Assignment 3

Part 1

Team Members

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Reading Images

```
[ ] import cv2
    from google.colab.patches import cv2_imshow
    import numpy as np
    from matplotlib import pyplot as plt
    import math
    from zipfile import ZipFile
```

```
[ ] %%shell
    gdown 1Ixr3-nKsI1rJUH6SCWQd8noysUAp6YGd
```

```
Downloading...
From: https://drive.google.com/uc?id=1Ixr3-nKsI1rJUH6SCWQd8noysUAp6YGd
To: /content/Images.zip
100% 1.64M/1.64M [00:00<00:00, 102MB/s]
```

```
[ ] with ZipFile("/content/Images.zip", 'r') as file:
      file.extractall(path="/content/")
```

```
[ ] img1_l = cv2.imread("/content/l1.png", cv2.IMREAD_GRAYSCALE)
    img1_r = cv2.imread("/content/r1.png", cv2.IMREAD_GRAYSCALE)
    img2_l = cv2.imread("/content/l2.png", cv2.IMREAD_GRAYSCALE)
    img2_r = cv2.imread("/content/r2.png", cv2.IMREAD_GRAYSCALE)
    img3_l = cv2.imread("/content/l3.png", cv2.IMREAD_GRAYSCALE)
    img3_r = cv2.imread("/content/r3.png", cv2.IMREAD_GRAYSCALE)
```

```
[ ] def plot_img(fig, rows, cols, id, img, title):
      ax = fig.add_subplot(rows, cols, id)
      imgplot = plt.imshow(img, cmap='gray', vmin=0, vmax=255)
      ax.set_title(title)
      ax.tick_params(left = False, right = False ,
                     labelleft = False , labelbottom = False, bottom = False)
```

```
[ ] fig = plt.figure(figsize=(15,15))
    plot_img(fig, 3, 2, 1, img1_l, 'image 1 - left')
    plot_img(fig, 3, 2, 2, img1_r, 'image 1 - right')
    plot_img(fig, 3, 2, 3, img2_l, 'image 2 - left')
    plot_img(fig, 3, 2, 4, img2_r, 'image 2 - right')
    plot_img(fig, 3, 2, 5, img3_l, 'image 3 - left')
    plot_img(fig, 3, 2, 6, img3_r, 'image 3 - right')
```

image 1 - left

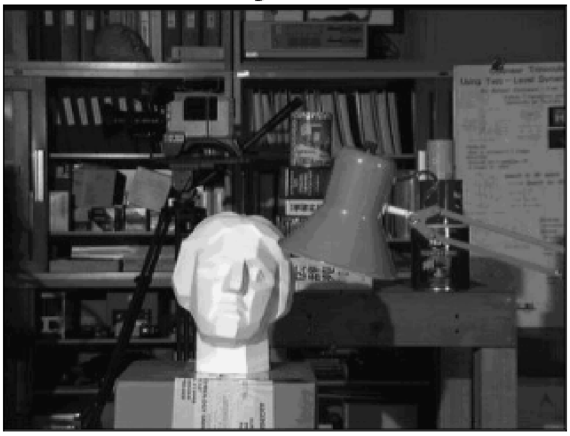


image 1 - right

image 2 - left

image 2 - right

image 3 - left

image 3 - right

Block Matching

- Matching each pixel in the left image to a pixel in the right image.
- Since there is no rectification needed --> only need to match the row in the left image with its equivalent in the right image.
- Calculate the disparity in two ways:

> 1. using the cost as the Sum of Absolute Differences (SAD)
> 2. using Sum of Squared Difference (SSD)

- For windows of sizes --> w = 1, 5 and 9.
- Produce 6 maps: 2 maps for each window size, once using SAD and the other using SSD.

get_windows_lists()

This function takes:

left image, the right image

scanline: row number

pixel_in_left: column number of the pixel in the left image for which we want to find a match in the right image

w: window size

offset: the range of pixels to search for the match in

Pad the right image and the left image with the floor of half the window size, so we get left_padded and right_padded.

Then, we get the window with the center pixel [scanline, pixel_in_left] from the left image and all windows from the right image with center [scanline, x] for x = all columns of the right image.

