Systems Lab: Systems of ODEs in MATLAB

In this lab, you will write your own ODE system solver for the Heun method (aka the Improved Euler method), and compare its results to those of `ode45`.

You will also learn how to save images in MATLAB.

Opening the m-file lab4.m in the MATLAB editor, step through each part using cell mode to see the results. Compare the output with the PDF, which was generated from this m-file.

There are four (4) exercises in this lab that are to be handed in on the due date of the lab. Write your solutions in a separate file, including appropriate descriptions in each step. Save the m-files and the pdf-file for Exercise 4 and submit them on Quercus.

Contents

Student Information

```
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Student Number: 1009669404
```

Exercise 1

Objective: Write your own ODE system solver using the Heun/Improved Euler Method and compare it to `ode45`.

Details: Consider the system of 2 ODEs:

`x1'=f(t,x1,x2), x2'=g(t,x1,x2)`

This m-file should be a function which accepts as variables (t0,tN,x0,h), where t0 and tN are the start and end points of the interval on which to solve the ODE, h is the stepsize, and x0 is a vector for the initial condition of the system of ODEs `x(t0)=x0`. Name the function solvesystem_<UTORid>.m (Substitute your UTORid for UTORid). You may also want to pass the functions into the ODE the way `ode45` does (check MATLAB labs 2 and 3).

Your m-file should return a row vector of times and a matrix of approximate solution values (the first row has the approximation for `x1` and the second row has the approximation for `x2`).

Note: you will need to use a loop to do this exercise. You will also need to recall the Heun/Improved Euler algorithm learned in lectures.

```
%{

function [x,y]=solvesystem_krisanti(f,g, t0, tN, x0, h) %f,start, endpoint, initial condition, stepsize

x=t0:h:tN;
%disp(x);
size(x)
y=zeros(2, length(x));
y(1,1)=x0(1);
```

```
y(2,1)=x0(2);
num=size(x);
for i=1:(num(2)-1)
    kf=f(x(i),y(1,i),y(2,i)); %y(tn+1) = y(n) + (1/2) * h * (f(tn,yn) + f(tn+h, yn+h*f(tn,yn))
    kg=g(x(i),y(1,i),y(2,i));
    x1=y(1,i)+h.*kf;
    x2=y(2,i)+h.*kg;
    y(1, i+1)=(kf+f(x(i+1),x1,x2)).*(1/2).*h   +  y(1,i);
    y(2, i+1)=(kg+g(x(i+1),x1,x2)).*(1/2).*h   +  y(2,i);
end
[ox,oy]=ode45(@(t,y) [f(t, y(1), y(2)); g(t, y(1), y(2))], [t0,tN] ,x0);
%plot(x,y)

%legend("mine", "ode45")
end

%x1' = x1/2 - 2*x2, x2' = 5*x1 - x2
%with initial condition x(0)=(1,1).
%Use your method from Exercise 1 to approximate the solution from t=0 to t=4*pi with step size h=0.05.

%}
```

Exercise 2

Objective: Compare Heun with an exact solution

Details: Consider the system of ODEs

`x1' = x1/2 - 2*x2, x2' = 5*x1 - x2`

with initial condition `x(0)=(1,1)`.

Use your method from Exercise 1 to approximate the solution from `t=0` to `t=4*pi` with step size `h=0.05`.

Compute the exact solution (by hand) and plot both phase portraits on the same figure for comparison.

Your submission should show the construction of the inline function, the use of your Heun's method to obtain the solution, a construction of the exact solution, and a plot showing both. In the comments, include the exact solution.

Label your axes and include a legend.

```
x1=@(t,x1, x2) x1/2-2*x2
x2=@(t,x1,x2) 5*x1 - x2
[x,y]=solvesystem_krisanti(x1, x2, 0, 4*pi, [1,1], 0.05)
plot(y(1,:),y(2,:))
legend('Heun solution to 2 system ODE')
xlabel('x1(t)')
ylabel('x2(t)')
%exact solution for system with initial condition x(0)=(1 1) is
%c1=1/20
%c2=17/(20sqrt(151))
%s=sqrt(151)
%o=sqrt(151)/4
%x1=c1exp(-t/4).*(3cos(ot)-s*sin(ot)+c2*exp(-t/4)*(s*cos(ot)+3*sin(ot))
%x1=c1exp(-t/4).*(20*cos(ot)+c2*exp(-t/4)*(20*sin(ot))
```

```
x1 =
```

```
  function_handle with value:

    @(t,x1,x2)x1/2-2*x2

x2 =

  function_handle with value:

    @(t,x1,x2)5*x1-x2

ans =

     1   252

x =

  Columns 1 through 7

         0    0.0500    0.1000    0.1500    0.2000    0.2500    0.3000

  Columns 8 through 14

    0.3500    0.4000    0.4500    0.5000    0.5500    0.6000    0.6500

  Columns 15 through 21

    0.7000    0.7500    0.8000    0.8500    0.9000    0.9500    1.0000

  Columns 22 through 28

    1.0500    1.1000    1.1500    1.2000    1.2500    1.3000    1.3500

  Columns 29 through 35

    1.4000    1.4500    1.5000    1.5500    1.6000    1.6500    1.7000

  Columns 36 through 42

    1.7500    1.8000    1.8500    1.9000    1.9500    2.0000    2.0500

  Columns 43 through 49

    2.1000    2.1500    2.2000    2.2500    2.3000    2.3500    2.4000

  Columns 50 through 56

    2.4500    2.5000    2.5500    2.6000    2.6500    2.7000    2.7500

  Columns 57 through 63

    2.8000    2.8500    2.9000    2.9500    3.0000    3.0500    3.1000

  Columns 64 through 70

    3.1500    3.2000    3.2500    3.3000    3.3500    3.4000    3.4500

  Columns 71 through 77

    3.5000    3.5500    3.6000    3.6500    3.7000    3.7500    3.8000
```

