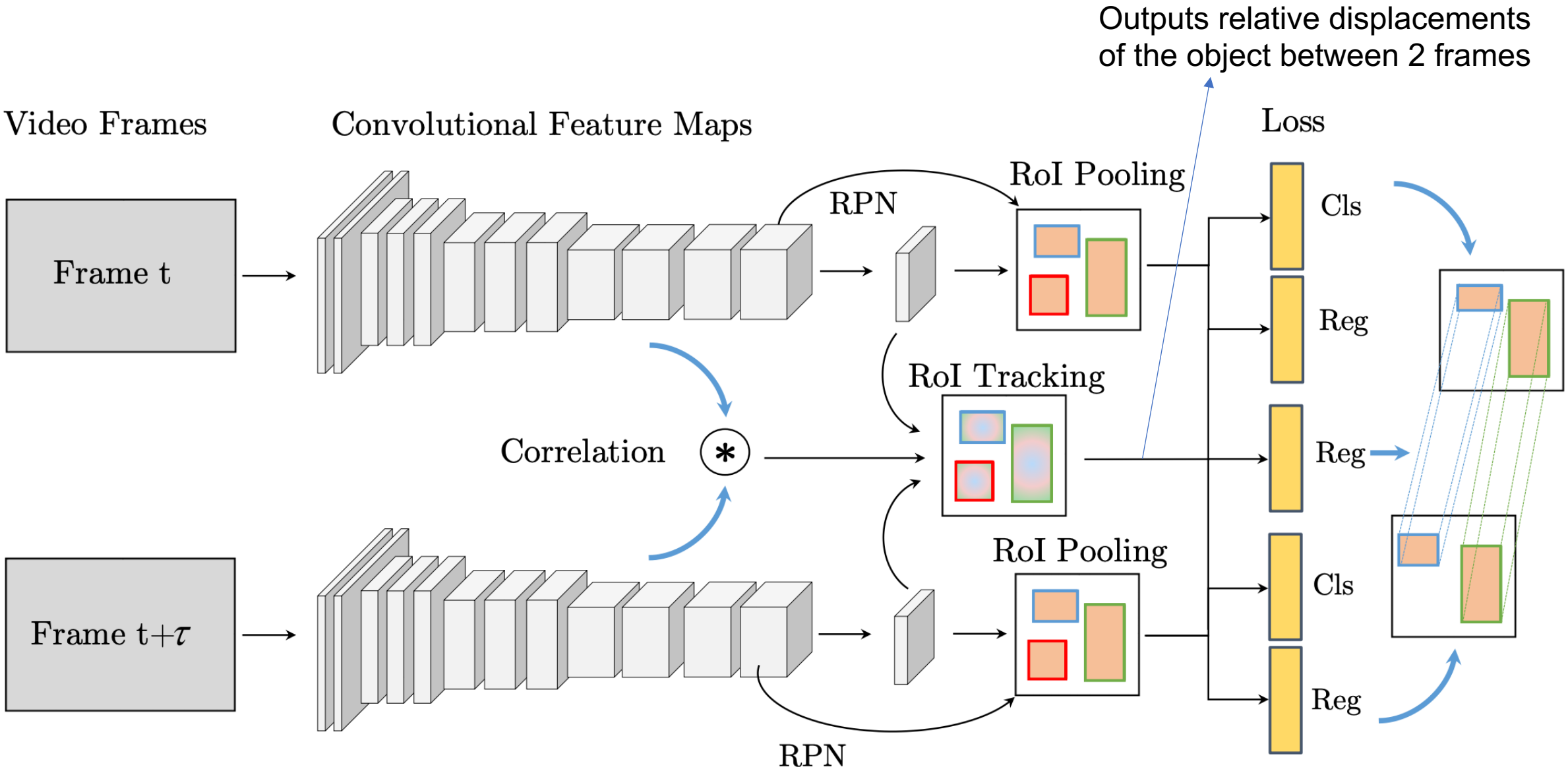
Overall Architecture



Overall Architecture



Overall Architecture



Objective function

$$L(\{p_i\},\{b_i\},\{\Delta_i\}) = \frac{1}{N}\sum_{i=1}^{N} L_{cls}(p_{i,c^*})$$

→ Classification loss (foreground/background)