The function returns the window from the left image in ‘left’ and an array of all windows from the right image in ‘right’.

This will be used later to calculate the cost of matching each pixel from the left image to all pixels in the same scanline from the right image.

```
[ ]  def get_windows_lists(left_img, right_img, scanline, pixel_in_left, w, offset):
       left_padded = np.pad(left_img, w//2, 'constant')
       right_padded = np.pad(right_img, w//2, 'constant')
       left = left_padded[scanline : scanline + w , pixel_in_left : pixel_in_left + w]
       right = []
       start = 0 if pixel_in_left < offset else pixel_in_left - offset
       end =  pixel_in_left + 1 if pixel_in_left != right_img.shape[1] - 1 else right_img.shape[1]
       for r in range(start, end):
         right.append(right_padded[scanline : scanline + w , r : r + w])

       return left, np.asarray(right, dtype=np.int32)
```

normalize()

This function is used to normalize the image:

```
[ ]  def normalize(A, max):
       return ((A - np.min(A)) * max) // (np.max(A) - np.min(A))
```

calculate_SAD()

This function takes a window from the left image and a window from the right image and calculate the SAD of them, where

$$SAD = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} |f(x,y) - g(x,y)|$$

```
[ ]  def calculate_SAD(left, right):
       return np.sum(np.abs(np.array(left - right, dtype=int)))
```

calculate_SSD()

This function takes a window from the left image and a window from the right image and calculate the SSD of them, where

$$SSD = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} |f(x,y) - g(x,y)|^2$$

```
[ ]  def calculate_SSD(left, right):
       return np.sum(np.square(np.array(left - right, dtype=int)))
```

block_matching()

This function takes:

left image, right image

disparity_method: calculate_SAD or calculate_SSD

best_val_method: the method by which we get the best value of the calculated costs

w: window size

offset: the range of pixels to search for the match in

For each pixel in the left image, calculate the cost of matching with each pixel in the same scanline from the right image. It uses get_windows_lists() to get the corresponding windows, calculate the cost, get the index of the minimum cost, the difference between the offset and this index is the disparity. Then, returns the disparity matrix containing the disparity of the matching pairs.

```
[ ]  def block_matching(left_img, right_img, disparity_method, best_val_method, w, offset):
       disp = np.zeros(left_img.shape)
       for sl in range(left_img.shape[0]):
         for l in range(left_img.shape[1]):
           left, right = get_windows_lists(left_img, right_img, sl, l, w, offset)
           vals = []
           for r in right:
             vals.append(disparity_method(left.flatten(), r.flatten()))
           disp[sl][l] = len(vals) - best_val_method(np.array(vals))

       disp = normalize(disp, 255)
       return np.array(disp)
```