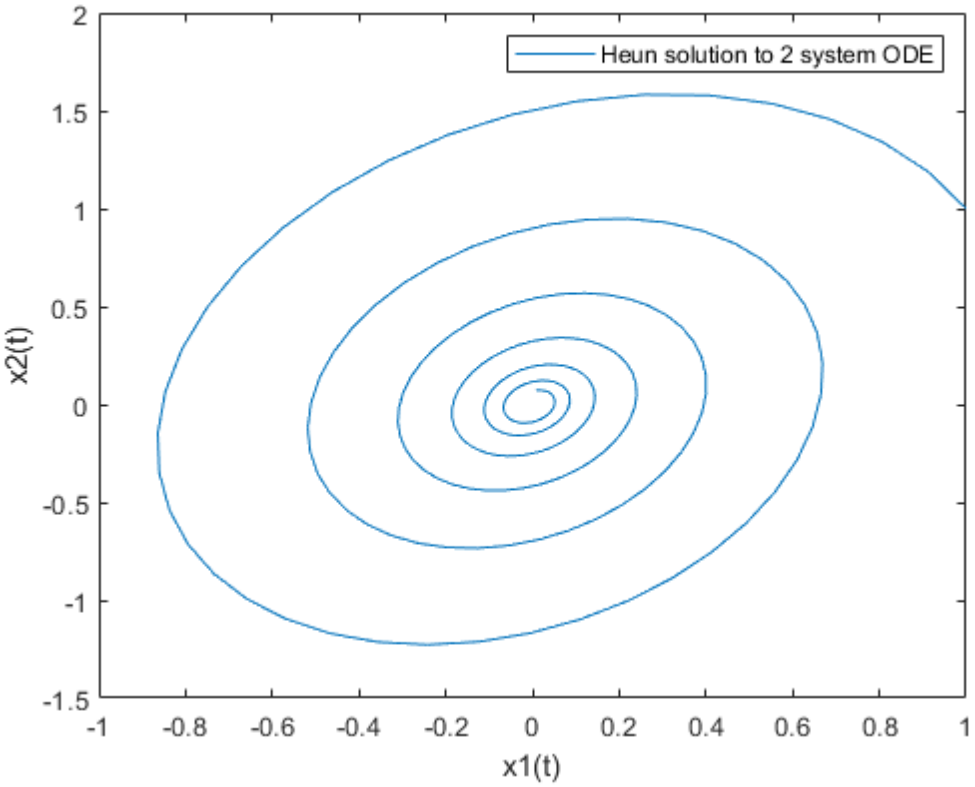
```
Columns 78 through 84

  3.8500    3.9000    3.9500    4.0000    4.0500    4.1000    4.1500

Columns 85 through 91

  4.2000    4.2500    4.3000    4.3500    4.4000    4.4500    4.5000

Columns 92 through 98

  4.5500    4.6000    4.6500    4.7000    4.7500    4.8000    4.8500

Columns 99 through 105

  4.9000    4.9500    5.0000    5.0500    5.1000    5.1500    5.2000

Columns 106 through 112

  5.2500    5.3000    5.3500    5.4000    5.4500    5.5000    5.5500

Columns 113 through 119

  5.6000    5.6500    5.7000    5.7500    5.8000    5.8500    5.9000

Columns 120 through 126

  5.9500    6.0000    6.0500    6.1000    6.1500    6.2000    6.2500

Columns 127 through 133

  6.3000    6.3500    6.4000    6.4500    6.5000    6.5500    6.6000

Columns 134 through 140

  6.6500    6.7000    6.7500    6.8000    6.8500    6.9000    6.9500

Columns 141 through 147

  7.0000    7.0500    7.1000    7.1500    7.2000    7.2500    7.3000

Columns 148 through 154

  7.3500    7.4000    7.4500    7.5000    7.5500    7.6000    7.6500

Columns 155 through 161

  7.7000    7.7500    7.8000    7.8500    7.9000    7.9500    8.0000

Columns 162 through 168

  8.0500    8.1000    8.1500    8.2000    8.2500    8.3000    8.3500

Columns 169 through 175

  8.4000    8.4500    8.5000    8.5500    8.6000    8.6500    8.7000

Columns 176 through 182

  8.7500    8.8000    8.8500    8.9000    8.9500    9.0000    9.0500

Columns 183 through 189
```

```
    9.1000    9.1500    9.2000    9.2500    9.3000    9.3500    9.4000

  Columns 190 through 196

    9.4500    9.5000    9.5500    9.6000    9.6500    9.7000    9.7500

  Columns 197 through 203

    9.8000    9.8500    9.9000    9.9500   10.0000   10.0500   10.1000

  Columns 204 through 210

   10.1500   10.2000   10.2500   10.3000   10.3500   10.4000   10.4500

  Columns 211 through 217

   10.5000   10.5500   10.6000   10.6500   10.7000   10.7500   10.8000

  Columns 218 through 224

   10.8500   10.9000   10.9500   11.0000   11.0500   11.1000   11.1500

  Columns 225 through 231

   11.2000   11.2500   11.3000   11.3500   11.4000   11.4500   11.5000

  Columns 232 through 238

   11.5500   11.6000   11.6500   11.7000   11.7500   11.8000   11.8500

  Columns 239 through 245

   11.9000   11.9500   12.0000   12.0500   12.1000   12.1500   12.2000

  Columns 246 through 252

   12.2500   12.3000   12.3500   12.4000   12.4500   12.5000   12.5500


y =

  Columns 1 through 7

    1.0000    0.9141    0.8087    0.6869    0.5519    0.4072    0.2565
    1.0000    1.1856    1.3387    1.4563    1.5367    1.5788    1.5826

  Columns 8 through 14

    0.1035   -0.0482   -0.1949   -0.3334   -0.4606   -0.5737   -0.6705
    1.5490    1.4797    1.3772    1.2447    1.0862    0.9059    0.7088

  Columns 15 through 21

   -0.7491   -0.8081   -0.8465   -0.8640   -0.8607   -0.8371   -0.7944
    0.4998    0.2843    0.0674   -0.1457   -0.3501   -0.5411   -0.7147

  Columns 22 through 28

   -0.7340   -0.6578   -0.5680   -0.4670   -0.3575   -0.2422   -0.1241
   -0.8670   -0.9951   -1.0966   -1.1696   -1.2133   -1.2272   -1.2118
```

```
Columns 29 through 35

 -0.0061    0.1092    0.2191    0.3210    0.4129    0.4927    0.5589
 -1.1683   -1.0982   -1.0040   -0.8884   -0.7547   -0.6066   -0.4478

Columns 36 through 42

  0.6103    0.6460    0.6656    0.6689    0.6565    0.6288    0.5870
 -0.2824   -0.1144    0.0521    0.2132    0.3653    0.5050    0.6293

Columns 43 through 49

  0.5324    0.4666    0.3914    0.3088    0.2210    0.1302    0.0385
  0.7356    0.8220    0.8868    0.9291    0.9485    0.9449    0.9192

Columns 50 through 56

 -0.0517   -0.1386   -0.2199   -0.2941   -0.3595   -0.4148   -0.4589
  0.8724    0.8062    0.7226    0.6240    0.5132    0.3930    0.2665

Columns 57 through 63

 -0.4911   -0.5109   -0.5182   -0.5131   -0.4959   -0.4676   -0.4288
  0.1369    0.0073   -0.1193   -0.2399   -0.3519   -0.4528   -0.5405

Columns 64 through 70

 -0.3810   -0.3253   -0.2633   -0.1967   -0.1270   -0.0562    0.0143
 -0.6132   -0.6697   -0.7090   -0.7306   -0.7344   -0.7208   -0.6905

Columns 71 through 77

  0.0827    0.1474    0.2070    0.2603    0.3061    0.3435    0.3719
