Second-Order Lab: Second-Order Linear DEs in MATLAB

In this lab, you will learn how to use `iode` to plot solutions of second-order ODEs. You will also learn to classify the behaviour of different types of solutions.

Moreover, you will write your own Second-Order ODE system solver, and compare its results to those of `iode`.

Opening the m-file lab5.m in the MATLAB editor, step through each part using cell mode to see the results.

There are seven (7) exercises in this lab that are to be handed in on the due date of the lab.

Contents

Student Information

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```

Iode for Second-Order Linear DEs with constant coefficients

In the `iode` menu, select the `Second order linear ODEs` module. It opens with a default DE and a default forcing function `f(t) = cos(2t)`. The forcing function can be plotted along with the solution by choosing `Show forcing function` from the `Options` menu.

Use this module to easily plot solutions to these kind of equations.

There are three methods to input the initial conditions:

Method 1. Enter the values for `t0`, `x(t0)`, and `x'(t0)` into the `Initial conditions` boxes, and then click `Plot solution`.

Method 2. Enter the desired slope `x'(t0)` into the appropriate into the `Initial conditions` box, and then click on the graph at the point `(t0,x(t0))` where you want the solution to start.

Method 3. Press down the left mouse button at the desired point `(t0,x(t0))` and drag the mouse a short distance at the desired slope `x'(t0)`. When you release the mouse button, `iode` will plot the solution.

Growth and Decay Concepts

We want to classify different kinds of behaviour of the solutions. We say that a solution:

`grows` if its magnitude tends to `infinity` for large values of `t`, that is, if either the solution tends to $+\infty$ or $-\infty$,

`decays` if its magnitude converges to `0` for large values of `t`,

`decays while oscillating` if it keeps changing sign for large values of `t` and the amplitude of the oscillation tends to zero,

`grows while oscillating` if it keeps changing sign for large values of `t` and the amplitude of the oscillation tends to infinity.