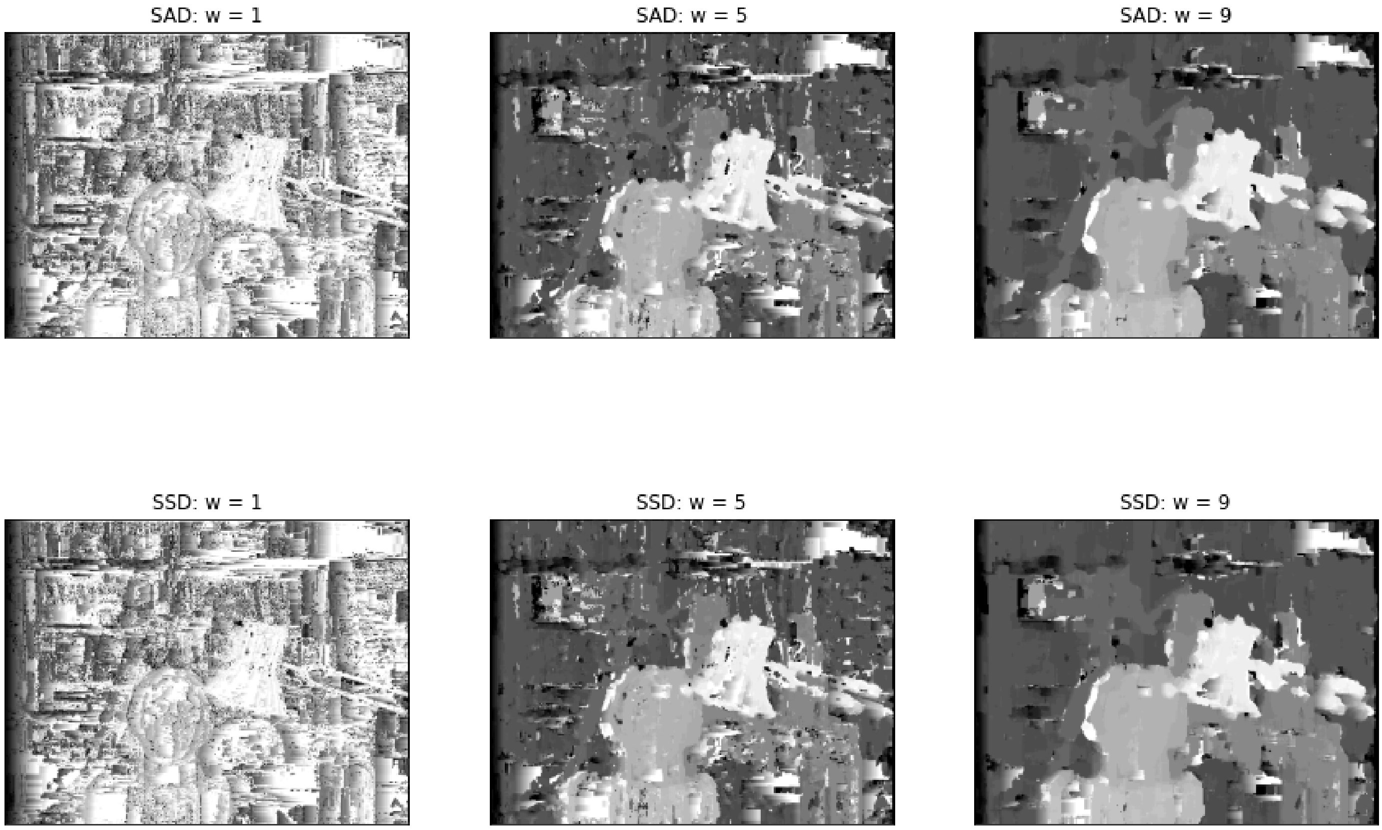
Plotting the disparity

We get the disparity map for each image at windows of size = 1, 5, 9 using both SAD and SSD as cost functions.

```
[ ]  def plot_SAD_SSD_disp(title, img_SAD_1, img_SAD_5, img_SAD_9, img_SSD_1, img_SSD_5, img_SSD_9):
       fig = plt.figure(figsize=(15,10))
       print(title)
       plot_img(fig, 2, 3, 1, np.abs(img_SAD_1), 'SAD: w = 1')
       plot_img(fig, 2, 3, 2, np.abs(img_SAD_5), 'SAD: w = 5')
       plot_img(fig, 2, 3, 3, np.abs(img_SAD_9), 'SAD: w = 9')
       plot_img(fig, 2, 3, 4, np.abs(img_SSD_1), 'SSD: w = 1')
       plot_img(fig, 2, 3, 5, np.abs(img_SSD_5), 'SSD: w = 5')
       plot_img(fig, 2, 3, 6, np.abs(img_SSD_9), 'SSD: w = 9')
```