 -0.6447   -0.5848   -0.5126   -0.4301   -0.3395   -0.2431   -0.1434

Columns 78 through 84

  0.3909    0.4001    0.3997    0.3898    0.3710    0.3439    0.3094
 -0.0428    0.0563    0.1517    0.2410    0.3225    0.3943    0.4551

Columns 85 through 91

  0.2684    0.2221    0.1717    0.1185    0.0639    0.0091   -0.0446
  0.5036    0.5390    0.5608    0.5689    0.5633    0.5446    0.5135

Columns 92 through 98

 -0.0959   -0.1436   -0.1868   -0.2244   -0.2559   -0.2805   -0.2979
  0.4710    0.4185    0.3574    0.2894    0.2163    0.1399    0.0620

Columns 99 through 105

 -0.3079   -0.3103   -0.3054   -0.2933   -0.2747   -0.2500   -0.2200
 -0.0153   -0.0904   -0.1615   -0.2270   -0.2855   -0.3358   -0.3770

Columns 106 through 112

 -0.1856   -0.1477   -0.1072   -0.0652   -0.0227    0.0192    0.0597
 -0.4082   -0.4290   -0.4392   -0.4388   -0.4280   -0.4074   -0.3777

Columns 113 through 119
```

```
    0.0978    0.1326    0.1634    0.1896    0.2107    0.2263    0.2361
   -0.3398   -0.2949   -0.2441   -0.1888   -0.1304   -0.0704   -0.0103

  Columns 120 through 126

    0.2401    0.2384    0.2311    0.2185    0.2011    0.1794    0.1539
    0.0487    0.1050    0.1574    0.2048    0.2462    0.2808    0.3079

  Columns 127 through 133

    0.1255    0.0948    0.0627    0.0298   -0.0029   -0.0347   -0.0650
    0.3270    0.3380    0.3407    0.3353    0.3221    0.3017    0.2746

  Columns 134 through 140

   -0.0929   -0.1180   -0.1396   -0.1575   -0.1711   -0.1804   -0.1852
    0.2417    0.2040    0.1624    0.1180    0.0719    0.0252   -0.0209

  Columns 141 through 147

   -0.1855   -0.1815   -0.1732   -0.1611   -0.1454   -0.1268   -0.1056
   -0.0653   -0.1071   -0.1454   -0.1792   -0.2080   -0.2312   -0.2483

  Columns 148 through 154

   -0.0824   -0.0578   -0.0326   -0.0071    0.0178    0.0418    0.0641
   -0.2592   -0.2636   -0.2618   -0.2538   -0.2400   -0.2209   -0.1970

  Columns 155 through 161

    0.0844    0.1022    0.1171    0.1289    0.1374    0.1424    0.1439
   -0.1691   -0.1379   -0.1043   -0.0690   -0.0329    0.0030    0.0380

  Columns 162 through 168

    0.1420    0.1368    0.1285    0.1174    0.1037    0.0880    0.0706
    0.0712    0.1019    0.1294    0.1532    0.1728    0.1878    0.1980

  Columns 169 through 175

    0.0519    0.0325    0.0128   -0.0067   -0.0256   -0.0434   -0.0597
    0.2033    0.2037    0.1992    0.1902    0.1769    0.1598    0.1393

  Columns 176 through 182

   -0.0742   -0.0866   -0.0967   -0.1042   -0.1091   -0.1113   -0.1108
    0.1160    0.0906    0.0636    0.0359    0.0080   -0.0195   -0.0457

  Columns 183 through 189

   -0.1077   -0.1021   -0.0943   -0.0844   -0.0728   -0.0598   -0.0456
   -0.0703   -0.0925   -0.1121   -0.1285   -0.1415   -0.1508   -0.1563

  Columns 190 through 196

   -0.0308   -0.0156   -0.0004    0.0144    0.0286    0.0417    0.0534
   -0.1580   -0.1559   -0.1502   -0.1411   -0.1289   -0.1140   -0.0967

  Columns 197 through 203

    0.0637    0.0721    0.0787    0.0833    0.0857    0.0861    0.0844
   -0.0776   -0.0571   -0.0358   -0.0142    0.0072    0.0280    0.0475
```

```
Columns 204 through 210

  0.0808    0.0754    0.0683    0.0598    0.0501    0.0395    0.0282
  0.0654    0.0814    0.0950    0.1061    0.1143    0.1197    0.1221

Columns 211 through 217

  0.0165    0.0047   -0.0070   -0.0181   -0.0286   -0.0381   -0.0465
  0.1216    0.1182    0.1121    0.1035    0.0927    0.0800    0.0657

Columns 218 through 224

 -0.0536   -0.0592   -0.0633   -0.0658   -0.0667   -0.0660   -0.0638
  0.0502    0.0339    0.0172    0.0005   -0.0158   -0.0313   -0.0457

Columns 225 through 231

 -0.0601   -0.0551   -0.0489   -0.0417   -0.0337   -0.0251   -0.0161
 -0.0586   -0.0698   -0.0792   -0.0864   -0.0914   -0.0941   -0.0945

Columns 232 through 238

 -0.0070    0.0021    0.0109    0.0192    0.0268    0.0337    0.0395
 -0.0927   -0.0888   -0.0828   -0.0751   -0.0657   -0.0551   -0.0434

Columns 239 through 245

  0.0443    0.0480    0.0504    0.0515    0.0514    0.0501    0.0477
 -0.0310   -0.0181   -0.0052    0.0076    0.0198    0.0313    0.0418

Columns 246 through 252

  0.0442    0.0397    0.0344    0.0284    0.0219    0.0151    0.0080
  0.0510    0.0588    0.0650    0.0695    0.0722    0.0732    0.0725
```