Example

```
t = 0:0.1:10;

% Example 1
figure();
y1 = exp(t);
plot(t,y1)

% Annotate the figure
xlabel('t');
ylabel('f_1(t)');
title('The function e^t grows');
legend('f_1(t)=e^t');



% Example 2
figure();
y2 = -exp(-t);
plot(t,y2)

% Annotate the figure
xlabel('t');
ylabel('f_2(t)');
title('The function -e^{-t} decays');
legend('f_2(t)=-e^{-t}');



% Example 3
figure();
y3 = exp(-t);
plot(t,y3)

% Annotate the figure
xlabel('t');
ylabel('f_3(t)');
title('The function e^{-t} decays');
legend('f_3(t)=e^{-t}');
```

```
% Example 4
figure();
y4 = exp(-t).*cos(t);
plot(t,y4)

% Annotate the figure
xlabel('t');
ylabel('f_4(t)');
title('The function e^{-t}cos(t) decays while oscillating');
legend('f_4(t)=e^{-t}*cos(t)');



% Example 5
figure();
y5 = exp(t).*sin(2*t);
plot(t,y5)

% Annotate the figure
xlabel('t');
ylabel('f_5(t)');
title('The function e^{t}sin(2t) grows while oscillating');
legend('f_5(t)=e^{t}sin(2t)');

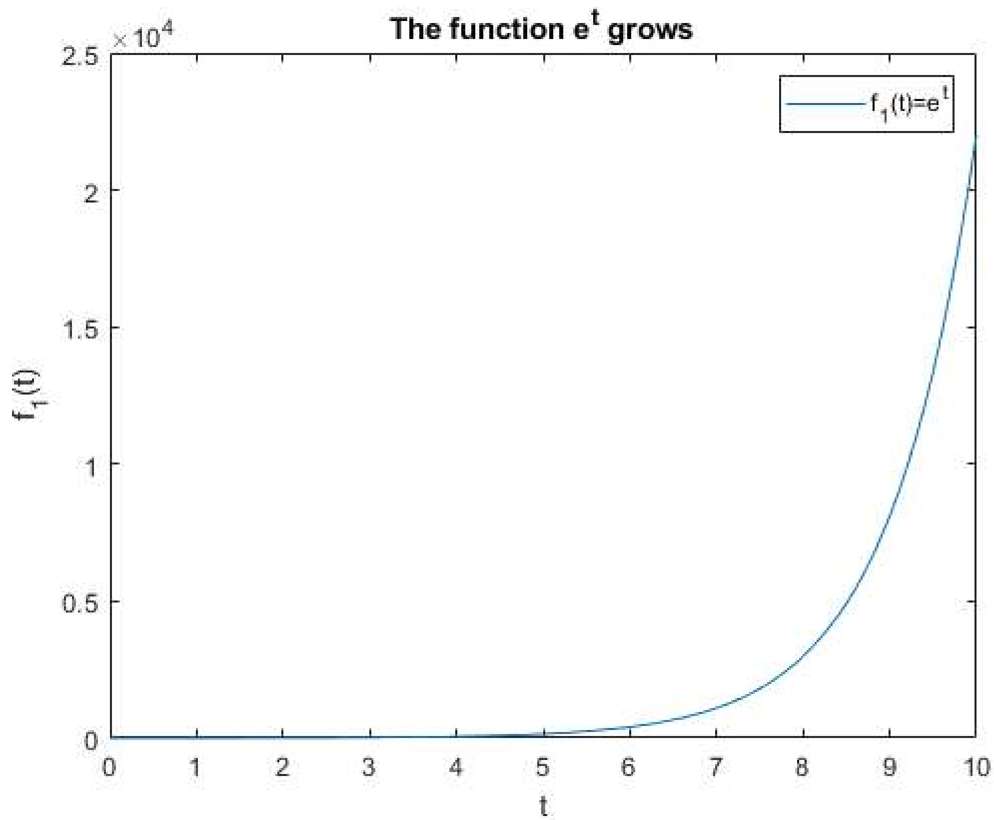


% Example 6
figure();
y6 = sin(3*t);
plot(t,y6)

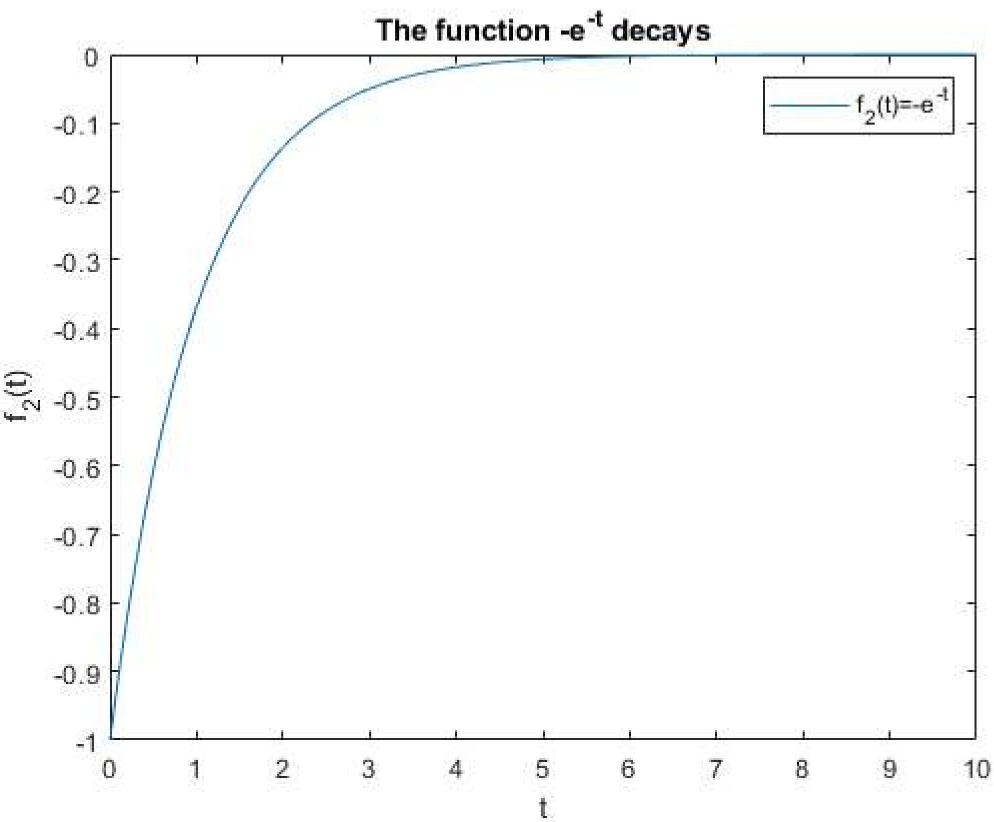
% Annotate the figure
xlabel('t');
ylabel('f_6(t)');
title('The function sin(3t) neither decays nor grows, it just oscillates');
legend('f_6(t)=sin(3t)');



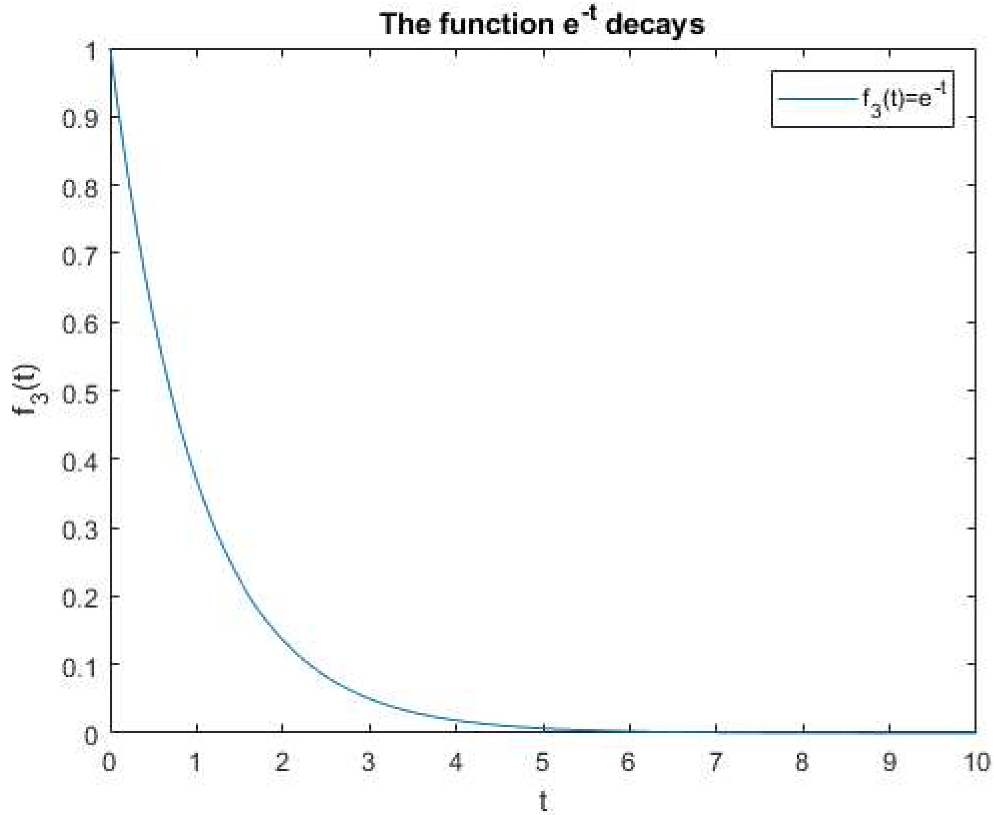
% |Remark.| A function which |grows while oscillating| doesn't |grow|,
% because it keeps changing sign, so it neither tends to $+\infty$ nor to
% $-\infty$.
```