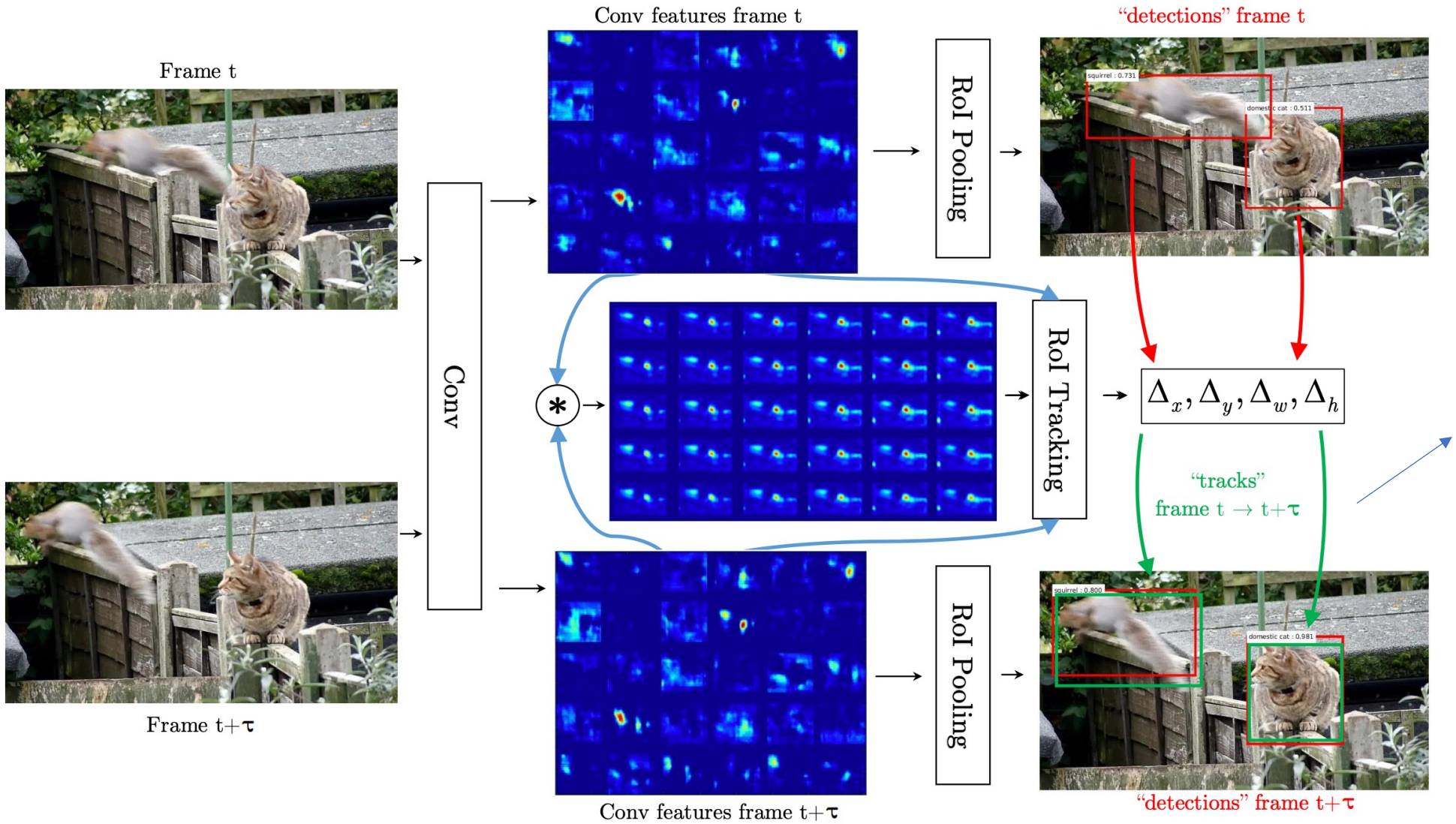
$$+\lambda\frac{1}{N_{fg}}\sum_{i=1}^{N}[c_i^* > 0]L_{reg}(b_i, b_i^*)$$

→ Bounding box regression cost (for both frames)

$$+\lambda\frac{1}{N_{tra}}\sum_{i=1}^{N_{tra}} L_{tra}(\Delta_i^{t+\tau}, \Delta_i^{*,t+\tau}).$$

→ Regression offsets

Inference



Bounding boxes for t+τ is computed using boxes for frame t and the predicted offsets

Self-supervised Tracking

- To learn a feature extractor without any supervision

- Self-supervised learning via cycle consistency

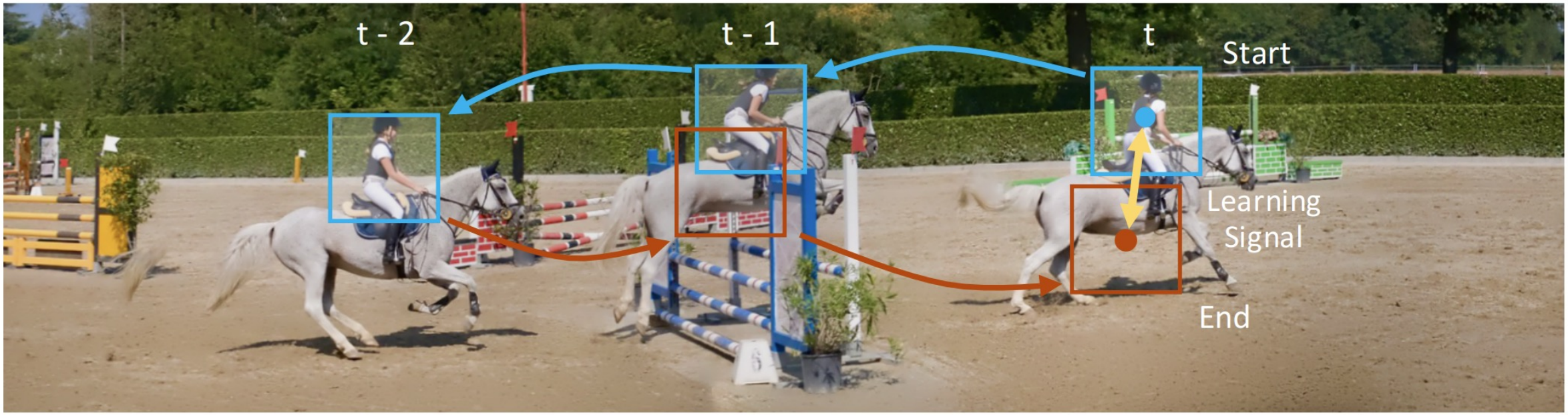


Figure 2: **A Cycle in Time.** Given a video, tracking along the sequence formed by a cycle in time can be self-supervised: the target is simply the beginning of the cycle. The yellow arrow between the start and end represents the differentiable learning signal.