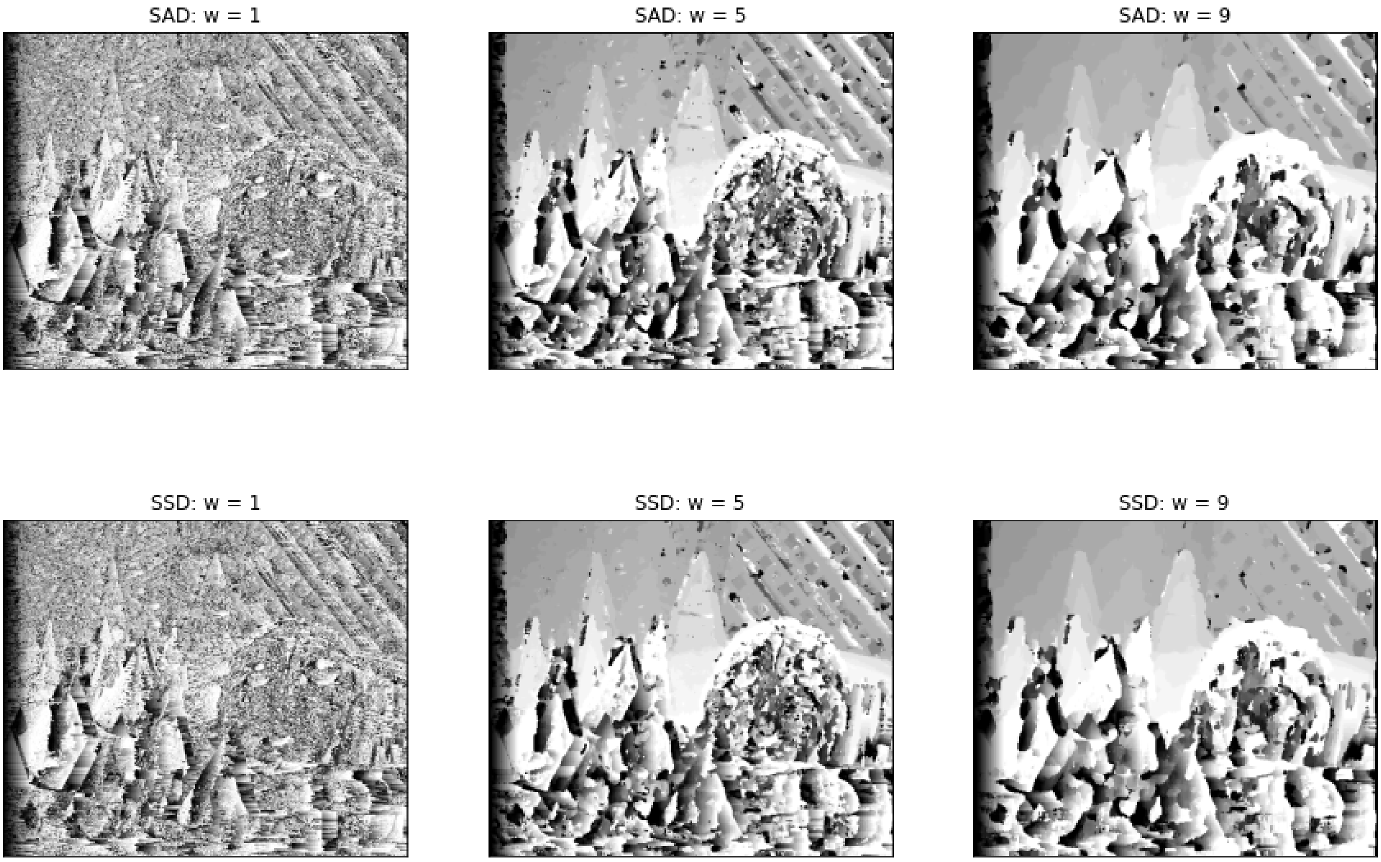
Results of the first image

```
[ ]  disp_img1_SAD_1 = block_matching(img1_l, img1_r, calculate_SAD, np.argmin, 1, 30)
     disp_img1_SAD_5 = block_matching(img1_l, img1_r, calculate_SAD, np.argmin, 5, 30)
     disp_img1_SAD_9 = block_matching(img1_l, img1_r, calculate_SAD, np.argmin, 9, 30)
     disp_img1_SSD_1 = block_matching(img1_l, img1_r, calculate_SSD, np.argmin, 1, 30)
     disp_img1_SSD_5 = block_matching(img1_l, img1_r, calculate_SSD, np.argmin, 5, 30)
     disp_img1_SSD_9 = block_matching(img1_l, img1_r, calculate_SSD, np.argmin, 9, 30)

     plot_SAD_SSD_disp('Image 1 using SAD and SSD', disp_img1_SAD_1, disp_img1_SAD_5,
                       disp_img1_SAD_9, disp_img1_SSD_1, disp_img1_SSD_5, disp_img1_SSD_9)
```