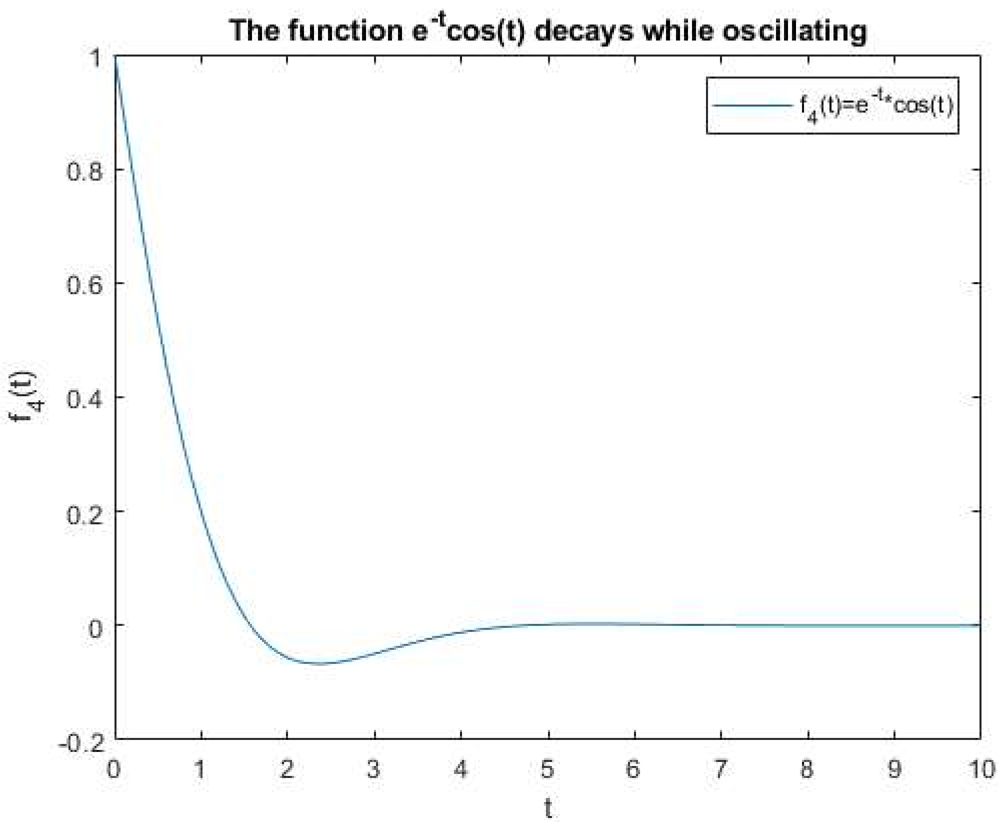


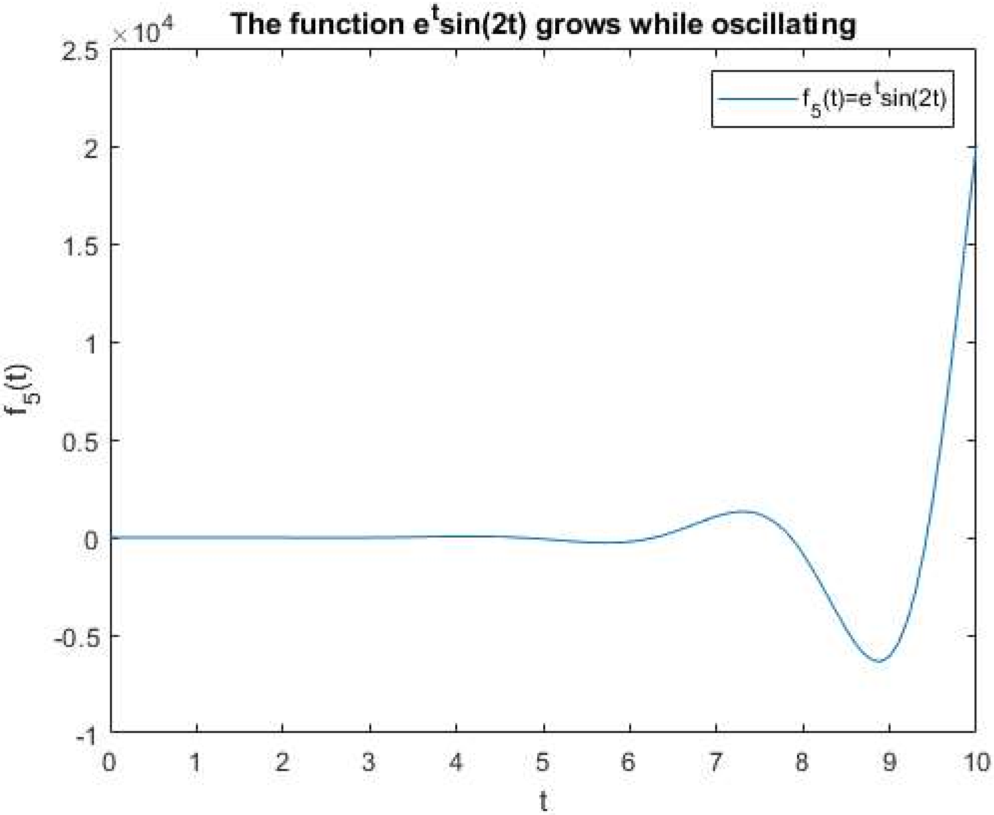
The function e^t grows
×10^4
f_1(t)=e^t
f_1(t)
t