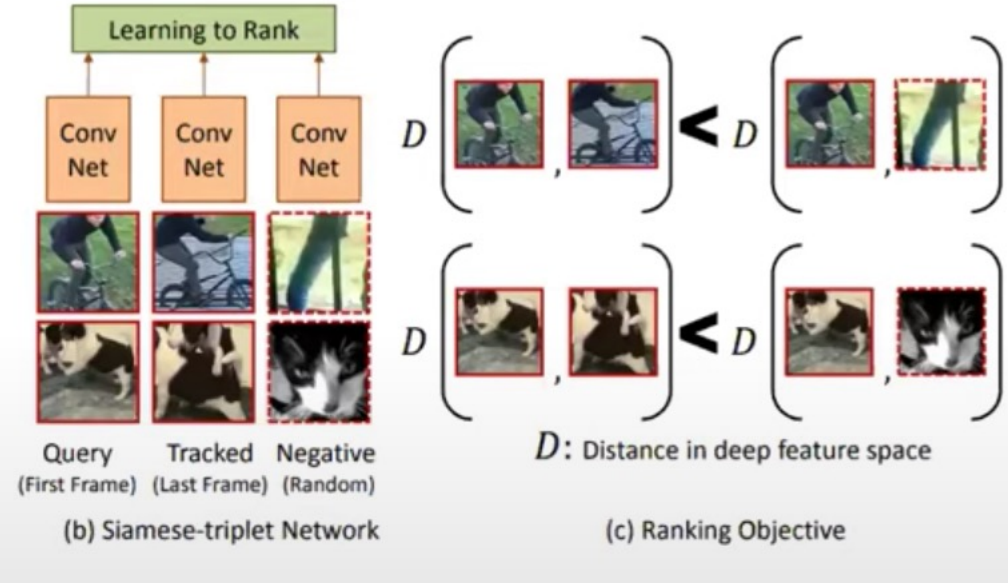
Previous Self-Supervised Correspondence Learning

- Tracking-based Similarity Learning

(a) Prepare Query (Positive & Negative)