Exercise 3

Objective: Compare your method with Euler's Method (from `iode`).

Details: Use `iode` to plot the solution for the same problem with the same step size as on Exercise 2.

Compare your solution on exercise 2, the exact solution from exercise 2 and the approximation using Euler's method. Plot the solution for Euler's method and make note of any differences.

```
%euler method using euler.m
fe=@(t,x) [x1(t, x(1), x(2)); x2(t, x(1), x(2))];
t=0:0.05:4*pi
eu=euler(fe, [1;1], t)
plot(eu(1,:), eu(2,:))
%ex(1,:)

%exact solution
c1=1/20
c2=17/(20*sqrt(151))
s=sqrt(151)
o=sqrt(151)/4
ex1=c1.*exp(-t./4).*(3.*cos(o.*t)-s.*sin(o.*t))+c2.*exp(-t./4).*(s.*cos(o.*t)+3.*sin(o.*t))
ex2=c1.*exp(-t./4).*(20.*cos(o.*t))+c2.*exp(-t./4).*(20.*sin(o*t))

%plotting it all together
plot(eu(1,:), eu(2,:),y(1,:),y(2,:),'--', ex1,ex2, 'x')
legend("euler method", "Heun method", "exact solution")
xlabel('x1(t)')
ylabel('x2(t)')


%Both Heun and exact solution shows the 0,0 point. From the initial condition,
%the exact solution and Heun method approaches 0,0 by moving counterclockwise while the
```

```
%The Euler method did not reach that point, it has a larger error and
%overshoots in the beginning.
```

```
t =

  Columns 1 through 7

         0    0.0500    0.1000    0.1500    0.2000    0.2500    0.3000

  Columns 8 through 14

    0.3500    0.4000    0.4500    0.5000    0.5500    0.6000    0.6500

  Columns 15 through 21

    0.7000    0.7500    0.8000    0.8500    0.9000    0.9500    1.0000

  Columns 22 through 28

    1.0500    1.1000    1.1500    1.2000    1.2500    1.3000    1.3500

  Columns 29 through 35

    1.4000    1.4500    1.5000    1.5500    1.6000    1.6500    1.7000

  Columns 36 through 42

    1.7500    1.8000    1.8500    1.9000    1.9500    2.0000    2.0500

  Columns 43 through 49

    2.1000    2.1500    2.2000    2.2500    2.3000    2.3500    2.4000

  Columns 50 through 56

    2.4500    2.5000    2.5500    2.6000    2.6500    2.7000    2.7500

  Columns 57 through 63

    2.8000    2.8500    2.9000    2.9500    3.0000    3.0500    3.1000

  Columns 64 through 70

    3.1500    3.2000    3.2500    3.3000    3.3500    3.4000    3.4500

  Columns 71 through 77

    3.5000    3.5500    3.6000    3.6500    3.7000    3.7500    3.8000

  Columns 78 through 84

    3.8500    3.9000    3.9500    4.0000    4.0500    4.1000    4.1500

  Columns 85 through 91

    4.2000    4.2500    4.3000    4.3500    4.4000    4.4500    4.5000

  Columns 92 through 98

    4.5500    4.6000    4.6500    4.7000    4.7500    4.8000    4.8500
```

```
Columns 99 through 105

    4.9000    4.9500    5.0000    5.0500    5.1000    5.1500    5.2000

Columns 106 through 112

    5.2500    5.3000    5.3500    5.4000    5.4500    5.5000    5.5500

Columns 113 through 119

    5.6000    5.6500    5.7000    5.7500    5.8000    5.8500    5.9000

Columns 120 through 126

    5.9500    6.0000    6.0500    6.1000    6.1500    6.2000    6.2500

Columns 127 through 133

    6.3000    6.3500    6.4000    6.4500    6.5000    6.5500    6.6000

Columns 134 through 140

    6.6500    6.7000    6.7500    6.8000    6.8500    6.9000    6.9500

Columns 141 through 147

    7.0000    7.0500    7.1000    7.1500    7.2000    7.2500    7.3000

Columns 148 through 154

    7.3500    7.4000    7.4500    7.5000    7.5500    7.6000    7.6500

Columns 155 through 161

    7.7000    7.7500    7.8000    7.8500    7.9000    7.9500    8.0000

Columns 162 through 168

    8.0500    8.1000    8.1500    8.2000    8.2500    8.3000    8.3500

Columns 169 through 175

    8.4000    8.4500    8.5000    8.5500    8.6000    8.6500    8.7000

Columns 176 through 182

    8.7500    8.8000    8.8500    8.9000    8.9500    9.0000    9.0500

Columns 183 through 189

    9.1000    9.1500    9.2000    9.2500    9.3000    9.3500    9.4000

Columns 190 through 196

    9.4500    9.5000    9.5500    9.6000    9.6500    9.7000    9.7500

Columns 197 through 203

    9.8000    9.8500    9.9000    9.9500   10.0000   10.0500   10.1000

Columns 204 through 210
```

```
    10.1500   10.2000   10.2500   10.3000   10.3500   10.4000   10.4500

  Columns 211 through 217

    10.5000   10.5500   10.6000   10.6500   10.7000   10.7500   10.8000

  Columns 218 through 224

    10.8500   10.9000   10.9500   11.0000   11.0500   11.1000   11.1500

  Columns 225 through 231

    11.2000   11.2500   11.3000   11.3500   11.4000   11.4500   11.5000

  Columns 232 through 238

    11.5500   11.6000   11.6500   11.7000   11.7500   11.8000   11.8500

  Columns 239 through 245

    11.9000   11.9500   12.0000   12.0500   12.1000   12.1500   12.2000

  Columns 246 through 252

    12.2500   12.3000   12.3500   12.4000   12.4500   12.5000   12.5500


eu =

  Columns 1 through 7

    1.0000    0.9250    0.8281    0.7117    0.5785    0.4318    0.2749
    1.0000    1.2000    1.3712    1.5097    1.6122    1.6762    1.7003

  Columns 8 through 14

    0.1118   -0.0538   -0.2179   -0.3767   -0.5263   -0.6632   -0.7842
    1.6840    1.6278    1.5329    1.4018    1.2375    1.0441    0.8261

  Columns 15 through 21

   -0.8864   -0.9674   -1.0254   -1.0589   -1.0673   -1.0502   -1.0083
    0.5887    0.3377    0.0790   -0.1813   -0.4370   -0.6820   -0.9104

  Columns 22 through 28

   -0.9425   -0.8543   -0.7460   -0.6201   -0.4796   -0.3280   -0.1687
   -1.1170   -1.2967   -1.4455   -1.5597   -1.6368   -1.6748   -1.6731

  Columns 29 through 35

   -0.0056    0.1574    0.3165    0.4679    0.6079    0.7334    0.8413
   -1.6316   -1.5514   -1.4345   -1.2836   -1.1025   -0.8954   -0.6673

  Columns 36 through 42