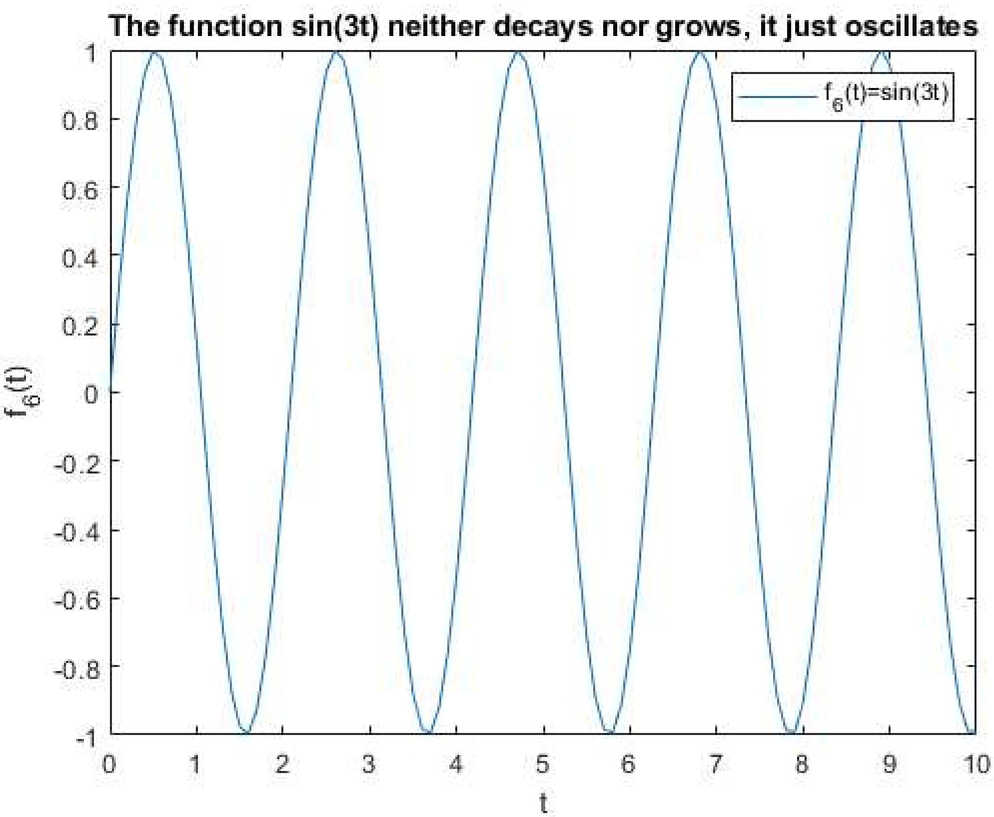


The function -e^-t decays
f_2(t)=-e^-t
f_2(t)
t


The function e-t decays
f3(t)=e-t
f3(t)
t
0 1 2 3 4 5 6 7 8 9 10
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


The function e-tcos(t) decays while oscillating
f4(t)=e-t*cos(t)
f4(t)
t
0 1 2 3 4 5 6 7 8 9 10
-0.2 0 0.2 0.4 0.6 0.8 1





Exercise 1

Objective: Use `iode` to solve second-order linear DEs. And classify them.

Details: Consider the ODE:

```
4y'' + 4 y' + 17 y = 0
```

(a) Use `iode` to plot six (6) numerical solutions of this equation with "random" initial data (use Method 3 above) and press-and-drag at various initial points, with some of the slopes being positive and some negative)

Use only initial points in the part of the window where `0<t<1` and `-1<x<1` and take all initial slopes between `-3` and `+3`.

Change the window to `[0,10]x[-3,3]`. Attach a cropped screenshot to your answers file.

(b) Based on the results of (a), state what percentage of solutions decay, grow, grow while oscillating, or decay while oscillating.

(c) Solve the DE and write the exact solution. Explain why this justifies your answer in (b).

```
%answer on docs
```

Exercise 2

Consider the ODE:

```
y'' + sqrt(3) y' - y/4 = 0
```

Repeat (a), (b), (c) from Exercise 1 with this DE.

```
%answer on docs
```

Exercise 3

Consider the ODE:

```
y'' + sqrt(3) y' + y/4 = 0
```

Repeat (a), (b), (c) from Exercise 1 with this DE.

```
%answer on docs
```

Example

Consider the ODE:

```
y'' + 2y' + 10y = 0
```

The solution is

```
y(t) = e^(-t) ( c1 cos(3t) + c2 sin(3t) )
```

From this, it is easy to see that all solutions decay while oscillating.

Similarly, for the equation

```
y'' - 2y' + 10y = 0
```

The solution is

```
y(t) = e^t ( c3 cos(3t) + c4 sin(3t) )
```

which grows while oscillating.

Exercise 4

Consider the fourth-order ODE:

```
y'''' + 2 y''' + 6 y'' + 2 y' + 5 y = 0
```

(a) Find the general solution for this problem. You can use MATLAB to find the roots of the characteristic equation numerically with `roots`.