(b) Obtain Positive via Hand-craft tracker

(c) Use Siamese-Triplet Network

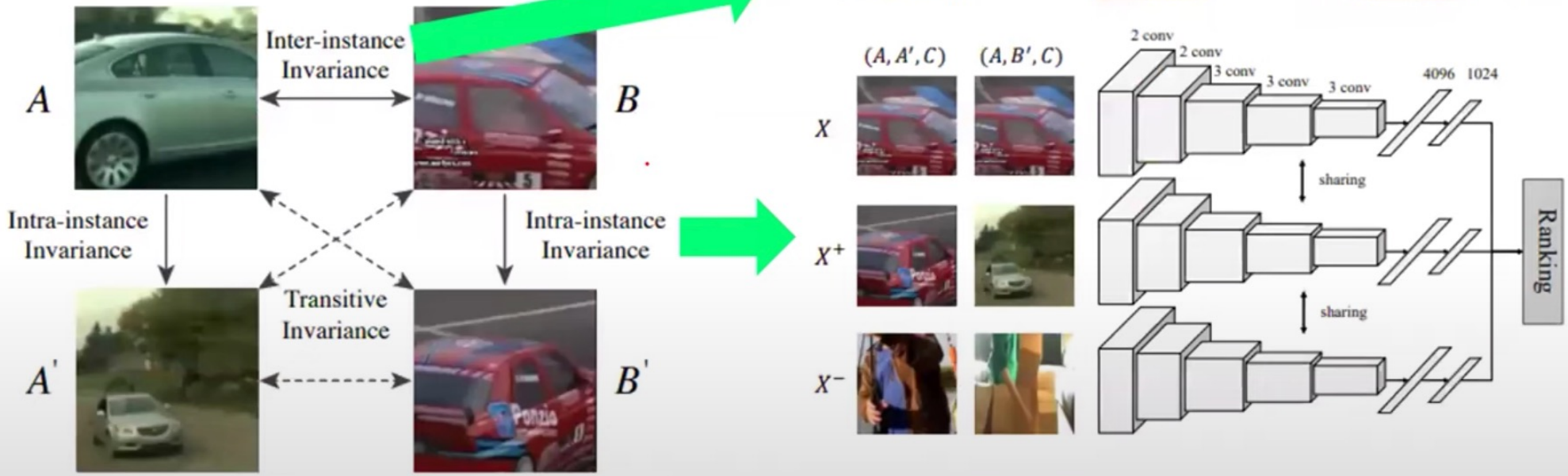


https://www.youtube.com/watch?v=5g8-8Hz5x08

https://arxiv.org/pdf/1903.07593.pdf

Previous Self-Supervised Correspondence Learning

- Tracking-based Similarity Learning
- Transitive Invariance Learning



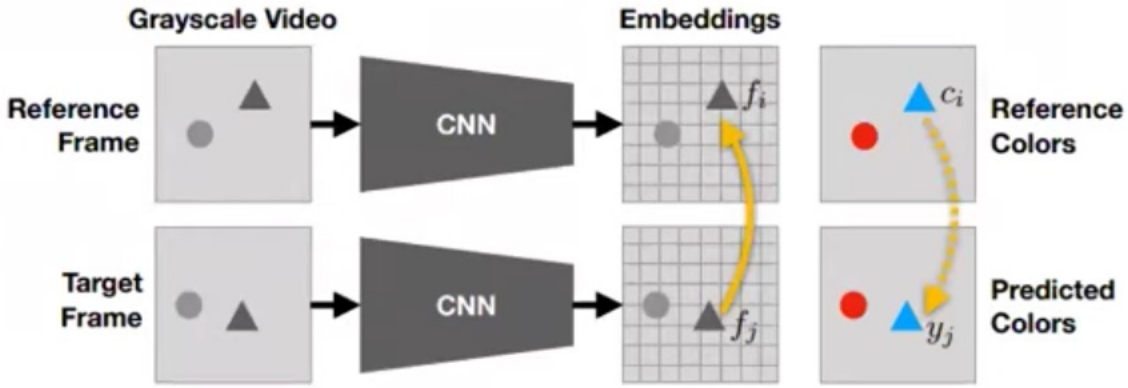
X. Wang, K. He, and A. Gupta. Transitive invariance for selfsupervised visual representation learning. In ICCV, 2017

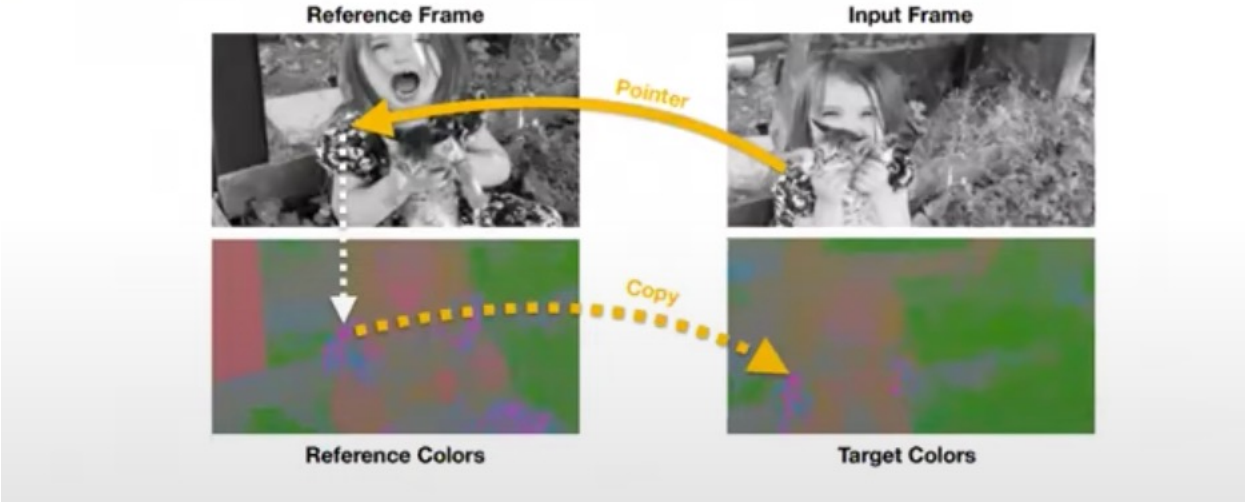
https://arxiv.org/pdf/1708.02901.pdf

Previous Self-Supervised Correspondence Learning

- Tracking-based Similarity Learning
- Transitive Invariance Learning
- Colorization

- (a) Find similar features
- (b) Copy colors from reference frame
- (c) Learn CNN using colorization loss