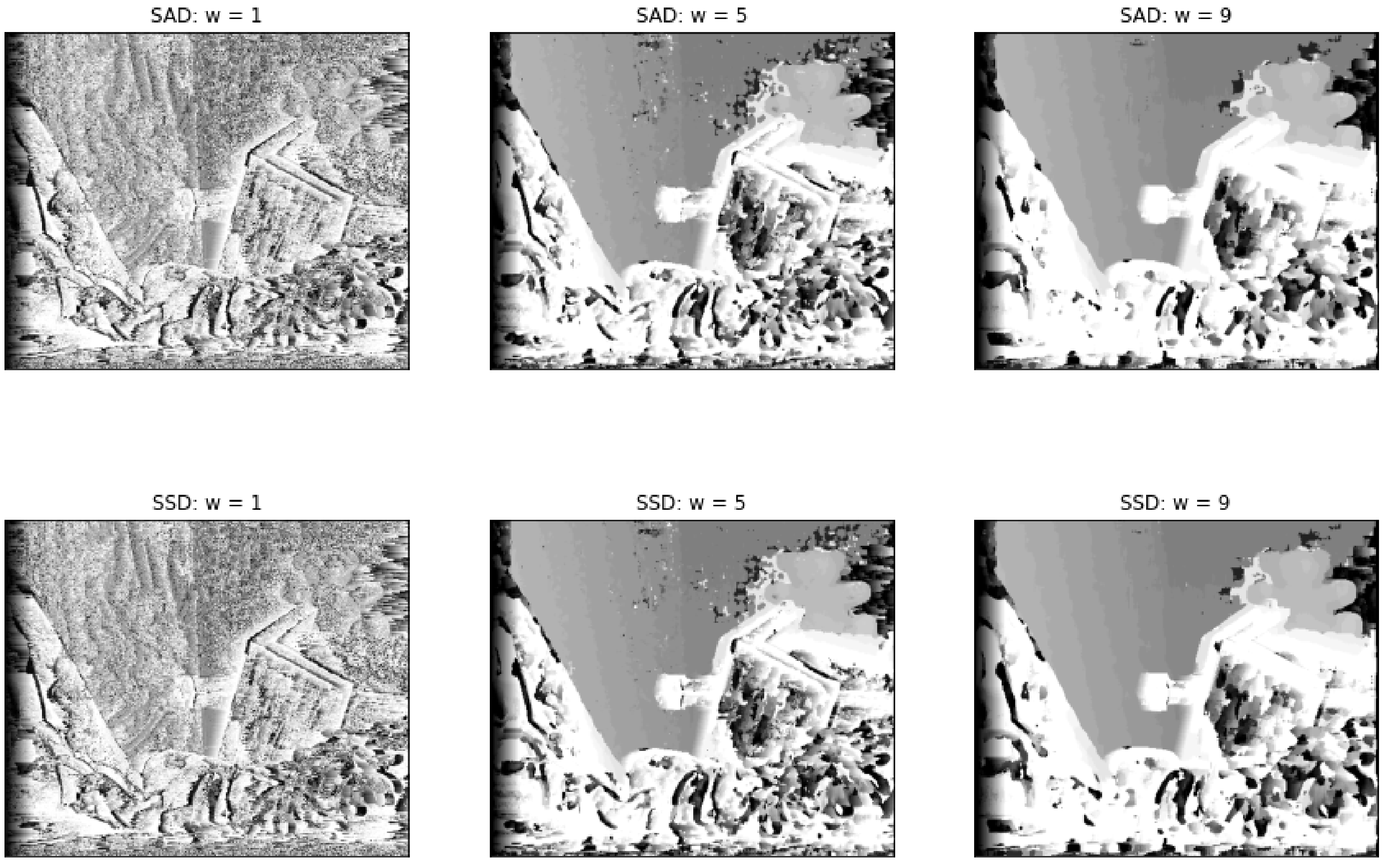
Results of the second image

```
disp_img2_SAD_1 = block_matching(img2_l, img2_r, calculate_SAD, np.argmin, 1, 30)
disp_img2_SAD_5 = block_matching(img2_l, img2_r, calculate_SAD, np.argmin, 5, 30)
disp_img2_SAD_9 = block_matching(img2_l, img2_r, calculate_SAD, np.argmin, 9, 30)
disp_img2_SSD_1 = block_matching(img2_l, img2_r, calculate_SSD, np.argmin, 1, 30)
disp_img2_SSD_5 = block_matching(img2_l, img2_r, calculate_SSD, np.argmin, 5, 30)
disp_img2_SSD_9 = block_matching(img2_l, img2_r, calculate_SSD, np.argmin, 9, 30)

plot_SAD_SSD_disp('Image 2 using SAD and SSD', disp_img2_SAD_1, disp_img2_SAD_5,
                  disp_img2_SAD_9, disp_img2_SSD_1, disp_img2_SSD_5, disp_img2_SSD_9)
```