(b) Predict what percentage of solutions with random initial data will grow, decay, grow while oscillating, and decay while oscillating. Explain.

```
p = [1 2 6 2 5];
r = roots(p)
p = [1 2 10];
r = roots(p)
syms a b t y r c d
q=a*exp(-t)*cos(2*t)
w=b*exp(-t)*(1i*sin(2*t))
y=c*cos(t)
r=d*(1i*sin(t))
simplify(q+w+y+r)
```

```
r =

  -1.0000 + 2.0000i
  -1.0000 - 2.0000i
   0.0000 + 1.0000i
   0.0000 - 1.0000i


r =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i


q =

a*cos(2*t)*exp(-t)


w =

b*sin(2*t)*exp(-t)*1i


y =
```

```
c*cos(t)


r =

d*sin(t)*1i


ans =

c*cos(t) + d*sin(t)*1i + a*exp(-t)*(2*cos(t)^2 - 1) + b*exp(-t)*cos(t)*sin(t)*2i
```

Exercise 5

Objective: Classify equations given the roots of the characteristic equation.

Details: Your answer can consist of just a short sentence, as `grows` or `decays while oscillating`.

Consider a second-order linear constant coefficient homogeneous DE with `r1` and `r2` as roots of the characteristic equation.

Summarize your conclusions about the behaviour of solutions for randomly chosen initial data when.

(a) `0 < r1 < r2`

(b) `r1 < 0 < r2`

(c) `r1 < r2 < 0`

(d) `r1 = alpha + beta i` and `r2 = alpha - beta i` and `alpha < 0`

(e) `r1 = alpha + beta i` and `r2 = alpha - beta i` and `alpha = 0`

(f) `r1 = alpha + beta i` and `r2 = alpha - beta i` and `alpha > 0`

Numerical Methods for Second-Order ODEs

One way to create a numerical method for second-order ODEs is to approximate derivatives with finite differences in the same way of the Euler method.

This means that we approximate the first derivative by:

```
y'(t[n]) ~ ( y[n] - y[n-1] ) / h
```

and

```
y''(t[n])  ~ ( y'(t[n+1]) - y'(t[n]) ) / h ~ ( y[n+1] - 2 y[n] + y[n-1]
) / ( h^2 )
```

By writing these approximations into the ODE, we obtain a method to get `y[n+1]` from the previous two steps `y[n]` and `y[n-1]`.

The method for approximating solutions is:

1. Start with `y[0]=y0`

2. Then we need to get `y[1]`, but we can't use the method, because we don't have two iterations `y[0]` and `y[-1]`(!!). So we use Euler to get

```
y[1] = y0 + y1 h
```

y1 is the slope given by the initial condition

3. Use the method described above to get `y[n]` for `n=2,3,...`.

Exercise 6

Objective: Write your own second-order ODE solver.

Details: Consider the second-order ODE

```
y'' + p(t) y' + q(t) y = g(t)
```

Write a second-order ODE solver using the method described above.

This m-file should be a function which accepts as variables (t0,tN,y0,y1,h), where t0 and tN are the start and end points of the interval on which to solve the ODE, y0, y1 are the initial conditions of the ODE, and h is the stepsize. You may also want to pass the functions into the ODE the way `ode45` does (check MATLAB lab 2). Name the function DE2_<UTORid>.m.

Note: you will need to use a loop to do this exercise.

```
%function [x,y]=DE2_krisanti(t0,tN,y0,y1,h, p,q,g)
```

Exercise 7

Objective: Compare your method with `iode`

Details: Use `iode` to plot the solution of the ODE `y'' + exp(-t/5) y' + (1-exp(-t/5)) y = sin(2*t)` with the initial conditions `y(0) = 1, y'(0) = 0`

Use the window to `[0,20]x[-2,2]` Without removing the figure window, plot your solution (in a different colour), which will be plotted in the same graph.