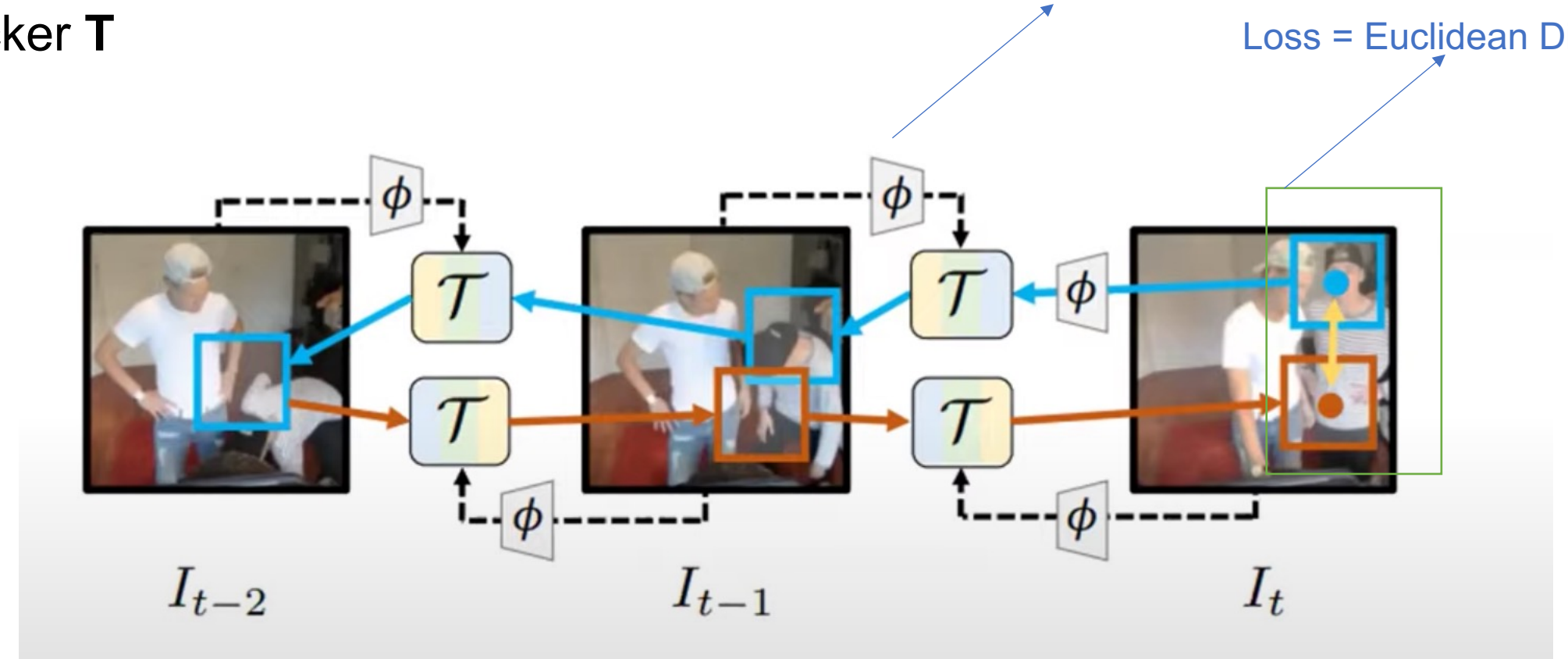




https://arxiv.org/pdf/1806.09594.pdf

Cycle-in-Time

- Feature extractor **Φ**
- Tracker **T**



https://arxiv.org/pdf/1903.07593.pdf

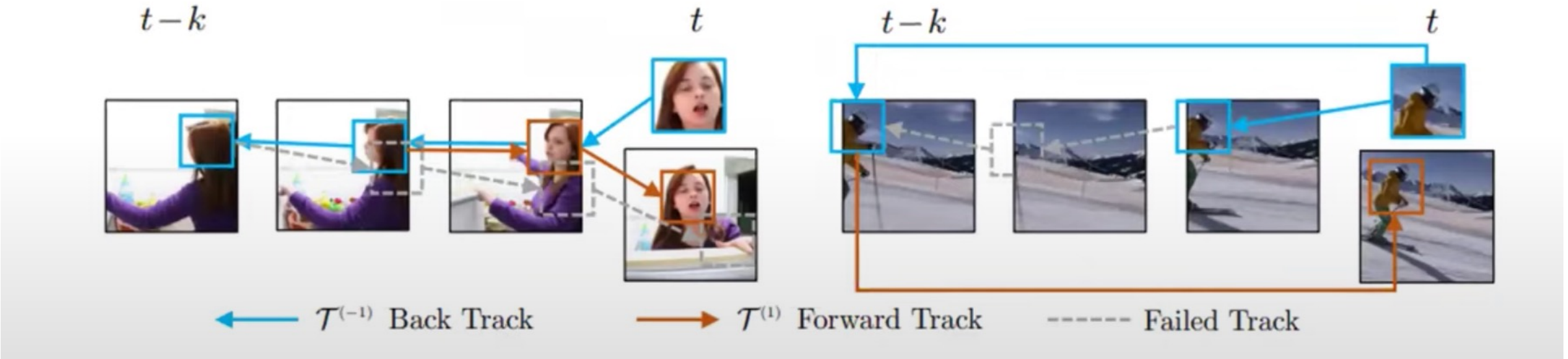
Multiple Cycles / Skip Cycle