Results of the third image

```
[ ] disp_img3_SAD_1 = block_matching(img3_l, img3_r, calculate_SAD, np.argmin, 1, 30)
    disp_img3_SAD_5 = block_matching(img3_l, img3_r, calculate_SAD, np.argmin, 5, 30)
    disp_img3_SAD_9 = block_matching(img3_l, img3_r, calculate_SAD, np.argmin, 9, 30)
    disp_img3_SSD_1 = block_matching(img3_l, img3_r, calculate_SSD, np.argmin, 1, 30)
    disp_img3_SSD_5 = block_matching(img3_l, img3_r, calculate_SSD, np.argmin, 5, 30)
    disp_img3_SSD_9 = block_matching(img3_l, img3_r, calculate_SSD, np.argmin, 9, 30)

    plot_SAD_SSD_disp('Image 3 using SAD and SSD', disp_img3_SAD_1, disp_img3_SAD_5,
                      disp_img3_SAD_9, disp_img3_SSD_1, disp_img3_SSD_5, disp_img3_SSD_9)
```



Dynamic Programming

Calculate disparity using a dynamic programming algorithm to get the minimum cost of matching a whole row in the image. Consider two scanlines $I_l(i)$ and $I_r(j)$. Pixels in each scanline may be matched, or skipped (considered to be occluded in either the left or right image). Let d_{ij} be the cost associated with matching pixel $I_l(i)$ with pixel $I_r(j)$. Here we consider a squared error measure between pixels given by:

$$d_{ij} = \frac{(I_l(i) - I_r(j))^2}{\sigma^2}$$

where σ is some measure of pixel noise. The cost of skipping a pixel (in either scanline) is given by a constant c_0 . For the experiments here we will use σ = 2 and c_0 = 1. Given these costs, we can compute the optimal (minimal cost) alignment of two scanlines recursively as follows:

$$1. D(1,1) = d_{11}$$

$$2. D(i,j) = min(D(i-1,j-1) + d_{ij}, D(i-1,j) + c_0, D(i,j-1) + c_0)$$

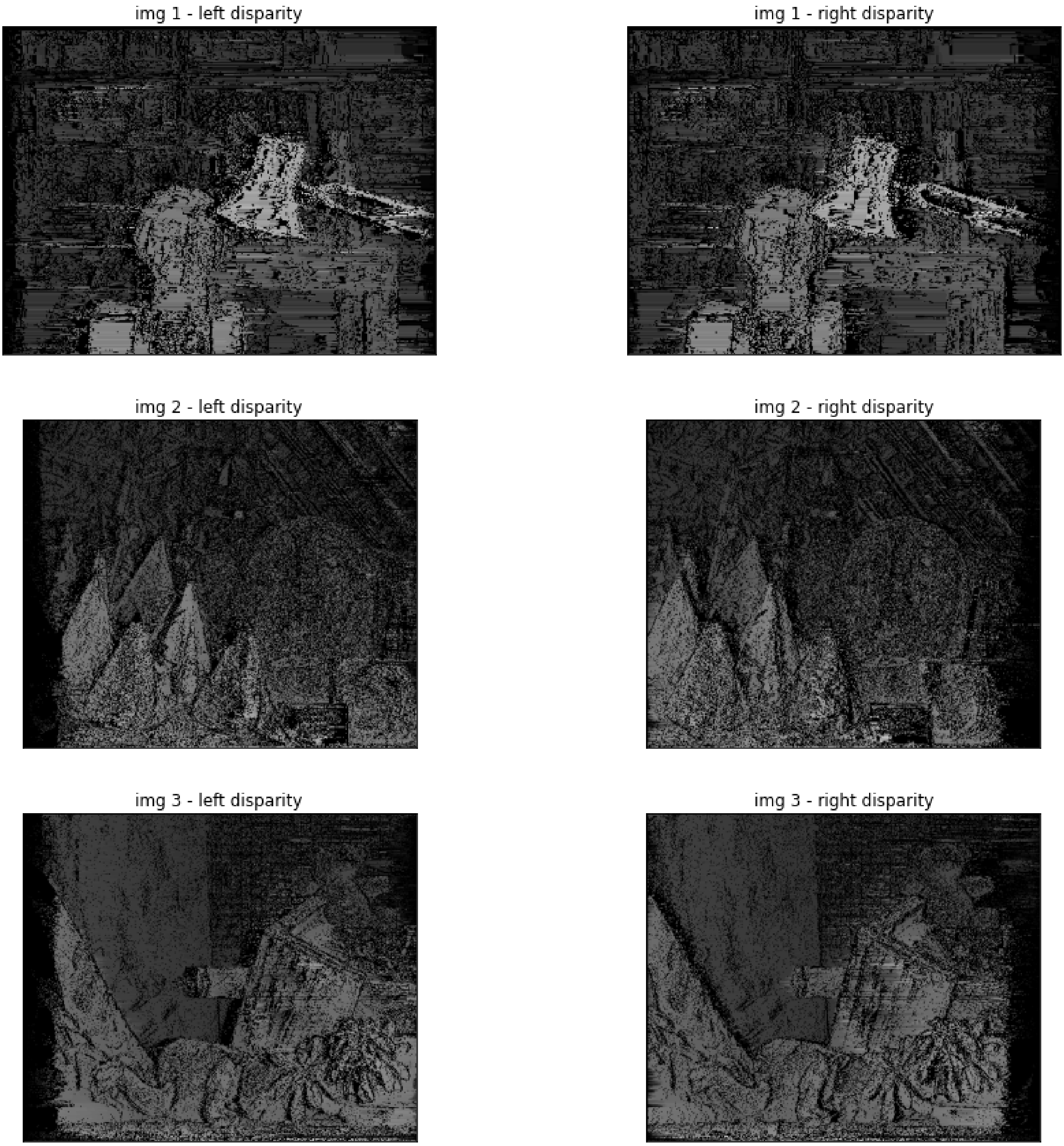
The intermediate values are stored in an N-by-N matrix, D. The total cost of matching two scanlines is D(N, N). Note that this assumes the lines are matched at both ends (and hence have zero disparity there). This is a reasonable approximation provided the images are large relative to the disparity shift. You can find the disparity map of matching the left image to the right or vice versa at the same time or only calculate one of them. Given the cost matrix D we find the optimal alignment by backtracking. In particular, starting at (i, j) = (N, N), we choose the minimum value of D from (i - 1, j - 1), (i - 1, j), (i, j - 1). Selecting (i - 1, j) corresponds to skipping a pixel in I_l , so the left disparity map of i is zero. Selecting (i, j - 1) corresponds to skipping a pixel in I_r, and the right disparity map of j is zero. Selecting (i - 1, j - 1) matches pixels (i, j), and therefore both disparity maps at this position are set to the absolute difference between i and j.

```
[ ]  def dynamic_programming(left_img, right_img, sigma, c0):
       n_cols = left_img.shape[1]
       left_disp = np.zeros(left_img.shape)
       right_disp = np.zeros(right_img.shape)
       for sl in range(left_img.shape[0]):
         D = np.zeros((n_cols, n_cols))
         D[0][0] = compute_d(int(left_img[sl][0]), int(right_img[sl][0]), sigma)
         for k in range(1, n_cols):
           D[0][k] = D[0][k-1] + c0
           D[k][0] = D[k-1][0] + c0
         for i in range(1, n_cols):
           for j in range(1, n_cols):
             cost = compute_d(int(left_img[sl][i]), int(right_img[sl][j]), sigma)
             D[i][j] = min(D[i-1][j-1] + cost, D[i-1][j] + c0, D[i][j-1] + c0)

         # backtracking
         i, j = n_cols - 1, n_cols - 1
         while i != 0 and j != 0:
           if D[i][j] == D[i-1][j-1] + compute_d(int(left_img[sl][i]), int(right_img[sl][j]), sigma):
             left_disp[sl][i] = abs(i - j)
             right_disp[sl][j] = abs(i - j)
             i -= 1
             j -= 1
           elif D[i][j] == D[i-1][j] + c0:
             left_disp[sl][i] = 0
             i -= 1
           elif D[i][j] == D[i][j-1] + c0:
             right_disp[sl][j] = 0
             j -= 1

       return normalize(left_disp, 255), normalize(right_disp, 255)
```

```python
def compute_d(Il, Ir, sigma):
  return ((Il - Ir)**2) / (sigma**2)
```

```python
left_disp_1, right_disp_1 = dynamic_programming(img1_l, img1_r, 2, 1)
left_disp_2, right_disp_2 = dynamic_programming(img2_l, img2_r, 2, 1)
left_disp_3, right_disp_3 = dynamic_programming(img3_l, img3_r, 2, 1)
```

```python
fig = plt.figure(figsize=(15,15))
plot_img(fig, 3, 2, 1, left_disp_1, 'img 1 - left disparity')
plot_img(fig, 3, 2, 2, right_disp_1, 'img 1 - right disparity')
plot_img(fig, 3, 2, 3, left_disp_2, 'img 2 - left disparity')
plot_img(fig, 3, 2, 4, right_disp_2, 'img 2 - right disparity')
plot_img(fig, 3, 2, 5, left_disp_3, 'img 3 - left disparity')
plot_img(fig, 3, 2, 6, right_disp_3, 'img 3 - right disparity')
```