    0.9290    0.9946    1.0365    1.0537    1.0459    1.0132    0.9565
   -0.4236   -0.1702    0.0870    0.3418    0.5881    0.8202    1.0325

  Columns 43 through 49

    0.8772    0.7771    0.6587    0.5248    0.3786    0.2236    0.0635
```

```
    1.2200    1.3782    1.5036    1.5931    1.6447    1.6571    1.6301

  Columns 50 through 56

   -0.0979   -0.2568   -0.4094   -0.5521   -0.6815   -0.7945   -0.8886
    1.5645    1.4618    1.3245    1.1559    0.9601    0.7417    0.5060

  Columns 57 through 63

   -0.9614   -1.0113   -1.0371   -1.0382   -1.0147   -0.9671   -0.8966
    0.2585    0.0053   -0.2478   -0.4947   -0.7295   -0.9467   -1.1412

  Columns 64 through 70

   -0.8049   -0.6942   -0.5672   -0.4268   -0.2765   -0.1198    0.0396
   -1.3083   -1.4441   -1.5455   -1.6100   -1.6362   -1.6235   -1.5723

  Columns 71 through 77

    0.1978    0.3511    0.4959    0.6287    0.7465    0.8463    0.9260
   -1.4838   -1.3601   -1.2044   -1.0202   -0.8120   -0.5848   -0.3439

  Columns 78 through 84

    0.9835    1.0176    1.0275    1.0130    0.9744    0.9128    0.8295
   -0.0953    0.1554    0.4020    0.6388    0.8601    1.0607    1.2359

  Columns 85 through 91

    0.7267    0.6067    0.4725    0.3272    0.1743    0.0175   -0.1396
    1.3815    1.4941    1.5710    1.6106    1.6119    1.5749    1.5005

  Columns 92 through 98

   -0.2931   -0.4395   -0.5753   -0.6972   -0.8024   -0.8884   -0.9533
    1.3906    1.2478    1.0755    0.8779    0.6597    0.4261    0.1827

  Columns 99 through 105

   -0.9954   -1.0138   -1.0081   -0.9785   -0.9256   -0.8509   -0.7560
   -0.0647   -0.3104   -0.5483   -0.7729   -0.9789   -1.1613   -1.3160

  Columns 106 through 112

   -0.6433   -0.5155   -0.3756   -0.2269   -0.0730    0.0824    0.2356
   -1.4392   -1.5281   -1.5805   -1.5954   -1.5724   -1.5120   -1.4158

  Columns 113 through 119

    0.3831    0.5213    0.6469    0.7570    0.8490    0.9207    0.9705
   -1.2861   -1.1260   -0.9394   -0.7307   -0.5049   -0.2674   -0.0239

  Columns 120 through 126

    0.9972    1.0001    0.9793    0.9352    0.8690    0.7822    0.6770
    0.2200    0.4583    0.6854    0.8959    1.0849    1.2479    1.3811

  Columns 127 through 133

    0.5558    0.4216    0.2775    0.1270   -0.0263   -0.1788   -0.3269
    1.4813    1.5462    1.5743    1.5649    1.5184    1.4359    1.3194
```

```
Columns 134 through 140

 -0.4670   -0.5958   -0.7104   -0.8079   -0.8861   -0.9432   -0.9778
  1.1717    0.9964    0.7976    0.5802    0.3492    0.1102   -0.1311

Columns 141 through 147

 -0.9891   -0.9769   -0.9416   -0.8839   -0.8053   -0.7077   -0.5934
 -0.3690   -0.5978   -0.8122   -1.0069   -1.1776   -1.3200   -1.4309

Columns 148 through 154

 -0.4651   -0.3260   -0.1792   -0.0285    0.1228    0.2710    0.4125
 -1.5077   -1.5486   -1.5527   -1.5199   -1.4510   -1.3477   -1.2126

Columns 155 through 161

  0.5441    0.6626    0.7652    0.8495    0.9135    0.9558    0.9752
 -1.0488   -0.8604   -0.6517   -0.4278   -0.1941    0.0440    0.2808

Columns 162 through 168

  0.9716    0.9448    0.8956    0.8252    0.7353    0.6281    0.5061
  0.5105    0.7279    0.9277    1.1052    1.2563    1.3773    1.4654

Columns 169 through 175

  0.3722    0.2296    0.0818   -0.0678   -0.2156   -0.3581   -0.4919
  1.5187    1.5358    1.5164    1.4610    1.3710    1.2486    1.0966

Columns 176 through 182

 -0.6139   -0.7211   -0.8111   -0.8817   -0.9312   -0.9586   -0.9632
  0.9188    0.7194    0.5032    0.2752    0.0410   -0.1938   -0.4238

Columns 183 through 189

 -0.9449   -0.9042   -0.8421   -0.7600   -0.6600   -0.5444   -0.4161
 -0.6434   -0.8475   -1.0311   -1.1901   -1.3206   -1.4195   -1.4847

Columns 190 through 196

 -0.2780   -0.1335    0.0140    0.1609    0.3039    0.4395    0.5644
 -1.5145   -1.5082   -1.4662   -1.3894   -1.2797   -1.1397   -0.9729

Columns 197 through 203

  0.6758    0.7710    0.8478    0.9044    0.9394    0.9520    0.9420
 -0.7831   -0.5750   -0.3535   -0.1239    0.1084    0.3378    0.5589

Columns 204 through 210

  0.9097    0.8558    0.7816    0.6890    0.5801    0.4576    0.3243
  0.7665    0.9556    1.1217    1.2611    1.3703    1.4468    1.4888

Columns 211 through 217

  0.1836    0.0386   -0.1071   -0.2501   -0.3869   -0.5144   -0.6295
  1.4955    1.4666    1.4029    1.3060    1.1782    1.0225    0.8428

Columns 218 through 224
```

```
   -0.7295   -0.8121   -0.8753   -0.9176   -0.9380   -0.9362   -0.9121
    0.6433    0.4287    0.2043   -0.0248   -0.2529   -0.4748   -0.6851

  Columns 225 through 231

   -0.8664   -0.8002   -0.7151   -0.6130   -0.4966   -0.3685   -0.2318
   -0.8789   -1.0515   -1.1990   -1.3178   -1.4052   -1.4591   -1.4782

  Columns 232 through 238

   -0.0897    0.0542    0.1968    0.3344    0.4640    0.5823    0.6867
   -1.4623   -1.4116   -1.3274   -1.2119   -1.0677   -0.8983   -0.7078

  Columns 239 through 245

    0.7747    0.8441    0.8934    0.9214    0.9276    0.9116    0.8741
   -0.5007   -0.2820   -0.0569    0.1693    0.3912    0.6035    0.8013

  Columns 246 through 252

    0.8158    0.7382    0.6432    0.5330    0.4103    0.2781    0.1393
    0.9797    1.1347    1.2625    1.3602    1.4254    1.4567    1.4534


c1 =

    0.0500


c2 =

    0.0692


s =

   12.2882


o =

    3.0721


ex1 =

  Columns 1 through 7

    1.0000    0.9145    0.8096    0.6885    0.5541    0.4102    0.2601

  Columns 8 through 14

    0.1077   -0.0434   -0.1898   -0.3282   -0.4554   -0.5689   -0.6662

  Columns 15 through 21

   -0.7456   -0.8055   -0.8451   -0.8639   -0.8620   -0.8401   -0.7989

  Columns 22 through 28

   -0.7401   -0.6655   -0.5770   -0.4773   -0.3689   -0.2545   -0.1369
```