- Multiple cycles for variation
- Skip cycle for occlusion

Mean region similarity

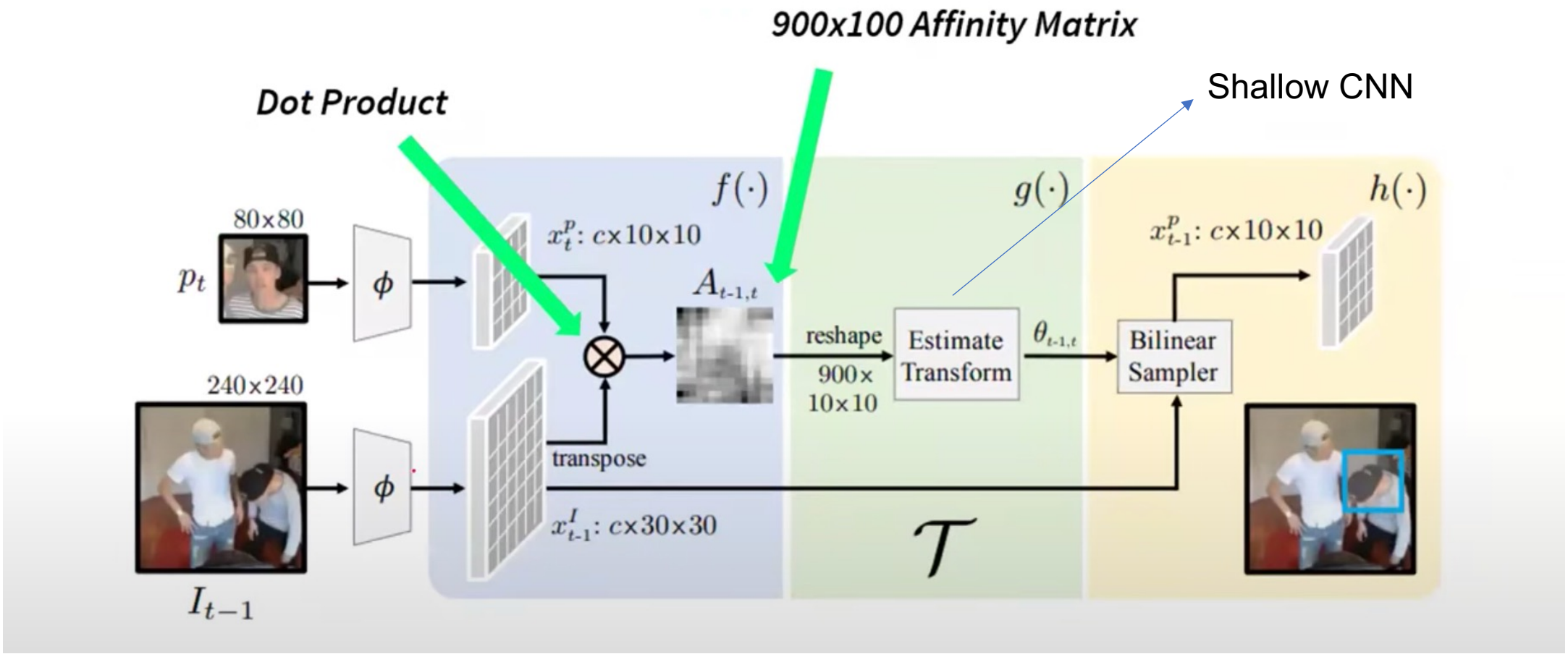
Contour based accuracy

Experiment	$\mathcal{J}$(Mean)	$\mathcal{F}$(Mean)
Ours	41.9	39.4
Ours without Skip Cycles	39.5	37.9