Comment on any major differences, or the lack thereof.

```
[t,y]=DE2_krisanti(0,20, 1,0,0.1,@(t) exp(-t/5), @(t)(1-exp(-t/5)), @(t) sin(2*t))
plot(t,y)
```

```
t =

  Columns 1 through 7

         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000

  Columns 8 through 14

    0.7000    0.8000    0.9000    1.0000    1.1000    1.2000    1.3000
```

```
Columns 15 through 21

  1.4000    1.5000    1.6000    1.7000    1.8000    1.9000    2.0000

Columns 22 through 28

  2.1000    2.2000    2.3000    2.4000    2.5000    2.6000    2.7000

Columns 29 through 35

  2.8000    2.9000    3.0000    3.1000    3.2000    3.3000    3.4000

Columns 36 through 42

  3.5000    3.6000    3.7000    3.8000    3.9000    4.0000    4.1000

Columns 43 through 49

  4.2000    4.3000    4.4000    4.5000    4.6000    4.7000    4.8000

Columns 50 through 56

  4.9000    5.0000    5.1000    5.2000    5.3000    5.4000    5.5000

Columns 57 through 63

  5.6000    5.7000    5.8000    5.9000    6.0000    6.1000    6.2000

Columns 64 through 70

  6.3000    6.4000    6.5000    6.6000    6.7000    6.8000    6.9000

Columns 71 through 77

  7.0000    7.1000    7.2000    7.3000    7.4000    7.5000    7.6000

Columns 78 through 84

  7.7000    7.8000    7.9000    8.0000    8.1000    8.2000    8.3000

Columns 85 through 91

  8.4000    8.5000    8.6000    8.7000    8.8000    8.9000    9.0000

Columns 92 through 98

  9.1000    9.2000    9.3000    9.4000    9.5000    9.6000    9.7000

Columns 99 through 105

  9.8000    9.9000   10.0000   10.1000   10.2000   10.3000   10.4000

Columns 106 through 112

 10.5000   10.6000   10.7000   10.8000   10.9000   11.0000   11.1000

Columns 113 through 119
```

```
   11.2000   11.3000   11.4000   11.5000   11.6000   11.7000   11.8000

  Columns 120 through 126

   11.9000   12.0000   12.1000   12.2000   12.3000   12.4000   12.5000

  Columns 127 through 133

   12.6000   12.7000   12.8000   12.9000   13.0000   13.1000   13.2000

  Columns 134 through 140

   13.3000   13.4000   13.5000   13.6000   13.7000   13.8000   13.9000

  Columns 141 through 147

   14.0000   14.1000   14.2000   14.3000   14.4000   14.5000   14.6000

  Columns 148 through 154

   14.7000   14.8000   14.9000   15.0000   15.1000   15.2000   15.3000

  Columns 155 through 161

   15.4000   15.5000   15.6000   15.7000   15.8000   15.9000   16.0000

  Columns 162 through 168

   16.1000   16.2000   16.3000   16.4000   16.5000   16.6000   16.7000

  Columns 169 through 175

   16.8000   16.9000   17.0000   17.1000   17.2000   17.3000   17.4000

  Columns 176 through 182

   17.5000   17.6000   17.7000   17.8000   17.9000   18.0000   18.1000

  Columns 183 through 189

   18.2000   18.3000   18.4000   18.5000   18.6000   18.7000   18.8000

  Columns 190 through 196

   18.9000   19.0000   19.1000   19.2000   19.3000   19.4000   19.5000

  Columns 197 through 201

   19.6000   19.7000   19.8000   19.9000   20.0000