Bonus

A good way to interpret your solution is to plot the alignment found for single scan line. Display the alignment by plotting a graph of Il (horizontal) vs Ir (vertical). Begin at D(N, N) and work backwards to find the best path. If a pixel in Il is skipped, draw a horizontal line. If a pixel in Ir is skipped, draw a vertical line. Otherwise, the pixels are matched, and you draw a diagonal line. The plot should end at (1, 1)

```
[ ]  def get_alignment(left_img, right_img, sl, sigma, c0):
       n_cols = left_img.shape[1]
       D = np.full(np.array((n_cols, n_cols)), np.inf)
       D[0][0] = compute_d(int(left_img[sl][0]), int(right_img[sl][0]), sigma)
       for i in range(1, n_cols):
         for j in range(1, n_cols):
           cost = compute_d(int(left_img[sl][i]), int(right_img[sl][j]), sigma)
           D[i][j] = min(D[i-1][j-1] + cost, D[i-1][j] + c0, D[i][j-1] + c0)

       # backtracking
       i, j = n_cols - 1, n_cols - 1
       points = []
       while i != 0 and j != 0:
         if D[i][j] == D[i-1][j-1] + compute_d(int(left_img[sl][i]), int(right_img[sl][j]), sigma):
           i -= 1
           j -= 1
         elif D[i][j] == D[i-1][j] + c0:
           i -= 1
         else:
           j -= 1
         points.append([i,j])

       return np.array(points)
```

```
[ ]  points = get_alignment(img1_l, img1_r, 100, 2, 1)
     plt.plot((points[:,0]), (points[:,1]))
```

```
[<matplotlib.lines.Line2D at 0x7ff06fc74f10>]
```