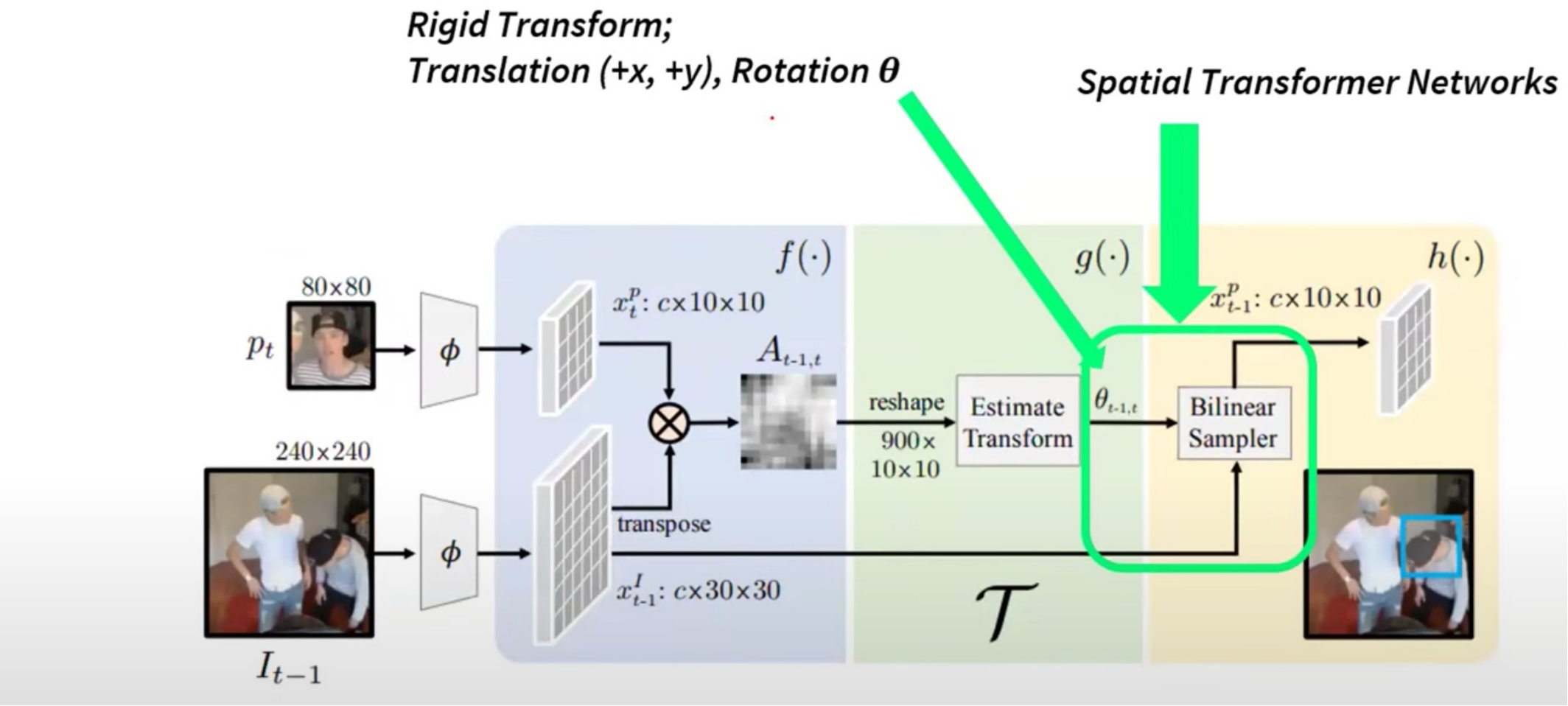
https://arxiv.org/pdf/1903.07593.pdf

Differentiable Tracker



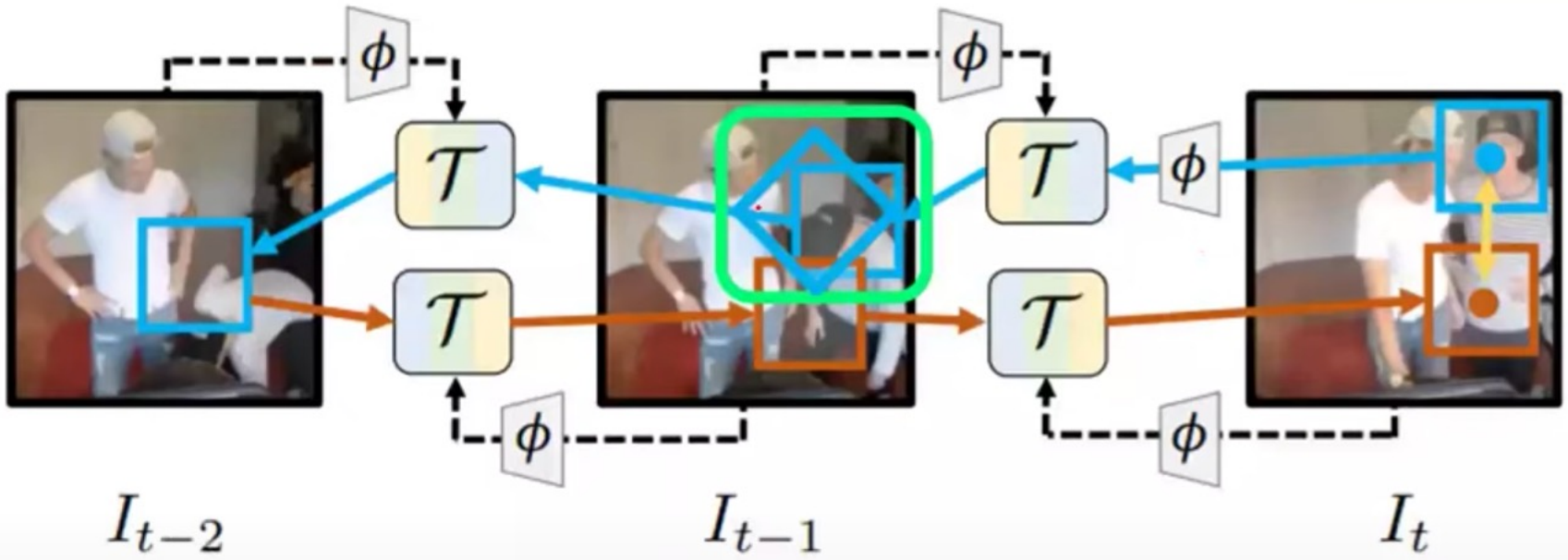
https://arxiv.org/pdf/1903.07593.pdf

Differentiable Tracker



https://arxiv.org/pdf/1506.02025.pdf

Training



https://arxiv.org/pdf/1903.07593.pdf

Results : Video Object Segmentation

model	Supervised	$\mathcal{J}$(Mean)	$\mathcal{F}$(Mean)
Identity		22.1	23.6
Random Weights (ResNet-50)		12.4	12.5
Optical Flow (FlowNet2) [22]		26.7	25.2
SIFT Flow [39]		33.0	35.0
Transitive Inv. [74]		32.0	26.8
DeepCluster [8]		37.5	33.2
Video Colorization [69]		34.6	32.7
Ours (ResNet-18)		40.1	38.3
Ours (ResNet-50)		**41.9**	**39.4**
ImageNet (ResNet-50) [18]	✓	50.3	49.0
Fully Supervised [81, 7]	✓	55.1	62.1

https://arxiv.org/pdf/1903.07593.pdf

Results: Pose Propagation

model	Supervised	PCK@.1	PCK@.2
Identity		43.1	64.5
Optical Flow (FlowNet2) [22]		45.2	62.9
SIFT Flow [39]		49.0	68.6
Transitive Inv. [74]		43.9	67.0
DeepCluster [8]		43.2	66.9
Video Colorization [69]		45.2	69.6
Ours (ResNet-18)		57.3	78.1
Ours (ResNet-50)		**57.7**	**78.5**
ImageNet (ResNet-50) [18]	✓	58.4	78.4
Fully Supervised [59]	✓	68.7	92.1