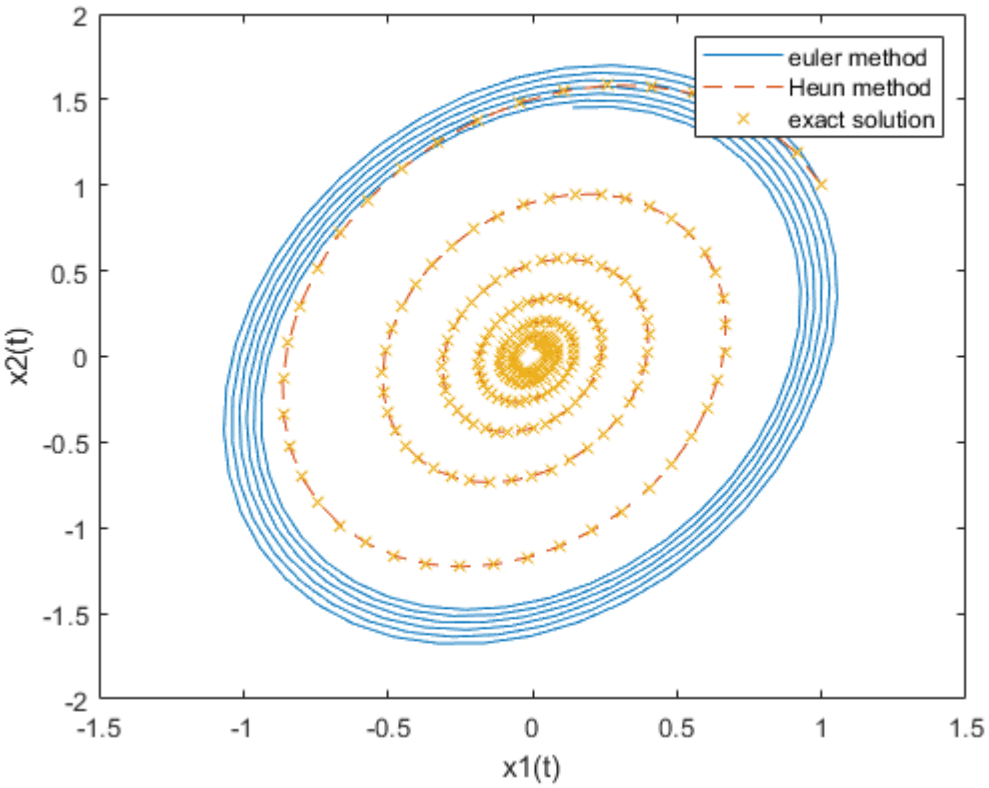
```
Columns 29 through 35

 -0.0191    0.0963    0.2066    0.3093    0.4022    0.4835    0.5514

Columns 36 through 42

  0.6047    0.6425    0.6644    0.6702    0.6601    0.6349    0.5954

Columns 43 through 49

  0.5429    0.4791    0.4056    0.3244    0.2376    0.1474    0.0560

Columns 50 through 56

 -0.0345   -0.1219   -0.2043   -0.2799   -0.3471   -0.4045   -0.4510

Columns 57 through 63

 -0.4858   -0.5084   -0.5185   -0.5162   -0.5019   -0.4762   -0.4400

Columns 64 through 70

 -0.3944   -0.3407   -0.2803   -0.2148   -0.1460   -0.0754   -0.0048

Columns 71 through 77

  0.0642    0.1299    0.1910    0.2461    0.2941    0.3341    0.3652

Columns 78 through 84

  0.3870    0.3992    0.4017    0.3948    0.3789    0.3544    0.3223

Columns 85 through 91

  0.2834    0.2388    0.1897    0.1374    0.0832    0.0284   -0.0257

Columns 92 through 98

 -0.0779   -0.1270   -0.1719   -0.2117   -0.2455   -0.2728   -0.2930

Columns 99 through 105

 -0.3058   -0.3111   -0.3091   -0.2998   -0.2838   -0.2615   -0.2337

Columns 106 through 112

 -0.2010   -0.1645   -0.1250   -0.0836   -0.0412    0.0011    0.0423

Columns 113 through 119

  0.0815    0.1178    0.1505    0.1789    0.2023    0.2205    0.2330

Columns 120 through 126

  0.2398    0.2408    0.2361    0.2260    0.2109    0.1912    0.1675

Columns 127 through 133

  0.1405    0.1109    0.0794    0.0468    0.0139   -0.0185   -0.0496

Columns 134 through 140
```

```
  -0.0788   -0.1055   -0.1290   -0.1490   -0.1649   -0.1766   -0.1839

Columns 141 through 147

  -0.1867   -0.1850   -0.1791   -0.1691   -0.1554   -0.1384   -0.1186

Columns 148 through 154

  -0.0965   -0.0727   -0.0477   -0.0223    0.0030    0.0277    0.0510

Columns 155 through 161

   0.0726    0.0920    0.1087    0.1225    0.1331    0.1403    0.1440

Columns 162 through 168

   0.1443    0.1412    0.1348    0.1255    0.1134    0.0990    0.0827

Columns 169 through 175

   0.0648    0.0458    0.0262    0.0065   -0.0129   -0.0315   -0.0489

Columns 176 through 182

  -0.0647   -0.0786   -0.0903   -0.0997   -0.1064   -0.1105   -0.1120

Columns 183 through 189

  -0.1107   -0.1069   -0.1007   -0.0923   -0.0819   -0.0699   -0.0565

Columns 190 through 196

  -0.0422   -0.0272   -0.0119    0.0032    0.0179    0.0319    0.0447

Columns 197 through 203

   0.0562    0.0661    0.0742    0.0803    0.0844    0.0865    0.0864

Columns 204 through 210

   0.0844    0.0804    0.0746    0.0673    0.0585    0.0486    0.0378

Columns 211 through 217

   0.0264    0.0146    0.0028   -0.0088   -0.0199   -0.0302   -0.0396

Columns 218 through 224

  -0.0479   -0.0548   -0.0602   -0.0641   -0.0664   -0.0671   -0.0663

Columns 225 through 231

  -0.0638   -0.0600   -0.0548   -0.0485   -0.0412   -0.0331   -0.0245

Columns 232 through 238

  -0.0154   -0.0063    0.0028    0.0116    0.0199    0.0275    0.0343

Columns 239 through 245

   0.0401    0.0449    0.0484    0.0508    0.0519    0.0518    0.0504
```

```
  Columns 246 through 252

    0.0479    0.0444    0.0399    0.0345    0.0285    0.0220    0.0151


ex2 =

  Columns 1 through 7

    1.0000    1.1850    1.3377    1.4553    1.5359    1.5786    1.5832

  Columns 8 through 14

    1.5507    1.4827    1.3817    1.2508    1.0939    0.9152    0.7195