y =

  Columns 1 through 7

    1.0000    1.0000    1.0000    1.0018    1.0069    1.0166    1.0318

  Columns 8 through 14
```

```
    1.0531    1.0806    1.1143    1.1534    1.1972    1.2444    1.2936

  Columns 15 through 21

    1.3429    1.3907    1.4349    1.4737    1.5051    1.5275    1.5393

  Columns 22 through 28

    1.5394    1.5268    1.5011    1.4621    1.4101    1.3457    1.2701

  Columns 29 through 35

    1.1848    1.0913    0.9918    0.8885    0.7835    0.6793    0.5780

  Columns 36 through 42

    0.4817    0.3925    0.3118    0.2410    0.1809    0.1320    0.0943

  Columns 43 through 49

    0.0676    0.0508    0.0430    0.0425    0.0477    0.0565    0.0669

  Columns 50 through 56

    0.0768    0.0842    0.0871    0.0840    0.0734    0.0544    0.0263

  Columns 57 through 63

   -0.0110   -0.0572   -0.1116   -0.1731   -0.2401   -0.3108   -0.3831

  Columns 64 through 70

   -0.4546   -0.5230   -0.5859   -0.6409   -0.6859   -0.7192   -0.7391

  Columns 71 through 77

   -0.7445   -0.7348   -0.7099   -0.6700   -0.6160   -0.5491   -0.4712

  Columns 78 through 84

   -0.3842   -0.2905   -0.1928   -0.0938    0.0037    0.0972    0.1841

  Columns 85 through 91

    0.2621    0.3294    0.3844    0.4261    0.4539    0.4678    0.4682

  Columns 92 through 98

    0.4560    0.4327    0.3998    0.3595    0.3139    0.2655    0.2167

  Columns 99 through 105

    0.1697    0.1268    0.0899    0.0605    0.0398    0.0287    0.0274

  Columns 106 through 112

    0.0356    0.0529    0.0779    0.1093    0.1452    0.1836    0.2221
```

```
  Columns 113 through 119

    0.2585    0.2904    0.3158    0.3325    0.3390    0.3339    0.3165

  Columns 120 through 126

    0.2864    0.2437    0.1891    0.1237    0.0492   -0.0326   -0.1191

  Columns 127 through 133

   -0.2079   -0.2962   -0.3812   -0.4602   -0.5306   -0.5899   -0.6363

  Columns 134 through 140

   -0.6681   -0.6843   -0.6841   -0.6677   -0.6354   -0.5883   -0.5278

  Columns 141 through 147

   -0.4560   -0.3752   -0.2878   -0.1967   -0.1047   -0.0145    0.0711

  Columns 148 through 154

    0.1498    0.2194    0.2781    0.3248    0.3586    0.3793    0.3871

  Columns 155 through 161

    0.3827    0.3674    0.3427    0.3106    0.2732    0.2329    0.1919

  Columns 162 through 168

    0.1526    0.1172    0.0874    0.0651    0.0512    0.0465    0.0514

  Columns 169 through 175

    0.0656    0.0884    0.1187    0.1549    0.1951    0.2371    0.2788

  Columns 176 through 182

    0.3175    0.3511    0.3772    0.3938    0.3992    0.3922    0.3718

  Columns 183 through 189

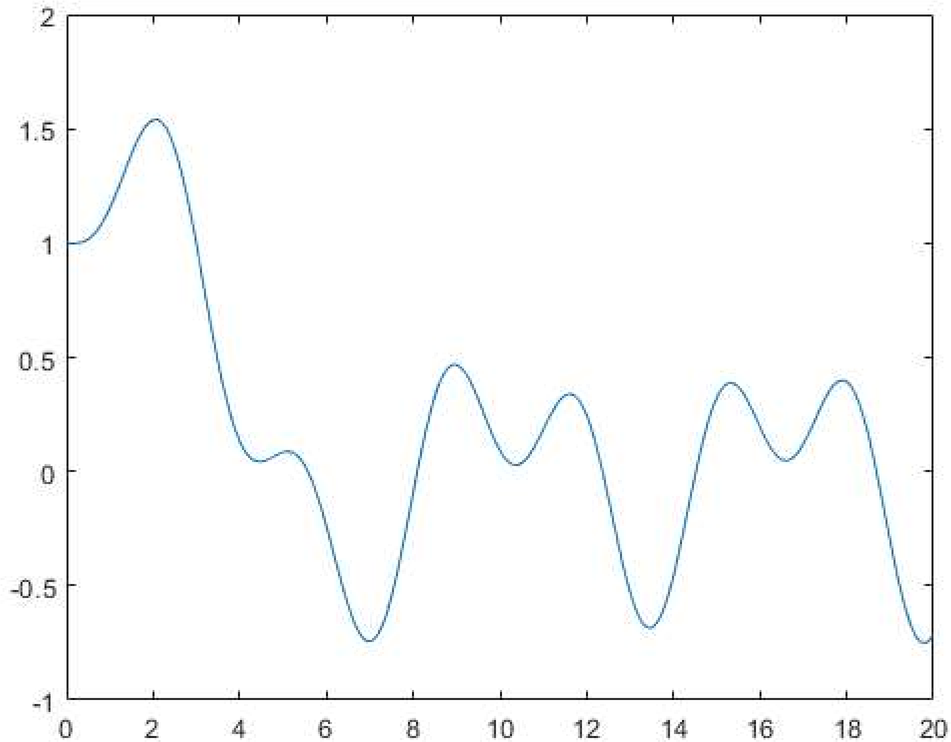
    0.3378    0.2902    0.2299    0.1579    0.0761   -0.0135   -0.1084

  Columns 190 through 196

   -0.2058   -0.3030   -0.3970   -0.4848   -0.5637   -0.6313   -0.6854

  Columns 197 through 201

   -0.7243   -0.7467   -0.7520   -0.7401   -0.7113
```



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