Probability of Correct Keypoint

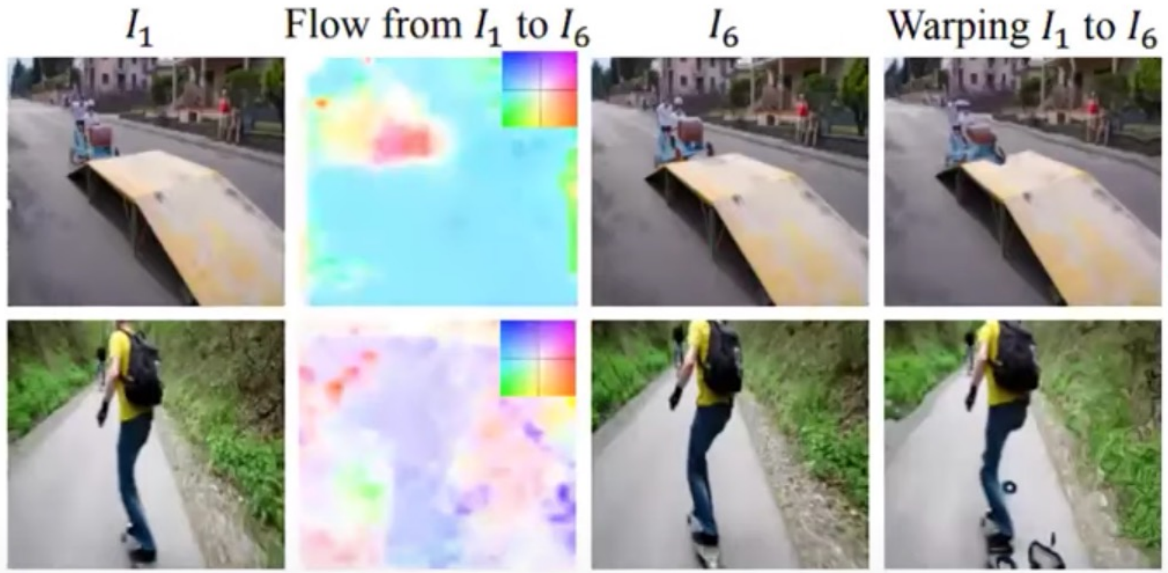
https://arxiv.org/pdf/1903.07593.pdf

Results: Human Part Labels Propagation

model	Supervised	mIoU	AP^r_{vol}
Identity		13.6	4.0
Optical Flow (FlowNet2) [22]		16.1	8.3
SIFT Flow [39]		21.3	10.5
Transitive Inv. [74]		19.4	5.0
DeepCluster [8]		21.8	8.1
Ours (ResNet-50)		**28.9**	**15.6**
ImageNet (ResNet-50) [18]	✓	34.7	16.1
Fully Supervised [85]	✓	37.9	24.1

https://arxiv.org/pdf/1903.07593.pdf

Results: Long-range Optical Flow



model	5-F	10-F
Identity	82.0	97.7
Optical Flow (FlowNet2) [22]	62.4	90.3
ImageNet (ResNet-50) [18]	64.0	79.2
Ours (ResNet-50)	**60.4**	**76.4**

https://arxiv.org/pdf/1903.07593.pdf

Next week

- Segmentation
  - + Graph-based segmentation
  - + Semantic segmentation
  - + Instance segmentation
  - + Panoptic segmentation
- 3D Deep Learning (Recognition and Segmentation)
  - + VoxNet
  - + PointNet
  - + MVCNN
  - + FoldingNet

- + : know the concept

References for next week

- Sz: Ch 6.4

- Maturana, Daniel, and Sebastian Scherer. "Voxnet: A 3d convolutional neural network for real-time object recognition." *2015 IEEE/RSJ international conference on intelligent robots and systems (IROS)*. IEEE, 2015.
- Su, Hang, et al. "Multi-view convolutional neural networks for 3d shape recognition." *Proceedings of the IEEE international conference on computer vision*. 2015.
- Qi, C. R., Su, H., Mo, K., & Guibas, L. J. (2017). Pointnet: Deep learning on point sets for 3d classification and segmentation. In *Proceedings of the IEEE conference on computer vision and pattern recognition* (pp. 652-660).
- Yang, Y., Feng, C., Shen, Y., & Tian, D. (2018). Foldingnet: Point cloud auto-encoder via deep grid deformation. In *Proceedings of the IEEE conference on computer vision and pattern recognition* (pp. 206-215).