  Columns 15 through 21

    0.5119    0.2973    0.0811   -0.1317   -0.3361   -0.5276   -0.7021

  Columns 22 through 28

   -0.8557   -0.9856   -1.0892   -1.1647   -1.2111   -1.2281   -1.2158

  Columns 29 through 35

   -1.1754   -1.1084   -1.0172   -0.9044   -0.7733   -0.6272   -0.4701

  Columns 36 through 42

   -0.3059   -0.1385    0.0279    0.1896    0.3429    0.4843    0.6110

  Columns 43 through 49

    0.7202    0.8098    0.8783    0.9245    0.9479    0.9486    0.9270

  Columns 50 through 56

    0.8843    0.8219    0.7419    0.6464    0.5382    0.4200    0.2949

  Columns 57 through 63

    0.1660    0.0364   -0.0908   -0.2128   -0.3268   -0.4303   -0.5212

  Columns 64 through 70

   -0.5976   -0.6582   -0.7019   -0.7280   -0.7365   -0.7275   -0.7017

  Columns 71 through 77

   -0.6602   -0.6042   -0.5354   -0.4558   -0.3675   -0.2728   -0.1740

  Columns 78 through 84

   -0.0736    0.0261    0.1227    0.2140    0.2980    0.3731    0.4375

  Columns 85 through 91

    0.4901    0.5299    0.5563    0.5691    0.5682    0.5540    0.5273

  Columns 92 through 98

    0.4888    0.4399    0.3818    0.3163    0.2449    0.1696    0.0922
```

```
Columns 99 through 105

  0.0146   -0.0615   -0.1343   -0.2021   -0.2635   -0.3173   -0.3623

Columns 106 through 112

 -0.3977   -0.4229   -0.4375   -0.4416   -0.4353   -0.4189   -0.3931

Columns 113 through 119

 -0.3587   -0.3168   -0.2685   -0.2152   -0.1581   -0.0987   -0.0384

Columns 120 through 126

  0.0212    0.0789    0.1333    0.1832    0.2277    0.2657    0.2965

Columns 127 through 133

  0.3196    0.3347    0.3416    0.3403    0.3311    0.3143    0.2906

Columns 134 through 140

  0.2607    0.2254    0.1858    0.1427    0.0974    0.0509    0.0044

Columns 141 through 147

 -0.0411   -0.0845   -0.1249   -0.1613   -0.1931   -0.2195   -0.2402

Columns 148 through 154

 -0.2547   -0.2629   -0.2648   -0.2604   -0.2500   -0.2340   -0.2129

Columns 155 through 161

 -0.1874   -0.1581   -0.1259   -0.0915   -0.0558   -0.0196    0.0161

Columns 162 through 168

  0.0505    0.0829    0.1126    0.1389    0.1613    0.1793    0.1927

Columns 169 through 175

  0.2013    0.2050    0.2038    0.1978    0.1873    0.1727    0.1545

Columns 176 through 182

  0.1330    0.1090    0.0831    0.0558    0.0279   -0.0000   -0.0272

Columns 183 through 189

 -0.0531   -0.0771   -0.0987   -0.1174   -0.1330   -0.1450   -0.1534

Columns 190 through 196

 -0.1579   -0.1587   -0.1557   -0.1491   -0.1393   -0.1264   -0.1108

Columns 197 through 203

 -0.0931   -0.0736   -0.0529   -0.0314   -0.0097    0.0116    0.0322

Columns 204 through 210
```

```
  0.0515    0.0691    0.0846    0.0978    0.1084    0.1162    0.1211

Columns 211 through 217

  0.1230    0.1220    0.1181    0.1116    0.1027    0.0915    0.0785

Columns 218 through 224

  0.0640    0.0483    0.0319    0.0151   -0.0016   -0.0178   -0.0332

Columns 225 through 231

 -0.0475   -0.0603   -0.0714   -0.0805   -0.0875   -0.0923   -0.0949

Columns 232 through 238

 -0.0951   -0.0931   -0.0890   -0.0829   -0.0750   -0.0655   -0.0548

Columns 239 through 245

 -0.0430   -0.0305   -0.0176   -0.0046    0.0082    0.0204    0.0319

Columns 246 through 252

  0.0424    0.0516    0.0593    0.0655    0.0700    0.0728    0.0738
```



Saving Images in MATLAB

To do the following exercises, you will need to know how to output graphics from MATLAB. Create a folder on your Desktop (or elsewhere) to contain the files generated by these exercises. Make this folder the "Current Folder" in the left side of the main MATLAB window. This will ensure that the files output by MATLAB end up in the folder you created.

To save an image of a phase portrait, use the following steps:

1. Get the phase portrait looking the way you want in the `iode` window.

2. Leaving `iode` open, switch to the main MATLAB window.

3. Type the command `print -dpng -r300 'filename.png'` in the command window.

This command will create a PNG graphic called `filename.png` in the current folder. The `-dpng` option tells MATLAB to output the graphic in PNG format; MATLAB also allows output in other formats, such as BMP, EPS, PNG and SVG. The `-r300` option tells MATLAB to set the resolution at 300 dots per inch and can be adjusted if you wish.

Exercise 4

Objective: Analyze phase portraits.

Details: Compile the results of the following exercises into a single document (e.g. using a word processor) and export it to PDF for submission on Quercus.

For each of the first-order systems of ODEs 4.1 to 4.10 below, do the following exercises:

(a) Generate a phase portrait for the system (centre the graph on the equilibrium point at (0,0)). Include a few trajectories.

(b) Classify the equilibrium on asymptotic stability, and behaviour (sink, source, saddle-point, spiral, center, proper node, improper node) - check table 3.5.1 and figure 3.5.7. Classify also as for clockwise or counterclockwise movement, when relevant.

(c) Compute the eigenvalues of the matrix (you do not need to show your calculations). Using the eigenvalues you computed, justify part (b).

To avoid numerical error, you should use Runge-Kutta solver with a step size of `0.05`. Change the display parameters, if necessary, to best understand the phase portrait.

4.1. `dx/dt = [2 1; 1 3] x`

4.2. `dx/dt = [-2 -1; -1 -3] x`

4.3. `dx/dt = [-4 -6; 3 5] x`

4.4. `dx/dt = [4 6; -3 -5] x`

4.5. `dx/dt = [0 -1; 1 -1] x`

4.6. `dx/dt = [0 1; -1 1] x`

4.7. `dx/dt = [2 8; -1 -2] x`

4.8. `dx/dt = [-2 -8; 1 2] x`

4.9. `dx/dt = [-8 5; -13 8] x`

4.10. `dx/dt = [8 -5; 13 -8] x`

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